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Using some methods for estimating the survival times of patients infected with Covid-19 utilizing new two parameters Lindley distribution NTPLD

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Abstract

The aim of this paper is the needs to analyze the survival time for patients with Covide-19b who remains in the hospital until death, so it is necessary to study the survival times and estimate the reliability. The problem of finding the best distribution that fits the data is the key idea to analyze the data accurately. Consequently, the misspecifying of the distribution that fit the data leads to poor quality inference criteria of the phenomenon, also leads to unreliable reliability estimations. Many data of sciences areas are of different probability distributions depending on the nature of the phenomenon within the studied communities Some of the data are represented simple phenomena that cope with a unique probability distribution, and some of which are very complex and heterogeneous systems that force the researchers to use probability distributions fits the behavior of this random phenomenon. Many works in the field of reliability, failure and survival times, and the function of reliability (survival) follows some common distributions such as the Exponential distribution, Weibull distribution and other distributions. In this paper we introduced the survival function that follows an important distribution in survival modeling, that is called the Lindley distribution with two Parameters, taking into account two forms of this distribution, one of them we proposed based on different forms of the probability density function and finding the survival function for the distribution and compared to other distributions using several methods of estimation including the Maximum Likelihood Estimator (MLE), (percentiles estimators) by using Monte Carlo simulation experiments and comparing using the Integrated Mean Square Error (IMSE), (-2ln L) and AIC to achieve the best estimate of survival function among the distributions, as well as a real data analysis

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conducted for the survival times for patients with COVID-19 stay in hospital until death. The proposed distribution fitted the data very well in the Maximum Likelihood method compared with the other distribution.

Keywords: Maximum Likelihood, Percentiles, Lindley Distribution, Survival function.

1. Introduction

The study of reliability (survival) has many benefits and has many uses in practical life, by studying the reliability of many components and units such as electrical appliances, equipment and others that are beneficial to the individual and society, knowing the reliability of each machine in any laboratory or facility makes it possible to predict the total optimal number For working and idle machines at any time, in addition to many other benefits, including comparing the reliability of the current product with the reliability of the previous product in order to know the extent of development or deterioration in the product, in addition to the importance of reliability in protecting and warding off danger from human life, as it is important in solving problems Survival theory and life tables analysis.

2. Reliability (Survival) Function

It is the probability that the experimental unit will remain operating for at least time t. Since t>0, that is, if T is the random variable that represents the survival time of the experimental unit, then R(t) represents the probability of the item remaining working for a future period (Al-Yasiri, 2007, 4). Reliability can be considered as a fixed measure, and it is also defined as a function of the system (vehicle) continuity of work with the lapse of a periodic time of t, and it is also defined as a measure of the ability or ability of any part of a particular system or a system as a whole to work with full validity without stopping.
In theory, reliability can be defined as the probability that a device will operate under the specific conditions for consumer use of this device (Stapelberg: 2009 P43-45)[5].
So if we symbolize the reliability function (survival) with the symbol R(t), then:

$$R(t) = p(T>t) = 1-P(T\leq t) = \int_t^{\infty} f_T(t)dt \quad ; t\geq 0 \tag{2.1}$$

where $f_T(t)$: represents the probability-of-failure density function (p.d.f) for the random variable t. If $F_T(t)$represents the cumulative density function of failure (C.D.F) for the random variable t, we will have:

$$F_T(t) = 1-R(t) \tag{2.2}$$

where

$$F_T(t)+R(t)=1$$

3. Two Parameters Lindley Distribution

The two-parameter Lindley distribution (1TPLD) was proposed by (Shanker et al, 2013)[4] as a model for studying survival times and reliability studies. It is a continuous probability distribution resulting from mixing the Gamma(2,b) distribution with the Exponential(b) distribution.
The probability density function of the Lindley distribution can be defined with two parameters by the following formula:

$$f\,(t;a,b)=\frac{b^2}{b+a}\,(1+at)\;\;e^{-bt}\;;t>0,\;b>0\;,a>-b \tag{3.1}$$

Where :
b scale parameter
a shape parameter).
The cumulative distribution function (CDF) for a distribution is known by the following formula:

$$F\,(t;b)=1-\left[\frac{b+a+abt}{(b+a)}\right]\;e^{-bt}\qquad;\;t>0,\;b>0,a>-b \tag{3.2}$$

And the reliability function (survival) of a Lindley distribution with two parameters, as shown by the following formula:

$$R\,(t)=\;\left[\frac{b+a+abt}{(b+a)}\right]\;e^{-bt}\qquad;\;t>0,\;b>0,a>-b \tag{3.3}$$

The risk function is defined by the following formula:

$$h\,(t)=\frac{b^2\,(1+at)}{b+a+abt} \tag{3.4}$$

(Shanker et al, 2013, 363-364)[4]
In 2016 (Shanker & Sharma) proposed a new developed formula for the two-parameter Lindley distribution, and the probability density function of the two-parameter Lindley distribution (TPLD2) is known as the following formula:

$$f\,(t;a,b)=\frac{b^2}{ab+1}\;\;(a+t)\,e^{-bt}\qquad;t>0,\;b>0,ab>-1 \tag{3.5}$$

The cumulative distribution function of the Lindley distribution is defined by the two most common and recently used parameters (TPLD2) with the following formula:

$$F\,(t;b)=1-\left[\frac{1+ab+bt}{ab+1}\right]e^{-bt}\qquad;\;t>0,\;b>0,ab>-1 \tag{3.6}$$

The reliability function (survival) of the Lindley distribution is known by the two parameters (TPLD2) as shown by the following formula:

$$R\,(t)=\;\left[\frac{1+ab+bt}{(ab+1)}\right]e^{(-bt)}\qquad;\;t>0,\;b>0,ab>-1 \tag{3.7}$$

The risk function of the Lindley distribution can be defined by the two parameters (TPLD2), as shown by the following formula:

$$h\,(t)=\frac{1+ab+bt}{ab^2+tb^2} \tag{3.8}$$

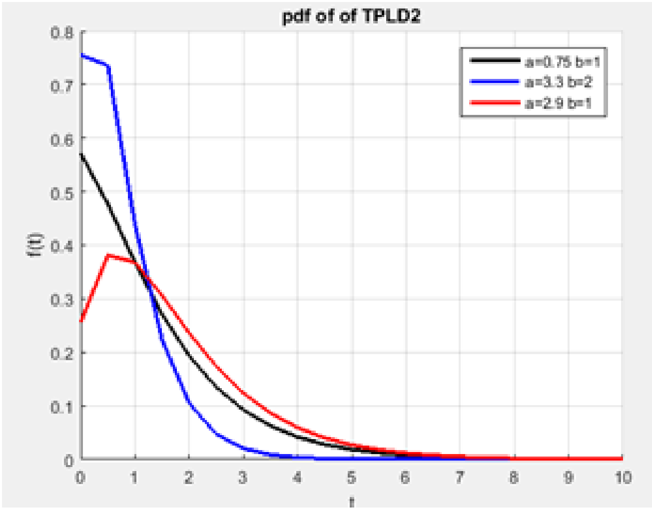

Figure 1: Behavior of the probability density function curve (TPLD2)

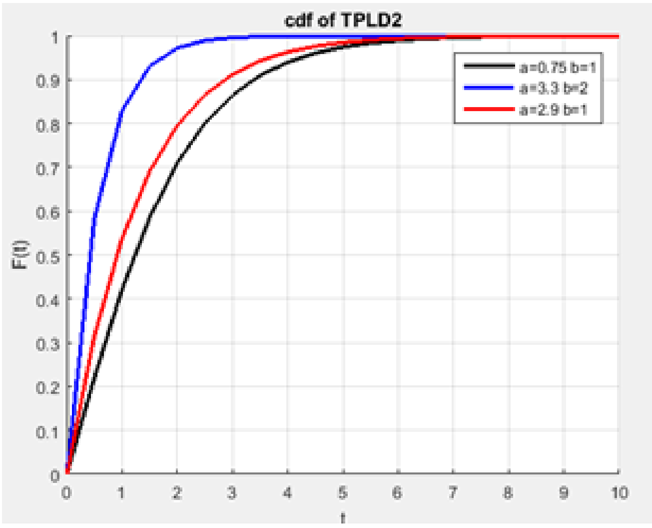

Figure 2: Behavior of the curve of the cumulative distribution function of the TPLD2 distribution.

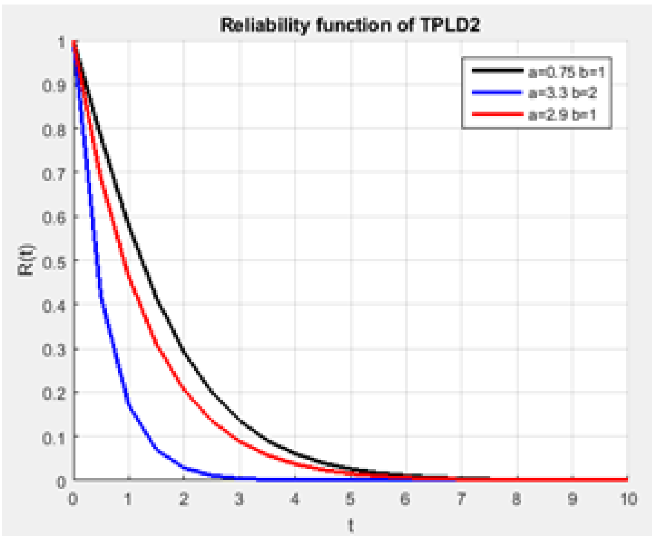

Figure 3: Behavior of the reliability (survival) function curve of the TPLD2 distribution.

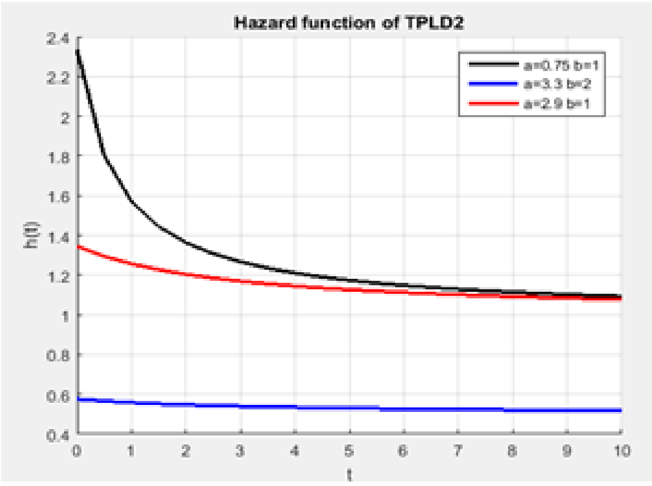

Figure 4: Behavior of the risk function curve of the TPLD2 distribution.

4. A proposed new formula for the two-parameter Lindley distribution (NTPLD)

(New type of two parameter Lindley distribution)
The researcher suggested another formula for the Lindley distribution with the two parameters, as it was found that it is possible to suggest an alternative formula for the Lindley distribution with the two parameters,
Assuming the density density in an exponential distribution, it is defined by the following formula :

$$f_1(t) = \frac{1}{b}\, e^{-\frac{t}{b}} \qquad , \qquad t>0$$

And the probability density function of the gamma distribution with the two parameters (2, b) is defined by the following formula :

$$f_2(t) = \frac{1}{b^2}\, t \; e^{-\frac{t}{b}} \qquad , \qquad t>0$$

And ues mixture formula

$$f(t;a,b) = p\; f_1(t) + (1-p)\, f_2(t)$$

Assuming that $p = \frac{b}{b+a}$
Substitute into mixture formula shown :

$$f_{NTPLD}(t;a,b) = \frac{ba}{ba+1}\frac{1}{b}\, e^{-\frac{t}{b}} + \left(1-\frac{ba}{ba+1}\right)\frac{1}{b^2}\, t\, e^{-\frac{t}{b}}$$
$$= \frac{e^{-\frac{t}{b}}}{ab+1}\left(a+\frac{t}{b^2}\right)$$

which is shown in the following formula:

$$f_{NTPLD}(t;a,b) = \frac{1}{ab+1}\left(a+\frac{t}{b^2}\right) e^{-\frac{t}{b}} \qquad ;t>0,\ b>0,\ ab>-1 \tag{4.1}$$

as that :
a : Shape Parameter
b : Scale Parameter

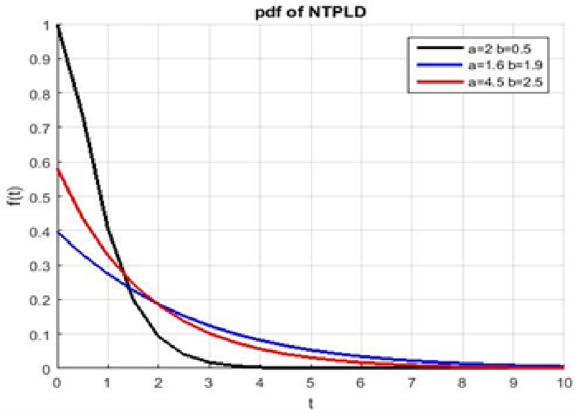

Figure 5: behavior of the curve of the probability density function of the distribution (NTPLD)

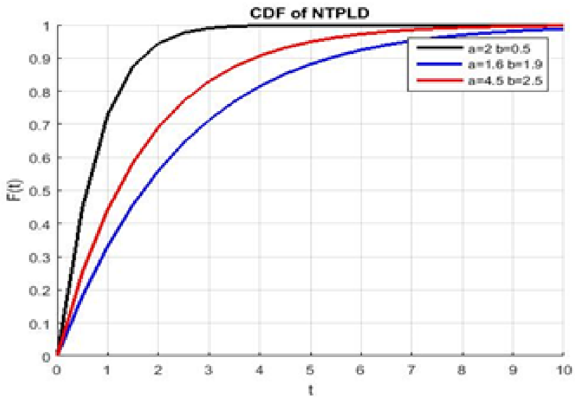

Figure 6: Behavior of the cumulative Density Function Distribution (NTPLD) Curve

And the cumulative distribution function F(t) for the new Lindley distribution (NTPLD) is as follows:

$$F_{NTPLD}\ (t)=1-\left[\frac{b+t+ab^2}{b\,(ab+1)}\right]e^{-\frac{t}{b}} \tag{4.2}$$

Figure (6) shows the behavior of the NTLPD curve at different values of the measurement parameter b and the parameter of Figure a.

The rth moment about origin of the two-parameter LD has been obtained as :

$$E(T^r) = \mu_r=\frac{1}{ab+1}\ \left[ab^{r+1}\ +_{r+1}+b^r+_{r+2}\right] \quad , r= 1,2,...$$

The mean and variance of (NTPLD) respectively, are given by:

$$E\,(T)=\frac{ab^2+2b}{ab+1} \qquad , \qquad Var\,(T)=\frac{2ab^3+6b^2}{ab+1}-\left(\frac{ab^2+2b}{ab+1}\right)^2$$

the mode of the distribution is given by :

$$t_{mode}=b-ab^2$$

The (NTPLD) Reliability (survival) function is defined by the following formula:

$$R(t)\ =\ \left[\frac{b+t+ab^2}{b\,(ab+1)}\right]e^{-\frac{t}{b}} \tag{4.3}$$

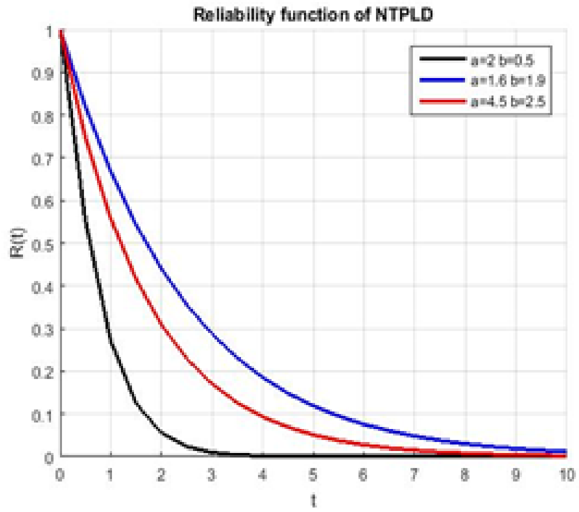

Figure 7: Behavior of the Reliability Function Curve (Survival) for the NTPLD Distribution

Figure (7) shows the behavior of the reliability curve (survival) curve for the proposed distribution (NTPLD) at different values of measurement parameter b and parameter of Figure a. So, write the risk function for the distribution (NTPLD) (Hazard function) as follows

$$h(t) = \frac{ab^2+t}{b(b+t+ab^2)} \tag{4.4}$$

5. Weibull Distribution (WD)

This distribution was derived by the Swedish scientist (Waloddi Weibull, 1939) and is one of the most important probability distributions in the study of reliability and survival functions and modeling of failure times for electrical devices and equipment and location.
A random variable T has a Weibull distribution if its probability density function is as follows:

$$f\,(t;\alpha,\lambda) = \alpha\lambda t^{\alpha-1}\,\exp(-\lambda t^{\alpha}); t > 0 \tag{5.1}$$

$\alpha>0$ is the shape parameter and $\lambda>0$ is the scale parameter, and T represents the random variable time to failure.
If t$\sim$Weibull(α,λ) , then its Cumulative Distribution Function is:

$$F\,(t,\alpha,\lambda) = \int_0^t f\,(u)\,du = 1-\exp(-\lambda t^{\alpha})\ ;\ t\geq 0 \tag{5.2}$$

The reliability function (survival) of the distribution is defined by the following formula:

$$R\,(t) = \exp(-\lambda t^{\alpha}) \quad ; \ t>0,\ \alpha>0, \lambda>0 \tag{5.3}$$

The risk function of the distribution **:**

$$h\,(t) = \alpha\lambda t^{\alpha-1} \quad t>0,\ \alpha>0, \lambda>0 \tag{5.4}$$

(A.Loganathan & M. Uma : 2017 ,83-84)[1]
(M. M. Mohie El-Din; M. Nagy, 2017, 98)[3]

6. Reliabilty Estimation Methods of (NTPLD)

6.1. Maximum Likelihood Method

The method of Maximum Likelihood was presented for the first time by the researcher (Fisher, 1922), and this method is one of the important methods of estimating because it gives estimators that have good properties such as invariant, efficiency, adequacy and sometimes consistency, and the principle of this method lies in finding the parameter estimator. (Parameters) which makes the possibility function at its Maximum end, and that the formulation of the possibility function was through a set of ideas that included the parameter (Parameter) and (Consistency) and (Efficiency) in addition to (estimation). (Hamsa, 2008, 153)

Likelihood function is a common probability function for p random variables that are used in estimating the parameters:

$$L{=}L(\theta_1,\ \theta_2,\ \ldots,\ .\theta_p) \tag{6.1}$$

And that (P) of the equations resulting from the partial derivative of the logarithm function of the possibility and equal to zero as follows:

$$\left.\begin{array}{c} \frac{\partial LogL}{\partial \theta_1}{=}0 \\ \frac{\partial LogL}{\partial \theta_2}{=}0 \\ \vdots \\ \vdots \\ \frac{\partial LogL}{\partial \theta_P}{=}0 \end{array}\right\} \tag{6.2}$$

(Chaojun & Yan, 2020, 1-3)

And by solving equations 21 , we get the capabilities of the maximum potential (MLEs):

If we have a random sample $(x_1, x_2, \ldots, x_n($ from the (NTPLD) distribution with a probability density function as in equation (4.1), then the probability function can be written as follows:

$$L = \prod_{i=1}^{n} f(t_i, a, b)$$

$$L = \left(\frac{1}{ab+1}\right)^n \prod_{i=1}^{n} \left(a{+}\frac{t_i}{b^2}\right)\ e^{-\frac{\sum_{i=1}^{n} t_i}{b}} \tag{6.3}$$

By taking the natural logarithm of both sides of Equation 16, we get:

$$\ln\ L\ =\ -n\ln{(ab+1)} +\ \sum_{i=1}^{n} \ln\left(a{+}\frac{t_i}{b^2}\right) - \frac{\sum_{i=1}^{n} t_i}{b} \tag{6.4}$$

To obtain the estimators of b and a, we take the first partial derivative of the formula (5.2) with respect to (b,a) and set it equal to zero, we get the following:

$$\frac{\partial \ln L}{\partial b} = \frac{-an}{ab+1} - \sum_{i=1}^{n} \frac{2\frac{t_i}{b^3}}{a+\frac{t_i}{b^2}} + \frac{\sum_{i=1}^{n} t_i}{b^2}$$

$$= \frac{-an}{ab+1} - \sum_{i=1}^{n} \frac{2t_i}{b^3\left[\frac{ab^2+t_i}{b^2}\right]} + \frac{\sum_{i=1}^{n} t_i}{b^2}$$

$$\therefore \frac{\partial \ln L}{\partial \widehat{b}} = \frac{\sum_{i=1}^{n} t_i}{\widehat{b}^2} - \frac{\widehat{a}n}{\widehat{a}\widehat{b}+1} - \sum_{i=1}^{n} \frac{2\sum_{i=1}^{n} t_i}{\widehat{b}(\hat{a}\widehat{b}^2 + t_i)} = 0 \tag{6.5}$$

$$\frac{\partial \ln L}{\partial a} = \frac{-bn}{ab+1} + \sum_{i=1}^{n} \frac{1}{a+\frac{t_i}{b^2}}$$

$$\frac{\partial \ln L}{\partial a} = \frac{-bn}{ab+1} + \sum_{i=1}^{n} \frac{1}{\frac{ab^2+t_i}{b^2}}$$

$$\therefore \frac{\partial \ln L}{\partial \widehat{a}} = \sum_{i=1}^{n} \frac{\widehat{b}^2}{\widehat{a}\widehat{b}^2+t_i} - \frac{\widehat{b}n}{\widehat{a}\widehat{b}+1} = 0 \tag{6.6}$$

We note from equations (6.5) and (6.6) that they cannot be solved by the usual analytical methods because they are non-linear equations, so they will be solved using several iterative methods by using the function (fsolve) in Matlab for the purpose of finding the estimator of $\widehat{b}_{mle}$ and $\widehat{a}_{mle}$. Then the estimator for the (reliability) function to survive the NTLDD is as follows:

$$\widehat{R}_{mle}(t_i) = \frac{(\widehat{b}_{mle}+t_i+\widehat{a}_{mle}\widehat{b}^2_{mle})}{\widehat{b}_{mle}(\widehat{a}_{mle}\widehat{b}_{mle}+1)}\, e^{-\frac{t_i}{\widehat{b}_{mle}}} \tag{6.7}$$

6.2. Percentile estimators' method

The method of partial estimators (PEM) was first proposed by the English scientist (Kao, 1959) by graphical approximation of the best unbiased linear estimators. The idea of this method is to find the possibility function of the distribution by equating the cumulative distribution function of the probability distribution (CDF) with a nonparametric estimator of the function The cumulative distribution (Wi) which takes different forms, and then find the estimator of the parameter (parameters) by taking the partial derivative of the resulting potential function for each parameter to be estimated and set it equal to zero and solving the resulting equations to get the partial estimators of the parameters.

If $(x_1, x_2, \ldots, x_n)$ are distributed according to the (NTPLD) distribution, then the ordered sample $x_{1(1)}, x_{2(2)}, \ldots, x_{n(n)}$, represents the ordered statistics of the sample, then:

$$Wi = F(t_i) \tag{6.8}$$

$$Wi = \frac{i}{n+1}$$

$$Wi = \frac{i-\frac{3}{8}}{n+\frac{1}{4}} \tag{6.9}$$

(Yadav et al, 2020, 133-134)
When applying Equation (6.4), it results in:

$$Wi \ = 1- \ \frac{(b+t+ab^2)}{b\,(ab+1)} e^{-\frac{t}{b}} \tag{6.10}$$

By taking the natural logarithm of both sides of the equation ((2.2)-(6.10)), we get the following:

$$\ln Wi \ = \ln\left(1- \ \frac{(b{+}t{+}ab^2)}{b\,(ab{+}1)} e^{-\frac{t}{b}}\right)$$

$$\ln Wi \ - \ln\left(1- \ \frac{(b{+}t{+}ab^2)}{b\,(ab{+}1)} e^{-\frac{t}{b}}\right) = 0 \tag{6.11}$$

By squaring both sides of equation (6.6) and taking the sum, we get:

$$\sum_{i=1}^{n}\left(\ln Wi \ - \ln\left(1- \ \frac{(b{+}t{+}ab^2)}{b\,(ab{+}1)} e^{-\frac{t}{b}}\right)\right)^2 = 0 \tag{6.12}$$

In order to obtain an estimator (PEM) for parameter (b) and (a (we derive equation (6.7))) with respect to (b) once and (a) again and set them equal to zero, as follows:

$$\sum_{i=1}^{n}\left[\ln(W_i) - \ln\left(1- \ \frac{(b+t+ab^2)\,e^{-\frac{t}{b}}}{b\,(ab{+}1)}\right)\right]$$

$$\frac{b\,(ab{+}1)\left(-\frac{t\left(b+t+ab^2\right)}{b^2} e^{-\frac{t}{b}} + e^{-\frac{t}{b}}\,(1{+}2ab)\right) - (b{+}t{+}ab^2)\,e^{-\frac{t}{b}}\,2ab}{b^2(ab{+}1)^2\left(1- \ \frac{(b+t+ab^2)e^{-\frac{t}{b}}}{b(ab+1)}\right)} = 0 \tag{6.13}$$

$$\sum_{i=1}^{n}\left[\ln(Wi \ ) - \ln\left(1- \ \frac{(b{+}t{+}ab^2)\,e^{-\frac{t}{b}}}{b\,(ab{+}1)}\right)\right] \ \frac{\left(b^2 e^{-\frac{t}{b}}\right) - (b{+}t{+}ab^2)\left(e^{-\frac{t}{b}}\right)^2 b^2}{b^2(ab{+}1)^2\left(1- \ \frac{(b+t+ab^2)e^{-\frac{t}{b}}}{b(ab+1)}\right)} = 0 \tag{6.14}$$

We note from equation (6.13) and (6.14) that they cannot be solved by the usual analytical methods because they are non-linear equations, so one of the numerical iterative methods will be used by using the function (fsolve) in MatLab for the purpose of finding an estimator of b^_per and a^_per. Thus, the moment estimator for the approximate reliability (survival) function of the NTLDD distribution is as follows:

$$\widehat{R}_{per}(t) \ = \ \frac{(\widehat{b}_{per}{+}t{+}\widehat{a}_{per}\widehat{b}_{per}^2)}{\widehat{b}_{per}(\widehat{a}_{per}\widehat{b}_{per}{+}1)} \ e^{-\frac{t}{\widehat{b}_{per}}} \tag{6.15}$$

7. Applied side

Data for patients infected with the Corona virus were taken from Al-Diwaniyah General Hospital - Epidemiological Diseases Unit, which represents the times of stay until death due to Corona disease inside the hospital from the patients who were hospitalized by the number of (60) patients who fell asleep and then died as a result of infection with the Corona virus, and after appropriate work was done for the data Using the χ^2 statistic for good fit using Matlab program, the following results were obtained:

Table 1: Results of the data fit test results for the new two-parameter Lindley distribution

Distribution	χ^2_c	χ^2_t	Sig.	Decision
NTPLD	1.167	7.92	0.214	Accept H_0

We note from Table (1) that the calculated value of χ^2_c (1.167) is less than the tabular value of χ^2_t which is (7.82), and the probability value Sig $= 0.214$ is greater than the level of significance (0.05), which means that no rejection The null hypothesis means that the real data are distributed according to the new Lindley distribution with two parameters .
Then a comparison was made between the new two-parameter Lindley distribution and the two-parameter Lindley distribution proposed by Shanker, 2016)) and Weibull Distribution for the purpose of showing the best distribution when applying the real data, and the results were as in Table (2) :

Table 2: the results of the four tests of goodness of fit, which were applied to the real data

Distribution	Parameters estimation		$-2\ln L$	**AIC**	**AICc**	**BIC**	**HQIC**
NTPLD	**2.89**	**1.12**	**12.77462**	**15.26337**	**14.45333**	**14.56556**	**5.89771**
TPLD1	**3.55**	1.55	**27.77013**	**29.46891**	**31.18320**	**30.01287**	**8.29668**
TPLD2	**3.15**	**1.34**	**27.35527**	**29.40746**	**31.12199**	**30.00188**	**8.26438**
WD	**3.99**	**1.78**	**136.06271**	**137.97556**	**138.22788**	**141.79789**	**12.55564**

Table (2) shows that the new proposed Lindley distribution achieved the lowest criteria (-2ln L, AIC , AICc BIC HQIC) compared to the other distributions (TPLD1, TPLD2, WD), which indicates that the real data fit the new proposed distribution more than the remaining distributions
After showing the priority of NTPLD distribution to represent real data, and in light of this, the survival function will be estimated using the (Mat Lab) program using the Maximum Likelihood method and the estimation method, as the results of the estimation are as in Table (3) below:

Table 3: Survival Function Values Estimated According To the Maximum Likelihood Method and the Percentile Method for NTPLD Distribution

t_i	S_Real	S_MLE	S_Per
0.10	0.98810	0.98759	0.96556
0.10	0.94605	0.94384	0.90340
0.11	0.92415	0.92110	0.89187
0.11	0.92182	0.91870	0.87619
0.12	0.91618	0.91284	0.87468
0.19	0.87009	0.86517	0.86171
0.20	0.86138	0.85617	0.81516
0.22	0.85071	0.84516	0.81439
0.22	0.84762	0.84198	0.76449
0.24	0.83782	0.83188	0.76296
0.24	0.83435	0.82831	0.76237
0.29	0.80340	0.79648	0.74696
0.34	0.77102	0.76326	0.73457
0.37	0.75648	0.74837	0.72725
0.38	0.74892	0.74063	0.69394
0.39	0.74572	0.73736	0.67807
0.39	0.74490	0.73653	0.67747
0.39	0.74249	0.73406	0.66451
0.40	0.73703	0.72848	0.66161
0.44	0.71416	0.70514	0.65711
0.49	0.68878	0.67928	0.63812
0.54	0.66288	0.65294	0.61955
0.57	0.65002	0.63988	0.60246
0.57	0.64834	0.63818	0.60188
0.63	0.61955	0.60901	0.59741
0.76	0.55732	0.54618	0.51678
0.78	0.54946	0.53826	0.49750
0.83	0.53061	0.51931	0.48004

0.84	0.52671	0.51539	0.45845
0.92	0.49404	0.48263	0.44356
0.98	0.47147	0.46003	0.43322
0.98	0.46920	0.45777	0.42189
1.00	0.46166	0.45023	0.40767
1.03	0.45278	0.44137	0.38086
1.03	0.45028	0.43887	0.37309
1.08	0.43218	0.42082	0.36862
1.09	0.43118	0.41983	0.35506
1.14	0.41474	0.40346	0.33928
1.19	0.39802	0.38684	0.31174
1.23	0.38396	0.37288	0.29345
1.36	0.34592	0.33522	0.28243
1.49	0.31079	0.30055	0.25689
1.53	0.30092	0.29083	0.22717
1.75	0.25185	0.24265	0.20133
1.76	0.24873	0.23960	0.19988
1.78	0.24627	0.23719	0.18133
1.79	0.24280	0.23379	0.16984
1.91	0.22083	0.21232	0.16541
1.99	0.20572	0.19758	0.16161
2.12	0.18572	0.17811	0.15580
2.19	0.17418	0.16691	0.11670
2.23	0.16917	0.16204	0.09650
2.30	0.15928	0.15245	0.08073
2.75	0.10910	0.10397	0.07589
3.02	0.08666	0.08241	0.06345
3.07	0.08281	0.07871	0.06229
3.26	0.07074	0.06715	0.04822
3.34	0.06599	0.06261	0.04733
3.89	0.04071	0.03850	0.02530
6.54	0.00387	0.00363	0.00310

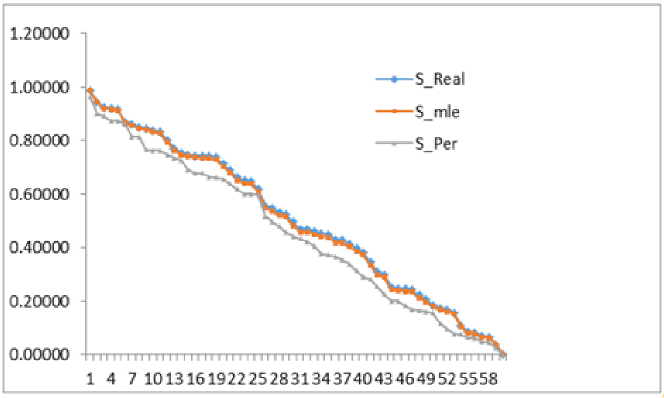

Figure 8: Behavior of the Reliability Function Curve (Survival) for the NTPLD Distribution

Figure (1) the true survival function curve estimated according to the Maximum Likelihood Method and the Percentiles estimator's method
It is clear from Table (3) and Figure (2) that the estimated values of the survival function according to the Maximum Likelihood Method converge to the real values. The survival function values clearly contradict time, and this corresponds to the behavior of this function being decreasing with time. And the survival function curve estimated according to the Maximum Likelihood method converges with the survival function curve for real data at the default parameters (a = 2.9 and b = 1)

8. Results discussion

By analyzing the time-to-death data for people infected with Coronavirus by estimating the survival function for NTPLD distribution using two methods of estimation which are the Maximum Likelihood method and the Percentiles estimators' method, it was found that the true data fit the NTPLD distribution and that the NTPLD distribution is better than the TPLS1, TPLD2 distribution. Through the values of the survival function, it was found that the estimation of the survival function according to the Maximum Likelihood method was better than the estimation of the survival function according to the Percentiles estimators method, as it is more close to the real data function, and the values of the survival function for the two methods are decreasing with time. As we note that the longer the patient stays in the hospital, the less likely he will survive.

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