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Comparison between some estimation methods for an intuitionistic fuzzy semi-parametric logistic regression model with practical application about covid-19

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Abstract

In this paper, the intuitionistic fuzzy set and the triangular intuitionistic fuzzy number were displayed, as well as the intuitionistic fuzzy semi-parametric logistic regression model when the parameters and the dependent variable are fuzzy and the independent variables are crisp. Two methods were used to estimate the model on fuzzy data representing Coronavirus data, which are the suggested method and The Wang et al method, through the mean square error and the measure of goodness-of-fit, the suggested estimation method was the best.

Keywords: intuitionistic fuzzy set, the triangular intuitionistic fuzzy number, fuzzy data, mean square error, goodness-of-fit

1. Introduction

Fuzzy data is one of the forms of fuzzy logic. This logic originated in 1965 by Lotfi Zadeh in the University of California through the increase in complexity in engineering systems increases uncertainty. Fuzzy logic based on the theory of fuzzy sets is the best solution to complex systems; the fuzzy set defined as the set of elements that have the membership function, which are real numbers

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within the belong to membership function [0, 1]. The variable X have membership function for any element in a set A as follow[1]:

$$M_A(X) : X \to [0,1] \tag{1.1}$$

The fuzzy set used in linear regression is called fuzzy linear regression, researcher Tanaka was the first studied the model of fuzzy linear regression in year 1980, which aims to model an inaccurate or ambiguous phenomenon using fuzzy parameters; this is developed in year 1982 from Tanaka, uejima and Asai [2].

Researchers Cheng & Lee study the nonparametric fuzzy regression model in year 1999 through proposed model to nonparametric fuzzy regression consisting of an independent variable (explanatory) crisp and a fuzzy dependent variable, it was called the non-parametric fuzzy regression model because of the lack of conditions for the parametric model from error distribution and others [3].

One of the regression models is Intuitionistic fuzzy semi-parametric logistic regression model consists of two parts. first the parameterize part and the other the non-parameterize part which representing smoothing function, this model has become more applied and interesting in recent years on the fuzzy data set; and in this model the parameters and the dependent variable y are fuzziness but the explanatory variables are crisp.

Sometimes the dependent variable cannot be defined in two cases only, response, non-response or success, and failure, or more than two responses, especially in medical studies; like to measure the severity of the disease or pain in patients, the terms (low, very low, medium, high, very high) are also used.

In this case, the dependent variable represent the intuitive triangular fuzzy number, this leads the Intuitionistic fuzzy semi-parametric logistic regression model will be used to deal with the conditions of uncertainty.

Many researchers have studied fuzzy data with logistic regression model. [15] proposed an algorithm to estimate the parameters of the logistic regression model of the fuzzy category, Where described the effects of explanatory variables on response variables. [14] researchers proposed a new hybrid approach based on fuzzy logic and logistic regression analysis.

[10] studied the comparison between the fuzzy logistic regression model and the ordinal logistic regression. [4] studied the fuzzy logistic regression model through the effect of fuzzy inputs (explanatory variables) and fuzzy outputs described by linguistic variables. Researcher (Ahmadini) [1] proposed a new model for intuitive fuzzy logistic regression to deal with inaccurate parameters that contain degrees of vagueness and hesitation at the same time.

2. Intuitionistic Fuzzy Set

Let X be a universal set, that fuzzy set $\acute{A}$ in X is defined as a set of ordered pairs, as each pair represents the element and the degree of membership of its according to the following equation [6]

$$\acute{A} = \{(x, \mu_{\acute{A}}(x)) \ ; \ x \in X\}. \tag{2.1}$$

We can express intuitionistic fuzzy set A in X through a group arranged in three ranks, as the three ranks are represented (the element, the belong degree of element to the intuitionistic fuzzy set, the non-belong degree of element to intuitionistic fuzzy set) according to the following equation [12]:

$$A = \{(x, \mu_A(x), \nu_A(x)) ; x \in X\} \tag{2.2}$$

In addition, the belonging degree falls between zero and one$\mu_A : X \to [0,1]$, the non-belonging degree is $\nu_A : X \to [0,1]$.

Definition 2.1. *Let* $(\acute{l}_a > l_a > 0; \acute{r}_a > r_a > 0)$, *the triangular intuitionistic fuzzy number TIFN is* $\overline{A} = (a; l_a, r_a; \acute{l}_a, \acute{r}_a)$ *; when the membership function* $\mu_A(x_i)$, *non-membership function* $\nu_A(x_i)$ *to the triangular intuitionistic fuzzy number TIFN as in the following formulas [5]:*

$$\mu_A(x_i) = \begin{cases} \frac{x_i - a + l_a}{l_a} & if \quad a - l_a \le x_i < a \\ 1 & if \quad x_i = a \\ \frac{a + r_a - x_i}{r_a} & if \quad a < x_i \le a + r_a \\ 0 & Otherwise \end{cases}$$
$$\nu_A(x_i) = \begin{cases} 1 - \frac{x_i - a + \acute{l}_a}{\acute{l}_a} & if \quad a - \acute{l}_a \le x_i < a \\ 0 & if \quad x_i = a \\ 1 - \frac{a + \acute{r}_a - x_i}{\acute{r}_a} & if \quad a < x_i \le a + \acute{r}_a \\ 1 & Otherwise \end{cases} \tag{2.3}$$

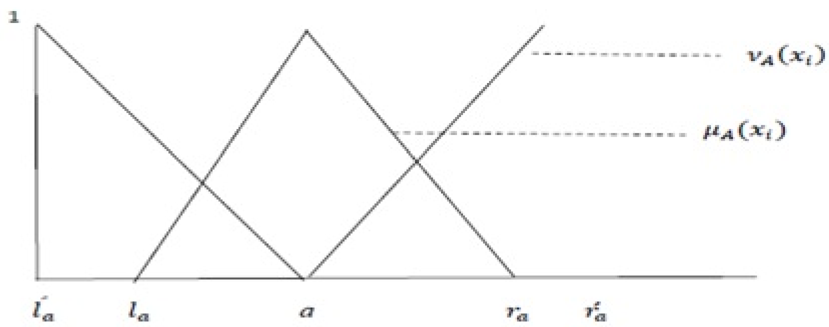

Figure 1: Represents membership function, non-membership function to TIFN [6]

Lemma 2.2. *If there are two numbers, the triangular intuitionistic fuzzy number* $\overline{A} = (a; l_a, r_a; \acute{l}_a, \acute{r}_a)$ *and* $\overline{B} = (b; l_b, r_b; \acute{l}_b, \acute{r}_b)$ *have the following properties, then some arithmetic operations on the two numbers are as follows [7]:*

1. $\overline{A} \bigoplus \overline{B} = \left(a + b; l_a + l_b, r_a + r_b;\ \acute{l}_a + \acute{l}_b, \acute{r}_a + \acute{r}_b\right)$

2. $\overline{A} \ominus \overline{B} = \left(a - b; l_a + r_b, r_a + l_b;\ \acute{l}_a + \acute{r}_b, \acute{r}_a + \acute{l}_b\right)$

3. $\overline{A} \otimes \overline{B} \approx \left(a.b; al_b + bl_a, ar_b + br_a; a\acute{l}_b + b\acute{l}_a, a\acute{r}_b + b\acute{r}_a\right)$

4. $\overline{A} \oslash \overline{B} \approx \left(a/b; \frac{ar_b + bl_a}{b^2}, \frac{al_b + br_a}{b^2};\ \frac{a\acute{r}_b + b\acute{l}_a}{b^2}, \frac{a\acute{l}_b + b\acute{r}_a}{b^2}\right)$

5. $\lambda \otimes \overline{A} = \left(\lambda a; \lambda l_a, \lambda r_a;\ \lambda \acute{l}_a, \lambda \acute{r}_a\right)$ *for all* $\lambda > 0$
   $\lambda \otimes \overline{A} = \left(\lambda a; -\lambda l_a, -\lambda r_a;\ -\lambda \acute{l}_a, -\acute{\lambda} r_a\right)$ *for all* $\lambda < 0$

3. Intuitionistic Fuzzy Semi Parametric Logistic Regression Model

The semi-parametric logistic regression model consists of the effect of non-linear variables and on the binary- response dependent variable y, when the dependent variable y and the parameter $(\beta_0, \beta_1 \ldots\ldots \beta_k)$ are triangular intuitionistic fuzzy number, explanatory variables $(X_1, X_2, \ldots, X_K)$ and the smooth function variable $\mathrm{g}(t_i)$ are crisp variables, the response function to intuitionistic fuzzy semi parametric logistic regression model is [8].

$$\overline{p_i} = p_r\left(y_i = \frac{1}{-} =, t\right) = \frac{e^{\beta_0+\beta_1 X_{i1}+\ldots+\beta_k X_k+g(t_i)}}{1+e^{\beta_0+\beta_1 X_{i1}+\ldots+\beta_k X_k+g(t_i)}} \tag{3.1}$$

Berkson find a logarithmic relationship to convert the response function $\overline{p_i}$ into a linear function as follow [4]:

$$V_i = logit\ \ (\overline{p_i}) = \ln\frac{\overline{p_i}}{1-\overline{p_i}} \tag{3.2}$$

It will become, the intuitionistic fuzzy semi parametric logistic regression model IFSPLRM as follows [7]:

$$V_i = \oplus_{j=1}^{k}(\otimes x_{\mathrm{ij}}) \oplus g(t_i) \oplus \epsilon_i, \tag{3.3}$$

where $i = 1, 2, \ldots, n$. The observation, $j = 1, 2, \ldots\ldots k$ the number of explanatory variables x_{ij}. A vector of unknown parameters which representing the triangular intuitionistic fuzzy parameters$\beta_j = (b_j; l_{b_j}, r_{b_j}; l'_{b_j}, r'_{b_j})$, that is estimated by using ordinary least squares method [11] $g(t_i)$ smooth function when estimate it represents $\widehat{g}(t_i) = ((t_i); l_{t_i}, r_{t_i}; l'_{t_i}, r'_{t_i})$. ϵ_i Error term.

When we estimate $g(t_i)$, we use Nadaraya Watson estimator it can be expressed as follows [9]:

$$\widehat{g}(t) = \sum_{i=1}^{n}\left[\frac{k_h(t-T_i)}{\sum_{j=1}^{n} k_h(t-T_j)}\right] V_i = \sum_{i=1}^{n} W(t_j) V_i \tag{3.4}$$

where $W(t_j)$ are the Weights to Nadaraya Watson estimator and the sum of a series weights is equal to one,
$\sum_{i=1}^{n} w_{\mathrm{hi}}(t_j) = 1$, $k(.)$ Kernel function and we use Gaussian kernel function, h smoothing parameter (bind width) and we use silverman's rule of thumb bind width, refined plug bind width [9]. The fuzzy logit function can be found to the fuzzy output (dependent variable) which represent the triangular intuitionistic fuzzynumber $\overline{p_i} = (\overline{p_{i1}}; l_{\overline{p_i}}, r_{\overline{p_i}}; l'_{\overline{p_i}_i}, r'_{\overline{p_i}_i})$ depending on point 2, 4 from arithmetic operations of intuitionistic as follow [7]:

$$V_i = \ln\frac{\overline{p_i}}{1-\overline{p_i}} = \left(v_i;\ l_{v_i}, r_{v_i}; l'_{v_i}, r'_{v_i}\right) \tag{3.5}$$

when

$$
\begin{aligned}
v_i &= \ln\left(\frac{\overline{p_i}}{1-\overline{p_i}}\right)\\
l_{v_i} &= \ln\frac{\overline{p_i}\left(1+l_{\overline{p_i}}\right)+\left(1-\overline{p_i}\right)l_{\overline{p_i}}}{\left(1-\overline{p_i}\right)^2} = ln\frac{\overline{p_i}+l_{\overline{p_i}}}{\left(1-\overline{p_i}\right)^2}\\
r_{v_i} &= \ln\frac{\overline{p_i}\left(1+r_{\overline{p_i}}\right)+\left(1-\overline{p_i}\right)r_{\overline{p_i}}}{\left(1-\overline{p_i}\right)^2} = \ln\frac{\overline{p_i}_i+r_{\overline{p_i}}}{\left(1-\overline{p_i}\right)^2}\\
l'_{v_i} &= \ln\frac{\overline{p_i}\left(1+l'_{\overline{p_i}}\right)+\left(1-\overline{p_i}\right)l'_{\overline{p_i}}}{\left(1-\overline{p_i}\right)^2} = ln\frac{\overline{p_i}_i+l'_{\overline{p_i}}}{\left(1-\overline{p_i}\right)^2}\\
r'_{v_i} &= \ln\frac{\overline{p_i}\left(1+r'_{\overline{p_i}}\right)+\left(1-\overline{p_i}_i\right)r'_{\overline{p_i}}}{\left(1-\overline{p_i}\right)^2} = ln\frac{\overline{p_i}+r'_{\overline{p_i}}}{\left(1-\overline{p_i}\right)^2}.
\end{aligned}
$$

This leads the logit function to the Intuitionistic fuzzy semi-parametric logistic regression model as follows:

$$
V_i = \ln\frac{\overline{p_i}}{1-\overline{p_i}} = (\ln\left(\frac{\overline{p_i}}{1-\overline{p_i}}\right); \ln\frac{\overline{p_i}+l_{\overline{p_i}}}{\left(1-\overline{p_i}\right)^2}, \ln\frac{\overline{p_i}+r_{p_i}}{\left(1-\overline{p_i}\right)^2}; \ln\frac{\overline{p_i}+l'_{\overline{p_i}}}{\left(1-\overline{p_i}\right)^2}, \ln\frac{\overline{p_i}+r'_{\overline{p_i}}}{\left(1-\overline{p_i}_i\right)^2}. \tag{3.6}
$$

We can estimate the Intuitionistic fuzzy semi-parametric logistic regression model through two methods.

4. The suggest method

In this method, the fuzzy semi-parametric logistic regression model is estimated in two stages, the first stage is to estimate the parameter part and the second stage is to estimate the non-parametric portion through the smoothing function according to the following formula:

$$
\widehat{g}\left(t\right) = \sum_{i=1}^{n} w_i\left(t_j\right) \otimes V_i \tag{4.1}
$$

$w_i\left(t_j\right)$ matrix of estimator weights to Kernel smoothing, thus the Intuitionistic fuzzy logistic regression model estimator is.

$$
\widehat{V_i} = \left(\oplus_{j=1}^{p} \otimes x_{\mathrm{ij}}\right) + \sum_{i=1}^{n} w_i\left(t_j\right) \otimes V_i \tag{4.2}
$$

When the dependent variable represent the fuzzy intuitionistic triangular number

$$
(\ V_i = \left(v_i;\ l_{v_i}, r_{v_i}; l'_{v_i}, r'_{v_i}\right),
$$

the estimated outputs as follows:

$$
\begin{aligned}
\widehat{v_i} &= \left(\oplus_{j=1}^{p}\ b_j \otimes x_{\mathrm{ij}}\right) + \left(\sum_{i=1}^{n} w_i\left(t_j\right) \otimes v_i\right) \\
l_{\widehat{v_i}} &= \left(\oplus_{j=1}^{p}\ l_{b_j} \otimes x_{\mathrm{ij}}\right) + \left(\sum_{i=1}^{n} w_i\left(t_j\right) \otimes l_{v_i}\right) \\
r_{\widehat{v_i}} &= \left(\oplus_{j=1}^{p}\ r_{b_j} \otimes x_{\mathrm{ij}}\right) + \left(\sum_{i=1}^{n} w_i\left(t_j\right) \otimes r_{v_i}\right) \\
l'_{\widehat{v_i}} &= \left(\oplus_{j=1}^{p}\ l'_{b_j} \otimes x_{\mathrm{ij}}\right) + \left(\sum_{i=1}^{n} w_i\left(t_j\right) \otimes l'_{v_i}\right) \\
r'_{\widehat{v_i}} &= \left(\oplus_{j=1}^{p}\ r'_{b_j} \otimes x_{\mathrm{ij}}\right) + \left(\sum_{i=1}^{n} w_i\left(t_j\right) \otimes r'_{v_i}\right).
\end{aligned}
\tag{4.3}
$$

By using matrices, we can write the estimated outputs as follows:

$$
\underline{\widehat{V}} = \left(X\widehat{\beta} + W\underline{V}; X\underline{L} + W\underline{L_V}, X\underline{R} + W\underline{R_V}; X\underline{R\acute{L}} +\ W\underline{\acute{L}_V},\ X\underline{\acute{R}} +\ W\underline{\acute{R}_V}\right) \tag{4.4}
$$

where

$$
X = \begin{bmatrix} x_{11} & x_{12} & . & . & . & x_{1p} \\ x_{21} & x_{22} & . & . & . & x_{2p} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_{n1} & x_{n2} & . & . & . & x_{\mathrm{np}} \end{bmatrix} \quad W = \begin{bmatrix} w_{11} & w_{12} & . & . & . & w_{1n} \\ w_{21} & w_{22} & . & . & . & w_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ w_{n1} & w_{n2} & . & . & . & w_{\mathrm{nn}} \end{bmatrix}
$$

$$
\underline{\tilde{\beta}}\beta = \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ . \\ b_j \end{bmatrix}, \quad \underline{L} = \begin{bmatrix} l_{b_1} \\ l_{b_2} \\ . \\ . \\ . \\ l_{b_n} \end{bmatrix}, \quad \underline{R} = \begin{bmatrix} r_{P_1} \\ r_{P_2} \\ . \\ . \\ . \\ r_{P_n} \end{bmatrix}, \quad \underline{\acute{L}} = \begin{bmatrix} l'_{P_1} \\ l'_{P_2} \\ . \\ . \\ . \\ l'_{P_n} \end{bmatrix}, \quad \underline{\acute{R}} = \begin{bmatrix} r'_{P_1} \\ r'_{P_2} \\ . \\ . \\ . \\ r'_{P_n} \end{bmatrix}
$$

$$
V = \begin{bmatrix} v_1 \\ v_2 \\ . \\ . \\ . \\ v_n \end{bmatrix}, \quad L_V = \begin{bmatrix} l_{v_1} \\ l_{v_2} \\ . \\ . \\ . \\ l_{v_n} \end{bmatrix}, \quad R_V = \begin{bmatrix} r_{v_1} \\ r_{v_2} \\ . \\ . \\ . \\ r_{v_n} \end{bmatrix}, \quad L'_V = \begin{bmatrix} l'_{v_1} \\ l'_{v_2} \\ . \\ . \\ . \\ l'_{v_n} \end{bmatrix}, \quad R'_V = \begin{bmatrix} r'_{v_1} \\ r'_{v_2} \\ . \\ . \\ . \\ r'_{v_n} \end{bmatrix}
$$

5. The Wang et al method

Wang et al 2007 presented a method to estimate The Intuitionistic fuzzy logistic regression model, in this method the nonparametric portion can be estimated through the smoothing function according to the following formula [13, 7]:

$$
\widehat{g}\left(t\right) = \sum_{i=1}^{n} w_i\left(t_j\right) \otimes \left(V_i - \oplus_{j=1}^{k}(\otimes x_{\mathrm{ij}})\right) \tag{5.1}
$$

Depending on the nonparametric portion of equation (4.4), the Intuitionistic fuzzy logistic regression model estimator will be as follows:

$$\widehat{V_i} = \left(\sum_{i=1}^{n} w_i(t_j) \otimes V_i\right) \oplus \left(\oplus_{j=1}^{k} \ \otimes x_{\text{ij}}{}^*\right) \tag{5.2}$$

When $x_{\text{ij}}{}^* = x_{\text{ij}} - \sum_{i=1}^{n} w_i\,(t_j)x_{\text{ij}}$. Thus, the estimated fuzzy output (dependent variable) can found as follow:

$$\begin{aligned}
\widehat{v_i} &= (\sum_{i=1}^{n} w_i\,(t_j) \otimes v_i) \oplus (\oplus_{j=1}^{k} \ b_j \otimes x_{\text{ij}}{}^*) \\
l_{\widehat{v_i}} &= (\sum_{i=1}^{n} w_i\,(t_j) \otimes l_{v_i}) \oplus (\oplus_{j=1}^{k} \ l_{b_j} \otimes s_{\text{ji}}x_{\text{ij}}{}^*) \ominus (\oplus_{j=1}^{k} \ r_{b_j} \otimes (1 - s_{\text{ji}})x_{\text{ij}}{}^*) \\
r_{\widehat{v_i}} &= (\sum_{i=1}^{n} w_i\,(t_j) \otimes r_{v_i}) \oplus (\oplus_{j=1}^{k} \ r_{b_j} \otimes s_{\text{ji}}x_{\text{ij}}{}^*)) \ominus (\oplus_{j=1}^{k} \ l_{b_j} \otimes (1 - s_{\text{ji}})x_{\text{ij}}{}^*) \\
\acute{l}_{\widehat{v_i}} &= (\sum_{i=1}^{n} w_i\,(t_j) \otimes \acute{l}_{v_i}) \oplus (\oplus_{j=1}^{k} \ \acute{l}_{b_j} \otimes s_{\text{ji}}x_{\text{ij}}{}^*) \ominus (\oplus_{j=1}^{k} \ \acute{r}_{b_j} \otimes (1 - s_{\text{ji}})x_{\text{ij}}{}^*) \\
\acute{r}_{\widehat{v_i}} &= (\sum_{i=1}^{n} w_i\,(t_j) \otimes \acute{r}_{v_i}) \oplus (\oplus_{j=1}^{k} \ \acute{r}_{b_j} \otimes s_{\text{ji}}x_{\text{ij}}{}^*) \ominus (\oplus_{j=1}^{k} \ \acute{l}_{b_j} \otimes (1 - s_{\text{ji}})x_{\text{ij}}{}^*)
\end{aligned} \tag{5.3}$$

where $s_{ji} = \begin{cases} 1 & x_{\text{ij}}{}^* \geq 1 \\ 0 & x_{\text{ij}}{}^* < 0 \end{cases}.$

The estimated fuzzy output (dependent variable) which represents the fuzzy intuitionistic triangular number as follow:

$$\begin{aligned}
\underline{\widehat{V}} = \Big(&W\underline{V} + X^*\underline{\widehat{\beta}}; W\underline{L_V} + X_s^*\underline{L} - X_{1-s}^*\underline{R}, W\underline{R_V} + X_s^*\underline{R} - X_{1-s}^*\underline{L}; W\underline{\acute{L}_V} + X_s^*\underline{\acute{L}} - X_{1-s}^*\underline{\acute{R}}, W\underline{\acute{R}_V} \\
&+ X_s^*\underline{\acute{R}} - X_{1-s}^*\underline{\acute{L}}\Big)
\end{aligned} \tag{5.4}$$

where

$$X^* = \begin{bmatrix} x_{11}^* & x_{12}^* & . & . & . & x_{1k}^* \\ x_{21}^* & x_{22}^* & . & . & . & x_{2k}^* \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_{n1}^* & x_{n2}^* & . & . & . & x_{\text{nk}}^* \end{bmatrix}, \quad X_s^* = \begin{bmatrix} s_{11}x_{11}^* & s_{12}x_{12}^* & . & . & . & s_{1k}x_{1k}^* \\ s_{21}x_{21}^* & s_{22}x_{22}^* & . & . & . & s_{2k}x_{2k}^* \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ s_{n1}x_{n1}^* & s_{n2}x_{n2}^* & . & . & . & s_{\text{nk}}x_{\text{nk}}^* \end{bmatrix}$$

$$X_{(1-s)}^* = \begin{bmatrix} (1 - s_{11})\,x_{11}^* & (1 - s_{12})x_{12}^* & . & . & . & (1 - s_{1k})x_{1k}^* \\ (1 - s_{21})x_{21}^* & s_{22}x_{22}^* & . & . & . & s_{2k}x_{2k}^* \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ (1 - s_{n1})x_{n1}^* & (1 - s_{n2})x_{n2}^* & . & . & . & (1 - s_{\text{nk}})x_{\text{nk}}^* \end{bmatrix}$$

6. Mean Square Error And Goodness of Fit

The comparison between the estimation methods of the intuitionistic fuzzy semi-parametric logistic regression model is made through mean squares error MSE and the measure goodness of fit $S\left(\widehat{V}, V\right)$ based on Euclidean distance according to the following formulas [16]:

$$S\left(\widehat{V}, V\right) = \frac{1}{n}\sum_{i=1}^{n} s^{*}\left(\widehat{V},V\right) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+d2\left(\widehat{V},V\right)} \tag{6.1}$$

$$\text{MSE}\,(V) = \frac{1}{n}\sum_{i=1}^{n} d2\left(\widehat{V},V\right) \tag{6.2}$$

When $d2\left(\widehat{V},V\right)$ represents the Euclidean distance between V and $\widehat{V}$, the relationship between the mean square error and the measure goodness of fit is inverse [16].

7. Application

Covid-19 one from the set of viruses that cause infections in the respiratory system of humans, which include colds and are usually fatal, such as severe acute respiratory syndrome and the Middle East respiratory syndrome. Coronaviruses belong to the straight coronavirus family within the Coronavirus family within the order of nest viruses.

We application the Intuitionistic fuzzy semi-parametric logistic regression model on the data Covid-19, where The fuzzy dependent variable represented the incidence ratio of Covid-19 virus, as the percentage of infection was divided into five cases by the doctors, which are (very low, low, medium, high, very high) as follow :

Table 1: Triangular Intuitionistic Fuzzy Number

Cases	***Triangular Intuitionistic Fuzzy Number***
Very low	***(0.00,0.05,0.173,0.2,0.25)***
Low	***(0.2,0.25,0.313,0.4,0.45)***
Medium	***(0.4,0.45,0.497,0.55,0.60)***
High	***(0.55,0.60,0.667,0.75,0.80)***
Very high	***(0.75,0.80,0.824,0.95,1)***

A sample was taken from 30 people infected with the Coronavirus, and the most important factors identified by the infected people, which represent the independent variables, are (X_1 sex of the patient where male X_1=2 and female X_1=1 , X_2 patient's age, X_3 white blood cells, X_4 Urea, and X_5 D-dimer represent non parametric variable), as in the following table:

After estimating the Intuitionistic fuzzy semi-parametric logistic regression model on the real data, in the case of the dependent variable, it represents a triangular intuitionistic fuzzy number through two methods of estimation the suggested method and the second method is the Wang et al method; the mean squares error and the measure goodness of fit for the model estimation methods were according to the following table 3:

Table 2: the data set of covid-19

$\bar{P}_i$	X_1	X_2	X_3	X_4	X_5
LOW	*1.0*	*70.0*	*5.6*	*137.0*	*784.0*
Medium	*2.0*	*49.0*	*7.9*	*45.0*	*7274.0*
High	*1.0*	*55.0*	*20.5*	*47.0*	*2230.0*
Very High	*2.0*	*92.0*	*9.2*	*164.0*	*3864.0*
Very High	*2.0*	*75.0*	*12.8*	*46.0*	*1744.0*
LOW	*2.0*	*76.0*	*13.8*	*155.0*	*2197.0*
Low	*2.0*	*85.0*	*7.4*	*55.0*	*1218.0*
Very LOW	*2.0*	*33.0*	*6.2*	*45.0*	*450.0*
High	*2.0*	*57.0*	*22.0*	*43.0*	*13076.0*
Medium	*2.0*	*70.0*	*11.7*	*80.0*	*8000.0*
High	*2.0*	*67.0*	*9.2*	*45.0*	*1100.0*
High	*1.0*	*75.0*	*12.8*	*46.0*	*1744.0*
Low	*1.0*	*55.0*	*7.2*	*39.0*	*260.0*
Low	*1.0*	*65.0*	*9.3*	*45.0*	*500.0*
Medium	*1.0*	*60.0*	*9.6*	*18.0*	*482.0*
Very LOW	*1.0*	*70.0*	*7.4*	*51.0*	*1900.0*
High	*1.0*	*63.0*	*7.6*	*69.0*	*10700.0*
High	*2.0*	*65.0*	*8.3*	*58.0*	*403.0*
Very LOW	*2.0*	*27.0*	*8.6*	*23.0*	*554.0*
Low	*1.0*	*50.0*	*11.1*	*139.0*	*502.0*
Low	*1.0*	*43.0*	*5.2*	*45.0*	*2203.0*
Medium	*1.0*	*45.0*	*10.4*	*65.0*	*2000.0*
Very LOW	*1.0*	*70.0*	*10.9*	*103.0*	*945.0*
Medium	*1.0*	*33.0*	*8.9*	*40.0*	*430.0*
Low	*1.0*	*38.0*	*17.5*	*72.0*	*445.0*
High	*1.0*	*56.0*	*11.9*	*60.0*	*1685.0*
Very LOW	*1.0*	*57.0*	*11.0*	*70.0*	*2805.0*
High	*1.0*	*20.0*	*15.7*	*44.0*	*454.0*
Very High	*1.0*	*60.0*	*8.3*	*72.0*	*1084.0*
Very LOW	*1.0*	*30.0*	*5.0*	*15.0*	*1316.0*

Table 3: Mean Square Error & Goodness of Fit

Estimation Methods	*Nadaraya Watson*		$MSE(V)$	$S(\widehat{V}, V)$
	Kernel Function	*Bind Width*		
The Suggest Method	*Gaussian*	*Silverman's Rule h=681.524*	*0.9406777*	*0.5734781*
		Refined Plug h=44.616	*0.9257968*	*0.5766637*
The Wang Et Al Method	*Gaussian*	*Silverman's Rule h=681.524*	*3.017323*	*0.2677759*
		Refined Plug h=44.616	*3.012533*	*0.2698827*

8. Conclusion

By applying the Intuitionistic fuzzy semi-parametric logistic regression model to the Coronavirus data, the suggested method for estimating the model was better than the Wang et al method. In addition, the smoothing parameter that was calculated through refined plug bind width a better method than Silverman's rule bandwidth, and from Table No. 2, the value of the independent variable D-dimer had a significant impact on infection with the Coronavirus.

References

[1] A.A.H. Ahmadini, *A novel technique for parameter estimation in intuitionistic fuzzy logistic regression model*, Ain Shams Engin. J. 13(1) (2022) 101518.

[2] H.T. Asai, S. Tanaka and K. Uegima, *Linear regression analysis with fuzzy model*, IEEE Trans. Syst. Man. Cybern. 12 (1982) 903–907.

[3] C.-B. Cheng and E.S. Lee, *Nonparametric fuzzy regression-k-NN and kernel smoothing techniques*, Comput. Math. Appl. 38(3-4) (1999) 239–251.

[4] Y. Gao and L. Qiujun, *A fuzzy logistic regression model based on the least squares estimation*, Comput. Appl. Math. 37(3) (2018) 3562–3579.

[5] T. Garai, D. Chakraborty and T.K. Roy, *Possibility-necessity-credibility measures on generalized intuitionistic fuzzy number and their applications to multi-product manufacturing system*, Granular Comput. 3(4) (2018) 285–299.

[6] D. Guha and D. Chakraborty, *A theoretical development of distance measure for intuitionistic fuzzy numbers*, Int. J. Math. Math. Sci. 2010 (2010).

[7] G. Hesamian and M.G. Akbari, *Semi-parametric partially logistic regression model with exact inputs and intuitionistic fuzzy outputs*, Appl. Soft Comput. 58 (2017) 517–526.

[8] M. Kim, Y. Song, B. Li and D. Micciancio, *Semi-parallel logistic regression for GWAS on encrypted data*, BMC Medical Genom. 13(7) (2020) 1–13.

[9] W.K. Härdle, M. Müller, S. Sperlich and A. Werwatz, *A Nonparametric and Semiparametric Models*, Springer Science and Business Media, 2004.

[10] M. Namdari, M. Namdari, A. Abadi, S.M. Taheri, M. Rezaei, N. Kalantari and N. Omidvar, *Effect of folic acid on appetite in children: Ordinal logistic and fuzzy logistic regressions*, Nutrition 30(3) (2014) 274–278.

[11] P. Speckman, *Kernel smoothing in partial linear models*, J. Royal Statist. Soc. Ser. B 50(3) (1988) 413–436.

[12] E. Szmidt and J. Kacprzyk, *Using intuitionistic fuzzy sets in group decision making*, Control Cybern. 31 (2002) 1055–1057.

[13] N. Wang, W.-X. Zhang and C.-L. Mei, *Fuzzy nonparametric regression based on local linear smoothing technique*, Inf. Sci. 177(18) (2007) 3882–3900.

[14] S.-J. Wu, S.L. Lin, D.B. Wu and H.L. Ma, *Predicting risk parameters using intelligent fuzzy/logistic regression*, Int. Conf. Artific. Intell. Comput. Intell. IEEE, vol. 3, 2010, pp. 397–401.

[15] M.-S. Yang and H.-M. Chen, *Fuzzy class logistic regression analysis*, Int. J. Uncert. Fuzz. Knowledge-Based Syst. 12(6) (2004) 761–780.

[16] J. Ye, *Multicriteria group decision-making method using the distances-based similarity measures between intuitionistic trapezoidal fuzzy numbers*, Int. J. Gen. Syst. 41(7) (2012) 729–739.

[17] A.L. Zadehi, *Fuzzy sets*, Inf. Control 8 (1965) 338–353.