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Dissipation coefficient from low-momentum and pole contribution warm inflation

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Abstract

The conventional models of warm inflation include the two-stage field interaction mechanism where the inflation field interacts with other intermediate fields (bosonic and fermionic fields), which are coupled with other fields themselves. These heavy intermediate fields decay to light degrees of freedom. During these two-stage renormalizable interactions under adiabatic conditions and close to the thermal equilibrium state, the dissipative effects are produced and cause alterations in the inflationary dynamics. Interaction between the inflation, intermediate and radiation fields lead to the formation of a thermal bath during inflation, and therefore the effect of thermal and radiative corrections should also be addressed. In this work, by focusing on the regime in which the intermediate field mass is greater than temperature, a relationship is obtained for the dissipation coefficient based on the super-symmetric and numerical calculation models, which is originated from both real and virtual modes of the intermediate field. Moreover, it is indicated that the contribution of on-shell modes decay to the bosonic and fermionic radiation fields is more than expected. It is also demonstrated that there are strong dissipative effects in the validity range of a perturbative analysis.

Keywords: warm inflation, dissipation coefficients, on-shell and off-shell modes.
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1 Introduction

In most physical problems, studying the dynamic evolution of a system coupled with the surrounding environment is interesting for us. The main focus is commonly on the average properties of the system that can be expressed in terms of simple dynamic variable and environment properties. However, in most of these cases, the fluctuating behavior of these average properties is also notable. The non-equilibrium quantum field theory benefits from an efficient and robust formulation to explore many systems associated with one or multiple fundamental fields evolving in a given environment. The non-equilibrium dynamics are generally very complicated, but a simple expression can be presented for particular systems. For example, when the environment is close to the thermal equilibrium and weakly perturbed by studied system, if the system evolution slowly occur compared to the typical environment response to local disturbance, the problem is simplified considerably. In these cases, the environment can always be considered

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in a local equilibrium state, described by a simple set of macroscopic variables, namely its temperature, which can calculated the back-reaction on the system dynamic. As presented, disturbances and interactions lead to a nonlocal term in the effective equation of motion governing the studied system, indicating the field energy conversion in the system and its transfer into the environment. Ultimately, the environment is perturbed under adiabatic condition and therefore, the perturbed environment quickly reaches a new equilibrium state. The dynamically studied system usually returns to equilibrium by experiencing the dissipation coefficient - the nonlocal term in quantum field theory (QFT). This coefficient is calculated in many theoretical works [15, 23, 12, 5]. Morikawa presented a Langevin-like effective equation of motion for system by applying the closed time path formulations [4, 13, 3]. Although the situation described above may be so constrained, it occurs in many physical systems, particularly cosmological ones. There is more evidence of cosmic microwave background, based on which the very early universe experienced a local thermal equilibrium a long time ago. A notable example of the slowly evolving system interacting with its surrounding environment is the inflation field, which is assumed to be vacuum energy deriving an accelerated expansion of the early universe in a slow-roll regime [31]. The inflation provided on astonishing successful pattern for resolving the standard model of cosmology problems. In the standard picture of inflation, the inflation field interaction with other fields has no crucial role during inflationary expansion, which cause the formation of a thermodynamic super-cooled phase during inflation that requires a separate reheating phase to enter the radiation-dominant stage. An alternative picture of cold (standard) inflation is to examine a dynamic realization of inflation in which the inflation field interacts with other fields during the inflationary regime. In this case, the dissipation effects became more critical and play a vital role in inflationary system dynamics [28, 14, 11, 27, 29, 1]. In the warm inflationary dynamic, the radiation is simultaneously produced with inflationary expansion due to the interaction of fields. To maintain the flatness of inflationary potential, which is necessary for inflation, these interactions must be weak. If the light radiation fields with masses of $m_\sigma < T$ are directly coupled with inflation, they produce large thermal correction on the inflationary potential and prevent inflation. To avoid these conditions, a conventional method is the inflation coupling with heavy intermediate fields, which are massed through the vacuum expectation value, vev, in their interaction with the inflationary filed. Field interactions are according to the common patterns of super-symmetric models with the associated super-potential. The common models of warm inflation include two-stage field interaction mechanisms, in which the inflationary scalar filed is coupled with heavy intermediate fields that are coupled with other bosonic and fermionic fields themselves. The intermediate field is a heavy field that decay to light filed, called radiation field. Although these two-stage interactions are seemingly more complicated than direct ones, they have outstanding feature. For example, in a regime where the mass of intermediate fields is greater than temperature (low-temperature regime), while the radiation fields are light, it can be indicated that the leading order contribution provide an acceptable estimate of the dissipation coefficient while not sufficient alone to calculate this coefficient in the high-temperature regime. The dissipation coefficient for the two-stage interaction mechanism in a low-temperature regime is first calculated by Moss and Xiong [1]. The heavy intermediate filed, massed by inflationary fields during the interaction, are substantially in the ground state, and their associated quantum correction can be canceled based on super-symmetric models. According to super-symmetric models, the critical benefit of using two-stage interaction mechanism is that even for higher order terms in the large perturbative coupling, quantum correction to the effective potential of these terms can be controlled effectively to maintain the potential flatness, a prerequisite for inflation. The super-symmetric dose not cancel the temporally non-quantum effects, which are the same dissipation terms with a vital role in inflationary dynamics. In the majority of models studied so far, the emphasis has been on the virtual modes of the intermediate field because it is imagined that in this temperature regime $m_\chi \gg T$, the on-shell production is suppressed because of Boltzmann factor $e^{-m_\chi/T}$. In this work, an expression is presented for the dissipation coefficient by repeating the pervious calculation for dissipation coefficient in a low-temperature regime of warm super-symmetric inflation and considering a case in which the intermediate fields, which decay to the final-state radiation of bosonic and fermionic fields, include both on-shell and off-shell production. The equation of motion is presented for the inflation field in the next section. Then, section 3 introduces the super-symmetric potential and interaction terms. In section 4, the constraints associated with the field coupling, and thermal and radiative corrections are examined. In section 5 the equation of dissipation coefficient is determined by low-momentum and pole approximation for low-temperature regimes. Finally, section 6 describes the effects of the approximate expression on warm inflationary models.

2 langeving- like equation for dynamical scalar field

In order to observe how the dissipation term and its fluctuation appear in the general framework of QFT, we employ the real-time formulation, particularly a specific form developed by Schwinger and Keldysh for the first time [2, 19]. Here the system is evolved in a closed time path, and since it is in a non-equilibrium state, the formulation is known by “in-in”. Consequently, the effective equation of motion governing the field ϕ for the system up to the first order is expressed as follows:

$$(\sigma^2 + m^2)\phi + v'(\phi) + \int d^4x' \sum_R (x, x')\, \phi(x) = \xi(x) \tag{2.1}$$

The above relationship is a Langevin-like equation for quantum field dynamics, where the auxiliary field $\xi(x)$ is interpreted as Gaussian stochastic noise. The important feature of the effective equation of motion in Keldysh formalism is that its causality is always explicit and is well considered by the retarded self-energy term, $\sum_R$, in Eq. (2.1). The second term on the left side of the above equation is an imaginary term depending on the self-energy and indicating the energy conversion from the inflation field to radiation and the same dissipation coefficient. If the time scales of the system evolution can be separated so that the context field varies slowly ($\dot{\phi}/\phi \ll \tau^{-1}, \dot{T}/T \ll \tau^{-1}$), the nonlocal terms can be localized [26, 25]. With this assumption that the self-energy is expressing the response time scale, it is concluded that:

$$\tau = \frac{2\omega_j(p)}{Im \sum_j(p)} \tag{2.2}$$

If the context field slowly varies, the adiabatic approximation of the effective equation of motion for this field include a linear dissipation term as follows:

$$\ddot{\varphi} + 3H\dot{\varphi} + \Upsilon_\phi\dot{\varphi} + \frac{\partial v_{eff}}{\partial \varphi} = 0, \tag{2.3}$$

where Υ is the dissipation coefficient expressed as:

$$\Upsilon = -\int d^4x' \sum_R (x, x')\,(t, t')\,, \tag{2.4}$$

where $\sum_R$ represent the retarded correlation that depends on the specific form of field interaction terms.

3 Interaction terms

The dissipation term resulting from the intermediate fields is determined for interactive structure introduced in following. The scalar field ϕ can be an example of the inflation field during cosmological inflation. The scalar and fermionic fields χ and ψ_χ gain mass interacting with the inflation field, while the bosonic and fermionic radiation fields σ and ψ_σ remain light fields. In the interaction terms, the coupling between the inflationary scalar field with a non-zero background and the light fermions can be prohibited because of $U(1)$ global symmetry conversely, the coupling between the inflationary scalar field and light scalar fields can be prevented in the super-symmetric model. The applied generic super-symmetric model is expressed by the following super-potential, which is a generic structure based on the renormalizable model and follows the simple assumption that all degrees of freedom are not directly coupled with the inflation field::

$$W = \frac{g}{2}\mathbf{\Phi}\mathbf{X}^2 + \frac{h_i}{2}\mathbf{X}\mathbf{Y}_i^2 \tag{3.1}$$

The Lagrangian density, expressing the interaction between inflation field and the scalar component of super-field in the super-symmetric model, is defined as follows:

$$l_{scalar} = \frac{1}{2}g^2\phi^2|\chi|^2 + \frac{g^2}{4}|\chi|^4 + \frac{h_i}{2}\frac{g\phi}{\sqrt{2}}\left(\chi\sigma_i^{\dagger^2} + \chi^\dagger\sigma_i^2\right) + \frac{h_ih_j}{4}\sigma_i^2\sigma_j^{\dagger^2} + h_i^2|\chi|^2|\sigma_i|^2. \tag{3.2}$$

The interactions of both fermionic fields ψ_χ, ψ_σ with the inflation field are determined as follows:

$$-l_{fermion} = \frac{g\phi}{\sqrt{2}}\bar{\psi}_\chi P_L\psi_\chi + h_i\chi\bar{\psi}_{\sigma_i}P_R\psi_{\sigma_i} + \frac{h_i}{2}\sigma_i\bar{\psi}_{\sigma_i}P_L\psi_\chi + h.c, \tag{3.3}$$

where h_i and g are the coupling coefficients and for simplicity: $h_i = h$. Also $P_{R,L} = \frac{(1\pm\gamma_5)}{2}$ represent the chiral projection operators. Two-stage interaction of the inflation field with χ and σ_i fields leads to the thermal bath formation during the inflationary regime, and as a result, the finite temperature effects become significant. Since the intermediate field mass is $\frac{g\phi}{\sqrt{2}} = m_{\chi,\psi_{\chi'}}$, this mass will be greater than the radiation bath temperature during the inflationary phase if the inflation field vev is large enough. Consequently, the contribution of above fields in thermal loop correction will be Boltzmann-suppressed, and these thermal correction can be neglected. However, the masses

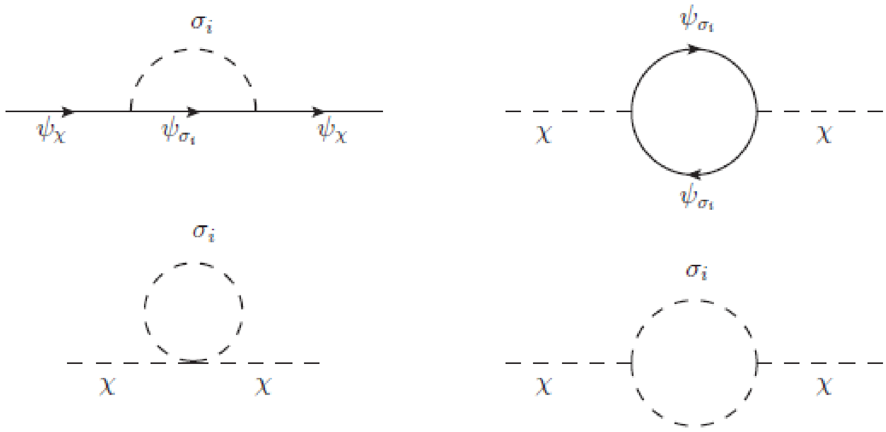

Figure 1: Feynman diagrams that contributes to the self-energies of fields χ, ψ_χ.

of fields χ and ψ_χ receive radiative corrections due to interactions with light fields; this is demonstrated in Feynman diagrams: (Fig. 1).

Among the interaction terms associated with the super-symmetric models, this study only focuses on values of interaction in which the leading contribution from vacuum polarization are mostly of the order of one-loop processes. Interactions vertices with terms of two-loop order are neglected in the low-temperature regime; for example, two-loop order terms result from field self-interaction or bi-quadratic interaction, which are lower than the one-loop order processes. From Feynman diagrams for the interaction of inflation field with intermediate fields (scalar and fermionic), the following equation will be obtained for the dissipation coefficient [1, 2, 20, 22]:

$$\Upsilon = \frac{2}{T}\left(\frac{g^2}{2}\right)^2 \int \frac{d^4p}{(2\pi)^4}[\rho^2_{\chi\to 2\sigma} + \rho^2_{\chi\to 2\psi_\sigma}]n_B(p_0)(1+n_B(p_0))\frac{g^2}{2T}\int \frac{d^4p}{(2\pi)^4} tr[\rho^2_{\psi_\chi}]n_f(1-n_f), \tag{3.4}$$

where ρ_χ and ρ_{ψ_χ} are the spectral function and fundamental quantities required for calculating the dissipation coefficient. Also,, n_B and n_f are Bose-Einstein and Fermi-Dirac distribution function, respectively. The behavior of the dissipation coefficient during various interaction temperature and regimes is determined by a trade-off between spectral function and thermal occupation numbers.

4 Thermal and radiative correction

In the low-temperature regime, the light scalar fields σ_i must be maintained. Although thermal correction are applied on the masses of these fields because of self-coupling, the self-coupling coefficient must be: $\lambda_\sigma = h^2 \ll m^2_\chi/T^2$. The corrected mass of the radiation field is expressed as:

$$m^2_{R\sigma} \approx m^2_\sigma + \lambda_\sigma \frac{T^2}{3}. \tag{4.1}$$

Therefore, it is required the coupling coefficient to be: $h \ll 1/2$. Thermal correction and the non-zero potential energy deriving the inflation result in the super-symmetry breaking and mass splitting of the super-field **X**:

$$\Delta m^2_\chi = \left| m^2_{R,\chi} - m^2_{R,\psi_\chi} \right| = \frac{g^2+h^2}{3}T^2 \tag{4.2}$$

The term Δm^2_χ can lead to a mass correction on the light scalar field σ of one-loop order as follows:

$$\delta m^2_\sigma \sim \frac{h^2\phi^2}{(4\pi)^2}\left(\frac{\Delta m^2_\chi}{m^2_\chi}\right) \tag{4.3}$$

The low-temperature regime is defined as; $m_{R,\sigma}, m_{R,\psi_\sigma} \ll T \ll m_{R,\chi}, m_{R\psi_\chi}$; this means the regime is low-temperature relative to the heavy intermediate field while being high-temperature relative to radiation fields. Therefore, two inequalities $\delta m_\sigma^2 \ll T^2$ and $m_\sigma \ll T$ must be considered in calculating the dissipation coefficient. The temperature correction, resulting from the self-energies of the light field on the effective mass of the heavy field, can be neglected. In similar fashion for light fermionic fields ψ_σ, whose loop radiative corrections, including thermal ones, are suppressed by masses of the heavy field despite being in a high-temperature regime. The thermal correction on the masses of bosonic and fermionic fields χ and ψ_χ are defined by considering the normalization of the coupling in super-potential as follows:

$$m_{R\chi}^2 = m_\chi^2 + \left(4\lambda_\chi + 3h^2\right) T^2 N_Y/24 \approx m_\chi^2 \tag{4.4}$$

$$m_{R\psi_\chi}^2 = m_{\psi_\chi}^2 + h^2 T^2 N_Y/24 \approx m_{\psi_\chi}^2 \tag{4.5}$$

For $T \ll m_\chi$, these thermal correction are of order T^2 and significantly smaller than radiative correction, thus being neglected. However, in the lower-left diagram of Fig. 1, the light scalar loop σ_i dose not cause any thermal correction of order T^2, but the contribution of this diagram will appear as divergent in the mass term. This contribution can be reabsorbed in the mass terms using the renormalization procedure, but it is discussed in more detail due to its significant effects on calculation [1, 20, 9, 16, 6].

4.1 Radiative correction to the two-point function

The two-point function associated with the scalar field χ will have radiative corrections concerning the interaction between the scalar and light fields and, consequently, the spectral function related to this field is rewritten in the following form by separating the imaginary and real parts of the field self-energy:

$$\rho_\chi = \frac{2Im\sum_\chi(p_0,\mathbf{p})}{\left[-p_0^2 + m_\chi^2 + Re\sum_\chi(p)\right]^2 + \left[Im\sum_\chi(p)\right]^2} \tag{4.6}$$

Contrary to the thermal mass correction, the self-energy of field χ is also influenced by the σ loop. In the adiabatic approximation based on which all calculation are performed, the imaginary part of this curve represents the decay width Γ_χ, that is inversely proportional to the response time scale: $\Gamma_\chi \to 1/\tau$. However, the real part of self-energy determined in [30, 10, 21] is determined as follows:

$$Re\left[\sum_\chi\right] = -\frac{h^2 N_Y m_\chi^2}{32\pi^2}\left[\frac{1}{\epsilon} - 2\frac{\pi}{2} + Ln\left|\frac{4\pi\kappa^2}{p_0^2 - p^2}\right|\right] - I_T(p_0,p), \tag{4.7}$$

where $I_T(p_0,p)$ is:

$$I_T(p_0,p) = -\frac{h^2 N_Y m_\chi^2}{32\pi^2}\left[\frac{1}{p}\int_0^\infty dKn(K)Ln\left(\frac{K_p - p_0^2K^2 + (p_0^2-p^2)/2}{K_p - p_0^2K^2 - (p_0^2-p^2)/2}\right)^2\right]. \tag{4.8}$$

The first term in Eq. (4.7) is independent of T, has a momentum- independent divergence, and only requires mass renormalization. However, the dependency of the integral term in $I_T(p_0,p)$ to the momentum is obvious. By calculating this integral in a low-temperature regime based on $p_0 \leq p$, $p \leq T$, it is concluded that:

$$Re\left[\sum_\chi\right] \approx \frac{-m_\chi^2}{16}\frac{T}{p}h^2 N_Y. \tag{4.9}$$

The spectral function in Eq. (4.6) result from one-loop diagrams, thus, Eq. (4.9) indicates a regime in which the perturbation theory is valid. The numerical calculation of the dissipation coefficient reveals that the most predominant contribution to the dissipation coefficient is attributed to modes of momentum as $p \leq T$. Therefore, even though Eq. (4.9) exhibits a strong dependence on the momentum, it is predicted that the perturbation theory will be fail for $h^2 N_Y \leq 1$. This strong dependency of the self-energy on the momentum precludes accurately predicting this term‘s contribution to the dissipation coefficient. This result can be an important tip for determining the validity range of perturbation theory in our calculation. Finally, although the thermal mass correction for large values of N_Y are not significant for determining the dissipation coefficient, this correction mode applies an upper limit to $h\sqrt{N_Y}$, which must be included in the calculating the dissipation coefficient.

5 Dissipation coefficient

In Eq.(3.4) defined for the dissipation coefficient, the contribution from the decay of both modes χ, ψ_χ is presented, but calculation have indicated that the contribution of fermionic modes during the low-temperature regime is suppressed due to symmetry breaking. This temperature regime is the range considered in this study, and therefore, the contribution from the decay of the scalar field χ to radiation fields σ, ψ_σ will only be considered for calculating the dissipation coefficient. It should be noted that both bosonic and fermionic forms of field χ have positive contribution to the dissipation coefficient, and for this reason, the time local radiative correction are canceled by the super-symmetric model [32, 18]. In order to calculate dissipation coefficient, a spectral function is required. The decay width $\Gamma_\chi(p_0, \mathbf{p})$ in terms of the self-energy's imaginary part as follows:

$$\Gamma_\chi(p_0, \mathbf{p}) = \frac{Im \sum_\chi(\mathbf{p})}{2\omega_j(\mathbf{p})}. \tag{5.1}$$

Is substituted into Eq. (4.6). Finally, the decay width resulting from $\chi \to \sigma\sigma$ of one-loop order is obtained as follow:

$$\begin{aligned}\Gamma_{\chi\to\sigma\sigma} = &\frac{h^2 N_Y}{64\pi}\frac{m_\chi^2}{\omega_p}\left[\frac{\omega_+ - \omega_-}{\mathbf{p}} + \frac{T}{\mathbf{p}} Ln\left(\frac{1-e^{\frac{-\omega_+}{T}}}{1-e^{\frac{-\omega_-}{T}}}\frac{1-e^{\frac{-p_0-\omega_-}{T}}}{1-e^{\frac{-p_0-\omega_+}{T}}}\right)\right]\theta(p_0^2 - \mathbf{p}^2 - 4m_\sigma^2)\\ &+\frac{h^2 N_Y}{64\pi}\frac{m_\chi^2}{\omega_p}\left[\frac{T}{\mathbf{p}} Ln\left(\frac{1-e^{\frac{-\omega_+}{T}}}{1-e^{\frac{-\omega_-}{T}}}\frac{1-e^{\frac{-p_0-\omega_-}{T}}}{1-e^{\frac{-p_0-\omega_+}{T}}}\right)\theta(p_0^2 - \mathbf{p}^2)\right].\end{aligned} \tag{5.2}$$

The first term of the above equation is obtained for $p_0 = \omega(\mathbf{p}-k)+\omega(Kk)$ in $p_0^2-\mathbf{p}^2 > (4m_\sigma^2)$, and decay of off-shell particles of field χ to on-shell particles of radiation filed. Also, the second term is defined for $p_0 = \omega(p-k) - \omega(k)$ and Landau damping. In similar fashion, for decay of $\chi \to \psi_\sigma\psi_\sigma$:

$$\begin{aligned}\Gamma_{\chi\to\psi_\sigma\psi_\sigma} = &\frac{h^2 N_Y}{32\pi}\frac{-p_0^2+\mathbf{p}^2}{\omega_p}\left[1 - \frac{4m_\chi^2}{p_0^2-\mathbf{p}}\right]\left\{\frac{\omega_+ - \omega_-}{\mathbf{p}} + \frac{T}{\mathbf{p}} Ln\left(\frac{1+e^{\frac{-\omega_+}{T}}}{1+e^{\frac{-\omega_-}{T}}}\frac{1+e^{\frac{-p_0-\omega_-}{T}}}{1+e^{\frac{-p_0-\omega_+}{T}}}\right)\right\}\\ &\times\theta(p_0^2 - \mathbf{p}^2 - 4m_\sigma^2) + \frac{h^2 N_Y}{32\pi}\frac{-p_0^2+\mathbf{p}^2}{\omega_p}\left[1 - \frac{4m_\chi^2}{p_0^2-\mathbf{p}}\right]\frac{T}{\mathbf{p}} Ln\left(\frac{1+e^{\frac{-\omega_+}{T}}}{1+e^{\frac{-\omega_-}{T}}}\frac{1+e^{\frac{-p_0-\omega_-}{T}}}{1+e^{\frac{-p_0-\omega_+}{T}}}\right)\\ &\times\theta(p_0^2 - \mathbf{p}^2 - 4m_\sigma^2),\end{aligned} \tag{5.3}$$

where

$$\omega_\pm = \sqrt{k_\pm^2 + m_\chi^2} \text{ and } k_\pm^2 = \frac{1}{4}\left\{|P| \pm p_0\left(\frac{4m_\chi^2}{-p_0^2+p^2}\right)^{1/2}\right\}^2.$$

5.1 low-energy and low three-momentum approximation

In most calculation steps for determining the dissipation coefficient in a low-temperature regime, the expression has been obtained using low-momentum and energy approximation with maximum contribution from virtual modes, $p_0, |p| \ll m_{R\chi} m_{R\psi_\chi}$. In this regime, by neglecting the thermal correction on the mass, the spectral function will become $\rho_\chi \approx \frac{4\Gamma_\chi}{m_\chi^3} using (p_0^2 - \omega_p^2)^2 \approx m_\chi^4$ approximation. By substituting Eq. (5.2), (5.3) into the spectral function, and then into (4.3), and calculating the integral of the dissipation coefficient , we have:

$$\Upsilon = \frac{h^4 N_Y^2 g^2 T^3}{4m_\chi^2 \pi^5}\left(\frac{A}{4} + B\frac{T^4}{m_\chi^4} + C\frac{T^2}{m_\chi^2}\right) \tag{5.4}$$

Constant in the above equation, $A \approx 1.65 \times 10^{-3}, B \approx 1.69 \times 10^{-3}, C \approx 3.87 \times 10^{-3}$ arise from evaluation of the momentum and energy integral in Eq. (4.3). The obtained dissipation coefficient in $m_\chi \ll T$ is according to [1, 10]. In Eq. (5.4) it is observed that the dissipation coefficient changes by increasing the effective coefficient $h\sqrt{N_Y}$ .Also, Γ_χ increases by increasing the effective coupling value. In the limit of perturbative analysis $h\sqrt{N_Y} \leq 1$, by applying a proper correction [32, 18], the dissipation coefficient is defined as:

$$\Upsilon = \frac{T^3}{\phi^2}\frac{1}{8\pi^5}\left(\frac{Ah^4 N_Y^2}{4 + 4\beta h^2 N_Y} + \frac{Bh^4 N_Y^2}{1 + \beta h^2 N_Y}\frac{T^4}{m_\chi^4} + \frac{Ch^4 N_Y^2}{1 + \beta h^2 N_Y}\frac{T^4}{2m_\chi^4}\right) \tag{5.5}$$

In low- momentum approximation, because of $m_\sigma/T \ll 1$, the dissipation is rewritten as:

$$\Upsilon = \frac{T^3}{\phi^2}\left(C_\phi + C'_\phi\frac{T^4}{m_\chi^4} + C''_\phi\frac{T^4}{m_\chi^4}\right) \tag{5.6}$$

where $C_\phi \approx 0.02h^2N_Y, C'_\phi \approx 0.018h^2N_Y$, and $C''_\phi \approx 3.08h^2N_Y$. In $T \ll m_\chi$ where the inflation field can decay to particles of radiation field through virtual modes, by applying Eq. (5.5) to the dissipation coefficient with condition $m_\sigma \ll T$, we have: $C_\phi \propto h^2N_Y$; while based on Eq. (5.4), Υ is proportional to $h^4N_Y^2$. The third term in Eq. (5.6) relates to the fermion loop contribution that becomes sub-leading in the low-temperature regime and can be neglected. In the limit of perturbative analysis, $h^2N_Y \leq 1$ for $N_\chi = 1$, that means at least one intermediate field interacting with the inflation field, dissipation constant is $C_\phi \leq 1$.

5.2 pole approximation

In the low-temperature regime, the contribution from real χ modes to the dissipation coefficient is neglected due to the Boltzmann suppression factor ($e^{-m_\chi/T}$), and these modes have entered the high-temperature regime [21, 8, 32, 18]. However, numerical calculation [24] show that in the low-temperature regime, if the effective coupling coefficient is set to be small enough, the contribution of these modes to the dissipation coefficient will be dominant in the limit of perturbative analysis. Therefore, the spectral function is determined based on pole approximation $p_0 \approx \omega_p$, associated with on-shell modes where $\omega_p = \sqrt{m_{R,\chi}^2 + \mathbf{p}^2}$. This function is then substituted into Eq. (4.3). In doing so, the equation of dissipation coefficient will be inversely proportional to the decay width.

$$\Upsilon \approx \frac{1}{T}\left(\frac{g^2}{\mathbf{2}}\right)^2 \phi^2 \sum_{i=1}^{2}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{1}}{\Gamma_{\chi_i}} n_B(n_B+1) \tag{5.7}$$

For on-shell χ modes, the contributions relating to the decay of intermediate field to final states of scalar and fermionic fields are same $\Gamma_{\chi\to\sigma\sigma} = \Gamma_{\chi\to\psi_\chi\psi_\chi}$. In equations (5.2),(5.3) for $m_\chi \geq T$ and $m_\sigma \ll T$ with the approximation of $p_0 \approx \omega_p$, the decay width is expressed as:

$$\Gamma_{\chi\to\sigma\sigma} = \frac{h^2N_Y}{32\pi}\frac{m_\chi^2}{\omega_p}\left(\frac{1}{2} + e^- m_\chi/2T\right) + O(\frac{p}{T}) \tag{5.8}$$

Above equation shows that thermal corrections in the mentioned limits become subdominant. However, the contribution of thermal corrections is revealed for momentum higher than temperature. By selecting small values of the effective coupling coefficient, $h\sqrt{N_Y}$, the decay width Γ_χ will also take small values. In, $m_\chi \gg T$, the second term of Eq. (5.8) can be neglected with a good approximation in calculating the momentum integral:

$$\Upsilon \approx \frac{g^2}{h^2N_Y}\frac{64}{T}\int_0^\infty \frac{p^2dp}{2\pi}\frac{1}{\omega_p} n_B(n_B+1) \tag{5.9}$$

By substituting the approximation $n_B(n_B+1) \approx e^{-m_\chi/T}e^{-p^2/m_\chi T}$ into the above equation, the integral of momentum term in Eq. (5.9) can be expressed as:

$$\int_0^\infty \frac{p^2dp}{p_0}e^{-p^2/m_\chi T} \approx \sqrt{\frac{\pi m_\chi}{2T}} \tag{5.10}$$

Thus, in the pole approximation, the dissipation coefficient is defined by the following equation:

$$\Upsilon \approx \frac{16}{\sqrt{2\pi}}\frac{g^2}{h^2N_Y}\sqrt{\frac{m_\chi}{T^3}}e^{-m_{\chi/T}} \tag{5.11}$$

In the low-temperature regime, the complete form of the dissipation equation by adding the contribution of on-shell modes to that of virtual modes will be as follows:

$$\Upsilon \approx \frac{16}{\sqrt{2\pi}}\frac{g^2}{h^2N_Y}\sqrt{\frac{m_\chi}{T^3}}e^{-m_{\chi/T}} + \frac{2T^3g^2}{m_\chi^2}\left(0.02 + 0.01\frac{T^4}{m_\chi^4}\right)h^2N_Y \tag{5.12}$$

An interesting result obtained from this equation is that it reveals the contribution of on-shell modes of field χ that decays to the modes σ and ψ_σ is more predominant that the contribution of virtual modes, despite the Boltzmann

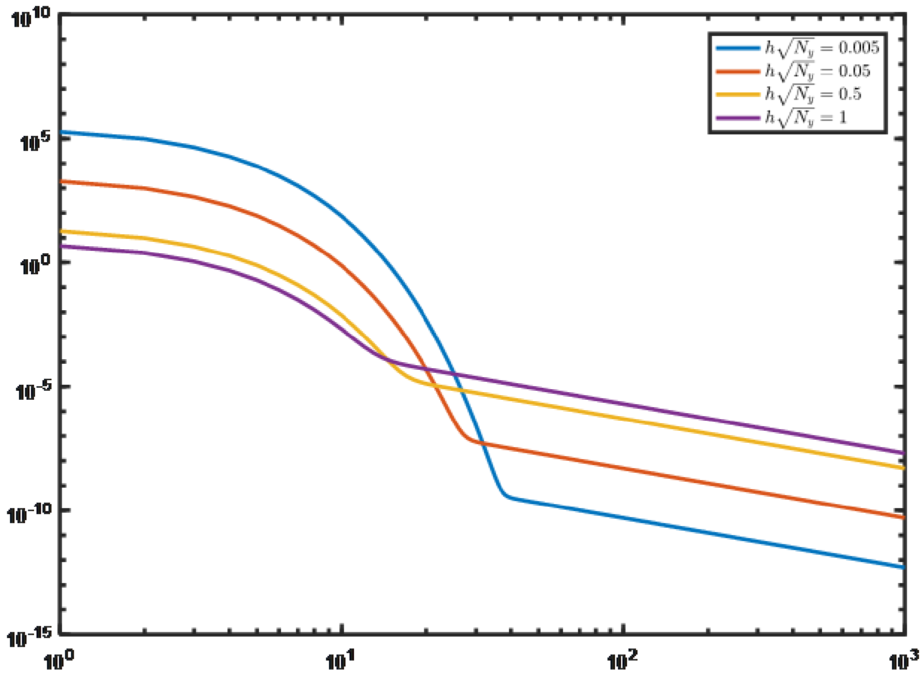

Figure 2: Dissipation coefficient as a function of m_χ/T for different values of $h\sqrt{N_Y}$

suppression factor. When $h\sqrt{N_Y}$ has small values, Γ_χ is also small. Fig. 2 demonstrates the complete relationship of the dissipation coefficient Eq. (5.12) where there is a strong dissipation coefficient for $m_\chi \geq T$ and $0.005 \leq h\sqrt{N_Y} \leq 1$.

In different analyses of the warm inflation dynamics with low-momentum approximation performed so far, a complete regime of inflation with sufficient length requires $C_\phi \geq 10^7$; this means a very large number of intermediate fields for virtual modes, while the contribution of on-shell modes to the dissipation coefficient can be of this order for a single filed and there is no need to very small effective coefficient. However, it should be noted that in the studied regimes, C_ϕ dosen't remain constant for a long time and it is change. but it can be a good motivation for detailed investigation of warm inflation dynamics in low-temperature regime.

6 Conclusion

This paper extended the pervious analysis performed to determine the dissipation coefficient in the warm cosmic inflation based on super-symmetric models. Our primary focus was on a temperature regime where the intermediate field mass was higher than the thermal bath temperature. In the low-temperature regime, the intermediate fields have greater masses than the temperature scale, while the radiation fields are in the high-temperature regime, and the estimations of the dissipation coefficient are sufficient only up to the one-loop order of coefficient. According to previous analyses, the dissipation coefficient results from the two-stage scalar interactions in this temperature regime. Our results indicate that the dissipation factor is significantly dependent on the on or off-shell feature of the intermediate field which the analytical expression of the dissipation coefficient is determined for both of them. Consequently, for the case with large values of mass, m_χ, and coupling coefficients, the dissipation resulting from virtual modes of the field χ is predominant in the low-momentum approximation. In $m_\sigma \ll T$, the dissipation coefficient term is valid only for small values of Yukawa coupling coefficient, h, even if the number of produced radiation fields N_Y is high. The main point in this calculation process is that the perturbative analysis limit $h^2 N_Y \leq 1$ is considered; otherwise, the radiative correction on χ two-point function cannot be perturbatively resumed. The result of using this limit $h^2 N_Y \leq 1$, implies that a large number of light fields dosen't necessarily result in the dissipation coefficient increment. Since C_ϕ in Eq. (5.6) is independent of the coupling coefficient betwwen intermediate and inflation fields, models of inflation may be obtained in which the increased number of χ fields cause an increase in the dissipation coefficient for $g^2 N_\chi < 1$. On the other hand, for smaller masses and coupling coefficient $h^2 N_Y \ll 1$, the contribution of on-shell χ modes are predominant to the dissipation coefficient. In $m_\sigma \ll T$, the dissipation coefficient is inversely proportional to the coupling coefficient $\Upsilon \propto 1/h^2 N_Y$. Indeed, these result indicated that the contribution of on-shell modes, which is obtained in the ploe contribution, can be more significant than that obtained by low-momentum approximation. These results make warm inflation models possible where a few fields directly interact with the inflationary field.

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