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Soft ideal theory and applications

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Abstract

A soft model gives a vital mathematical tool for handling uncertainties and vague concepts. Soft models have been introduced. We give a new concept and investigate the soft ψ closed, soft $\alpha\psi$ closed, and soft $I_{\alpha\psi}$ closed set in soft topological structure It is also shown their properties. We provide some illustrated examples.

Keywords: Soft ψ-closed, soft $\alpha\psi$ closed, soft $I_{\alpha\psi}$-closed
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1 Introduction

In this paper, soft and model gives a vital mathematical tool for handling uncertainties and vague concepts. Soft and model have been introduced in [10]. Applied set theory to topological structure see [15] they introduced some concepts like soft topological structure, soft interior, soft closure, soft subspaces and studied some of their properties. The soft ideal was introduced see [13]. The concept of soft I-open sets and investigated their properties see [1]. The reader for example the works in [2, 4, 7, 11, 12]. We define a new soft ψ-closed, soft $\alpha\psi$-closed, soft $I_{\alpha\psi}$-closed set concept in soft ideal topological structure and study their properties.

2 Preliminaries

In this paper, λ is a non-empty set (initial universal) and the parameters of set. The set of power denoted by $P(\lambda)$ and the soft sets denoted by $S(\lambda)$.

Definition 2.1. [10] A pair of soft set is written as (M, δ) over (λ, δ), where $M : \delta \to P(\lambda)$.

Definition 2.2. [18] A soft set (M, δ) its complement is defined as $(M, \delta)^c = (M^c, \delta)$, where $M^c(\alpha) = (M(\alpha))^c = \lambda\backslash M(\alpha)$, for all $\alpha \in \delta$. Clearly, we have $(\tilde{\phi})^c = \tilde{\lambda}$ and $(\tilde{\lambda})^c = \tilde{\phi}$.

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Definition 2.3. [15] Let be a soft sets collection over λ, then it was stated a soft topological structure on λ if,

(a) $\tilde{\phi}, \tilde{\lambda}$ are members of τ.
(b) Any two soft sets intersection in $\tau \in \tau$.
(c) Any number of soft sets union in $\tau \in \tau$..

Soft topological structure is written as (λ, τ, δ) over λ. The soft open sets -τ is stated as a member of τ.

Proposition 2.4. [6]. Let a soft topological structure (λ, τ, δ) and $(M, \delta), (H, \delta)$, be two soft sets. Then:

(a) $i(i(M, \delta)) = i(M, \delta)$.
(b) $(H, \delta)\tilde{\supseteq}(M, \delta)$, then $i(H, \delta)\tilde{\supseteq}i(M, \delta)$.
(c) $c(c(M, \delta)) = c(M, \delta)$.
(d) $(H, \delta)\tilde{\supseteq}(M, \delta)$, then $c(H, \delta)\tilde{\supseteq}c(M, \delta)$.

Definition 2.5. [5]. A collection of null soft sets is defined as a soft idea I over λ if the following are hold:

(a) $(M, \delta)\tilde{\in}I,\ (M, \delta)\tilde{\supset}$, then $(M, \delta)\tilde{\in}I$
(b) $(M, \delta)\tilde{\in}I,\ (H, \delta)\tilde{\supset}$, then $(H, \delta)\tilde{\cup}(M, \delta)\tilde{\in}I$

$(\lambda, \tau, \delta, I)$ is defined as soft ideal topological structure.

Definition 2.6. [3] If soft ideal topological structure $(\lambda, \tau, \delta, I)$, then the function $(M, \delta)^*(i, \tau) = \tilde{\cap}(\lambda_\epsilon \in: Ux_\epsilon\tilde{\cap}(M, \delta)) \notin I, \forall x_\epsilon \in \tau$ is soft local of (M, δ).

Definition 2.7. [5] A soft ideal topological structure $(\lambda, \tau, \delta, I)$, then the soft closure operator is defined by $c^*(M, \delta) = (M, \delta)\tilde{\cup}(M, \delta)^*$.

Lemma 2.8. [5] A soft ideal topological structure $(\lambda, \tau, \delta, I)$ and $(H, \delta), (F, \delta)$, be two soft sets. Then:

(a) $(H, \delta)\tilde{\supset}(M, \delta) \Longrightarrow (H, \delta)^*\tilde{\supset}(M, \delta)^*$.
(b) $((M, \delta)\tilde{\cup}(H, \delta))^* = (M, \delta)^*\tilde{\cup}(H, \delta)^*$.
(c) $c(M, \delta)\tilde{\supset}(M, \delta)^*, (M, \delta)^*\tilde{\supset}((M, \delta)^*)^*$.
(d) (M, δ) soft open.
(e) $(M, \delta)\tilde{\cap}(H, \delta)\tilde{\in}I \Longrightarrow (H, \delta)^*\tilde{\cap}(M, \delta) = \tilde{\phi}$.
(f) $(M, \delta)^*$ is soft closed.
(g) $(M, \delta)\tilde{\supset}(M, \delta)^*$.

Proposition 2.9. [5] A soft ideal topological structure $(\lambda, \tau, \delta, I)$ and $(H, \delta), (F, \delta)$, be two soft sets. Then:

(a) $c^*(\tilde{\phi}) = \tilde{\phi}$ *and* $c^*(\tilde{\lambda}) = \tilde{\lambda}$.
(b) $c^*(M, \delta)\tilde{\supseteq}(M, \delta)$ *and* $c^*(c^*(M, \delta)) = c^*(M, \delta)$.
(c) If $(M, \delta)\tilde{\subset}(H, \delta)$, then $c^*(H, \delta)\tilde{\supset}c^*(M, \delta)$.
(d) $c^*(M, \delta)\tilde{\cup}c^*(H, \delta) = c^*((M, \delta)\tilde{\cup}(H, \delta))$.

Definition 2.10. A subset (M, δ) of a soft topological structure $(\lambda, \tau, \delta, I)$ is called

(a) soft α-open [2] if $(M, \delta) \subseteq i(c(i(M, \delta)))$,
(b) soft semi-open [4] if $(M, \delta) \subseteq c(i(M, \delta))$,
(c) soft generalized α-closed [16] if $\alpha c(M, \delta) \subseteq (U, \delta)$ where $(U, \delta) \supseteq (M, \delta)$ is soft α-open.

Definition 2.11. (M, δ) is a subset of a soft ideal topological structure $(\lambda, \tau, \delta, I)$ is defined as

(a) soft $I_{\pi g}$-closed [14], if $(U, \delta) \supseteq (M, \delta)^*$, where $(U, \delta) \supseteq (M, \delta)$ and (U, δ) is soft π-open,
(b) soft τ^*-closed [17], if $(M, \delta)\tilde{\supseteq}(M, \delta)^*$,
(c) soft τ^*-dense-in-itself [17], if $(M, \delta)^*\tilde{\supseteq}(M, \delta)$.

3 Soft $I_{\alpha\psi}$-Closed Sets

Definition 3.1. Let $(\lambda, \tau, \delta, I)$ be a soft ideal topological structure then the soft ψ-closed if $(U, \delta) \supseteq sc(M, \delta)$ where $(U, \delta) \supseteq (M, \delta)$ and (U, δ) is soft sg-open.

Definition 3.2. Let $(\lambda, \tau, \delta, I)$ be a soft ideal topological structure then the soft $\alpha\psi$-closed if $(U, \delta) \supseteq \psi c(M, \delta)$, where $(U, \delta) \supseteq (F, \delta)$ and (U, δ) is soft α-open.

Definition 3.3. Let $(\lambda, \tau, \delta, I)$ be a soft ideal topological structure then the soft $I_{\alpha\psi}$-closed if $(U, \delta) \supseteq (M, \delta)^*$, where $(U, \delta) \supseteq (M, \delta)$ and (U, δ) is soft $\alpha\psi$-open.

Theorem 3.4. Let $(\lambda, \tau, \delta, I)$ be a soft topological structure then

(a) Each soft α-closed set is a soft $\alpha\psi$-closed set.
(b) Each soft semi-closed set is a soft $\alpha\psi$-closed set.
(c) Each soft ψ-closed set is a soft $\alpha\psi$-closed set.
(d) Each soft α-closed set is a soft $\alpha\psi$-closed set.

Proof .
(a) Let (M, δ) be a α-closed set (soft), $(M, \delta) = \alpha c(M, \delta)$. Let $(U, \delta) \supseteq (M, \delta), (U, \delta)\alpha-$open(soft). That (M, δ) is α-closed(soft), $\psi c(M, \delta) \subseteq (U, \delta)$. This shows that (M, δ) is a $\alpha\psi$-closed set (soft).

(b) Let (M, δ) be a (soft) semi-closed set, $(M, \delta) = sc(M, \delta)$. Let $(M, \delta) \subseteq (U, \delta), (U, \delta)$ be soft $\alpha-$ open. (M, δ) semi-closed(soft), $\psi c(M, \delta)(U, \delta) \subseteq sc(M, \delta)$. Then (M, δ) is a soft $\alpha\psi$-closed set.

(c) Let (U, δ) be a α-open set (soft) that $(H, \delta \subseteq (U, \delta))$. Each soft α-open set is soft sh-open. Then $\psi c(M, \delta) \subseteq sc(M, \delta) \subseteq (U, \delta)$. Then (M, δ) is a $\alpha\psi$-closed set (soft).

(d) Let the set (U, δ) be a (soft) α-open that $(U, \delta) \supseteq (M, \delta)$. Then $(U, \delta) \supseteq \alpha c(M, \delta) \supseteq \psi c(M, \delta)$, that (M, δ) is a $\alpha\psi$-closed set (soft). $\square$

The opposite of the theory does not have to be correct as the illustrated examples

Example 3.5. Let $\lambda = \{a, b\}$, $\delta = \{e_1, e_2\}$, here

$$
\begin{aligned}
&(M,\delta)_1 = \{(e_1,\phi),(e_2,\phi)\}, && (M,\delta)_2 = \{(e_1,\phi),(e_2,\{a\})\}\\
&(M,\delta)_3 = \{(e_1,\phi),(e_2,\{b\})\}, && (M,\delta)_4 = \{(e_1,\phi),(e_2,\{a,b\})\}\\
&(M,\delta)_5 = \{(e_1,\{a\}),(e_2,\{b\})\}, && (M,\delta)_6 = \{(e_1,\{a\}),(e_2,\{a\})\}\\
&(M,\delta)_7 = \{(e_1,\{a\}),(e_2,\{b\})\}, && (M,\delta)_8 = \{(e_1,\phi),(e_2,\{a,b\})\}\\
&(M,\delta)_9 = \{(e_1,\{b\}),(e_2,\phi)\}, && (M,\delta)_{10} = \{(e_1,\{b\}),(e_2,\{a\})\}\\
&(M,\delta)_{11} = \{(e_1,\{b\}),(e_2,\{b\})\}, && (M,\delta)_{12} = \{(e_1,\{b\}),(e_2,\{a,b\})\}\\
&(M,\delta)_{13} = \{(e_1,\{a,b\}),(e_2,\phi)\}, && (M,\delta)_{14} = \{(e_1,\{a,b\}),(e_2,\{a\})\}\\
&(M,\delta)_{15} = \{(e_1,\{a,b\}),(e_2,\{b\})\}, && (M,\delta)_{16} = \{(e_1,\{a,b\}),(e_2,\{a,b\})\}.
\end{aligned}
$$

(a) Let $\tau = \{\tilde{\phi}, \tilde{\lambda}, (M, \delta)_2, (M, \delta)_3, (M, \delta)_4, (M, \delta)_{16}, \}$. Clearly, $(M, \delta)_3$ is a soft $\alpha\psi$-closed set but is not a soft α-closed set.
(b) Let $\tau = \{\tilde{\phi}, \tilde{\lambda}, (M, \delta)_2, (M, \delta)_3, (M, \delta)_4, (M, \delta)_{16}, \}$. Clearly, $(M, \delta)_8$ is a $\alpha\psi$-closed set (soft) but is not a soft ψ-closed set.
(c) Let $\tau = \{\tilde{\phi}, \tilde{\lambda}, (M, \delta)_2, (M, \delta)_3, (M, \delta)_4, (M, \delta)_{16}, \}$. Clearly, $(M, \delta)_6$ is a $\alpha\psi$-closed set (soft).
(d) Let $\tau = \{\tilde{\phi}, \tilde{\lambda}, (M, \delta)_2, (M, \delta)_3, (M, \delta)_4, (M, \delta)_{16}, \}$. Clearly, $(M, \delta)_5$ is a $\alpha\psi$-closed set (soft).

Theorem 3.6. Let $(\lambda, \tau, \delta, I)$ be a soft ideal topological structure, then each soft $I_{\alpha\psi}$-closed set is a soft $I_{\pi g}$-closed set.

Proof . Let (U, δ) be a soft π-open set that $(M, \delta) \subseteq (U, \delta)$. Each set of π-open (soft) is soft open, and each open set (soft) is $\alpha\psi$-open set (soft). Here (M, δ) is $I_{\alpha\psi}$-closed set (soft), then $(M, \delta)^* \subseteq (U, \delta)$. Then (M, δ) is $I_{\pi g}$-closed set(soft). $\square$

The opposite of the theory does not have to be correct as the illustrated examples

Example 3.7. Let $\tau = \{\tilde{\phi}, \tilde{\lambda}, (M,\delta)_{1,2,3,4,16}\}$ and $I = \{\tilde{\phi}, \{(e_1,\phi),(e_2,\{a\})\}\}$. Clearly $(M,\delta)_5$ is not a soft $I_{\pi g}$-closed set but it is an $I_{\alpha g}$-closed set.

Theorem 3.8. Union of two closed sets is a soft $I_{\alpha\psi}$-closed set.

Proof . Let $(M,\delta),(H,\delta)$ soft $I_{\alpha\psi}$ be closed sets in (λ,δ). Let (U,δ) be a soft $\alpha\psi$-open set, $(U,\delta) \supseteq (M,\delta) \cup (H,\delta)$. Then $(U,\delta) \supseteq (M,\delta)$ and $(U,\delta) \supseteq (H,\delta)$. That (M,δ) and (H,δ) are $I_{\alpha\psi}$-closed sets (soft). We have $(U,\delta) \supseteq (M,\delta)^*$, $(U,\delta) \supseteq (H,\delta)^*$.Then $(M,\delta)^* \cup (H,\delta)^* = (U,\delta \supseteq ((M,\delta) \cup (H,\delta))^*$. Therefore $(M,\delta) \cup (H,\delta)$ is a soft $I_{\alpha\psi}$-closed set. □

Theorem 3.9. If (λ,τ,δ,I) is a soft ideal topological structure and $(\lambda,\delta) \supseteq (M,\delta)$. Each are equivalent:

(a) (M,δ) is a soft $I_{\alpha\psi}$-closed set.
(b) $(U,\delta) \supseteq c^*(M,\delta)$, where $(U,\delta) \supseteq (M,\delta),(U,\delta)$, is soft $\alpha\psi$-open in (λ,δ).
(c) All $\lambda_g \in c^*(M,\delta), \alpha\psi c(\{\lambda\}) \cap (F,\delta) \neq \phi$.
(d) $c^*(M,\delta)\backslash(M,\delta)$ contains no null soft $\alpha\psi$-closed set.
(e) $(M,\delta)^*\backslash(M,\delta)$ contains no null soft $\alpha\psi$-closed set.

Proof .
(a)$\Longrightarrow$(b): Suppose that (M,δ) is an $I_{\alpha\psi}$-closed set (soft), then $(U,\delta) \supseteq (M,\delta)^*$, that $(U,\delta),(U,\delta) \supseteq \delta$, is $\alpha\psi$-open (soft) in (λ,δ) and so $(U,\delta) \supseteq c^*(M,\delta) = (M,\delta) \cup (M,\delta)^*$ that $(U,\delta) \supseteq (M,\delta)$ and it is $\alpha\psi$-open in (λ,δ).

(b)$\Longrightarrow$(c): Suppose that λ_g is a member of $c^*(M,\delta)$. If $\alpha\psi c(\{\lambda_g\}) \cap (M,\delta) = \phi$, then $(\lambda,\delta)\backslash\alpha\psi c(\{\lambda_g\}) \supseteq (M,\delta)$, by (b) $(\lambda,M)\backslash\alpha\psi c(\{\lambda_g\}) \supseteq c^*(M,\delta)$ which is a contradiction, because λ_g is a member of $c^*(M,\delta)$.

(c)$\Longrightarrow$(d): Suppose that $c^*(M,\delta)\backslash(M,\delta),(H,\delta) \supseteq (H,\delta)\alpha\psi$-closed(soft) and $\lambda_g \in (H,\delta)$. Since $(\lambda,\delta)\backslash(M,\delta) \supseteq (H,\delta)$ and (H,δ) is $\alpha\psi$-closed (soft), $(\lambda,\delta)\backslash(H,\delta) \supseteq (M,\delta), \alpha\psi c(\{\lambda_g\}) \cap (M,\delta) = \phi$, since λ_g is a member of $c^*(M,\delta)$, by (c) $\alpha\psi c(\{\lambda_g\}) \cap (M,\delta) \neq \phi$. Therefore $c^*(M,\delta)\backslash(M,\delta)$ contains no null soft $\alpha\psi$-closed set.

(d)$\Longrightarrow$(e): Since $c^*(M,\delta)\backslash(M,\delta) = ((M,\delta) \cup (M,\delta)^*)\backslash(M,\delta) = ((M,\delta) \cup (M,\delta)^*) \cap (M,\delta)^c = ((M,\delta) \cap (M,\delta)^c) \cup ((M,\delta)^* \cap (M,\delta)^c) = ((M,\delta)^* \cap (M,\delta)^c) = ((M,\delta)^*\backslash(M,\delta)), (M,\delta)^*\backslash(M,\delta)$ no null $\alpha\psi$-closed set (soft).

(e)$\Longrightarrow$(a): Let $(U,\delta) \supseteq (M,\delta)$ where is a $(U,\delta)\alpha\psi$-open set (soft). Therefore $(\lambda,\delta)\backslash(F,\delta) \supseteq (\lambda,\delta)\backslash(U,\delta)$ and so $(M,\delta)^* \cap ((\lambda,\delta)\backslash(M,\delta)) \supseteq (M,\delta)^* \cap ((\lambda,\delta)\backslash(U,\delta)) = (M,\delta)^*\backslash(M,\delta)$. Therefore $(M,\delta)^* \cap (\lambda,\delta)\backslash(U,\delta) = (M,\delta)^*\backslash(M,\delta)$. The set $(M,\delta)^*$ is always closed, so the soft $(M,\delta)^*$ is a $\alpha\psi$-closed set and so the soft set is -closed contained in. Therefore, $(M,\delta)^* \cap (\lambda,\delta)\backslash(U,\delta) = \phi$ and hence $(U,\delta) \supseteq (M,\delta)^*$. Then (M,δ) is a $I_{\alpha\psi}$-closed set (soft). □

Theorem 3.10. A soft ideal topological structure (λ,τ,δ,I) for each $(M,\delta) \in I$ is an $I_{\alpha\psi}$-closed set (soft).

Proof . Consider $(U,\delta) \supseteq (M,\delta)$ where (U,δ) is a $\alpha\psi$-open set (soft). Since $(M,\delta)^* = \phi$ for each $(M,\delta) \in I$, then $c^*(M,\delta) = (M,\delta)^* \cup (M,\delta) = (M,\delta) \subseteq (U,\delta)$. Therefore, by Theorem 3.9 (M,δ) is -closed set (soft). □

Theorem 3.11. Let (λ,τ,δ,I) be an ideal topological structure (soft). Then $(M,\delta)^*$ always is a $I_{\alpha\psi}$-closed set (soft) for each subset (M,δ) of (λ,δ).

Proof . Consider $(U,\delta) \supseteq (M,\delta)^*$ that (U,δ) is a $\alpha\psi$-open set (soft). Then $(M,\delta)^* \supseteq ((M,\delta)^*)^*$, we have $(U,\delta) \supseteq ((M,\delta)^*)^*$ that $(U,\delta) \supseteq (M,\delta)^*,(U,\delta)$ is a soft $\alpha\psi$-open set. Then $(M,\delta)^*$ is a soft $I_{\alpha\psi}$-closed set. □

Theorem 3.12. Let (λ,τ,δ,I) be an ideal topological structure (soft). Then each soft $I_{\alpha\psi}$-closed, soft $\alpha\psi$-open set is a soft τ^*-closed set.

Proof . Suppose that (M,δ) is a soft $I_{\alpha\psi}$-closed set. If (M,δ) is a $\alpha\psi$-open set (soft) and $(M,\delta) \supseteq (M,\delta)$. Then $(M,\delta) \supseteq (M,\delta)^*$. Then (M,δ) is a τ^*-closed set (soft). □

Corollary 3.13. Let (λ,τ,δ,I) be an ideal topological structure (soft) and (M,δ) be a soft $I_{\alpha\psi}$-closed set. The following statements are equivalent:

(a) (M,δ) is a τ^*-closed set (soft).
(b) $c^*(M,\delta)\backslash(M,\delta)$ is a $\alpha\psi$-closed set (soft).
(c) $(M,\delta)^*\backslash(M,\delta)$ is a $\alpha\psi$-closed set (soft).

Proof . (a)$\Longrightarrow$(b): If (M,δ) is soft τ^*-closed, that $(M,\delta)\supseteq(M,\delta)^*$, and so $c^*(M,\delta)\backslash(M,\delta)=((M,\delta)\cup(M,\delta)^*)\backslash(M,\delta)=\phi$. Then $c^*(M,\delta)\backslash(M,\delta)$ is $\alpha\psi$-closed set (soft).

(b)$\Longrightarrow$(c) $c^*(M,\delta)\backslash(M,\delta)=(M,\delta)^*\backslash(M,\delta)$ and so $(M,\delta)^*\backslash(M,\delta)$ is soft $\alpha\psi$-closed set.

(c)$\Longrightarrow$(a) If the set $(M,\delta)^*\backslash(M,\delta)$ is $\alpha\psi$-closed (soft). Since the set (M,δ) is $I_{\alpha\psi}$-closed (soft), by Theorem 3.9, $(M,\delta)^*\backslash(M,\delta)=\phi$ and the set (M,δ) is soft τ^*-closed (soft). □

Theorem 3.14. The ideal topological structure (soft) (λ,τ,δ,I) and (M,δ) is soft *-dense-in-itself, tends to (M,δ) is $\alpha\psi$-closed (soft).

Proof . Suppose (M,δ) is soft *-dense-in-itself, soft $I_{\alpha\psi}$-closed subset (λ,δ). Let $(U,\delta)\supseteq(M,\delta)$ where $(U,\delta)\alpha$-open (soft). Each α-open set (soft) will be $\alpha\psi$-open (soft). Then by Theorem 3.9 $(U,\delta)\supset c^*(M,\delta)$ that (U,δ) soft $\alpha\psi$-open and $(U,\delta)\supseteq(M,\delta)$. Since (M,δ) is soft *-dense-in-itself then $c(M,\delta)=c^*(M,\delta)$, Each ψ-closed (soft) is closed set (soft). Therefore $(U,\delta)\supseteq\psi c(M,\delta)$ that $(U,\delta)\supseteq(M,\delta)$ and (U,δ) soft α-open. Then the soft (M,δ) is $\alpha\psi$-closed. □

Theorem 3.15. Soft ideal topological structure $(\lambda,\tau,\delta,I),(\lambda,\delta)\supseteq(M,\delta)$. The soft $(M,\delta)I_{\alpha\psi}$-closed if and only if $(M,\delta)=(H,\delta)\backslash(N,\delta)$, where (H,δ) is soft τ^*-closed and (N,δ) contains no null soft $\alpha\psi$-closed set.

Proof . If is soft -closed then by (Theorem 3.9 (e)), the soft set $\alpha\psi$-closed $(N,\delta)=(M,\delta)^*\backslash(M,\delta)$ contains no null. If $(H,\delta)=c^*(M,\delta)$, then (H,δ) is soft τ^*-closed such that, $(H,\delta)\backslash(N,\delta)=((M,\delta)\cup(M,\delta)^*)\backslash((M,\delta)^*\backslash(M,\delta))=((M,\delta)\cup(M,\delta)^*)\cap((M,\delta)^*\cap(M,\delta)^c)^c=((M,\delta)\cup(M,\delta)^*)\cap(((M,\delta)^*)^c\cup(M,\delta))=((M,\delta)\cup(M,\delta)^*)\cap((M,\delta)\cup(M,\delta)^*)^c=((M,\delta)\cup((M,\delta)^*\cap((M,\delta)^*)^c)=(M,\delta)$.

Conversely, suppose that $(m,\delta)=(H,\delta)\backslash(N,\delta)$, where (H,δ) is soft τ^*-closed and (N,δ) contains no null soft $\alpha\psi$-closed set. Let (U,δ) be a soft $\alpha\psi$-open set that $(U,\delta)\supseteq(M,\delta)$. Then $(U,\delta)\supseteq(H,\delta)\backslash(N,\delta)$ which implies that $(N,\delta)\supseteq(H,\delta)\cap((\lambda,\delta)\backslash(U,\delta))$. Now $(H,\delta)\supseteq(M,\delta)$ and $(H,\delta)\supseteq(H,\delta)^*$, then $(H,\delta)^*\supseteq(M,\delta)^*$ and so $(N,\delta)\supseteq(H,\delta)\cap((\lambda,\delta)\backslash(U,\delta))\supseteq(H,\delta)^*\cap((\lambda,\delta)\backslash(U,\delta))\supseteq(M,\delta)^*\cap((\lambda,\delta)\backslash(U,\delta))$. By hypothesis, since $(M,\delta)^*\cap((\lambda,\delta)\backslash(U,\delta))=\phi$, we have $(U,\delta)\supseteq(M,\delta)^*$. Then the soft (M,δ) is $I_{\alpha\psi}$-closed. □

Theorem 3.16. A soft ideal topological structure (λ,τ,δ,I) if (λ,δ) contains (M,δ) and (H,δ) that $c^*(M,\delta)\supseteq(H,\delta)\supseteq(M,\delta)$ and the soft (M,δ) is $I_{\alpha\psi}$-closed tends to (H,δ) is soft $I_{\alpha\psi}$-closed.

Proof . (M,δ) is soft $I_{\alpha\psi}$-closed then by Theorem 3.9 (d), $c^*(M,\delta)\backslash(M,\delta)$ contains no null soft $\alpha\psi$-closed set. Since $c^*((H,\delta)\backslash(H,\delta)\supseteq c^*(M,\delta)\backslash(M,\delta)$ and so $c^*((H,\delta)\backslash(H,\delta)$contains no null soft $\alpha\psi$-closed set. Hence (H,δ) is soft $I_{\alpha\psi}$-closed. □

Corollary 3.17. Let a soft ideal topological structure (λ,τ,δ,I) if (λ,δ) contains (M,δ) and (H,δ) that $(M,\delta)^*\supseteq(H,\delta)\supseteq(M,\delta)$ and (M,δ) is soft $I_{\alpha\psi}$-closed, then (M,δ) and (H,δ) are soft $\alpha\psi$-closed.

Proof . Let (λ,δ) contains (M,δ) and (H,δ) that $(M,\delta)^*\supseteq(H,\delta)\supseteq(M,\delta)$ which $c^*(M,\delta)\supseteq(M,\delta)^*\supseteq(H,\delta)\supseteq(M,\delta)$ and (M,δ) is soft $I_{\alpha\psi}$-closed by Theorem 3.16, (H,δ) is soft $I_{\alpha\psi}$-closed. Since $(F,\delta)^*\supseteq(G,\delta)\supseteq(F,\delta)$, $(M,\delta)^*=(H,\delta)^*$ and so (M,δ) and (H,δ) are soft *-dense-in-itself by Theorem 3.14, (M,δ) and (H,δ) are soft $\alpha\psi$-closed sets. □

Theorem 3.18. Let (λ,τ,δ,I) be a soft ideal topological structure and $(\lambda,\delta)\supseteq(M,\delta)$. Then (soft $I_{\alpha\psi}$-open) (M,δ) if and only if $i^*(M,\delta)\supseteq(H,\delta)$, whenever $(M,\delta)\supseteq(H,\delta)$ and (H,δ) is soft $\alpha\psi$-closed.

Proof . Suppose that (M,δ) is soft $I_{\alpha\psi}$-open. If $(H,\delta)\supseteq(H,\delta)$ and (H,δ) is soft $\alpha\psi$-closed, then $(\lambda,\delta)\backslash(H,\delta)\supseteq(M,\delta)$ and so $(H,\delta)\supseteq c^*(\lambda\backslash\delta)$ by Theorem 3.9. Therefore $(\lambda,\delta)\backslash c^*((\lambda,\delta)\backslash(M,\delta))\supseteq(H,\delta)=i^*(F,\delta)$. Then $i^*(M,\delta)\supseteq(H,\delta)$.

Conversely, let soft $\alpha\psi$-open set write as (U,δ) by such that $(U,\delta)\supseteq(\lambda,\delta)\backslash(H,\delta)$. Then $(M,\delta)\supseteq(\lambda,\delta)\backslash(U,\delta)$ and so $i^*(M,\delta)\supseteq(\lambda,\delta)\backslash(U,\delta)$. Therefore by Theorem 3.9, $(\lambda,\delta)\backslash(H,\delta)$ (soft $I_{\alpha\psi}$-closed). Then (M,δ) is soft $I_{\alpha\psi}$-open. □

Theorem 3.19. A soft ideal topological structure writes as (λ,τ,δ,I) and $(\lambda,\delta)\supseteq(M,\delta)$. If (M,δ) is soft $I_{\alpha\psi}$-open and $(M,\delta)\supseteq(H,\delta)\supseteq i^*(M,\delta)$, then (H,δ) is soft $I_{\alpha\psi}$-open.

Proof . A soft (M,δ) is $I_{\alpha\psi}$-open, then the soft $(\lambda,\delta)\backslash(M,\delta)$ is $I_{\alpha\psi}$-closed by Theorem 3.9, $c^*((\lambda,\delta)\backslash(M,\delta))\backslash((\lambda,\delta)\backslash(M,\delta))$ contains no null soft $\alpha\psi$-closed set. Since $i^*(H,\delta)\supseteq i^*(M,\delta)$ which tends to that $c^*((\lambda,\delta)\backslash(M,\delta))\supseteq c^*((\lambda,\delta)\backslash(M,\delta))$ and so $c^*((\lambda,\delta)\backslash(M,\delta))\backslash((\lambda,\delta)\backslash(M,\delta))\supseteq c^*((\lambda,\delta)\backslash(H,\delta))\backslash((\lambda,\delta)\backslash(H,\delta))$. The soft (H,δ) is $I_{\alpha\psi}$-open. □

Theorem 3.20. A soft ideal topological structure writes as by $(\lambda, \tau, \delta, I)$ and $\delta \subseteq \lambda$. Each are equivalent:

(a) (M, δ) is a soft-closed set.
(b) $(M, \delta) \cup ((\lambda, \delta) \backslash (M, \delta)^*)$ is a soft $I_{\alpha\psi}$-closed set.
(c) $(M, \delta)^* \backslash (M, \delta)$ is a soft $I_{\alpha\psi}$-closed set.

Proof . (a)$\Longrightarrow$(b): Let (M, δ) be a soft $I_{\alpha\psi}$-closed set, if (U, δ) is a soft $\alpha\psi$-open set, that $(U, \delta) \supseteq (M, \delta) \cup ((\lambda, \delta) \backslash (M, \delta)^*)$, then $(\lambda, \delta) \backslash ((M, \delta) \cup ((\lambda, \delta) \backslash (M, \delta)^*)) \supseteq (\lambda, \delta) \backslash (U, \delta) = (\lambda, \delta) \cap ((M, \delta) \cup ((M, \delta)^*)^c) = (M, \delta)^* \cap (M, \delta)^c = (M, \delta)^* \backslash (M, \delta)$. Since (M, δ) is soft $I_{\alpha\psi}$-closed set by Theorem 3.9 (e), then $(\lambda, \delta) \backslash (U, \delta) = \phi$ and so $(\lambda, \delta) = (U, \delta)$. Therefore $(U, \delta) \supseteq (M, \delta) \cup ((\lambda, \delta) \backslash (M, \delta)^*)$, then $(M, \delta) \cup ((\lambda, \delta) \backslash (\lambda, \delta) \supseteq (M, \delta)^*)$ and so $(\lambda, \delta) \supseteq (\lambda, \delta)^* \supseteq ((M, \delta) \cup ((\lambda, \delta) \backslash (M, \delta)^*))^* = (U, \delta)$. Then the soft $(M, \delta) \cup ((\lambda, \delta) \backslash (M, \delta)^*)$ is an $I_{\alpha\psi}$-closed set.

(b)$\Longrightarrow$(a): Suppose that the soft $(M, \delta) \cup ((\lambda, \delta) \backslash (M, \delta)^*)$ is an $I_{\alpha\psi}$-closed set. If the soft (H, δ) is a $\alpha\psi$-closed set, $(M, \delta)^* \backslash (M, \delta) \supseteq (H, \delta)$, then $(M, \delta)^* \supseteq (H, \delta)$ and $(\lambda, \delta) \backslash (M, \delta) \supseteq (H, \delta)$ which tends to $(H, \delta) \supseteq (\lambda, \delta) \backslash (M, \delta)^*$ and $(\lambda, \delta) \backslash (H, \delta) \supseteq (M, \delta)$. Therefore $(F, \delta) \cup ((\lambda, \delta) \backslash (H, \delta)) = (\lambda, \delta) \backslash (H, \delta) \supseteq (M, \delta) \cup ((\lambda, \delta) \backslash (M, \delta)^*)$ and $(\lambda, \delta) \backslash (H, \delta)$ is soft $\alpha\psi$-open. Since $(\lambda, \delta) \backslash (G, \delta) \supseteq ((F, \delta) \cup ((\lambda, \delta) \backslash (F, \delta)^*))^*$, which implies that $(\lambda, \delta) \backslash (H, \delta) \supseteq (M, \delta)^* \cup ((\lambda, \delta) \backslash (M, \delta)^*)^*$ and so $(\lambda, \delta) \backslash (H, \delta) \supseteq (M, \delta)^*$. Then, $(\lambda, \delta) \backslash (M, \delta)^* \supseteq (H, \delta)$. Since $(M, \delta)^* \supseteq (H, \delta)$, it follows that $(H, \delta) = \phi$. Then the soft (M, δ) is a $I_{\alpha\psi}$-closed set.

(b)$\Longrightarrow$(c): Since $(\lambda, \delta) \backslash ((M, \delta)^* \backslash (M, \delta)) = (\lambda, \delta) \cap (((M, \delta)^* \cap (M, \delta)^c)^c = (\lambda, \delta) \cap (((M, \delta)^*)^c \cup (M, \delta))) = ((\lambda, \delta) \cap ((M, \delta)^*)^c) \cup ((\lambda, \delta) \cap (M, \delta)) = (M, \delta) \cup ((\lambda, \delta) \backslash (M, \delta)^*)$. There is the triple box, which is known as the soft approximation space. □

4 Application of Soft Nano Topology

Definition 4.1. [9] Let U be the universe which is a null set and E is parameters set. Let soft relation R be equivalence on U. Then there is a triple soft space approximation. Suppose that $\lambda \supseteq U$.

(i) Let soft be lower approximation in relation to R and E set of parameters of all objects, and it is written as $(L_R(\lambda), E)$, equivalently $(L_R(\lambda), E) = \cup\{R(\lambda) : R(\lambda) \subseteq \lambda\}$, Where the class of $R(\lambda)$ are equivalence calculated by $\lambda \in U$.
(ii) Let soft λ be upper approximation in relation to R and E set of parameters of all objects, and it is written as $(H_R(\lambda), E)$, equivalently $(H_R(\lambda), E) = \cup\{R(\lambda) : R(\lambda) \cap \lambda = \phi\}$.
(iii) The soft λ be boundary region relation to R and the sets of all objects contains the sets of parameters E, it is neither outside nor inside λ in relation to R writes as $(R_r(\lambda), E)$, equivalently $(B_R(\lambda), E) = (H_R(\lambda), E) \backslash (L_R(\lambda), E)$.

Definition 4.2. [3] Let U be the universal set which is null set and E be the parameters. Let soft relation R be equivalence on U. Suppose that $\lambda \subset U$ and $\tau_R(\lambda) = \{U, \phi, (L_R(\lambda), E), (H_R(\lambda), E)(B_R(\lambda), E)\}$. $\tau_R(\lambda)$ is a soft topology on (U, E). Hence the soft nano topology in relation to λ is denoted as $\tau_R(\lambda)$. Soft nano open sets are soft nano topology elements and $(U, \tau_R(\lambda), E)$ is a soft nano topological structure.

The following example of a simple information system, summarized in Table 1

Example 4.3. Let $U = \{z_1, z_2, \cdots, z_8\}$ be the set of patients and $E = \{e_1, e_2, \cdots, z_5\}$ be the attributes (Chicken box disease), where e_1 loss of appetite, e_2 headache, e_3 raised pink, e_4 filled blisters, and e_5 vesicles.
Let a soft set $(X = (M, E))$ given in Table 1.

Case 1: From the above table, we have the set of knowledge bases is given by:

$$(L_R(\lambda), E)U \backslash R = \{\{z_1\}, \{z_{2,8}\}, \{z_{3,6}\}, \{z_4\}, \{z_5\}, \{z_7\}\}.$$

Let $\lambda = \{z_2, z_3, z_7, z_8\}$ be the set of patients having Chicken box. Then one can deduce that:

$$(L_R(\lambda), E) = \{z_2, z_7, z_8\}, \quad (H_R(\lambda), E) = \{z_2, z_3, z_6, z_7, z_8\}, \quad (R_R(\lambda), E) = \{z_3, z_6\}.$$

Then $\beta(\tau_R(\lambda)) = \{U, \{z_3, z_6\}, \{z_2, z_7, z_8\}\}$.

Step 1: If the attribute e_1 is removed, then, we get

$$U \backslash (R - e_1) = \{\{z_1\}, \{z_2, 8\}, \{z_{3,4,6}\}, \{z_{5,7}\}\}, (L_{R-e_1}(\lambda), E) = \{z_{2,8}\},$$

Table 1:

U	e_1	e_2	e_3	e_4	e_5	Chicken box
z_1	1	1	0	0	0	0
z_2	1	1	0	0	1	1
z_3	1	0	0	0	1	1
z_4	0	0	0	0	1	0
z_5	0	1	1	1	1	0
z_6	1	0	0	0	1	0
z_7	1	1	1	1	1	1
z_8	1	1	0	0	1	1

$$(H_{R-e_1}(\lambda), E) = \{z_{2,3,4,5,6,7,8}\}, (B_{R-e_1}(\lambda), E) = \{z_{3,4,5,6,7}\},$$

then

$$\beta(\tau_{(R-e_1)}(\lambda)) = \{U, \{z_{2,8}\}, \{z_{3,4,5,6,7}\}\} \neq \beta(\tau_R(\lambda)).$$

Hence, $e_1 \in$ Core (Chicken box).

Step 2: If the attribute e_2 is removed, then, we get

$$U\backslash(R - e_2) = \{\{z_1\}, \{z_{2,3,6,8}\}, \{z_4\}, \{z_5, z_7\}\},\ (L_{R-e_2}(\lambda), E) = \{z_7\},$$

$$(H_{R-e_2}(\lambda), E) = \{z_{2,3,6,7,8}\},\ (B_{R-e_2}(\lambda), E) = \{z_{2,3,6,8}\},$$

then

$$\beta(\tau_{(R-e_2)}(\lambda)) = \{U, \{z_7\}, \{z_{2,3,6,8}\}\} \neq \beta(\tau_R(\lambda)).$$

Hence, $e_2 \in$ Core (Chicken box).

Step 3: If the attribute e_3 is removed, then

$$U\backslash(R - e_3) = \{\{z_1\}, \{z_{2,8}\}, \{z_{3,6}\}, \{z_4\}, \{z_5\}, \{z_7\}\},\ (L_{R-e_3}(\lambda), E) = \{z_{2,7,8}\},$$

$$(H_{R-e_3}(\lambda), E) = \{z_{2,3,6,7,8}\},\ (B_{R-e_3}(\lambda), E) = \{z_{3,6}\}.$$

Thus

$$\beta(\tau_{(R-e_3)}(\lambda)) = \{U, \{z_{3,6}\}, \{z_{2,7,8}\}\} = \beta(\tau_R(\lambda)).$$

Hence, $e_3 \notin$ Core (Chicken box).

Step 4: If the attribute e_4 is removed, then

$$U\backslash(R - e_4) = \{\{z_1\}, \{z_{2,8}\}, \{z_{3,6}\}, \{z_4\}, \{z_5\}, \{z_7\}\},\ (L_{R-e_4}(\lambda), E) = \{z_{2,7,8}\},$$

$$(H_{R-e_4}(\lambda), E) = \{z_{2,3,6,7,8}\},\ (B_{R-e_4}(\lambda), E) = \{z_{3,6}\}.$$

Thus

$$\beta(\tau_{(R-e_4)}(\lambda)) = \{U, \{z_{3,6}\}, \{z_{2,7,8}\}\} = \beta(\tau_R(\lambda)).$$

Hence, $e_4 \notin$ Core (Chicken box).

Step 5: If the attribute is removed, then If the attribute e_5 is removed, then

$$U\backslash(R - e_5) = \{\{z_{1,2,8}\}, \{z_{3,6}\}, \{z_4\}, \{z_5\}, \{z_7\}\}.$$

Hence,

$$(L_{R-e_5}(\lambda), E) = \{z_7\},\ (H_{R-e_5}(\lambda), E) = \{z_{1,2,3,6,7,8}\},\ (B_{R-e_5}(\lambda), E) = \{z_{1,2,3,6,8}\}.$$

Thus

$$\beta(\tau_{(R-e_5)}(\lambda)) = \{U, \{z_7\}, \{z_{1,2,3,6,8}\}\} \neq \beta(\tau_R(\lambda)).$$

Hence, $e_5 \notin$ Core (Chicken box). Therefore, CORE (Chicken box)=$\{e_1, e_2, e_5\}$. From the above, we deduce that, loss of appetite, headache, and vesicles are the key attributes to say that the patients have Chicken box.

5 Conclusion

Soft set theory is a general method for solving problem of uncertainty. We have discussed the new concept of soft-closed, soft-closed and soft-closed set in soft ideal topological structure. We discussed the relationship between (soft-closed set) with (soft-closed), (soft semi-closed), (soft-closed), (soft-closed sets) and investigate some of their properties. Also, the problem of Chicken box is discussed using the notion of soft nano topology. It is shown that the notion of soft nano topology is advantageous to analyze the problem arising from real world situations.

The concept of soft nano topology is used to analyses the Chicken Box diseases. It is demonstrated that the concept of soft nano topology is useful in analyzing problems that arise in real-world.

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