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Analysis of Markovian queueing system with server failures, N-policy and second optional service

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Abstract

The current work details the behaviour of a finite Markovian queueing system with a vacation in which the server may face problems of breakdowns while in service. The repair process does start immediately after a breakdown which immediately resumes the service. During this period any new customer is allowed to join the system. Whenever the server finds nobody, the server goes on vacation and resumes service after N customers are accumulated. Meanwhile, it triggers pre-service called start-up. Further, we considered two types of repair facilities for the broken-down server with an optional probability. The server first provides essential service to all customers and the second optional service will be provided with a probability of "p". The customer may renege in the first phase of service. We adopted Runge-Kutta Method to find Transient state probabilities and computed various performance indices like the expected length of the system, the mean waiting time etc. We then performed the sensitivity analysis to explore the effect of different parameters.

Keywords: N-Policy, Second optional service, Start-up, Two types of repair facilities.

1. Introduction

Queuing theory is a tool of Operations Research to understand the dynamic pattern of the processes as well as for the performance evaluation of such systems. Recent eras have seen an increasing attention in queueing models due to their wide applications.

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Breakdowns may occur for any server on continuous use which leads to loss of production, goodwill etc. To ensure proper production, Queueing theory plays a vital role with respect to the repair of the machines to optimize the system. Vasantha Kumar et.al, Jain et.al [5, 6, 8, 9, 10, 20, 17, 15, 16, 21] have contributed significantly this domain.

In real life, many queueing systems come across the problems of customer's impatient behaviours. These situations will cause loss to the server end. Many researchers like R.O Al-Seedy et al., C.J.Ancker et.al [2, 3, 7, 9, 10, 22, 14, 13, 1, 18, 23, 24] are working towards optimizing the system by reducing customer impatient behaviour.

Vacation concept in Queuing models has been extensively detailed in Ke et al., Doshi and Tian and Zhang et al. [4, 11, 19] etc. where the server is assumed to remain idle and becomes completely unproductive.

To the best of our knowledge, the existing literature focus mainly on the study of queueing system in steady state fashion. From the practical point of view transient analysis is needed to deal with this type of models to study many real time situations.

In this paper, we present the transient analysis of Markovian queue with server vacation, breakdowns, customers 'impatience and second optional service in a finite capacity system. We have used R-K method to find transient state probability distributions.

The rest of this paper is organized as follows: In section 2, we give the model description. In section 3, we present the Transient state solution. In section 4, we present some system performance measures through numerical results and carried sensitivity analysis. Section 5 details final conclusions.

2. Model Description

We consider the Transient Analysis of Markovian Queue with Working Vacation, Server Failures and Customer Impatience with the following assumptions:

1. The capacity of the system is assumed as S(finite)
2. The mean arrival rate is λ.
3. The mean service rates for first essential service and second optional service are μ_1 and μ_2.
4. After serving all existing customers in the queue, the server initiates vacation. The server does pre service work called start-up until N customers are accumulated which follows an exponential distribution with mean 1/Ø. Once the number in the system reaches to N, the server renders normal and first essential service.
5. Customers who want additional service can choose the second optional service with a probability of 'P'.
6. Unforeseen failures may hit the server and leads to breakdowns. The server failure rates are taken as ξ_1 and α_1 in first essential service and second optional service modes. The broken down server can be repaired with a parameter ξ_2 in one repair facility with a probability of "a" whereas the second type repair facility rate is ξ_3 which can be used with a probability of "1-a".The repair rate at second optional service phase is assumed as α_2. It is also assumed that the process of repair will be started without any delay.

3. Notations

In this paper, we have used the following notations to denote transient probabilities for the system to be in various states:

$\pi_{0,i} = p$(i customers in the system when the server is in vacation state mode)
$\pi_{1,i} = p$(i customers in the system when the server is doing pre-service)
$\pi_{2,i} = p$(i customers in the system when the server is doing first essential service)
$\pi_{3,i} = p$(i customers in the system when the server is doing first essential service but broken down)
$\pi_{4,i} = p$(i customers in the system when the server is doing second optional service)
$\pi_{5,i} = p$(i customers in the system when the server isdoing second optional service but broken down)

The Transient state equations governing the various probabilities are detailed in the form of following Differential equations:

$$\lambda\pi_{0,0} = (1-p)\mu_1\pi_{2,1} + \mu_2\pi_{4,1} \tag{3.1}$$

$$\lambda\pi_{0,i} = \lambda\pi_{0,i-1},\ 1 \le i \le N-1 \tag{3.2}$$

$$(\lambda + \O)\pi_{1,N} = \lambda\pi_{0,N-1} \tag{3.3}$$

$$(\lambda + \O)\pi_{1,i} = \lambda\pi_{1,i-1},\ i \ge N+1 \tag{3.4}$$

$$(\lambda + \mu_1 + \xi_1)\pi_{2,i} = \lambda\pi_{2,i-1} + (1-p)\mu_1\pi_{2,i+1} + a\xi_2\pi_{3,i} + (1-a)\xi_3\pi_{3,i},\ 1 \le i \le N-1 \tag{3.5}$$

$$(\lambda + \mu_1 + \xi_1)\pi_{2,i} = \lambda\pi_{2,i-1} + (1-p)\mu_1\pi_{2,i+1} + a\xi_2\pi_{3,i} + (1-a)\xi_3\pi_{3,i} + \O\pi_{1,i}, i \ge N \tag{3.6}$$

$$(\lambda + a\xi_2 + (1-a)\xi_3)\pi_{3,i} = \lambda\pi_{3,i-1} + \xi_1\pi_{2,i} + (1-a)\xi_3\pi_{3,i},\ i \ge 1 \tag{3.7}$$

$$(\lambda + \mu_2 + \alpha_1)\pi_{4,i} = \lambda\pi_{4,i-1} + p\mu_1\pi_{2,i+1} + \mu_2\pi_{4,i+1} + \alpha_2\pi_{5,i},\ i \ge 1 \tag{3.8}$$

$$(\lambda + \alpha_2)\pi_{5,i} = \lambda\pi_{5,i-1} + \alpha_1\pi_{4,i},\ i \ge 1 \tag{3.9}$$

4. Generating functions

We have applied probability generating functions to obtain various performance measures as shown below:

$G_0(z) = \sum_{i=0}^{N-1}, G_1(z) = \sum_{i=N}^{\infty} p_{1,i}z^i, G_2(z) = \sum_{i=1}^{\infty} p_{2,i}z^i, G_3(z) = \sum_{i=1}^{\infty} p_{3,i}z^i, G_4(z) = \sum_{i=1}^{\infty} p_{4,i}z^i$ and $G_5(z) = \sum_{i=1}^{\infty} p_{5,i}z^i$

Multiplication of equation (3.1),(3.2),(3.3),(3.4),(3.5),(3.6),(3.7),(3.8),(3.9) by z^i and adding over i in respective limits, they give

$$G_0(Z) = \frac{(1-z^N)}{(1-z)} p_{0,0} \tag{4.1}$$

$$(\lambda(1-z) + \varnothing)G_1(Z) = (\lambda z^N)p_{0,0} \tag{4.2}$$

$$(\lambda(1-z) + \mu_1 + \xi_1 - (1-p)\mu_1/z)G_2(Z) = \varnothing G_1(Z) + (a\xi_2 + (1-a)\xi_3)G_3(Z) - (1-p)\mu_1\pi_{2,1} \tag{4.3}$$

$$(\lambda(1-z) + a\xi_2 + (1-a)\xi_3)G_3(Z) = \xi_1 G_2(Z) \tag{4.4}$$

$$(\lambda(1-z) + \mu_2 + \alpha_1 - \mu_2/z)G_4(Z) = p^{\mu_1/z}G_2(Z) - \varnothing G_1(1) + \alpha_2 G_5(Z) \tag{4.5}$$

$$(\lambda(1-z) + \alpha_2)G_5(Z) = \alpha_1 G_4(Z) \tag{4.6}$$

The total probability generating function $G(z)$ is given by

$$G(z) = \sum_{i=0}^{5} G_i(z) \tag{4.7}$$

The normalizing condition is

$$G(1) = \sum_{i=0}^{5} G_i(1) = 1 \tag{4.8}$$

From equations (4.1) to (4.8), we get

$$G_0(1) = N\pi_{0,0} \tag{4.9}$$

$$G_1(1) = \frac{\lambda}{\varnothing}\pi_{0,0} \tag{4.10}$$

$$G_2(1) = \frac{\lambda}{p\mu_1}\pi_{0,0} \tag{4.11}$$

$$G_3(1) = \frac{\xi_1}{(a\xi_2 + (1-a)\xi_3)} G_2(1) \tag{4.12}$$

$$G_4(1) = \frac{\xi_1}{(a\xi_2 + (1-a)\xi_3)} G_3(1) \tag{4.13}$$

$$G_5(1) = \frac{\alpha_1}{\alpha_2} G_4(1) \tag{4.14}$$

Probability that the server is neither in first compulsory service nor in second optional service is given by

$$G_0(1) + G_1(1) = 1 - \frac{\lambda}{\mu_1}\left(1 + \frac{\xi_1}{(a\xi_2 + (1-a)\xi_3)}\right) - \frac{\lambda}{\mu_2}\left(1 + \frac{\alpha_1}{\alpha_2}\right)$$

This gives

$$p_{0,0} = (1-\rho)\frac{\emptyset}{(\lambda + N\emptyset)} \tag{4.15}$$

Where $\rho = \left(\frac{\lambda}{\mu_1}\left(1+\frac{\xi_1}{(a\xi_2+(1-a)\xi_3)}\right) - \frac{\lambda}{\mu_2}\left(1+\frac{\alpha_1}{\alpha_2}\right)\right)$.

Let $\pi_0, \pi_1, \pi_2, \pi_3, \pi_4$ and π_5 be the probabilities that the server is in vacation, start-up, first compulsory service, first compulsory service with break down, second optional service and in second optional service with breakdown states respectively. Then,

$$\pi_0 = G_0(1) \tag{4.16}$$

$$\pi_1 = G_1(1) \tag{4.17}$$

$$\pi_2 = G_2(1) \tag{4.18}$$

$$\pi_3 = G_3(1) \tag{4.19}$$

$$\pi_4 = G_4(1) \tag{4.20}$$

$$\pi_5 = G_5(1) \tag{4.21}$$

4.1. Expected system length at different states of the server

Let L_0, L_1, L_2, L_3, L_4 and L_5 be the expected number of customers in the system when the server is in vacation, start-up, first compulsory service, first compulsory service in failure mode, second optional service and in second optional service in failure mode respectively. Solving the above equations, we get

$$L_0 = \sum_{i=0}^{N-1} i\pi_{0,i} = G_0'(1) = \frac{N(N-1)}{2}\pi_{0,0} \tag{4.22}$$

$$L_1 = \sum_{i=N}^{\infty} i\pi_{1,i} = G_1'(1) = \frac{\lambda(\lambda+N\phi)}{\phi^2}\pi_{0,0} \tag{4.23}$$

$$L_2 = \sum_{i=1}^{\infty} i\pi_{2,i} = G_2'(1) = \frac{1}{p\mu_{1s_1}}(\phi s_1 G_1' + \lambda(s_1+\xi_1)G_2(1)) \tag{4.24}$$

Where

$$s_1 = (a\xi_2 + (1-a)\xi_3) \tag{4.25}$$

$$L_3 = \sum_{i=1}^{\infty} i\pi_{3,i} = G_3'(1) = \frac{\xi_1}{s_1}(s_1 G_2' - \lambda G_2(1)) \tag{4.26}$$

$$L_4 = \sum_{i=1}^{\infty} i\pi_{4,i} = G_4'(1) = \frac{1}{2f'(1)}(p\mu_1 G_2'' - G_4(1) * f''(1)) \tag{4.27}$$

where

$$f(z) = \frac{(\lambda(1-z) + \mu_2 + \alpha_1) * (\lambda z(1-z) + \alpha_2 z) - \mu_2(\lambda(1-z) + \alpha_2) - \alpha_1\alpha_2 z}{(\lambda z(1-z) + \alpha_2 z)} \tag{4.28}$$

$$L_5 = \sum_{i=1}^{\infty} i\pi_{5,i} = G_5'(1) = \frac{\alpha_1\{G_4'(1)\alpha_2 + \lambda G_4(1)\}}{\alpha_2^2} \tag{4.29}$$

The expected number of customers in the system is given by

$$L(N) = L_0 + L_1 + L_2 + L_3 + L_4 + L_5 \tag{4.30}$$

4.2. Characteristic features of the system

Let E_0, E_1, E_2, E_3, E_4 and E_5 denote the expected length of vacation period, start-up period, first compulsory service period, first compulsory service in failure mode, second optional service and in second optional service in failure mode respectively. Then the expected length of a busy cycle is given by

$$E_c = E_0 + E_1 + E_2 + E_3 + E_4 + E_5 \tag{4.31}$$

The long run fractions of time the server is in different states are as follows:

$$\frac{E_i}{E_c} = \pi_i, \ i = 0, 1, ..., 5 \tag{4.32}$$

Mean length of vacation period is given by

$$E_0 = \frac{N}{\lambda} \tag{4.33}$$

Hence,

$$E_c = \frac{1}{(\lambda p_{0,0})} \tag{4.34}$$

4.3. Optimum control policy

We govern the optimal value of N that minimizes the long run average cost of an $M/M/1$ queueing system with N-policy and second optional service. To fix the optimal value of N, we consider the following linear cost structure.

Let $T(N)$ be the average cost per unit of time, then

$$T(N) = C_h L(N) + C_o\left(\frac{E_2}{E_c} + \frac{E_4}{E_c}\right) + C_m\left(\frac{E_S}{E_c}\right) + C_{b1}\left(\frac{E_3}{E_c}\right) + C_{b2}\left(\frac{E_5}{E_c}\right) + C_s\left(\frac{1}{E_c}\right) - C_r\left(\frac{E_0}{E_c}\right) \tag{4.35}$$

where

C_h =Holding cost per unit time for each customer present in the system,

C_o =Cost per unit time for keeping the server on and in operation,
C_m =Start-up cost per unit time,
C_s =Setup cost per cycle,
C_{b1} =Break down cost per unit time for the unavailable server in batch service mode,
C_{b2} =Break down cost per unit time for the unavailable server in individual service mode,
C_r =Reward per unit time as the server is doing secondary work in vacation.

To fix the optimal operating N-policy, minimize $T(N)$ in equation (4.35).
An approximate value of the optimal threshold N^* can be found by solving the equation

$$\frac{dT_1(N)}{dN}\,|_{N=N^*}=0 \tag{4.36}$$

A computational algorithm translated in MATLAB is used to obtain the optimum values.

4.4. Sensitivity Analysis

The variations of monetary and non-monetary parameters on the optimal threshold N^*, mean number of jobs in the system and minimum expected cost are given in the following tables:
We perform this analysis by assuming
$\lambda = 0.092, \mu_1 = 5.628, \mu_2 = 4.87, \alpha_1 = 0.001, \alpha_2 = 0.002, \xi_1 = 0.003, \xi_2 = 0.004, \xi_3 = 0.005, a = 0.4, p = 0.25$ and $\phi = 21$
and
$C_h = 5, C_o = 20, C_m = 15, C_S = 3000, C_{b1} = 8, C_{b2} = 9$ and $C_r = 5$

Effect of variation in the monetary and non-monetary parameters are displayed through the following tables 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18:

Table 1: Effect of λ

λ	0.51	0.52	0.53	0.54	0.55	0.56
N^*	10.6334	11.0992	11.5388	11.9552	12.3508	12.7273
$L(N^*)$	5.4252	5.8026	6.1827	6.5671	6.957	7.3533
$T(N^*)$	49.917	53.1002	56.234	59.3328	62.4085	65.4706

Table 2: Effect of μ_1

μ_1	5.63	5.64	5.65	5.66	5.67	5.68
N^*	10.1376	10.1374	10.1373	10.1371	10.1369	10.1368
$L(N^*)$	5.0487	5.0486	5.0484	5.0483	5.0482	5.0481
$T(N^*)$	46.6668	46.6675	46.6681	46.6687	46.6693	46.6699

5. Conclusion

We have analysed an Markovian Queue with Vacation, Server Failures and second optional service with an objective of finding the threshold value for N that which can minimize total cost.And also

- presented closed analytical expressions of various performance measures

Table 3: Effect of μ_2

μ_2	5.63	5.64	5.65	5.66	5.67	5.68
N^*	10.1376	10.1374	10.1373	10.1371	10.1369	10.1368
$L(N^*)$	5.0487	5.0486	5.0484	5.0483	5.0482	5.0481
$T(N^*)$	46.6668	46.6675	46.6681	46.6687	46.6693	46.6699

Table 4: Effect of α_1

α_1	0.0011	0.0012	0.0013	0.0014	0.0015	0.0016
N^*	10.1195	10.1011	10.0827	10.0642	10.0456	10.0269
$L(N^*)$	5.0616	5.0743	5.0869	5.0994	5.1119	5.1242
$T(N^*)$	46.7604	46.8544	46.9483	47.042	47.1356	47.2289

Table 5: Effect of α_2

α_2	0.0021	0.0022	0.0023	0.0024	0.0025	0.0026
N^*	10.1622	10.1837	10.2027	10.2196	10.2348	10.2484
$L(N^*)$	5.0262	5.0063	4.9885	4.9727	4.9585	4.9457
$T(N^*)$	46.499	46.352	46.222	46.1062	46.0028	45.9098

Table 6: Effect of ξ_1

ξ_1	0.0031	0.0032	0.0033	0.0034	0.0035	0.0036
N^*	10.1405	10.1433	10.1461	10.1488	10.1516	10.1544
$L(N^*)$	5.0521	5.0554	5.0587	5.062	5.0653	5.0686
$T(N^*)$	46.6688	46.6714	46.6739	46.6764	46.679	46.6815

Table 7: Effect of ξ_2

ξ_2	0.0041	0.0042	0.0043	0.0044	0.0045	0.0046
N^*	10.1363	10.1349	10.1336	10.1323	10.131	10.1297
$L(N^*)$	5.0469	5.045	5.0432	5.0414	5.0397	5.038
$T(N^*)$	46.6621	46.6581	46.6542	46.6504	46.6468	46.6432

Table 8: Effect of ξ_3

ξ_3	0.0051	0.0052	0.0053	0.0054	0.0055	0.0056
N^*	10.1356	10.1336	10.1316	10.1297	10.1279	10.1261
$L(N^*)$	5.046	5.0432	5.0406	5.038	5.0356	5.0332
$T(N^*)$	46.6601	46.6542	46.6486	46.6432	46.6381	46.6332

Table 9: Effect of θ

θ	22	23	24	25	26	27
N^*	10.1379	10.138	10.1381	10.1382	10.1383	10.1383
$L(N^*)$	5.0488	5.0487	5.0487	5.0486	5.0486	5.0486
$T(N^*)$	46.6658	46.6654	46.6651	46.6647	46.6644	46.6642

- performed the sensitivity analysis to know the impact of both monetary and non-monetary measures on performance measures and total cost.

Table 10: Effect of p

p	0.26	0.27	0.28	0.29	0.30	0.31
N^*	10.1486	10.1585	10.1677	10.1762	10.1842	10.1916
$L(N^*)$	5.0633	5.0767	5.089	5.1005	5.1111	5.121
$T(N^*)$	46.7122	46.7546	46.7938	46.8301	46.8639	46.8954

Table 11: Effect of a

a	0.41	0.42	0.43	0.44	0.45	0.46
N^*	10.1381	10.1385	10.1389	10.1393	10.1396	10.14
$L(N^*)$	5.0493	5.0498	5.0503	5.0508	5.0513	5.0518
$T(N^*)$	46.6673	46.6683	46.6694	46.6705	46.6716	46.6727

Table 12: Effect of C_r

C_r	6	7	8	9	10	11
N^*	10.1379	10.138	10.138	10.1381	10.1382	10.1383
$L(N^*)$	5.0489	5.0489	5.0489	5.049	5.049	5.0491
$T(N^*)$	45.722	44.7778	43.8336	42.8894	41.9452	41.001

Table 13: Effect of C_{b1}

C_{b1}	9	10	11	12	13	14
N^*	10.1386	10.1394	10.1403	10.1411	10.1419	10.1427
$L(N^*)$	5.0492	5.0496	5.0501	5.0505	5.0509	5.0513
$T(N^*)$	46.6702	46.6742	46.6781	46.6821	46.6861	46.6901

Table 14: Effect of C_{b2}

C_{b2}	10	11	12	13	14	15
N^*	10.13705	10.13632	10.13559	10.13485	10.13412	10.13339
$L(N^*)$	5.048441	5.04807	5.047699	5.047329	5.046958	5.046587
$T(N^*)$	46.67187	46.67753	46.68319	46.68884	46.6945	46.70016

Table 15: Effect of C_m

C_m	16	17	18	19	20	21
N^*	10.1379	10.138	10.138	10.1381	10.1382	10.1383
$L(N^*)$	5.0489	5.0489	5.0489	5.049	5.049	5.0491
$T(N^*)$	46.6666	46.667	46.6674	46.6678	46.6682	46.6687

Table 16: Effect of C_o

C_0	22	24	26	28	30	32
N^*	10.1374	10.137	10.1366	10.1362	10.1358	10.1354
$L(N^*)$	5.0486	5.0484	5.0482	5.048	5.0478	5.0476
$T(N^*)$	46.701	46.7359	46.7707	46.8055	46.8404	46.8752

And also Observed that there is

- slight increase in system length wrt the raise in the parameters $\lambda, \alpha_1, \xi_1, p, a, C_r, C_{b1}, C_m$ and

Table 17: Effect of C_h

C_h	6	7	8	9	10	11
N^*	9.2055	8.4771	7.8867	7.3951	6.9769	6.6152
$L(N^*)$	4.574	4.1991	3.892	3.6335	3.4111	3.2166
$T(N^*)$	51.4674	55.8473	59.8882	63.6475	67.1671	70.479

Table 18: Effect of C_s

C_s	3100	3200	3300	3400	3500	3600
N^*	10.3142	10.4877	10.6583	10.8263	10.9916	11.1546
$L(N^*)$	5.1381	5.2258	5.3119	5.3966	5.4798	5.5617
$T(N^*)$	47.5157	48.3509	49.1725	49.9812	50.7775	51.562

C_S;

- slight increase in total cost raise in the parameters $\lambda, \mu, \alpha_1, \xi_1, p, a, C_{b1}, C_{b2}, C_m, C_o, C_h$ and C_S; and
- slight increase in start-up threshold "N" value raise in the parameters $\lambda, \mu_2, \alpha_2, \xi_1, \phi, p, a, C_r, C_{b1}, C_m$ and C_S

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