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[0,1] Truncated half logistic- half logistic distribution

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Abstract

In the past few years, many methods have been proposed to generate new distributions. In this paper, we introduce [0,1] truncated half logistic - half logistic distribution ([0,1] THL-HLD). Some of important statistical properties of this distribution will be derived. These properties are rth raw moments function, hazard rate function, stress-strength function and Shannon entropy.

Keywords: Half logistic distribution, [0,1] truncated distributions, r-th moment, Shannon entropy, stress- strength, reliability and hazard rate function.

1. Introduction

This study gives brief information about the generated class of continuous distributions that based on interval [0,1] truncated distributions, named as [0,1] truncated THL-HL distributions, The reader is referred to the following for an overview of the compound of continuous distribution:
In (2018) Hosseini et al. [3] introduced the generalized odd Gamma-G family and discussed some of its statistical properties along with its applications. In (2019) Korkmaz [8] presented a new generated family of probability continuous distributions which is an extended form of the Weibull distribution by adding three extra parameters. In (2019) Jamal et al. [5] presented a new generated family of models called the Burr –X family. With any parent probability continuous distribution G, they define the corresponding Burr –X generated (BXG) distribution with one extra positive parameters. In 2018 (Mona Mustafa el Biely and Haithem M. Yousof)presented an expanded model for the Lumax distribution and derived some of its properties[2]. In 2019 researcher (Mhamed Ibrahim) introduced a new distribution called Frechet Extended Distribution And he derived its properties[4].

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The Half-logistic distribution (H-L) is introduced and studied by de Kank et al.[6] A random variable X has H-L distribution if the pdf and cdf. With two positive parameter (μ, σ) given respectively by,

$$f(x;\mu,\sigma)_{H-L} = \frac{2e^{\frac{-(x-\mu)}{\sigma}}}{\sigma\left(1+e^{\frac{-(x-\mu)}{\sigma}}\right)^2} 0 < x < \infty, \qquad x \geq \mu, \qquad \sigma > 0 \tag{1.1}$$

And the Cumulative distribution function (CDF) is [7],

$$F(x;\mu,\sigma)_{H-L} = \frac{1-e^{\frac{-(x-\mu_1)}{\sigma_1}}}{1+e^{\frac{-(x-\mu_1)}{\sigma_1}}} 0 < x < \infty, \qquad x \geq \mu, \qquad \sigma > 0 \tag{1.2}$$

2. [0, 1] Truncated L-Q Distributions

In this search, a new generated class of continuous distributions based on the interval [0, 1] truncated cumulative distribution function L-Q, named [0,1] truncated L-Q (symbolized by [0,1] TL-Q distributions, have been discussed.
Suppose that $Q(x)$ *and* $q(x)$ are any continuous baseline and pdf of random variable x. Also suppose that $L(.)$ *and* $l(.)$ represents, respectively, the cdf and pdf of any continuous distribution on the interval $[0,\infty)$. The proposed general formula of cdf for this class depends composing L with Q is[1],

$$F(x)_{TL-Q} = \frac{L[Q(x)] - L[0]}{L[1] - L[0]} \tag{2.1}$$

Now, let $L[0] = 0$ Then cdf in (2.1) can be rewritten as,

$$F(x)_{TL-Q} = \frac{L[Q(x)]}{L[1]} \tag{2.2}$$

And its associated pdf, $f(x)\frac{d}{dx}[F(x)]$ will be,

$$f(x)_{TL-Q} = \frac{l\ [Q(x)]\,q(x)}{L[1]} \tag{2.3}$$

3. [0,1] Truncated Half Logistic- Q Distribution

we introduce here a new generated family of [0,1] truncated based on Half logistic (HL) distribution. Let $L(.)$ *and* $l(.)$ that mentioned in (2.2) and (2.3), be the cdf and pdf of (HL) distribution [7] recall (1.1) and (1.2) with tow non negative parameters ($\mu = 0$, $\sigma > 0$).We have L(0)=0 So,

$$L[Q(x)] = \frac{1-e^{\frac{-Q(x)}{\sigma}}}{1+e^{\frac{-Q(x)}{\sigma}}}, \quad L[1] = \frac{1-e^{\frac{-1}{\sigma}}}{1+e^{\frac{-1}{\sigma}}}$$

And $l[Q(x)] = \frac{2e^{\frac{-Q(x)}{\sigma}}}{\sigma\left(1+e^{\frac{-Q(x)}{\sigma}}\right)^2}$

Then according to (2.2) and (2.3), the cdf and associated pdf for new family of distribution named [0,1] truncated Half logistic- Q (symbolized by [0,1] THL- Q) distribution will be,

$$F(x)_{THL-Q} = \frac{\left(1-e^{\frac{-Q(x)}{\sigma}}\right)\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1+e^{\frac{-Q(x)}{\sigma}}\right)\left(1-e^{\frac{-1}{\sigma}}\right)} \quad , \quad \sigma > 0, \quad x \geq 0 \tag{3.1}$$

and,

$$f(x)_{THL-Q} = \frac{2e^{\frac{-Q(x)}{\sigma}}\left(1+e^{\frac{-1}{\sigma}}\right)q(x)}{\sigma\left(1+e^{\frac{-Q(x)}{\sigma}}\right)^2\left(1-e^{\frac{-1}{\sigma}}\right)}\ ,\quad \sigma>0,\quad x\geq 0 \tag{3.2}$$

Before addressing some cases belonging to this family, it should be noted that we need to find the general expanded formula to $F(x)_{THL-Q}$ and $f(x)_{THL-Q}$ which is important for obtaining the basic statistical properties when dealing with some cases.
By using the following Formula[1],

$$e^{-u} = \sum_{i=0}^{\infty}\frac{(-1)^i}{i!}u^i \tag{3.3}$$

the equation (3.1) can be rewritten as,

$$F(x)_{THL-Q} = \frac{\left(1+e^{\frac{-1}{\sigma}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(Q(x)\right)^i\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(Q(x)\right)^i\right)} \tag{3.4}$$

And,

$$f(x)_{THL-Q} = \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)e^{\frac{-Q(x)}{\sigma}}q(x)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma\left(1+e^{\frac{-Q(x)}{\sigma}}\right)^2} = \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma}\left(1+e^{\frac{-Q(x)}{\sigma}}\right)^{-2}e^{\frac{-Q(x)}{\sigma}}q(x) \tag{3.5}$$

And by using the Formula [1],

$$(a+u)^{-n} = \sum_{i=0}^{\infty}\mathsf{C}_i^{-n}a^{-n-i}u^i \tag{3.6}$$

$f(x)_{THL-Q}$ can be rewritten as ,

$$f(x)_{THL-Q} = \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma}\sum_{k=0}^{\infty}\mathsf{C}_k^{-2}e^{\frac{-kQ(x)}{\sigma}}e^{\frac{-Q(x)}{\sigma}}q(x) = \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma}\sum_{k=0}^{\infty}\mathsf{C}_k^{-2}e^{\frac{-(1+k)Q(x)}{\sigma}}q(x)$$

Accorded (3.3) $f(x)_{THL-Q}$ will be,

$$\begin{aligned} f(x)_{THL-Q} &= \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\mathsf{C}_k^{-2}\left(\frac{(1+k)Q(x)}{\sigma}\right)^i q(x) \\ &= \frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\mathsf{C}_k^{-2}\sigma^{-(1+i)}(1+k)^i[Q(x)]^i q(x) \end{aligned} \tag{3.7}$$

4. [0,1] Truncated Half Logistic- Half Logistic Distribution

Suppose that $Q(x)$ *and* $q(x)$ in (3.4), (3.6) represent the cdf and pdf of the Half logistic distribution respectively with tow positive parameter λ *and* β, cdf and pdf as,

$$Q(x;\lambda\ ,\ \beta\ ) = \frac{1-e^{\frac{-(x-\lambda)}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}} \tag{4.1}$$

$$q\left(x;\lambda\ ,\ \beta\ \right)=\frac{2e^{\frac{-(x-\lambda)}{\beta}}}{\beta\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^{2}} \tag{4.2}$$

Now, according to (3.4), the cdf of new distribution named [0,1] truncated Half logistic- Half logistic (symbolized by [0,1] THL-HL) distribution will be,

$$F\left(x\right)_{THL-HL}=\frac{\left(1+e^{\frac{-1}{\sigma}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^{i}}{\sigma^{i}i!}\left(\frac{1-e^{\frac{-(x-\lambda)}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^{i}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^{i}}{\sigma^{i}i!}\left(\frac{1-e^{\frac{-(x-\lambda)}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^{i}\right)} \tag{4.3}$$

The pdf [0,1] THL-HL distribution can be obtained, according to (3.7) as,

$$f\left(x\right)_{THL-HL}=\left\{\frac{4\left(1+e^{\frac{-1}{\sigma}}\right)}{\beta\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\frac{(-1)^{i}}{i!}C_{k}^{-2}\sigma^{-(1+i)}(1+k)^{i}\left[\frac{1-e^{\frac{-(x-\lambda)}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right]^{i}\frac{e^{\frac{-(x-\lambda)}{\beta}}}{\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^{2}}\right\} \tag{4.4}$$

Now let,

$$I=\left[\frac{1-e^{\frac{-(x-\lambda)}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right]^{i}\frac{e^{\frac{-(x-\lambda)}{\beta}}}{\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^{2}}$$

According to (3.6) and by using the Formula,

$$(1-u)^{b}=\sum_{i=0}^{\infty}(-1)^{i}C_{i}^{b}u^{i} \tag{4.5}$$

I will be,

$$I=\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}C_{j}^{i}C_{m}^{-2}\sum_{n}^{\infty}C_{n}^{-i}\left(-1\right)^{j}e^{\frac{-(j+m+1)(x-\lambda)}{\beta}} \tag{4.6}$$

Then, according to (4.4) and (4.6), the expansion formula for the pdf of [0,1] THL-HL distribution can be obtained as,

$$f\left(x\right)_{THL-HL}=\left\{\begin{array}{c}\frac{4\left(1+e^{\frac{-1}{\sigma}}\right)}{\beta\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}C_{j}^{i}C_{m}^{-2}\sum_{n}^{\infty}C_{n}^{-i}C_{K}^{-2}\frac{(-1)^{i+j}}{i!}\sigma^{-(1+i)}\\ (1+k)^{i}e^{\frac{-(j+m+1)(x-\lambda)}{\beta}}\end{array}\right\} \tag{4.7}$$

The Reliability function, $R\left(x\right)=1-F\left(x\right)$, of [0,1]THL-HL distribution can be obtained as,

$$R\left(x\right)_{THL-HL}=1-\frac{\left(1+e^{\frac{-1}{\sigma}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^{i}}{\sigma^{i}i!}\left(\frac{1-e^{\frac{-(x-\lambda)}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^{i}\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^{i}}{\sigma^{i}i!}\left(\frac{1-e^{\frac{-(x-\lambda)}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^{i}\right)} \tag{4.8}$$

The hazard rate function, $H(x) = \frac{f(x)}{R(x)}$, of [0,1]THL-HL distribution can be obtained as,

$$H(x)_{THL-HL} = \frac{\left\{ \begin{array}{c} \frac{4\left(1+e^{\frac{-1}{\sigma}}\right)}{\beta\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty} C_j^i C_m^{-2} \sum_n^{\infty} C_n^{-i} C_K^{-2} \frac{(-1)^{i+j}}{i!}\sigma^{-(1+i)} \\ (1+k)^i e^{\frac{-(j+m+1)(x-\lambda)}{\beta}} \end{array} \right\}}{\left\{ 1 - \frac{\left(1+e^{\frac{-1}{\sigma}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(\frac{1-e^{\frac{-(x-\lambda)}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^i\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma^i i!}\left(\frac{1-e^{\frac{-(x-\lambda)}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^i\right)} \right\}} \tag{4.9}$$

5. properties of the [0,1] THL-HL distribution are given respectively as

The r-th Moment: The r-th moment of Half- logistic distribution (H-L), can be obtained from $\int_0^{\infty} x^r f(x)_{THL-HL} dx$. According to (4.7), therefore the r-th moment of [0,1] THL-HL distribution can be obtained as follows,

$$E(x^r)_{THL-HL} = \int_0^{\infty} x^r \left\{ \begin{array}{c} \frac{4\left(1+e^{\frac{-1}{\sigma}}\right)}{\beta\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty} C_j^i C_m^{-2} \sum_n^{\infty} C_n^{-i} C_K^{-2} \frac{(-1)^{i+j}}{i!}\sigma^{-(1+i)} \\ (1+k)^i e^{\frac{-(j+m+1)(x-\lambda)}{\beta}} \end{array} \right\} dx$$

$$= \left\{ \begin{array}{c} \frac{4\left(1+e^{\frac{-1}{\sigma}}\right)}{\beta\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty} C_j^i C_m^{-2} \sum_n^{\infty} C_n^{-i} C_K^{-2} \frac{(-1)^{i+j}}{i!}\sigma^{-(1+i)} \\ (1+k)^i \int_0^{\infty} x^r e^{\frac{-(j+m+1)(x-\lambda)}{\beta}} \end{array} \right\}$$

By using the formula $\int_0^{\infty} x^{\alpha-1} e^{-\beta x} dx = \frac{r(\alpha)}{\beta^{\alpha}}$, Thus the r-th moment of the THL-HL distribution, is given by,

$$E(x^r)_{THL-HL} = \left\{ \begin{array}{c} \frac{4\left(1+e^{\frac{-1}{\sigma}}\right)}{\beta\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty} C_j^i C_m^{-2} \sum_n^{\infty} C_n^{-i} C_K^{-2} \frac{(-1)^{i+j}}{i!}\sigma^{-(1+i)} \\ e^{(j+m+1)\frac{\lambda}{\beta}} (1+k)^i \frac{r(r+1)}{\left(\frac{j+m+1}{\beta}\right)^{r+1}} \end{array} \right\} \tag{5.1}$$

Depending on the particular $E(x^r)_{THL-HL}$; ($r = 1,2,3,4$), another properties of this distribution such as, $(mean\ \mu = E(x))$, $variance\ \left(var(x) = \sigma^2 = E(x^2) - (E(x))^2\right)$ coefficient of skewness $\left(SK = \frac{E(X-\mu)^3}{\sigma^3} = \frac{E(x^2)-3\mu E(x^2)+2\mu^2}{[\sigma^2]^{\frac{3}{2}}}\right)$ And coefficient of kurtosis $\left(Kr = \frac{E(X-\mu)^4}{\sigma^4} = \frac{E(x^4)-4\mu E(x^3)+6\mu^2 E(x^2)-3\mu^3}{\sigma^4}\right)$
Can be obtained, where,

$$E(x)_{THL-HL} = \left\{ \begin{array}{c} \frac{4\left(1+e^{\frac{-1}{\sigma}}\right)}{\beta\left(1-e^{\frac{-1}{\sigma}}\right)} \sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty} C_j^i C_m^{-2} \sum_n^{\infty} C_n^{-i} C_K^{-2} \frac{(-1)^{i+j}}{i!}\sigma^{-(1+i)} \\ e^{(j+m+1)\frac{\lambda}{\beta}} (1+k)^i \frac{r(2)}{\left(\frac{j+m+1}{\beta}\right)^2} \end{array} \right\}$$

$$E\left(x^2\right)_{THL-HL}=\left\{\begin{array}{c}\frac{4\left(1+e^{\frac{-1}{\sigma}}\right)}{\beta\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}C_j^iC_m^{-2}\sum_{n}^{\infty}C_n^{-i}C_K^{-2}\frac{(-1)^{i+j}}{i!}\sigma^{-(1+i)}\\ e^{(j+m+1)\frac{\lambda}{\beta}}\left(1+k\right)^i\frac{r(3)}{\left(\frac{j+m+1}{\beta}\right)^3}\end{array}\right\}$$

$$E\left(x^3\right)_{THL-HL}=\left\{\begin{array}{c}\frac{4\left(1+e^{\frac{-1}{\sigma}}\right)}{\beta\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}C_j^iC_m^{-2}\sum_{n}^{\infty}C_n^{-i}C_K^{-2}\frac{(-1)^{i+j}}{i!}\sigma^{-(1+i)}\\ e^{(j+m+1)\frac{\lambda}{\beta}}\left(1+k\right)^i\frac{r(3)}{\left(\frac{j+m+1}{\beta}\right)^3}\end{array}\right\}$$

$$E\left(x^4\right)_{THL-HL}=\left\{\begin{array}{c}\frac{4\left(1+e^{\frac{-1}{\sigma}}\right)}{\beta\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}C_j^iC_m^{-2}\sum_{n}^{\infty}C_n^{-i}C_K^{-2}\frac{(-1)^{i+j}}{i!}\sigma^{-(1+i)}\\ e^{(j+m+1)\frac{\lambda}{\beta}}\left(1+k\right)^i\frac{r(5)}{\left(\frac{j+m+1}{\beta}\right)^5}\end{array}\right\}$$

The Characteristic Function : The characteristic function of the THL-HL distribution can be obtained from,

$$E\left(\ e^{itx}\right)=\sum_{r=0}^{\infty}\frac{(it)^r}{r!}E(X^r)_{THL-HL}.$$

Therefor the characteristic function of the THL-HL distribution is given by,

$$\emptyset_X\left(t\right)_{THL-HL}=\left\{\begin{array}{c}\frac{4\left(1+e^{\frac{-1}{\sigma}}\right)}{\beta\left(1-e^{\frac{-1}{\sigma}}\right)}\sum_{r=0}^{\infty}\frac{(it)^r}{r!}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}C_j^iC_m^{-2}\sum_{n}^{\infty}C_n^{-i}C_K^{-2}\frac{(-1)^{i+j}}{i!}\sigma^{-(1+i)}\\ e^{(j+m+1)\frac{\lambda}{\beta}}\left(1+k\right)^i\frac{?(r+1)}{\left(\frac{j+m+1}{\beta}\right)^{r+1}}\end{array}\right\}\tag{5.2}$$

Shannon Entropy: The Shannon entropy is defined as a measure of uncertainty that play an important role with the information's theory. Mathematically, Shannon entropy define as an expected of ($-\ln f(x)$). Thus, the Shannon entropy of the THL-HL distribution can be obtained from $-\int_0^{\infty}\ln\left(f(x)_{tHL-HL}\right)f\left(x\right)_{H-L}\,dx$. Taking the natural logarithm of the pdf in (3.5), and accorded (4.1) and (4.2), we get.

$$ln\left(f\left(x\right)_{THL-HL}\right)=ln\left\{\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)e^{\frac{-Q(x)}{\sigma}}q\left(x\right)}{\left(1-e^{\frac{-1}{\sigma}}\right)\sigma\left(1+e^{\frac{-Q(x)}{\sigma}}\right)^{2}}\right\}$$

$$=ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right)-\frac{Q\left(x\right)}{\sigma}+\ln\left(q\left(x\right)\right)-ln\left(1+e^{\frac{-Q(x)}{\sigma}}\right)^{2}$$

$$=ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right)-\frac{\left(1-e^{\frac{-(x-\lambda)}{\beta}}\right)}{\sigma\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)}+\ln\left(\frac{2e^{\frac{-(x-\lambda)}{\beta}}}{\beta\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^{2}}\right)-ln\left(1+e^{\frac{-Q(x)}{\sigma}}\right)^{2}$$

$$=ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right)-\frac{\left(1-e^{\frac{-(x-\lambda)}{\beta}}\right)}{\sigma\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)}-\frac{(x-\lambda)}{\beta}+ln\left(\frac{2}{\beta}\right)-2ln\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)-2ln\left(1+e^{\frac{-Q(x)}{\sigma}}\right)$$

By using the following Formula[1],

- $ln\left(u\right)=\sum_{i=0}^{\infty}\frac{(-1)^{i}(u-1)^{i-1}}{i+1}$; $\quad 0<u\leq 2$

$$ln\left(f\left(x\right)_{THL-HL}\right)=\left\{\begin{array}{c} ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right)-\frac{\left(1-e^{\frac{-(x-\lambda)}{\beta}}\right)}{\sigma\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)}-\frac{(x-\lambda)}{\beta}+ln\left(\frac{2}{\beta}\right)\\ -2\sum_{i=0}^{\infty}\frac{(-1)^{i}\left(e^{\frac{-(x-\lambda)}{\beta}}\right)^{i-1}}{i+1}-2\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(e^{\frac{-Q(x)}{\sigma}}\right)^{k-1}}{k+1}\end{array}\right\} \tag{5.3}$$

Now Let, $I=\ 2\sum_{i=0}^{\infty}\frac{(-1)^{i}\left(e^{\frac{-(x-\lambda)}{\beta}}\right)^{i-1}}{i+1}$, $II=2\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(e^{\frac{-Q(x)}{\sigma}}\right)^{k-1}}{k+1}$ *and* $III=\frac{\left(1-e^{\frac{-(x-\lambda)}{\beta}}\right)}{\sigma\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)}$

$$I=\ 2\sum_{i=0}^{\infty}\frac{(-1)^{i}\left(e^{\frac{-(x-\lambda)}{\beta}}\right)^{i-1}}{i+1}=2\sum_{i=0}^{\infty}\frac{(-1)^{i}e^{\frac{-(i-1)(x-\lambda)}{\beta}}}{i+1}$$

According (3.3) will be,

$$I=2\sum_{i=0}^{\infty}\frac{(-1)^{i}}{i+1}e^{\frac{(i-1)\lambda}{\beta}}\sum_{j=0}^{\infty}\frac{(-1)^{j}}{j!}\frac{(i-1)^{i}}{\beta^{i}}x^{i}$$

$$II=2\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(e^{\frac{-Q(x)}{\sigma}}\right)^{k-1}}{k+1}=2\sum_{k=0}^{\infty}\frac{(-1)^{k}e^{\frac{-(K-1)Q(x)}{\sigma}}}{k+1}$$

According (3.3) will be,

$$II=2\sum_{k=0}^{\infty}\frac{(-1)^{k}}{k+1}\sum_{n=0}^{\infty}\frac{(-1)^{n}(k-1)^{n}}{n!\sigma^{n}}Q(x)^{n} \tag{5.4}$$

Now, according (4.1) we get,

$$Q(x)^n = \left(\frac{1-e^{\frac{-(x-\lambda)}{\beta}}}{1+e^{\frac{-(x-\lambda)}{\beta}}}\right)^n = \frac{\left(1-e^{\frac{-(x-\lambda)}{\beta}}\right)^n}{\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^n}$$

According (3.6) and (4.5) will be,

$$Q(x)^n = \sum_{z=0}^{\infty}(-1)^z \mathsf{C}_z^n e^{\frac{-z(x-\lambda)}{\beta}} \sum_{s=0}^{\infty} \mathsf{C}_s^{-n} e^{\frac{-s(x-\lambda)}{\beta}}$$

According (3.3) will be,

$$Q(x)^n = \sum_{z=0}^{\infty}(-1)^z \mathsf{C}_z^n e^{\frac{z\lambda}{\beta}} \sum_{m=0}^{\infty} \frac{(-1)^m z^m}{m!\beta^m} x^m \sum_{s=0}^{\infty} \mathsf{C}_s^{-n} e^{\frac{s\lambda}{\beta}} \sum_{t=0}^{\infty} \frac{(-1)^t s^t}{t!\beta^t} x^t \tag{5.5}$$

Substitute (5.5) in (5.4) we get,

$$II = \left\{ \begin{array}{c} 2\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} \sum_{n=0}^{\infty} \frac{(-1)^n (k-1)^n}{n!\sigma^n} \sum_{z=0}^{\infty} (-1)^z \mathsf{C}_z^n e^{\frac{z\lambda}{\beta}} \\ \sum_{m=0}^{\infty} \frac{(-1)^m z^m}{m!\beta^m} x^m \sum_{s=0}^{\infty} \mathsf{C}_s^{-n} e^{\frac{s\lambda}{\beta}} \sum_{t=0}^{\infty} \frac{(-1)^t s^t}{t!\beta^t} x^t \end{array} \right\}$$

Now, According (3.6) and (4.5) we get (III),

$$\begin{aligned} III = & \frac{\left(1-e^{\frac{-(x-\lambda)}{\beta}}\right)}{\sigma\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)} = \frac{1}{\sigma}\left(1-e^{\frac{-(x-\lambda)}{\beta}}\right)\left(1+e^{\frac{-(x-\lambda)}{\beta}}\right)^{-1} \\ = & \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^c \mathsf{C}_c^1 e^{\frac{-c(x-\lambda)}{\beta}} \sum_{b=0}^{\infty} \mathsf{C}_b^{-1} e^{\frac{-b(x-\lambda)}{\beta}} \\ = & \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^c \mathsf{C}_c^1 \sum_{b=0}^{\infty} \mathsf{C}_b^{-1} e^{\frac{-(c+b)(x-\lambda)}{\beta}} \end{aligned}$$

According (3.3) will be,

$$III = \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^c \mathsf{C}_c^1 \sum_{b=0}^{\infty} \mathsf{C}_b^{-1} e^{\frac{(c+b)\lambda}{\beta}} \sum_{f=0}^{\infty} \frac{(-1)^f}{f!}\left(\frac{c+b}{\beta}\right)^f x^f$$

Substitute (I), (II) and (III) in (5.3) we get,

$$ln\left(f(x)_{THL-HL}\right) = \left\{ \begin{array}{c} ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right) - \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^c \mathsf{C}_c^1 \sum_{b=0}^{\infty} \mathsf{C}_b^{-1} e^{\frac{(c+b)\lambda}{\beta}} \sum_{f=0}^{\infty} \frac{(-1)^f}{f!}\left(\frac{c+b}{\beta}\right)^f x^f \\ -\frac{(x-\lambda)}{\beta} + ln\left(\frac{2}{\beta}\right) - 2\sum_{i=0}^{\infty} \frac{(-1)^i}{i+1} e^{\frac{(i-1)\lambda}{\beta}} \sum_{j=0}^{\infty} \frac{(-1)^j}{j!} \frac{-(i-1)^i}{\beta^i} x^i \\ -2\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} \sum_{n=0}^{\infty} \frac{(-1)^n (k-1)^n}{n!\sigma^n} \sum_{z=0}^{\infty} (-1)^z \mathsf{C}_z^n e^{\frac{z\lambda}{\beta}} \\ \sum_{m=0}^{\infty} \frac{(-1)^m z^m}{m!\beta^m} x^m \sum_{s=0}^{\infty} \mathsf{C}_s^{-n-s} e^{\frac{s\lambda}{\beta}} \sum_{t=0}^{\infty} \frac{(-1)^t s^t}{t!\beta^t} x^t \end{array} \right\}$$

Now, the Shannon entropy of the THL-HL distribution can be obtained as:

$$SH_{THL-HL} =$$

$$-\int_0^\infty \left\{ \begin{array}{c} ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right) - \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^c C_c^1 \sum_{b=0}^{\infty} C_b^{-1} e^{\frac{(c+b)\lambda}{\beta}} \sum_{f=0}^{\infty}\frac{(-1)^f}{f!}\left(\frac{c+b}{\beta}\right)^f x^f \\ -\frac{(x-\lambda)}{\beta} + ln\left(\frac{2}{\beta}\right) - 2\sum_{i=0}^{\infty}\frac{(-1)^i}{i+1}e^{\frac{(i-1)\lambda}{\beta}}\sum_{j=0}^{\infty}\frac{(-1)^j}{j!}\frac{-(i-1)^i}{\beta^i}x^i \\ -2\sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}\sum_{n=0}^{\infty}\frac{(-1)^n(k-1)^n}{n!\sigma^n}\sum_{z=0}^{\infty}(-1)^z C_z^n e^{\frac{z\lambda}{\beta}} \\ \sum_{m=0}^{\infty}\frac{(-1)^m z^m}{m!\beta^m}x^m \sum_{s=0}^{\infty} C_s^{-n-s} e^{\frac{s\lambda}{\beta}} \sum_{t=0}^{\infty}\frac{(-1)^t s^t}{t!\beta^t}x^t \end{array} \right\} f(x)_{THL-HL}$$

Therefor the Shannon entropy of the THL-HL distribution in (3.2) will be,

$$SH_{THL-HL} = \left\{ \begin{array}{c} -ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right) + \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^c C_c^1 \sum_{b=0}^{\infty} C_b^{-1} e^{\frac{(c+b)\lambda}{\beta}} \sum_{f=0}^{\infty}\frac{(-1)^f}{f!}\left(\frac{c+b}{\beta}\right)^f \int_0^\infty x^f f(x)\,dx \\ +\frac{(x-\lambda)}{\beta} - ln\left(\frac{2}{\beta}\right) + 2\sum_{i=0}^{\infty}\frac{(-1)^i}{i+1}e^{\frac{(i-1)\lambda}{\beta}}\sum_{j=0}^{\infty}\frac{(-1)^j}{j!}\frac{-(i-1)^i}{\beta^i}\int_0^\infty x^i f(x)\,dx \\ +2\sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}\sum_{n=0}^{\infty}\frac{(-1)^n(k-1)^n}{n!\sigma^n}\sum_{z=0}^{\infty}(-1)^z C_z^n e^{\frac{z\lambda}{\beta}} \\ \sum_{m=0}^{\infty}\frac{(-1)^m z^m}{m!\beta^m}\int_0^\infty x^m f(x)\,dx \sum_{s=0}^{\infty} C_s^{-n-s} e^{\frac{s\lambda}{\beta}} \sum_{t=0}^{\infty}\frac{(-1)^t s^t}{t!\beta^t}\int_0^\infty x^t f(x)\,dx \end{array} \right\}$$

$$SH_{THL-HL} = \left\{ \begin{array}{c} -ln\left(\frac{2\left(1+e^{\frac{-1}{\sigma}}\right)}{\sigma\left(1-e^{\frac{-1}{\sigma}}\right)}\right) + \frac{1}{\sigma}\sum_{c=0}^{\infty}(-1)^c C_c^1 \sum_{b=0}^{\infty} C_b^{-1} e^{\frac{(c+b)\lambda}{\beta}} \sum_{f=0}^{\infty}\frac{(-1)^f}{f!}\left(\frac{c+b}{\beta}\right)^f E(X^f) \\ +\frac{(x-\lambda)}{\beta} - ln\left(\frac{2}{\beta}\right) 2\sum_{i=0}^{\infty}\frac{(-1)^i}{i+1}e^{\frac{(i-1)\lambda}{\beta}}\sum_{j=0}^{\infty}\frac{(-1)^j}{j!}\frac{-(i-1)^i}{\beta^i}E(X^i) \\ +2\sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}\sum_{n=0}^{\infty}\frac{(-1)^n(k-1)^n}{n!\sigma^n}\sum_{z=0}^{\infty}(-1)^z C_z^n e^{\frac{z\lambda}{\beta}} \\ \sum_{m=0}^{\infty}\frac{(-1)^m z^m}{m!\beta^m}E(X^m) \sum_{s=0}^{\infty} C_s^{-n-s} e^{\frac{s\lambda}{\beta}} \sum_{t=0}^{\infty}\frac{(-1)^t s^t}{t!\beta^t}E(X^t) \end{array} \right\}$$

Where, $E\left(X^f\right), E\left(X^i\right), E\left(X^m\right) andE\left(X^t\right)$ as in (5.1) with $(r = f, i, m, t)$

The Stress Strength: Let Y and X be the Stress Strength random variables that independent of each other follows respectively [0,1] THL-HL with different Parameters, then the Stress Strength can be obtained by,

$SS_{THL-HL} = P(Y < X) = \int_0^\infty f_X(x)_{THL-HL} F_y(x)\,dx$, where,

$$F_y(x) = \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)\left(1-\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma_1{}^i i!}\left(\frac{1-e^{\frac{-(x-\lambda_1)}{\beta_1}}}{1+e^{\frac{-(x-\lambda_1)}{\beta_1}}}\right)^i\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)\left(1+\sum_{i=0}^{\infty}\frac{(-1)^i}{\sigma_1{}^i i!}\left(\frac{1-e^{\frac{-(x-\lambda_1)}{\beta_1}}}{1+e^{\frac{-(x-\lambda_1)}{\beta_1}}}\right)^i\right)} \tag{5.6}$$

Now, in order to find the Stress Strength, we firstly expansion the $F_y(x)$ as follows,

Let $J = \left(\frac{1-e^{\frac{-(x-\lambda_1)}{\beta_1}}}{1+e^{\frac{-(x-\lambda_1)}{\beta_1}}}\right)^i = \left(1-e^{\frac{-(x-\lambda_1)}{\beta_1}}\right)^i\left(1+e^{\frac{-(x-\lambda_1)}{\beta_1}}\right)^{-i}$

According (3.6) and (4.5) will be,

$$J = \left(\sum_{k=0}^{\infty}(-1)^k C_k^i\, e^{\frac{-k(x-\lambda_1)}{\beta_1}}\right)\left(\sum_{j=0}^{\infty} C_j^{-i}\, e^{\frac{-j(x-\lambda_1)}{\beta_1}}\right)$$

$$= \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} C_j^{-i} C_k^{i} (-1)^k e^{\frac{(k+j)\lambda_1}{\beta_1}} e^{\frac{-(k+j)x}{\beta_1}}$$

According (3.3) will be,

$$J = \sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty} \frac{(-1)^{m+k}}{m!} C_j^{-i} C_k^{i} \left(\frac{k+j}{\beta_1}\right)^m e^{\frac{(k+j)\lambda_1}{\beta_1}} x^m$$

Substitute J in (5.6) we get,

$$F_y(x) = \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)\left(1-\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{m+k+i}}{m!K!i!\sigma_1{}^i} C_j^{-i} C_k^{i}\left(\frac{k+j}{\beta_1}\right)^m e^{\frac{(k+j)\lambda_1}{\beta_1}} x^m\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)\left(1+\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{m+k+i}}{m!K!i!\sigma_1{}^i} C_j^{-i} C_k^{i}\left(\frac{k+j}{\beta_1}\right)^m e^{\frac{(k+j)\lambda_1}{\beta_1}} x^m\right)}$$

$$F_y(x) = \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)}\left[\frac{2}{\left(1-\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{m+k+i}}{m!K!i!\sigma_1{}^i} C_j^{-i} C_k^{i}\left(\frac{k+j}{\beta_1}\right)^m e^{\frac{(k+j)\lambda_1}{\beta_1}} x^m\right)} - 1\right]$$

$$= \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)}\left[2\left(1-\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{m+k+i}}{m!K!i!\sigma_1{}^i} C_j^{-i} C_k^{i}\left(\frac{k+j}{\beta_1}\right)^m e^{\frac{(k+j)\lambda_1}{\beta_1}} x^m\right)^{-1} - 1\right]$$

Accorded (3.6) will

$$F_y(x) = \left\{\begin{matrix} \frac{2\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)}\left(\sum_{n=0}^{\infty} C_n^{-1}\left(\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{m+k+i}}{m!K!i!\sigma_1{}^i} C_j^{-i} C_k^{i}\left(\frac{k+j}{\beta_1}\right)^m e^{\frac{(k+j)\lambda_1}{\beta_1}}\right)^n\right) \\ x^{nm} - \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)} \end{matrix}\right\} \quad (5.7)$$

Therefor based on (5.7), the stress strength of the [0,1] THL-HL distribution can be obtained as,

$$SS_{THL-HL} = \left\{\begin{matrix} \frac{2\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)}\left(\sum_{n=0}^{\infty} C_n^{-1}\left(\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{m+k+i}}{m!K!i!\sigma_1{}^i} C_j^{-i} C_k^{i}\left(\frac{k+j}{\beta_1}\right)^m e^{\frac{(k+j)\lambda_1}{\beta_1}}\right)^n\right) \\ \int_0^{\infty} x^{nm} f_X(x)_{THL-HL} - \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)} \end{matrix}\right\}$$

$$= \left\{\begin{matrix} \frac{2\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)}\left(\sum_{n=0}^{\infty} C_n^{-1}\left(\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{m+k+i}}{m!K!i!\sigma_1{}^i} C_j^{-i} C_k^{i}\left(\frac{k+j}{\beta_1}\right)^m e^{\frac{(k+j)\lambda_1}{\beta_1}}\right)^n\right) \\ E\left(x^{nm}\right) - \frac{\left(1+e^{\frac{-1}{\sigma_1}}\right)}{\left(1-e^{\frac{-1}{\sigma_1}}\right)} \end{matrix}\right\}$$

Where, $E\left(X^{nm}\right)$ as in (5.1) with $(r = nm)$

6. Conclusion

In this search, we generated a class of continuous distributions, named [0,1] truncated L-Q distributions. This class is based on replacing the beta distribution in the beta-G family by the interval [0,1] truncated distributions.
The proposed Distribution are: [0,1] Truncated Half Logistic- Half Logistic Distribution.

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