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Interval model for calculating prospect cross efficiency of data envelopment analysis and providing a solution for its expansion in fuzzy model

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Abstract

Cross efficiency evaluation of data envelopment analysis (DEA) is an effective tool in measuring the performance of decision-making units. In general, in cross efficiency evaluation models, it is assumed that decision makers are completely rational, in which case they refrain from considering the risk attitude that plays an important role in the evaluation process. In order to fill this gap, cross efficiency evaluation in DEA was performed based on prospect theory. In the real world, many inputs and outputs are not known, which are called inaccurate data; what is expected is that even if one of the data is not accurate, the answer will probably not be accurate. To solve this problem, the present study presents models that are able to evaluate the prospect cross efficiency with interval data and proves the feasibility of the models by proving the theorems.

Keywords: Data Envelopment Analysis, Fuzzy numbers, Interval Efficiency, Performance evaluation, Prospect Cross Efficiency
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1. Introduction

Data envelopment analysis (DEA) is a mathematical programming technique proposed by Charnes et al. [9] which was then developed by Banker et al. [4] to evaluate the relative efficiency of decision-making units. Given that in the evaluation of DMUs by DEA models, more than one unit may

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be introduced as an efficient unit, numerous ranking models have been proposed by researchers, each with its own strengths and weaknesses. One of the ranking methods is cross efficiency evaluation, which was introduced by Sexton, Silkman, and Hogan [28] and has been widely accepted as a tool to distinguish DEA decision-making units. In fact, it is generally used to identify efficient decision-making units from among other units [10]. Unique ranking of decision-making units [12], elimination of unrealistic weight adjustments without any predetermined weight limits [3], and effective distinction between strong and weak performances among decision-making units [8] are the main advantages of cross efficiency evaluation. Given these advantages, cross efficiency evaluation is used in various programs, such as project ranking, task performance measurement in a cellular production system [14], sports ranking [31], and supplier selection in government procurement [15]. Despite the benefits listed, cross efficiency is not always applicable due to non-compliance of optimal weights [16]. In particular, the probability of having several optimal weights in performance evaluation leads to different sets of multiple efficiency values for each DMU. This may reduce the ability to differentiate cross efficiency evaluation. To solve this problem, Sexton et al. [28] and Doyle and Green [11] proposed well-known pessimistic and optimistic formulations as secondary goals for selecting a single answer from several optimal weights. The main idea of pessimistic formulation is to obtain an answer by minimizing the cross efficiency of other decision-making units, while keeping the performance of the decision-making unit under evaluation at a predetermined optimal level. In contrast, optimistic formulation maximizes the cross efficiency of other decision-making units, while maintaining the performance of the unit under evaluation. Therefore, based on this idea, many secondary objective models have been proposed for cross efficiency evaluation. For example, Liang et al. [22] developed the optimistic and pessimistic models of Doyle and Green [11] by introducing different secondary objective models based on deviations from the target output of each decision unit. These models can be used in various practical situations. Wang and Chin [31, 32, 34] then proposed alternative secondary objective models for fixed-scale efficiency by switching efficiency from the ideal point, which was used by Liang et al. [22]. Similar ideas were presented in Lim [33], in which a type of minimum or maximum secondary objective is included in the optimistic and pessimistic formulation of cross efficiency. However, as noted in Wang and Chin [31, 32, 34], an optimistic and pessimistic formulation does not guarantee a lasting result.

In fact, there is agreement that decision-making interactive theories play an important role in the decision-making process [5, 7, 21, 29]. However, these cross efficiency evaluation models assume that decision makers operate within the framework of the theory of expected utility. However, the theory of expected utility has several unknown effects, including the Allais paradox [2] and Ellsberg paradox [13]. Given the limitations of the theory of expected utility, Kahneman and Tversky [18] proposed the prospect theory that could take into account the irrational theories of risky decision makers. The prospect theory has three main conclusions: 1) decision makers show a risk-averse tendency for profit and a risky tendency for loss; 2) decision makers usually perceive profit and loss according to a reference point, and 3) decision makers are more sensitive to loss than profit. Since the prospect theory is largely consistent with real human behavior, the decision-making method based on prospect theory has recently become the focus of research [6, 7, 17, 19, 20, 25, 27, 30, 33, 35].

Moreover, given the world in which we live, most things that seem true are "relatively" true, and there are degrees of "uncertainty" about the accuracy of real phenomena. In other words, the real phenomena are not only black or white, but also somewhat "gray". As a result, real phenomena are always fuzzy and inaccurate, so fuzzy logic has been invented and widely used.

The theory of fuzzy sets was first proposed in 1965 by Professor Lotfi Askarzadeh, an Iranian scientist and professor at Berkeley University in the United States. Since its introduction, this theory has been greatly expanded and deepened and has found various applications in various fields. One of the important steps in modeling of fuzzy concepts is performing defuzzification operations [26]. Its various applications in ranking include Malmquist and other applications of DEA, which is the focus of a large volume of research. In the real world, many inputs and outputs are not known, including periodic, sequential, probabilistic, qualitative and other data that are called inaccurate data. What is expected is that if even one of the data is not accurate, the answer will probably not be accurate. To solve this problem, this paper presents models that are able to evaluate the prospect cross efficiency with interval data.

In the second part, a definition of prospect cross efficiency is provided. In the third part, the definition and some theorems are given in relation to the interval model. The fourth section includes a practical example in relation to the proposed model and the proposed method for ranking decision-making units, and finally, its conclusions are discussed.

2. Prospect cross efficiency

In DEA model, the cross efficiency evaluation is done in two stages, which include self-evaluation and congener evaluation. This method calculates the overall performance of each decision-making unit by considering its own weights as well as the weights of all other decision-making units. Since the prospect theory is presented as a descriptive theory for the decision-making behavior of a person at risk, it is considered as one of the most effective behavioral decision-making theories and includes the following important principles [18].

1. Obedience to the reference: The decision maker usually receives the results in the form of gain or loss in relation to a reference point. Thus, the prospect value curve of a decision maker is divided by the reference point into two parts of the gain and loss range.
2. Loss-aversion: The decision maker is more sensitive to loss than to gain [1]. In this case, the loss prospect value curve is steeper than the gain range.
3. Sensitivity reduction: The decision maker shows a risk-averse desire for gain and a risk-taking desire for loss. Correspondingly, the prospect value curve is concave in the gain range and convex in the loss range.

The meaning of the above three principles can be described by an S-shaped symmetric value curve [24]. The value function of this curve (prospect value function) is described as follows:

$$v(\Delta z) = \begin{cases} (\Delta z)^{\alpha}, \ \Delta z \geq 0 \\ -\theta(-\Delta z)^{\beta}, \ \Delta z < 0 \end{cases}$$

Where, $\Delta z = z - z_0$ is used to measure the value of the deviation of z from the reference point z_0. If ($\Delta z \geq 0$), then we will gain, otherwise we will lose ($\Delta z < 0$). The parameters α and β represent the change in the direction of the value function in the gain and loss area, respectively, where $0 < \alpha < 1$ and $0 < \beta < 1$. The variable θ is the loss-aversion coefficient and $\theta > 1$ indicates that the value function has a greater slope in the loss area than in the gain area.

In fact, the prospect theory shows that decision makers reflect their sensitivity to the status of the results in relation to the status quo (reference point). This means that they reflect whether the results are better or worse than the current situation.

The reference point can be considered 1: zero point, 2: average value, 3: median value, 4: worst value, and 5: best value. Liu et al. [24] used the worst and best values to obtain a cross efficiency evaluation matrix based on prospect theory, which is further explained in the next section.

From the decision makers' point of view, the worst DMU usually consumes the most input but produces the least output, and the best DMU gets the most output using the least possible input.

According to the prospect theory, a DMU is considered a gain if a DMU is higher than the worst DMU, and a loss if a DMU is below the best DMU.

For further discussion, consider n decision-making units, each of which uses m inputs to generate s outputs. Whenever the set of decision-making units is indexed as $N = \{1, 2, ..., n\}$, the set of input values as $M = \{1, 2, ..., m\}$, and the set of output values as $S = \{1, 2, ..., s\}$, x_{ik} and $y_{rk}(r \in S, k \in N, i \in m)$ indicate the inputs and outputs of DMU_k, respectively. Based on the above analysis, the prospect values are defined as follows.

Definition 2.1. *If the decision-making reference point is the worst DMU, then the gain value of the prospect corresponding to the i^{th} input and r^{th} output of DMU_k is defined as follows:*

$$V^+_{I_{ik}} = (x^-_i - x_{ik})^\alpha,\ V^+_{O_{rk}} = (y_{rk} - y^-_r)^\alpha \tag{2.1}$$

Where, $x^-_i = \max_k\{x_{ik}\}$ and $y^-_r = \min_k\{y_{rk}\}$ are the worst value of the I^{th} input and the worst value of the r^{th} output of all DMUs, respectively.

Definition 2.2. *If the decision-making reference point is the best DMU, then the loss value of the prospect corresponding to the i^{th} input and r^{th} output of DMU_k is defined as follows:*

$$V^-_{I_{ik}} = -\theta(x_{ik} - x^+_i)^\beta,\ V^-_{O_{rk}} = -\theta(y^+_r - y_{rk})^\beta, \tag{2.2}$$

Where, $x^+_i = \min_k\{x_{ik}\}$ and $y^+_r = \max_k\{y_{rk}\}$ are the best value of the i^{th} input and the best value of the r^{th} output of all DMU_s, respectively.

From the decision maker's point of view, he/she always chooses a unique set of input and output weights to maximize DMU_k gain as much as possible, as shown below:

$$\max \sum_{r=1}^{s} u_{rk} V^+_{O_{rk}} + \sum_{i=1}^{m} v_{ik} V^+_{I_{ik}} \tag{2.3}$$

Therefore, the gain model for cross efficiency evaluation can be constructed as follows:

$$\max \sum_{r=1}^{s} u_{rk}(y_{rk} - y^-_r)^\alpha + \sum_{i=1}^{m} v_{ik}(x^-_i - x_{ik})^\alpha \tag{2.4}$$

$$s.t.\ \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leq 0,\ j \in N,$$

$$\sum v_{ik} x_{ik} = 1,$$

$$\sum_{r=1}^{s} u_{rk} y_{rk} = E^*_{kk},$$

$$u_{rk}, v_{ik} \geq 0,\ r \in s,\ i \in M.$$

Where, $E^*_{kk} = \sum_{r=1}^{s} u^*_{rk} y_{rk}$ is known as CCR efficiency of DMU_k, which represents the optimal relative efficiency of DMU_k through self-assessment. If $E^*_{kk} = 1$ and all optimal weights of u^*_{rk} and v^*_{ik} are positive, then DMU_k is efficient. Otherwise it is inefficient.

In general, the decision maker always selects a single set of input and output weights to minimize DMU_k losses as follows:

$$\min \sum_{r=1}^{s} u_{rk} V^{-}_{O_{rk}} + \sum_{i=1}^{m} v_{ik} V^{-}_{I_{ik}} \tag{2.5}$$

Subsequently, the loss model of cross efficiency evaluation can be defined as follows:

$$\min \sum_{r=1}^{s} u_{rk}\theta(y_r^{+} - y_{rk})^{\beta} + \sum_{i=1}^{m} v_{ik}\theta(x_{ik} - x_i^{+})^{\beta} \tag{2.6}$$

$$s.t. \ \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leq 0, \ j \in N,$$

$$\sum v_{ik} x_{ik} = 1,$$

$$\sum_{r=1}^{s} u_{rk} y_{rk} = E^{*}_{kk},$$

$$u_{rk}, v_{ik} \geq 0, \ r \in s, \ i \in M.$$

Gaining more profit or avoiding more loss, i.e. maximum utility, is achieved when the decision-maker's perspective is considered, so by combining the gain model (2.4) and the loss model (2.6), a new model for cross efficiency evaluation is constructed as follows:

$$\max \lambda(\sum_{r=1}^{s} u_{rk}(y_{rk}-y_r^{-})^{\alpha} + \sum_{i=1}^{m} v_{ik}(x_i^{-}-x_{ik})^{\alpha}) - (1-\lambda)(\sum_{r=1}^{s} u_{rk}\theta(y_r^{+}-y_{rk})^{\beta} + \sum_{i=1}^{m} v_{ik}\theta(x_{ik}-x_i^{+})^{\beta}) \tag{2.7}$$

$$s.t. \ \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leq 0, \ j \in N,$$

$$\sum v_{ik} x_{ik} = 1,$$

$$\sum_{r=1}^{s} u_{rk} y_{rk} = E^{*}_{kk},$$

$$u_{rk}, v_{ik} \geq 0, \ r \in s, \ i \in M.$$

Which is called the prospect cross efficiency (PCE) model, and λ represents the relative importance of profit to loss and applies to $0 \leq \lambda \leq 1$.

In this optimization model, different λ values can be used as an indicator of diverse decision-making attitudes. For example, if $0 \leq \lambda \leq 0.5$, then the decision maker will focus more on losses than gains. If $\lambda = 0.5$, then the profit and loss factors are equally important to the decision maker. If $0.5 \leq \lambda \leq 1$, then the decision maker will focus his attention on profitable tendencies. During the decision-making process, the decision-maker can choose the appropriate λ according to his desires.

Also, the parameter α shows the degree of concavity of the value function in the gain area, and the large value of α indicates the steep slope of the gain curve; in such a case, the decision-maker tends to take risks. Thus, with a value of α tending to zero, the decision-maker's risk attitude is significantly risk-averted in the risk assessment process. Conversely, the parameter β indicates the degree of convexity of the value function in the loss region, in which case the decision maker becomes sensitive to losses and significantly conservative. Therefore, with a value of β that tends to zero, the decision maker becomes risk-taker in the evaluation process.

3. PCE interval model

The models considered so far all assume that the data of each DMU is definite, but this assumption is not always true; in the real world many inputs and outputs are not definite.

As a result, here, based on models (2.4) and (2.6), which are the gain and loss models, respectively, as well as their combined model, which were discussed in the second section, we present interval models that are able to calculate PCE in the interval form.

Assume that the number of DMU_s in question is n, each consuming m distinct inputs to produce s distinct outputs. Here inputs and outputs are considered as intervals. It is necessary to assume on the basis that in practice there are many cases in which decision makers cannot determine the amount of inputs and outputs decisively and accurately. In other words, if x_{ij}^L and x_{ij}^U are the lower and upper bounds for the input i of DMU_j, respectively, there will be $\tilde{x}_{ij} \in [x_{ij}^L, x_{ij}^U]$. If y_{ij}^L and y_{ij}^U are the lower and upper bounds for the r^{th} output of DMU_j, respectively, we will have $\tilde{y}_{ij} \in [y_{ij}^L, y_{ij}^U]$. Note that we always have: $x_{ij}^L \leq x_{ij}^U$ and $y_{ij}^L \leq y_{ij}^U$ and if $x_{ij}^L = x_{ij}^U$, the result is that the i^{th} input of DMU_j has a certain value. To evaluate DMU_k with interval data, the CCR model can be written as follows:

$$E_{kk} = \max \sum_{r=1}^{s} u_r \tilde{y}_{rk} \tag{3.1}$$

$$s.t. \ \sum_{i=1}^{m} v_i \tilde{x}_{ik} = 1,$$

$$-\sum_{i=1}^{m} v_i \tilde{x}_{ij} + \sum_{r=1}^{s} u_r \tilde{y}_{rj} \leq 0, \quad j = 1, ..., n,$$

$$u_r \geq 0 \qquad r = 1, ..., s,$$

$$v_i \geq 0 \qquad i = 1, ..., m.$$

Since the input and output values of the above model are interval, the value of the objective function, which is the relative efficiency, is interval that is obtained as follows.

Suppose the values of inputs and outputs for each DMU are in a certain interval. To evaluate DMU_k with interval data, we will have:

$$E_{kk}^u = \max \sum_{r=1}^{s} u_r y_{rk}^u \tag{3.2}$$

$$s.t. \ \sum_{i=1}^{m} v_i x_{ik}^l = 1,$$

$$-\sum_{i=1}^{m} v_i x_{ij}^{u} + \sum_{r=1}^{s} u_r y_{rj}^{l} \leq 0, \quad j = 1, ..., n \text{ and } j \neq k,$$

$$-v_i x_{ik}^{l} + u_r y_{rk}^{u} \leq 0,$$

$$u_r, v_i \geq 0 \qquad \forall r, i.$$

In other words, to calculate the upper bound of efficiency, we consider DMU_K under evaluation in the best conditions and all DMU_s in the worst conditions. Also, to calculate the lower bound of efficiency, we consider DMU_K under evaluation in the best conditions and all DMU_s in the worst conditions.

$$E_{kk}^{l} = \max \sum_{r=1}^{s} u_r y_{rk}^{l} \tag{3.3}$$

$$s.t. \ \sum_{i=1}^{m} v_i x_{ik}^{u} = 1,$$

$$-\sum_{i=1}^{m} v_i x_{ij}^{l} + \sum_{r=1}^{s} u_r y_{rj}^{u} \leq 0, \quad j = 1, ..., n \text{ and } j \neq k,$$

$$-v_i x_{ik}^{u} + u_r y_{rk}^{l} \leq 0,$$

$$u_r, v_i \geq 0 \qquad \forall r, i.$$

In this case, there will always be: $E_{kk} \in [E_{kk}^{l}, E_{kk}^{u}]$

Now that we know the definitions of efficiency values in the case of interval data, we can calculate the PCE as follows.

When the input and output values of the decision units are available not accurately but as values in an interval, the definitions in section two will be as follows:

Definition 3.1. *If the decision-making reference point is the worst DMU, then the gain value of the prospect corresponding to the i^{th} input and r^{th} output of DMU_K is defined as follows:*

$$V_{I_{ik}}^{+} = (\tilde{x}_i^{-} - \tilde{x}_{ik})^{\alpha}, \ V_{O_{rk}}^{+} = (\tilde{y}_{rk} - \tilde{y}_r^{-})^{\alpha}, \tag{3.4}$$

Where, $\tilde{x}_i^{-} = \max_k\{\tilde{x}_{ik}\}$ and $\tilde{y}_r^{-} = \min_k\{\tilde{y}_{rk}\}$ are the worst interval of i^{th} input and the worst interval of r^{th} output of all DMU_s, respectively.

Definition 3.2. *If the decision-maker's reference point is the best DMU, then the loss value of prospect corresponding to the i^{th} input and r^{th} output of DMU_K is defined as follows:*

$$V_{I_{ik}}^{-} = -\theta(\tilde{x}_{ik} - \tilde{x}_i^{+})^{\beta}, \ V_{O_{rk}}^{-} = -\theta(\tilde{y}_r^{+} - \tilde{y}_{rk})^{\beta}, \tag{3.5}$$

Where, $\tilde{x}_i^{+} = \min_k\{\tilde{x}_{ik}\}$ and $\tilde{y}_r^{+} = \max_k\{\tilde{y}_{rk}\}$ are the best interval of i^{th} input and the best interval of r^{th} output of all DMU_s, respectively.

From the decision maker's point of view, he/she always selects a unique set of input and output weights to maximize the DMU_k profits as follows:

$$\max \sum_{r=1}^{s} u_{rk} V^{+}_{O_{rk}} + \sum_{i=1}^{m} v_{ik} V^{+}_{I_{ik}} \tag{3.6}$$

Therefore, the gain model in cross efficiency evaluation for interval data can be constructed as follows:

$$\begin{aligned} z = \max \sum_{r=1}^{s} u_{rk}(\tilde{y}_{rk} - \tilde{y}^{-}_{r})^{\alpha} &+ \sum_{i=1}^{m} v_{ik}(\tilde{x}^{-}_{i} - \tilde{x}_{ik})^{\alpha} \\ s.t. \ \sum_{r=1}^{s} u_{rk}\tilde{y}_{rj} - \sum_{i=1}^{m} v_{ik}\tilde{x}_{ij} &\le 0, \ j \in N, \\ \sum_{i=1}^{m} v_{ik}\tilde{x}_{ik} &= 1, \\ \sum_{r=1}^{s} u_{rk}\tilde{y}_{rk} &= E^{*}_{kk}, \\ u_{rk}, v_{ik} \ge 0 \ r \in s, \ i \in M. \end{aligned} \tag{3.7}$$

Because the inputs and outputs are intermittent, the value of the objective function of the model (3.7) is obtained intermittently. With the help of the following models, the lower bound and the upper bound can be found.

To find an upper bound of the objective function of the model (3.7), the objective function must have its maximum value.

$$\begin{aligned} z^{u} = \max \sum_{r=1}^{s} u_{rk}(y^{u}_{rk} - y^{l}_{r})^{\alpha} &+ \sum_{i=1}^{m} v_{ik}(x^{u}_{i} - x^{l}_{ik})^{\alpha} \\ s.t. \ \sum_{r=1}^{s} u_{rk}y^{l}_{rj} - \sum_{i=1}^{m} v_{ik}x^{u}_{ij} &\le 0, \ j \in N, \ j \neq k \\ \sum_{r=1}^{s} u_{rk}y^{u}_{rk} - \sum_{i=1}^{m} v_{ik}x^{l}_{ik} &\le 0, \\ \sum_{i=1}^{m} v_{ik}x^{l}_{ik} &= 1, \\ \sum_{r=1}^{s} u_{rk}y^{u}_{rk} &= E^{*}_{kk}, \\ u_{rk}, v_{ik} \ge 0 \ r \in s, \ i \in M. \end{aligned} \tag{3.8}$$

To find a lower bound of the objective function model (3.7), the objective function must have its lowest value, so we have:

$$z^l = \max \sum_{r=1}^{s} u_{rk}(y^l_{rk} - y^u_r)^\alpha + \sum_{i=1}^{m} v_{ik}(x^l_i - x^u_{ik})^\alpha \tag{3.9}$$

$$\begin{aligned} s.t.\ & \sum_{r=1}^{s} u_{rk} y^u_{rj} - \sum_{i=1}^{m} v_{ik} x^l_{ij} \le 0,\ j \in N,\ j \neq k \\ & \sum_{r=1}^{s} u_{rk} y^l_{rk} - \sum_{i=1}^{m} v_{ik} x^u_{ik} \le 0, \\ & \sum_{i=1}^{m} v_{ik} x^u_{ik} = 1, \\ & \sum_{r=1}^{s} u_{rk} y^l_{rk} = E^*_{kk}, \\ & u_{rk}, v_{ik} \ge 0\ r \in s,\ i \in M. \end{aligned}$$

Theorem 3.3. *Models* (3.7)*,* (3.8) *and* (3.9) *are feasible.*

Proof . We show here the existence of a solution for model (3.9), the existence of a solution for the other two models is proved similarly:

Since we have $E^*_{kk} = \sum_{r=1}^{s} u^*_{rk} y_{rk}$, thus:

$$u_{rk} = \frac{u^*_{rk} y_{rk}}{y^l_{rk}}, \quad v_{ik} = \frac{x^u_{ik}}{\|x^u_{ik}\|^2}, \quad u_{rk}, v_{ik} \ge 0$$

By substituting the above values in the constraints of model (3.9), we will have:

1. $\sum_{i=1}^{m} v_{ik} x^u_{ik} = \frac{x^u_{ik}}{\|x^u_{ik}\|^2} x^u_{ik} = 1$
2. $\sum_{r=1}^{s} u_{rk} y^l_{rk} = \sum_{r=1}^{s} \frac{u^*_{rk} y_{rk}}{y^l_{rk}} y^l_{rk} = \sum_{r=1}^{s} u^*_{rk} y_{rk} = E^*_{kk}$
3.

$$\begin{aligned} \sum_{r=1}^{s} u_{rk} y^l_{rk} - \sum_{i=1}^{m} v_{ik} x^u_{ik} \le 0 &\leftrightarrow \sum_{r=1}^{s} \frac{u^*_{rk} y_{rk}}{y^l_{rk}} y^l_{rk} - \frac{x^u_{ik}}{\|x^u_{ik}\|^2} x^u_{ik} \le 0 \\ &\leftrightarrow \sum_{r=1}^{s} u^*_{rk} y_{rk} - 1 \le 0 \\ &\leftrightarrow \sum_{r=1}^{s} u^*_{rk} y_{rk} \le 1 \end{aligned}$$

The optimal value of the objective function E^*_{kk} is also limited for DMU_k because:

$$u^t_{rk} y_{rk} - v^t_{ik} x_{ik} \le 0 \rightarrow u^t_{rk} y_{rk} \le v^t_{ik} x_{ik} \rightarrow u^t_{rk} y_{rk} \le 1$$

Where, (u^t, v^t) are weights of DMU_k. Also, according to the above proof, the possible answers were true in the constraints. Thus, it is observed that model (3.9) is possible. □

In general, the decision maker always selects a single set of input and output weights to minimize DMU_K losses as follows:

$$\min \sum_{r=1}^{s} u_{rk} V^-_{O_{rk}} + \sum_{i=1}^{m} v_{ik} V^-_{I_{ik}} \tag{3.10}$$

Subsequently, the loss model of cross efficiency evaluation for interval data can be specified as follows:

$$z = \min \sum_{r=1}^{s} u_{rk} \theta(\tilde{y}_r^+ - \tilde{y}_{rk})^\beta + \sum_{i=1}^{m} v_{ik} \theta(\tilde{x}_{ik} - \tilde{x}_i^+)^\beta \tag{3.11}$$

$$s.t. \ \sum_{r=1}^{s} u_{rk} \tilde{y}_{rj} - \sum_{i=1}^{m} v_{ik} \tilde{x}_{ij} \leq 0, \ j \in N,$$

$$\sum v_{ik} \tilde{x}_{ik} = 1,$$

$$\sum_{r=1}^{s} u_{rk} \tilde{y}_{rk} = E^*_{kk},$$

$$u_{rk}, v_{ik} \geq 0 \ r \in s, \ i \in M.$$

In model (3.11), since all inputs and outputs are interval, the value of the objective function of model (3.11) is also obtained as an interval. With the help of the following models, one lower bound and one upper bound can be found for the objective function of model (3.11).

To find the upper bound of the objective function of model (3.11), the objective function must have its maximum value.

$$z^u = \min \sum_{r=1}^{s} u_{rk} \theta(y_r^l - y_{rk}^u)^\beta + \sum_{i=1}^{m} v_{ik} \theta(x_{ik}^l - x_i^u)^\beta \tag{3.12}$$

$$s.t. \ \sum_{r=1}^{s} u_{rk} y_{rj}^l - \sum_{i=1}^{m} v_{ik} x_{ij}^u \leq 0, \ j \in N, \ j \neq k$$

$$\sum_{r=1}^{s} u_{rk} y_{rk}^u - \sum_{i=1}^{m} v_{ik} x_{ik}^l \leq 0,$$

$$\sum v_{ik} x_{ik}^l = 1,$$

$$\sum_{r=1}^{s} u_{rk} y_{rk}^u = E^*_{kk},$$

$$u_{rk}, v_{ik} \geq 0 \ r \in s, \ i \in M.$$

Also, to find the lower bound of the objective function of model (3.11), the objective function must have its least value, so we have:

$$z^l = \min \sum_{r=1}^{s} u_{rk} \theta(y_r^u - y_{rk}^l)^\beta + \sum_{i=1}^{m} v_{ik} \theta(x_{ik}^u - x_i^l)^\beta \tag{3.13}$$

$$s.t.\ \sum_{r=1}^{s} u_{rk} y_{rj}^{u} - \sum_{i=1}^{m} v_{ik} x_{ij}^{l} \leq 0,\ j \in N,\ j \neq k$$
$$\sum_{r=1}^{s} u_{rk} y_{rk}^{l} - \sum_{i=1}^{m} v_{ik} x_{ik}^{u} \leq 0,$$
$$\sum v_{ik} x_{ik}^{u} = 1,$$
$$\sum_{r=1}^{s} u_{rk} y_{rk}^{l} = E_{kk}^{*},$$
$$u_{rk}, v_{ik} \geq 0\ r \in s,\ i \in M.$$

Theorem 3.4. *Models* (3.11)*,* (3.12) *and* (3.13) *are feasible.*

Proof . A possible answer for model (3.13) is as follows:

Since we have $E_{kk}^{*} = \sum_{r=1}^{s} u_{rk}^{*} y_{rk}$, there will be:

$$u_{rk} = \frac{u_{rk}^{*} y_{rk}}{y_{rk}^{l}},\quad v_{ik} = \frac{x_{ik}^{u}}{\|x_{ik}^{u}\|^{2}},\quad u_{rk}, v_{ik} \geq 0$$

By substituting the above values in the constraints of model (3.13), we will have:

1. $\sum_{i=1}^{m} v_{ik} x_{ik}^{u} = \frac{x_{ik}^{u}}{\|x_{ik}^{u}\|^{2}} x_{ik}^{u} = 1$
2. $\sum_{r=1}^{s} u_{rk} y_{rk}^{l} = \sum_{r=1}^{s} \frac{u_{rk}^{*} y_{rk}}{y_{rk}^{l}} y_{rk}^{l} = \sum_{r=1}^{s} u_{rk}^{*} y_{rk} = E_{kk}^{*}$
3.
$$\begin{aligned}\sum_{r=1}^{s} u_{rk} y_{rk}^{l} - \sum_{i=1}^{m} v_{ik} x_{ik}^{u} \leq 0 &\leftrightarrow \sum_{r=1}^{s} \frac{u_{rk}^{*} y_{rk}}{y_{rk}^{l}} y_{rk}^{l} - \frac{x_{ik}^{u}}{\|x_{ik}^{u}\|^{2}} x_{ik}^{u} \leq 0 \\ &\leftrightarrow \sum_{r=1}^{s} u_{rk}^{*} y_{rk} - 1 \leq 0 \\ &\leftrightarrow \sum_{r=1}^{s} u_{rk}^{*} y_{rk} \leq 1\end{aligned}$$

The optimal value of the objective function E_{kk}^{*} is also limited for DMU_k because:

$$u_{rk}^{t} y_{rk} - v_{ik}^{t} x_{ik} \leq 0 \rightarrow u_{rk}^{t} y_{rk} \leq v_{ik}^{t} x_{ik} \rightarrow u_{rk}^{t} y_{rk} \leq 1$$

Where, (u^t, v^t) are weights of DMU_k. Also, according to the above proof, the possible answers were true in the constraints. Similarly, the existence of feasible solutions for models (3.11) and (3.12) can be shown. □

By combining the gain model (3.7) and the loss model (3.11), the following model is obtained:

$$z = \max \lambda(\sum_{r=1}^{s} u_{rk}(\tilde{y}_{rk} - \tilde{y}_{r}^{-})^{\alpha} + \sum_{i=1}^{m} v_{ik}(\tilde{x}_{i}^{-} - \tilde{x}_{ik})^{\alpha}) - (1-\lambda)(\sum_{r=1}^{s} u_{rk}\theta(\tilde{y}_{r}^{+} - \tilde{y}_{rk})^{\beta} + \sum_{i=1}^{m} v_{ik}\theta(\tilde{x}_{ik} - \tilde{x}_{i}^{+})^{\beta}) \tag{3.14}$$

$$s.t.\ \sum_{r=1}^{s} u_{rk}\tilde{y}_{rj} - \sum_{i=1}^{m} v_{ik}\tilde{x}_{ij} \leq 0,\ j \in N,$$

$$\sum v_{ik}\tilde{x}_{ik} = 1,$$

$$\sum_{r=1}^{s} u_{rk}\tilde{y}_{rk} = E^{*}_{kk},$$

$$u_{rk}, v_{ik} \geq 0\ r \in s,\ i \in M.$$

Given that model (3.14) is expressed for interval data, therefore, with the help of the following models, a lower bound and an upper bound can be found for the objective function of model (3.14).

$$z^{u} = \max \lambda(\sum_{r=1}^{s} u_{rk}(y^{u}_{rk} - y^{l}_{r})^{\alpha} + \sum_{i=1}^{m} v_{ik}(x^{u}_{i} - x^{l}_{ik})^{\alpha}) - (1-\lambda)(\sum_{r=1}^{s} u_{rk}\theta(y^{l}_{r} - y^{u}_{rk})^{\beta} + \sum_{i=1}^{m} v_{ik}\theta(x^{l}_{ik} - x^{u}_{i})^{\beta}) \tag{3.15}$$

$$s.t.\ \sum_{r=1}^{s} u_{rk}y^{l}_{rj} - \sum_{i=1}^{m} v_{ik}x^{u}_{ij} \leq 0,\ j \in N,\ j \neq k$$

$$\sum_{r=1}^{s} u_{rk}y^{u}_{rk} - \sum_{i=1}^{m} v_{ik}x^{l}_{ik} \leq 0,$$

$$\sum v_{ik}x^{l}_{ik} = 1,$$

$$\sum_{r=1}^{s} u_{rk}y^{u}_{rk} = E^{*}_{kk},$$

$$u_{rk}, v_{ik} \geq 0\ r \in s,\ i \in M.$$

Its lower bound is:

$$z^{l} = \max \lambda(\sum_{r=1}^{s} u_{rk}(y^{l}_{rk} - y^{u}_{r})^{\alpha} + \sum_{i=1}^{m} v_{ik}(x^{l}_{i} - x^{u}_{ik})^{\alpha}) - (1-\lambda)(\sum_{r=1}^{s} u_{rk}\theta(y^{u}_{r} - y^{l}_{rk})^{\beta} + \sum_{i=1}^{m} v_{ik}\theta(x^{u}_{ik} - x^{l}_{i})^{\beta}) \tag{3.16}$$

$$s.t.\ \sum_{r=1}^{s} u_{rk}y^{u}_{rj} - \sum_{i=1}^{m} v_{ik}x^{l}_{ij} \leq 0,\ j \in N,\ j \neq k$$

$$\sum_{r=1}^{s} u_{rk}y^{l}_{rk} - \sum_{i=1}^{m} v_{ik}x^{u}_{ik} \leq 0,$$

$$\sum v_{ik}x^{u}_{ik} = 1,$$

$$\sum_{r=1}^{s} u_{rk} y_{rk}^{l} = E_{kk}^{*},$$

$$u_{rk}, v_{ik} \geq 0 \; r \in s, \;\; i \in M.$$

Theorem 3.5. *Models* (3.14)*,* (3.15) *and* (3.16) *are feasible.*

Proof . A possible answer for model (3.16) is as follows since $E_{kk}^{*} = \sum_{r=1}^{s} u_{rk}^{*} y_{rk}$.

$$u_{rk} = \frac{u_{rk}^{*} y_{rk}}{y_{rk}^{l}}, \quad v_{ik} = \frac{x_{ik}^{u}}{\|x_{ik}^{u}\|^{2}}, \quad u_{rk}, v_{ik} \geq 0$$

To prove the claim, we just put the above answer in the model constraints as follows and show that the constraints are true for them:

1. $\sum_{i=1}^{m} v_{ik} x_{ik}^{u} = \frac{x_{ik}^{u}}{\|x_{ik}^{u}\|^{2}} x_{ik}^{u} = 1$
2. $\sum_{r=1}^{s} u_{rk} y_{rk}^{l} = \sum_{r=1}^{s} \frac{u_{rk}^{*} y_{rk}}{y_{rk}^{l}} y_{rk}^{l} = \sum_{r=1}^{s} u_{rk}^{*} y_{rk} = E_{kk}^{*}$
3.

$$\begin{aligned} \sum_{r=1}^{s} u_{rk} y_{rk}^{l} - \sum_{i=1}^{m} v_{ik} x_{ik}^{u} \leq 0 &\leftrightarrow \sum_{r=1}^{s} \frac{u_{rk}^{*} y_{rk}}{y_{rk}^{l}} y_{rk}^{l} - \frac{x_{ik}^{u}}{\|x_{ik}^{u}\|^{2}} x_{ik}^{u} \leq 0 \\ &\leftrightarrow \sum_{r=1}^{s} u_{rk}^{*} y_{rk} - 1 \leq 0 \\ &\leftrightarrow \sum_{r=1}^{s} u_{rk}^{*} y_{rk} \leq 1 \end{aligned}$$

The optimal value of the objective function E_{kk}^{*} is also limited for DMU_k because:

$$u_{rk}^{t} y_{rk} - v_{ik}^{t} x_{ik} \leq 0 \rightarrow u_{rk}^{t} y_{rk} \leq v_{ik}^{t} x_{ik} \rightarrow u_{rk}^{t} y_{rk} \leq 1$$

Where, (u^t, v^t) are weights of DMU_k. Also, according to the above proof, the possible answers were true in the constraints. Similarly, the existence of feasible solutions for models (3.14) and (3.15) can be shown. □

Theorem 3.6. *if* $(u^*, v^*), (\hat{u}, \hat{v})$*, and* $(\bar{u}, \bar{v})$ *re optimal answers for models* (3.14)*,* (3.15) *and* (3.16)*, we will have* $z^l \leq z \leq z^u$*.*

Proof . For proving that $z^l \leq z$, it should be demonstrated that the optimal answer of model (3.16) is a possible answer for model (3.14).

It is assumed that $(\bar{u}, \bar{v})$ is the optimal answer for model (23), thus:

$$\sum_{r=1}^{s} \bar{u}_{rk} \tilde{y}_{rj} \leq \sum_{r=1}^{s} \bar{u}_{rk} y_{rj}^{u} \leq \sum_{i=1}^{m} \bar{v}_{ik} x_{ij}^{l} \leq \sum_{i=1}^{m} \bar{v}_{ik} \tilde{x}_{ij}^{l}$$

Therefore, according to the above relation, $\sum_{r=1}^{s} \bar{u}_{rk} \tilde{y}_{rj} \leq \sum_{i=1}^{m} \bar{v}_{ik} \tilde{x}_{ij}^{l}$ is established; therefore $(\bar{u}, \bar{v})$ is valid in the first constraint of model (3.14) and the other constraints are also similarly established. Thus, the optimal value of the objective function of model (3.16) is smaller than model (3.14). □

According to what was said and considering that the efficiency of each DMU is in the same interval, all DMU_s can be divided into the following three categories:

Category 1: DMU_s that are efficient in the best and worst evaluation conditions, in other words,

$$E^{++} = \{DMU_j | e_j^L = e_j^U = 1\}$$

Category 2: DMU_s that are efficient in the best evaluation conditions, but inefficient in the worst evaluation conditions, in other words:

$$E^{+} = \{DMU_j | e_j^L < 1,\ e_j^U = 1\}$$

Category 3: DMU_s that are inefficient in the best evaluation conditions. Obviously, these DMU_s are inefficient even in the worst evaluation conditions, that is,

$$E^{-} = \{DMU_j | e_j^U < 1\}$$

DMU_s belonging to E^{++} are efficient, DMU_s belonging to E^{-} are inefficient, and nothing can be said about DMU_s belonging to D^{+}.

It is observed that the efficiency of DMU_s is obtained as an interval.

To rank DMU_s, we use the general idea of fuzzy number ranking presented by Rahmani et al. [26] as follows:

First, two intervals of $[a, b]$ and $[\acute{a}, \acute{b}]$ are considered, where $a, \acute{a}, b, \acute{b} \in R$ and then placed in $m = \frac{a+b}{2}$ and $\acute{m} = \frac{\acute{a}+\acute{b}}{2}$. After that, the following steps are taken:

Step 1: if $m > \acute{m}$, then $[a, b] \succ [\acute{a}, \acute{b}]$.
Step 2: if $m = \acute{m}$ and $b > \acute{b}$, then $[a, b] \succ [\acute{a}, \acute{b}]$.
Step 3: if $m = \acute{m}$, $b = \acute{b}$, and $a > \acute{a}$, then $[a, b] \succ [\acute{a}, \acute{b}]$.
Step 4: if $m = \acute{m}$, $b = \acute{b}$, and $a = \acute{a}$, then $[a, b] \approx [\acute{a}, \acute{b}]$.

Note that we use the symbols $\succ$ and $\approx$ to indicate the relationship "larger" and "equality" of the intervals, respectively.

4. Practical example

Consider a set of four DMU_s that use 2 inputs to generate 2 outputs and the interval data related to them as shown in Table 1.

Table 1: Data on interval inputs and outputs

DMU_j	x_{1j}^L	x_{1j}^U	x_{2j}^L	x_{2j}^U	y_{1j}^L	y_{1j}^U	y_{2j}^L	y_{2j}^U
1	1	3	10	20	4	15	25	45
2	30	45	15	25	2	20	10	50
3	5	18	23	40	1	10	30	100
4	11	50	1	15	22	44	10	70

Using what has been said in Section 3, without losing generality, it is assumed that the decision maker will argue that the importance of profit and loss factors is the same (i.e., $\lambda = 0.5$). The other parameters α, β and θ in models (3.12) and (3.13) are 0.89, 0.92 and 2.25, respectively [18]. The upper and lower bounds of the efficiency scores obtained from models (3.15) and (3.16) for each of the DMU_s are shown in Table 2.

According to the data in Table 2, the lower and upper bounds of PCE are first determined for the four DMU_s. The lower bound is measured using model (3.16) and the upper bound is measured using model (3.15). As can be seen in Table 2, using the proposed method, the upper and lower bound sizes of efficiency were obtained and in the last column, the ranking results were calculated according to the ranking method proposed in the previous section. Using this method and according to what has been said, DMU_4 has the highest rank because its m value is greater than $\acute{m}$ of other DMU_s, and so DMU_1, DMU_2, and DMU_3 are ranked 2, 4, and 3, respectively.

Table 2: Data on the upper and lower bounds of interval PCE

DMU_j	z_j^l	z_j^u	**rank**
1	0.5180	0.8667	2
2	0.02718	0.5818	4
3	0.1135	0.5125	3
4	0.9777	1	1

5. Conclusion

DEA is used to evaluate the performance of DMU_s. In the evaluation of units using DEA models, an efficiency score is assigned to each unit, which is a number between zero and one. An efficient unit has an efficiency score of one, and the problem starts with more than one DMU having an efficiency score of one. This led to the need to use models that can distinguish efficient DMU_s. This concept was introduced in DEA as ranking. We know that DMU_s are rated according to the performance of each DMU compared to others, and each DMU with a higher performance is ranked better. One of the ranking methods of units under evaluation in DEA is cross efficiency evaluation method. In cross efficiency evaluation, a cross efficiency table is obtained and after arithmetic averaging, each unit with a higher average gets a better rank. Conventional cross efficiency evaluation models assume that decision-makers are perfectly rational and usually avoid considering the risk-taking attitude of the decision-maker, which plays an important role in the evaluation process. Given that the prospect theory can take into account the irrational psychological aspects of risk-taking decision-makers, this paper first examined PCE model, the most important advantage of which over other models is that it calculates efficiency considering decision-makers' risk attitude, which plays an important role in the evaluation process. By presenting theorems, efficiency was calculated for interval data, and also a numerical example was provided, in which four DMU_s that used 2 inputs to generate 2 outputs were studied. In this case, all inputs and outputs were included as values in an interval. Finally, using the proposed method, a lower bound and an upper bound were obtained for the efficiency of each of them, achieving more accurate results. Since the efficiency of DMU_s was obtained intermittently, a fuzzy number ranking method was used to rank the DMU_s. Ultimately, based on the proposed model, the evaluation of the performance of DMU_s for the case that the input and output data are not accurately expressed, but are presented as interval values, was done by DEA. Considering that in the real world, most of the data are inaccurate and ambiguous, it is suggested to expand the proposed model and the method of calculating the performance of DMU_s for interval and fuzzy data.

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