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Children's learning in math: The effects of an educational program

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Abstract

Data Envelopment Analysis (DEA) is a methodology to assess the performance of Decision Making Units (DMUs). In recent years, DEA has been used to measure the efficiency and the effectiveness of various organizations. In the educational systems, one of the most important issues is choosing the best teaching method among the various educational patterns to teach the difficult concepts to the students. For example, the fraction concept is one of the most difficult concepts in Mathematics. Learning this concept requires to a series of mathematical procedures consist of the cognitive and the skill-based procedures. Therefore, the important question that arises is which method is the best to teach the fraction concept? This study focuses on the effectiveness of the educational designs, Dick and Carey, Merrill and Control, on the students' performance of the elementary schools to understand the fraction concept. For this purpose, we use some educational designs to teach the fraction concept to the sixth-garde students of an elementary school in Tehran in 2018-2019 and suggest a DEA model to evaluate the effectiveness of the teaching design patterns on the students. Also, we present a statistical analysis for a more accurate comparison of the educational designs, Dick and Carey, Merrill and Control, on the students' performance.

Keywords: Data Envelopment Analysis, Target setting, Educational designs, Dick and Carey method, Merrill method
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1 Introduction

Data Envelopment Analysis (DEA) is a mathematical programming technique to evaluate the relative efficiency of Decision-Making Units (DMUs) with multiple inputs and multiple outputs [32, 1, 2, 3, 4]. Each unit gains an efficiency score between 0 and 1 by applying DEA models. The larger the efficiency score, the better performance the unit under evaluation has. A DMU is efficient if its efficiency score is equal to 1, otherwise, it is inefficient. DEA has witnessed widespread use in many application areas, such as electricity distribution, water utilities, health care, education, manufacturing, retailing, banking, environmental management and etc. Emrouznejad and Yang reported DEA studies from 1978 to the end of 2016 [5]. For more studies about DEA models, see [5, 6, 7].

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The main advantage of the DEA method is its ability to accommodate a multiplicity of inputs and outputs. It is also useful because it takes into consideration returns to scale in calculating efficiency, allowing for the concept of increasing or decreasing efficiency based on size and output levels. Some of the advantages of DEA are:

- no need to explicitly specify a mathematical form for the production function
- proven to be useful in uncovering relationships that remain hidden for other methodologies
- capable of handling multiple inputs and outputs
- capable of being used with any input-output measurement
- the sources of inefficiency can be analyzed and quantified for every evaluated unit

In traditional DEA models, the major goal was assessing the efficiency of units. Target setting is one of the main problems in organizations, which has become one of the attractive topics in DEA. [9, 10, 11] have introduced models for assessing targets based on DEA. For more studies about target setting in DEA, see [12, 13, 14, 15, 16, 17].

The teaching pattern is a description of the student's learning environment and the teacher's teaching behavior in using that pattern. A pattern, in fact, is the map of teaching to pay the attention to the interaction of students with a designed environment for optimal learning. Among the many educational patterns, no pattern can be considered bad or good per se, but the main goal is giving the priority of patterns in contrast with the others. The teaching patterns are the educational determined approaches which are included three characteristics.

1. Purposefulness: The teaching patterns are designed to grow and to develop critical thinking and a deep understanding of the contents.
2. Levelling: The teaching patterns include a series of steps or subsequent stages to reach the goal.
3. Foundations: The teaching patterns are based on the research and learning theories, teaching and motivation.

The presentation method has two types: descriptive and interrogative. The primary presentation includes stating the generalization, expressing the examples, the questions from the generalization and the questions from the examples. Moreover, the secondary presentation involves 1) mnemonics (describing and expanding the memory). A mnemonic is whatever helps a learner to remember something that she/he has learned before. 2) Providing the field (the teacher should match the teaching materials as much as possible with the real context which is used. One of the effective ways in this field is a discussion about the importance of educational topics. Presenting the educational materials in a story or a case study can improve the students understanding and their ability to match them with the previous learnings. 3) Attracting attention. 4) Providing a multi-presentation method. 5) Providing feedback. 6) Providing help.

The educational method is presented to improve two the following concepts:

Retention: It is the ability to remember information from long term memory. It means that how much students' learning is significant so that they can correctly answer the questions from the taught materials after three weeks.

Function: It includes the ability of students for solving mathematical problems which are the learning results, not the learning itself. In fact, it focuses on the students' potential, observable and measurable behavior. The educational designs refer to the systematic and feedback processes of learning and teaching that are programmed for the educational materials, the activities and the evaluation.

The fraction concept is one of the most important subjects in Mathematics at the elementary level. Learning the fraction involves a wide range of cognitive and skill-based procedures of Mathematics [18]. Researchers pointed out that understanding the fraction concept can have predictive features in learning the next mathematical topics such as algebra [18].

EminoEV Horstt used the component display theory for teaching 95 Japanese students. The acquired results showed an outstanding outcome in students' educational level and had a positive effect on the students' self-confidence. Moreover, it had a positive influence on the students' viewpoint about their own learning. Merrill investigated the component display theory pattern [19]. He pointed out that the teachers had to develop the modern educational patterns by employing their own creativity that develops growth opportunities and better educational quality. He believed that the cognitive structure involves learning and the learning improvement [20]. Further, Merrill stated that education should not merely be the information transferring using technology, but it should be interesting, efficient and effective [21]. Merrill's educational principle involves the most important elements of qualified education. National Mathematical Advisory platoon (nmap2008) stated that understanding the fraction concept and solving the fractional problems are the main goals of children's learning. Bailey and Kariz detected the important points for designing a blended learning environment and deduced that a framework for suitable designing of learning environments could be provided by using Merrill's principles education [18, 22].

Siegler did research in England and the United States and showed that the performance of the fifth and the sixth-grade students are connected to their performance in algebra and Mathematics in higher levels and the results emphasized the importance of learning the fraction concept in the fifth and the sixth grades [23]. Reigeluth considered the different perspectives of the educational theories in today's world and believed that the students learn at different speeds and they have various educational needs [24]. The teachers usually determine a certain amount of information and topics beforehand and educate them on a certain time horizon and unfortunately, the students who are weak and slow in learning the previous topics are fed with new ones and lose the opportunity to correct their mistakes. This makes it more difficult for the students to learn in the future. Torbyns did research in the United States, China and Belgium. Given that the knowledge of teacher in these three countries are not the same and the educational tools are not equal, it can be concluded that there is a relationship between understanding the fraction concept and the improvement in Mathematics [25].

Arya-oka did research with the aim of producing the educational materials in an effective interactional environment that can improve the learning results [27]. He used the Component Display Theory (CDT) for developing educational design and compared the learning results before and after using interactional educational material by employing the T-test. He showed that the learning results before and after using interactional educational material were different. Moreover, Ngugen did research on teaching the fraction concept to the students in Vietnam and showed that the students in four and five levels in the elementary schools learned the fraction concept but it was difficult to understand its application for them [28].

The Dick and Carey educational design model is a nine-step process for planning and designing effective learning initiatives. It includes all five stages of the ADDIE model but adds further depth and structure as well. It also has more focus on design and less focus on implementation than the ADDIE model, which builds in iterative development through ongoing revision of instruction. For more studies about the educational methods, see [28, 29, 30, 31].

Given the importance of the fraction concept, this study focuses on the effectiveness of the educational designs, Dick and Carey, Merrill and Control, on the students' performance in the fraction concept in sixth-grade elementary Mathematics. This study suggests a DEA model for comparing the learning results before and after using each of mentioned educational patterns. This method evaluates the efficiency of students and also provides a suitable target for inefficient students so that they can improve their performance by following the suggested targets. Also, we present a statistical analysis by applying a T-test for a more accurate comparison of the educational designs, Dick and Carey, Merrill and Control, on the students' performance.

The rest of this paper is organized as follows: Section 2 reviews some preliminaries and basic definitions in DEA and educational design patterns. In section 3, we explain how to use Merrill and Dick and Carey methods to teach the fraction concept to the sixth-grade elementary students and also, we suggest a DEA model evaluate the students' performance. A case study is provided in section 4. Section 5 concludes the paper.

2 Preliminaries and basic definitions

This section reviews some preliminaries in DEA and the teaching patterns, Dick and Carey and Merrill.

2.1 Background of DEA

Consider a system of n DMUs, denoted by $DMU_j, j = 1, \ldots, n$, where each unit consumes m different inputs to generate s different outputs. The i^{th} input and r^{th} output for DMU_j are denoted by x_{ij} and y_{rj}, respectively, for $i = 1, \ldots, m$ and $r = 1, \ldots, s$. Also, suppose that all the input and output values are nonnegative and at least one of them is non zero. The unit $DMU_o = (x_o, y_o)$ is the unit under evaluation. It should be noted that DEA models use the observed data and construct a set named Production Possibility Set (PPS). The frontier of PPS is the efficient frontier that envelopes all feasible production plans. Charnes et al. (1978) introduced the following PPS:

$$T_c = \left\{ (x, y) \mid x \geq \sum_{j=1}^{n} \lambda_j x_j, y \leq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0, j = 1, \ldots, n \right\} \tag{2.1}$$

Tone (2001) considered T_c and introduced the SBM model (2.2) to evaluate the efficiency score of units as follows:

$$\rho_o^* = \min \frac{1 - \frac{1}{m}\sum_{r=1}^{s} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}}$$

$$\begin{aligned} & s.t. \\ & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, && i = 1, \dots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, && r = 1, \dots, s, \\ & \lambda_j, s_r^+, s_i^- \geq 0, && j = 1, \dots, n, \;\; r = 1, \dots, s, \;\; i = 1, \dots, m, \end{aligned} \tag{2.2}$$

where s_i^{-*} and s_r^{+*} are the excesses of the i^{th} input and the shortage of the r^{th} output for all $i = 1, \dots, m$ and $r = 1, \dots, s$, respectively. The objective function of model (2.2) maximizes the reduction in the inputs and the increment in the outputs of DMU_o to measure the efficiency of this unit and determines an efficient unit located on the efficient frontier of T_c as the target unit.

Definition 2.1. A $DMU_o = (x_o, y_o)$ is called SBM-efficient if $\rho_o^* = 1$. This means that $s_i^{-*} = 0$ and $s_r^{+*} = 0$ for all $i = 1, \dots, m$ and $r = 1, \dots, s$, i.e. all input and output slacks are zero.

Suppose that $(\lambda^*, s^{-*}, s^{+*})$ is an optimal solution for model (2.2). The target unit for DMU_o was introduced as follows:

$$\begin{aligned} \hat{x}_{io} &= x_{io} - s_i^{-*}, \quad i = 1, \dots, m, \\ \hat{y}_{ro} &= y_{ro} + s_r^{+*}, \quad r = 1, \dots, s. \end{aligned} \tag{2.3}$$

It is clear that, if DMU_o is efficient, then $\hat{x}_{io} = x_{io}$ and $\hat{y}_{ro} = y_{ro}$, for all $i = 1, \dots, m$ and $r = 1, \dots, s$.

2.2 Background of the teaching patterns

This section summarizes the educational designs, Dick and Carey and Merrill.

2.2.1 Dick and Carey

The Dick and Carey instructional design model is a nine-step process for planning and designing effective learning initiatives. It adds more depth and structure than previous methods in the literature of educational designs. It also has more focus on the design and less focus on the implementation and builds in iterative development through ongoing revision of instruction. The nine steps of this design are as follows:

1. Identify instructional goals: The instructional goals should be defined with the organization's strategic goals and they should be clear on what learners will be able to do, or how they will behave after the teacher's initiative has been delivered and focus on the real-world skills and the behaviors.

2. Conduct instructional analysis: The teacher's instructional analysis determines the current state of skills and knowledge in her/his learning population and achieves this gap to his/her goals. This can be assessed through interviews, surveys, observation or different forms of testing depending on the nature of the skills.

3. Identify entry behaviors: In addition to analyzing the teacher's learning population's current level of knowledge, she/he also needs to understand their behaviors, traits, levels of motivation and other factors that will affect their learning journey. This information will help the teacher designs¬ appropriate learning methods.

4 . Write performance objectives: Learning objectives should be smart and should lay out tasks and processes that must be mastered and how they will be assessed. These may be known as "SWBAT" in education (Student Will Be Able To).

5. Develop criterion tests: To monitor both the progress and the effectiveness of an instruction, the teacher needs to develop criterion specific tests.

6. Develop instruction strategy: The teacher should define her/his instructional strategy. It should reflect her/his analysis and use appropriate learning theories.

7. Develop and select instructional materials: Materials, tools, exercises and delivery media should be decided when the teacher has defined her/his learning strategy. These may include face to face, group-based, facilitated or online learning materials.

8. Develop and conduct formative evaluation: Formative evaluation involves assessing how effectively the teacher has formulated her/his learning initiative. This can be obtained through review, focus groups, testing of segments and piloting her/his learning program.

9. Develop and conduct summative evaluation: Summative evaluation takes place when the teacher has delivered her/his initiative and is used to assess how effective it has been. Are teacher's participants satisfied with the program? Has the business noticed any benefits due to it?

2.2.2 Merrill

David Merrill introduced the Merrill teaching design. Merrill categorized the performance into three groups and believed that the students' practice had various requirements for showing the learning and the psychological activities which were essential for learning [21]. The three functions of Merrill's theory are remembering, application, detection and innovation.

Remembering: The function that the student searches him/herself memory so that the learned information is restored and is presented in the same way or in a new system.

Application: The function that the students employ the learned materials in a new situation.

Detection and innovation: The function is to access the material and the new collection. The content elements are the facts, the concepts, the method of teaching and the principles or the rules.

Merrill's teaching design pattern has been designed for cognitive subjects and determines the classification of the content elements of the educational materials and includes two main parts. The first part is a classification of the educational goals and the next is the presentation types that can provide optimal education by cooperation with each other. At the first, Merrill presents a classification of the educational goals which is called the application-content matrix. This Matrix displays four subjects (the facts, the concepts, the method of teaching and the principles or the rules) and three types of uses (remembering, application, detection and innovation). This educational design determines the types of the primary and the second presentation after clarifying the content type and the expected function. Then, it combines these two presentations and observes four pivotal principles (isolation, divergent, difficulty and matching). After that, it designs the whole educational method. It has been mainly planned for group learning. Based on this pattern, each education has two main elements as follows:

1. Generalization: It defines the concepts and describes the job processes and explains an element or a rule.

2. Sample or example: It is the generalization of a totality. Sample or example is one of the representations of this totality.

Merrill established five instructional design principles to achieve effective and efficient instruction as follows:

1. Topic-centeredness: Cognitive psychological researches show that if the students are involved with problem-solving, they learn far better. The principle of topic-centeredness itself involves three minor principles:

    a. Demonstration of the homework: When the problems which the students should solve are shown to them, the learning can happen easier.

    b. The level of homework: Problem-solving, doing the activities which are necessary for solving the problem, doing the activities which are in this task and the activities which makes this specific act.

    c. The sequence of the problem: It means that the problem is arranged from the easy ones to the most difficult.

2. Activation: The second principle that we use is the activation principle. In fact, with the pre-test, the teacher understands the students' knowledge about the fraction and starts her/his teaching from that point. In other words, materials are presented in a way that the students have the prerequisites for learning. Moreover, the teaching material would not be as same as the previous one. This means that the students are not on their own but the teacher provides a suitable cognitive structure and help them to relate the learned skills together and makes an appropriate schema by activating the previous knowledge and using the advance organizer, sharing the previous encounter, and supporting the cognitive structure.

3. Demonstration: The teachers try to demonstrate the method of solving problems to the students and employ three minor principles which are demonstration correspondence, users' guide and suitable medium.

4. Application: The students are asked to employ what they have learned in order to make their learning deeper.

5. Integration: Motivational factors help teachers to encourage students to learn. The most effective motivation is the enjoyment of learning. When the students use their learning intangible environment and employ them to solve problems, they would gain better motivation for learning. The researchers try to pose the problems that students are more faced with during their daily life. In order to do that, four main principles are used. They are reflection, criticizing the classmates, personal use, and public demonstration. The students are forced to reflect on the materials taught, consider their peers' solutions and are given tasks to understand the necessity of the use of the fraction concept. The teachers also give students the opportunity to show their learned skills to their peers.

2.2.3 Control

The Control group is the standard in which the comparisons are made in an experiment. Many experiments include a control group and one or more experimental groups. Therefore, the term "experiment" is used for the study designs that include a control group. In many ways, the control group and the experimental group are the same. The only difference between the control group and the experimental group is that the experimental group is exposed to the treatments or interventions thought to have an effect on the interested outcome while the control group is not. The researchers' ability to conclude a study can be strengthened by considering a control group. In other words, a researcher can determine the effectiveness of treatment under investigation on an experimental group in the presence of a control group. There are two different ways to manage a control group study:

1) Single-blind study: In this way, the researcher will know whether a particular subject is in the control group, but the subject will not know.

2) Double-blind study: In this way, neither the subject nor the researcher will know which treatment the subject is receiving.

In many situations, the researcher cannot affect the results by creating a control subject differently from an experimental subject and so, the double-blind study is preferable compared to the single-blind study.

3 The suggested methods

This section describes how to use the educational methods, of Merrill and Dick and Carey, to teach the fraction concept to sixth-grade elementary students. Then, we suggest a DEA method for assessing the performance of students.

3.1 The educational Methods

In this section, we explain the educational designs, Dick and Carey and Merrill for teaching the fraction concept.

3.1.1 Merrill

This section describes how to use the Merrill method to teach the fraction concept to sixth-grade elementary students.

Figure 1 shows the basic requirements for educating the fraction concept at the application level.

The sample of designing education using Merrill's pattern based on Component Display Theory includes seven steps as follows:

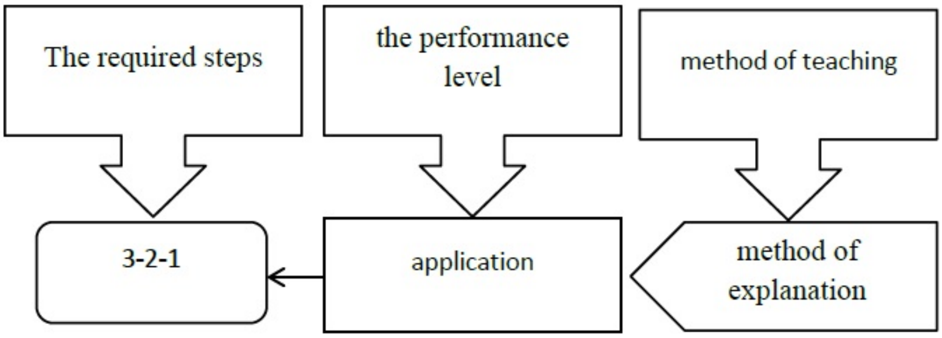


Figure 1: Presenting the basic requirements for educating the fraction concept in the application level.

Step 1. Interpreting the learners: Learners of the aforementioned education are students studying in the sixth level in elementary school. They learn how to divide shapes and objects into various equal parts in this lesson and then divide the surrounding objects approximately into equal parts and using them, they understand the fraction concept and it leads them to the mixed numbers. The important point in this lesson is that the students get familiar with the concept of fractions and learn that fractions and mixed numbers are two representations of one concept and that this idea can be represented in different ways.

Step 2. determining the topic of teaching: The teaching fraction in Merrill's categorization is placed in the "concept category". Concepts have elements including the name of the concept, the definition of the concept, the general group, examples and main characteristics. The word "fraction" is the name of this concept. It is defined as "fraction is made by dividing two integers." Various examples can be considered for it. Moreover, fraction structures can be described by examples.

Step 3. determining the function: In this step, the purpose is to teach students how to use this concept at the application level. It means that when the students learned fractions, they should be able to perform the following tasks.

- Understanding the fraction concept as a part from the whole.
- Dividing one shape or one object into equal parts.
- Showing the approximate amount of a fraction on a shape and vice versa.
- Understanding the concept of the mix numbers and their relation with the fraction.
- Converting a mixed number into a fraction and vice versa.
- Showing various representations of the fraction and the mixed number (number-graphic and verbal axis)
- Employing the fraction concept and mix number in solving surrounding problems.
- Stating and writing the taught mathematical concepts.

Step 4. determining the kinds of the primary presentation form: Step 1 (presenting the considered concepts), 2 (presenting examples for the concept), 3 (categorizing the presented examples appropriately and identifying examples of the concept under the study by learners) are described in Figure 1.

Step 5. determining the kinds of secondary presentation form: Considering the point that three kinds have been introduced in the primary presentation form; the secondary presentation kinds are as follows:

- Presenting generality (the secondary presentation form includes mnemonic aids, vocabulary definition, provide context, getting attention and multi-faceted presentation)
- Presenting examples (the secondary presentation form includes mnemonic aids, vocabulary definition, provide context, getting attention and multi-faceted presentation)
- Questions from the examples (the secondary presentation form includes mnemonic aids, vocabulary definition, providing context, getting attention and multi-faceted presentation).

Step 6. determining an educational formula: Considering the kinds of the required primary and the secondary presentation forms for teaching the fraction concept which was clarified previously, the teaching formula is designed.

Step 7. employing the rules of designing: As it was mentioned, in order to teach the fraction concept at the level of semantic reminder, four kinds of the primary presentation form can be used. Presenting examples, presenting generality, questioning the examples and questioning the generality. Regarding Table 2, in order to present generality, isolation rule is used and to present examples, isolation rule, divergent rule, difficulty rule and matching rule are employed.

Moreover, to question the examples, isolation rule, divergent rule, difficulty rule is utilized and to question the generality the isolation rule is used.

Now, we explain how to teach the fraction concept to elementary students by using Merrill's educational method.

Introducing the topic: This lesson should be started with the real things. A part of a cookie piece is taken and the teacher says what fraction of that cookie has been taken. But, as it is not clear how much, nothing can be written for that fraction. Now, the teacher should take a piece of bread or paper and divide it into equal parts. What fraction of the whole is each part of that piece of bread or paper? The teacher should take an apple and divide it into eight equal parts; what fraction of the whole is each part? In these examples, because the objects are divided into almost equal parts, the teacher can write a fraction for them.

After introducing the topic, the following activities should be considered so that students understand the topic as well as possible:

Activity 1 (dividing approximately)

At the first, students should choose a paper band or a piece of thread to make a favorite circle. Then, they should fold the paper or thread into the intended number and put it on the circumference of the circle and mark them to divide the circle into equal parts. Next, the students should explain the way that they divided it into equal parts. Explaining increases verbal communication among the students and enhances their interpretation skills. Each student should perform manipulative, graphic and verbal activities so as to reach the intended level.

Activity 2 (dividing into equal sizes)

The teacher should classify the students of the class, each group is given a number of equal rectangular papers (about 10). Then, members of the group are asked to think first about how to divide these papers among themselves individually. Next, they are asked to cooperatively present and explain their method for dividing it.

The students should try to divide each rectangle differently from their pier learners. After finishing the determined time, groups should stick the divided rectangles on the board. After that, students cooperatively omit the shapes which have similar divisions in a way that no attached shapes are similar on the board. Now, groups should discuss and explain their method of dividing.

Designer's note: Considering the steps of designing by Merrill's method, activity 1 includes the presentation sample, activity 2 includes the secondary presentation for getting attention cooperatively and the divergent rule is also observed.

Mix Number: The teacher starts by explaining and reminding the fraction concept. It is clear that the fraction of 5/7 means five parts out of seven equal ones. Now, the teacher should ask students: what is the meaning of 9/7? The teacher should pose the question in the class and discuss it. The ideas of the students should be taken into account carefully and reacted accordingly. Considering the point that this concept has been taught in the previous years, the teacher should start the lesson with some questions from the students. Fraction and mix number are two different representations of one number.

To write a number in the mix number representation, students should understand and determine the number of complete units. Students should come to the idea that the fraction and the mixed number are two representations of one concept. Therefore, moving from the fraction to the mixed number and vice versa is of paramount importance.

After that, the following activities should be considered so that students understand the topic as well as possible:

Activity 1 (better understanding between the measurement and the mixed number) students should be asked to measure the length of a pen or any other objects which are appropriate to teach this concept using suitable measurement tools (meter or ruler) and the acquired number should be shown with the unit of centimetre and millimetre (or meter and centimetre) in a fraction form, the number of complete units and a fraction of the complete unit and mix number. Moreover, by using a figure and 10-member groups (a complete unit) and one-member group, they should show the number related to the figure in a mixed number representation. Through this activity, students learn that they can also choose a complete unit in a two-dimensional continuous shape. It should be taken into account that the method of choosing a complete unit and the process of their correct interpretation refers to themselves. Figure 2 shows this activity by an example.

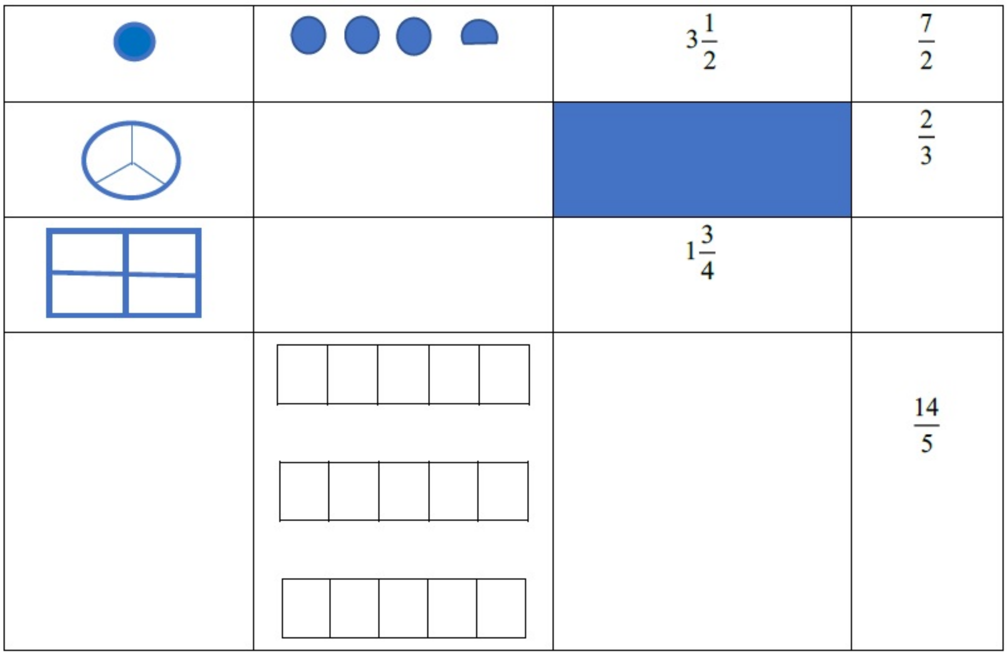


Figure 2: Students are asked to show the fraction and the mix number using various shapes. (questioning from the example). Students are asked to determine a group of the fraction numbers.(observing matching rule)

Activity 2 (a better understanding for choosing unit) students may have the problems with choosing the unit, so they can follow the proceeding steps:

1) A unit should be chosen that has been made from some continuous shapes.
2) Some of these units are drawn on cardboard and are cut with the help of students.
3) The students are provided with the units. (each group receives equal units)
4) The groups are asked to put some of the cardboards near each other and read the acquired fraction.
5) According to the unit, a fraction should be selected and then the groups are required to show them with their own units.
6) Now, considering the unit, the teacher should choose a fraction that is a complete unit and a part of a unit and groups are asked to show them by their own units.
7) The students are allowed to show the intended mix number with colouring and cutting a part of the unit and putting them beside each other.

The teacher should classify the students of the class into some groups and hold a competition among these groups. In the beginning, the first group should state a fraction and the second group should show it on the number axis, then the second group should state a fraction and the first group should show it on the number axis.

Determining the homework: At the end of the teaching, the activities in the textbook are chosen as the homework.

3.1.2 Dick and Carey educational model

This section describes how to use Dick and Carey method to teach the fraction concept to sixth-grade elementary students. The sample of the educational design for the concept of multiplication and division of the fractions using Dick and Carey's pattern is as follows:

1. Identifying the educational goals

   - Obtaining the ability to multiply and to divide the fractions and mixed numbers.
   - Obtaining the ability to multiply and to divide the fractions and mixed numbers by calculating the area, extensibility, shape and axis of numbers.

2. Educational analysis

   In this lesson, a new method (the area concept) are introduced to the students to calculate the multiplication of the fractions and the mixed numbers. Expressing the multiplication of the fractions and the mixed numbers using the area method increases the power of reasoning and visual understanding of a numerical phrase. Also, the different methods for dividing the fractions, which students have learned in the lower grades, are reminded again. The students also learn to display the division of the fractions graphically and calculate the common denominator (to perform the division operations) and they understand why we divide the numerator of the first fraction by the numerator of the second fraction, in the presence of the common denominators.

3. Learner features

   The students can use verbal skills for multiplying the fractions and then they can display them graphically. For example, for calculating one-third of one-fourth in the multiplication of the mixed numbers, students use the writing skill for showing the expansion of the mixed numbers and then then they present the expansions graphically, and then they are asked to do the multiply operations. This process develops the students' visual understanding of the multiplication concept and the multiplication of the mixed numbers. The students easily understand division 8 by 2, but they may have difficulty understanding division 8 by $\frac{1}{2}$. In these cases, the expression of the sentences such as "How many $\frac{1}{2}$ can display 8?" is effective in better understanding the division concept.

4. Functional goals

   - Multiplying the fractions (The area method)
   - Dividing a fractional number by an integer number using the drawing strategy.

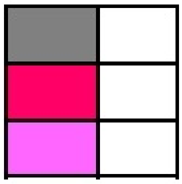
5. Criterion questions

   1. Calculate the product of $\frac{1}{3} \times \frac{1}{4}$?
   2. Obtain the quotients of $\frac{1}{2} \div 3$?

6. Educational strategies

   For example, we aim to obtain the product of $\frac{1}{3} \times \frac{1}{4}$ by using the area method.

   - The product of $\frac{1}{3} \times \frac{1}{4}$ can be expressed as the area of a quadrilateral whose length and width are $\frac{1}{3}$ and $\frac{1}{4}$, respectively.
   - At the first, draw a unit square. (For example, draw a square whose length is 12 cm.)
   - Divide one of the sides of the unit square (for example, its horizontal side) into three equal parts and show the value of $\frac{1}{3}$ on it.
   - Divide one of the adjacent sides of the selected side in the previous step (for example, its horizontal side) into four equal parts and show the value of $\frac{1}{4}$ on it.
   - Specify the area of the part of the shape whose length and width are $\frac{1}{3}$ and $\frac{1}{4'}$ respectively. (For example, by coloring)
   - The area of the colored part is equal to $\left(\frac{1}{3} \times \frac{1}{4}\right) = \frac{1}{12}$.
   - For example, we calculate the division $\left(\frac{1}{2} \div 3 = \frac{1}{6}\right)$ by using the following figure.
   - Given that $\frac{1}{2}$ is smaller than one, draw a complete unit square.
   - Show the value of $\frac{1}{2}$ on the figure. (By colouring)
   - Since we want to divide $\frac{1}{2}$ between 3 persons (3 groups), so divide $\frac{1}{2}$ into 3 equal parts on the figure. (The whole of the shape is divided into 6 equal parts in which 3 parts are coloured).
   - Each part of the 3 coloured parts shows the share of each person or the amount of each group.



- According to the figure, the result of the division $\left(\frac{1}{2} \div 3\right)$ is equal to $\frac{1}{6}$.

7. Designing and implementing the formative assessment.

   Find each of the following products or quotients.

$$
\begin{array}{cc}
4 \div \dfrac{2}{3} = & \dfrac{4}{9} \times \dfrac{4}{5} = \\
7 \div \dfrac{1}{4} = & 1\dfrac{1}{5} \times \dfrac{2}{3} =
\end{array}
$$

3.2 DEA method

In this section, we present a method based on the DEA technique to evaluate the students of some groups after using the educational design patterns. One of the main goals is evaluating the students according to the pre-test scores and the post-test scores before and after using the educational design patterns, respectively. After that, we aim to determine the suitable target for each student so that she/he can improve her/his performance by following the behavior of the target unit. For this purpose, the students of each group can be considered as the decision making units with two inputs (the pre-test score of the students (x_1), the pre-test score of the students with the retention after three weeks (x_2)) and two outputs (the post-test score of the students (y_1), the post-test score of the students with the retention after three weeks (y_2)).

We extend the SBM model (2.2) to achieve the mentioned goals, as follows:

$$
\begin{aligned}
&\rho_o^* = \min \frac{1}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}} \\
&s.t. \\
&\quad \sum_{j=1}^{n} \lambda_j x_{ij} = x_{io}, && i = 1, \ldots, m, \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, && r = 1, \ldots, s, \\
&\quad \lambda_j, s_r^+ \geq 0, && j = 1, \ldots, n, \quad r = 1, \ldots, s.
\end{aligned} \tag{3.1}
$$

Model (3.1) is the same as model 2 whose the first constraint is considered as an equation, because, the pre-test scores cannot be decreased. This model is the SBM model output oriented. Model (3.1) introduces a target for each inefficient student to increase the post-test scores of students after using the educational design patterns as much as possible.

Model (3.1) is solved for the students in the groups, Dick and Carey, Merrill and control and the efficiency score of each student (the objective value of model (3.1)) is determined. After that, a target unit is incorporated into each student as follows:

$$
\begin{aligned}
\hat{x}_{io} &= x_{io}, && i = 1, \ldots, m, \\
\hat{y}_{ro} &= y_{ro} + s_r^{+*}, && r = 1, \ldots, s.
\end{aligned} \tag{3.2}
$$

As we see in (5), the pre-test score of the target unit is the same as the score of the student under evaluation. But, the post-test score of the target unit may be greater than the score of the student under evaluation.

On the other hand, descriptive statistics examines the characteristics of the research sample and classifies and interprets demographic variables, indicators as well as the main variables of the research. Inferential statistical analysis infers the properties of a population, for example by testing hypotheses and deriving estimates. It is assumed that the observed data set is sampled from a larger population. This study is based on a descriptive strategy and investigates the relationship between the variables with different scales. Given that the relational hypothesis must be tested separately and individually, contrary to the scientific assumptions. Therefore, the researcher tests the hypotheses one by one or in order by applying an appropriate test. In the next section, the statistical results and the results obtained by DEA models are reported.

4 Results/ Findings

This section considers the fraction concept in Mathematics and assesses three groups of students for this concept after using the educational designs, i.e. Dick and Carey, Merrill and control. The first group, named Dick and Carey group, includes 31 students evaluated by Dick and Carey's educational design, the second group, named Merrill group, includes 30 students evaluated by Merrill's educational design and the third group, named the Control group, includes 32 students. The combination of sample members is reported in Table 1 and Figure 3. The pre-test and the post-test scores of students of these groups are reported in Table 2. Table 3 shows the pre-test and the post-test scores of the students with retention after three weeks.

Table 1: The combination of sample members

Method	Frequency	Percent
Merrill	32	34.4
Dick and Carey	31	33.3
Control	30	32.3
Total	93	100.0

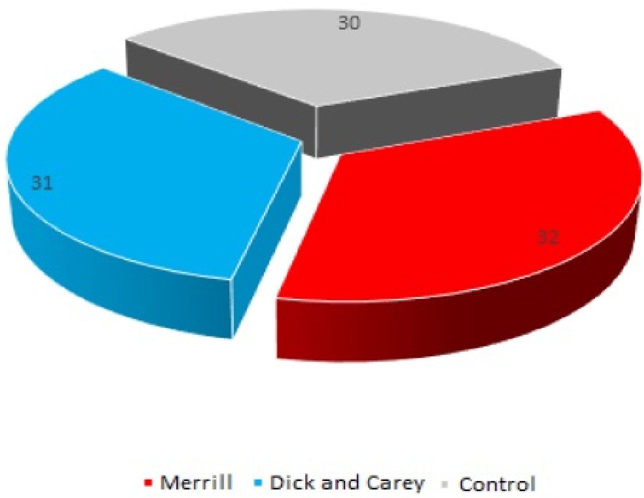


Figure 3: The combination of sample members.

In the following, we explain the obtained statistical results and the results of the DEA method.

4.1 Statistical results

This paper uses the parametric independent sample T-test. The validity of this test is based on the normality presupposition of observations. Therefore, the normality test, i.e. Kolmogorov-Smirnov test is used. The results are shown in Table 4.

Null hypothesis: The distribution of the data is normal.

Alternative hypothesis: The distribution of the data is not normal.

Table 2: The pre-test and the post-test scores of students in the fraction concept.

Dick and Carey			Merrill			Control		
student	Pre-test	Post-test	student	Pre-test	Post-test	student	Pre-test	Post-test
1	16	20	1	16.5	20	1	17	18
2	16	20	2	9.5	19	2	20	19
3	16.5	19	3	16	15	3	17	18
4	16.5	15	4	17.5	18	4	20	19
5	17	19.5	5	12.5	19	5	19	20
6	18	19.5	6	9	16	6	20	20
7	18	18.5	7	9.5	16	7	20	20
8	16	15	8	19	18	8	20	20
9	13.5	16.5	9	15.5	20	9	16	15
10	18.5	18.5	10	18.5	19	10	16	17
11	16.5	18.5	11	0.5	16	11	4	5
12	19	19	12	15.5	18	12	17	18
13	18.5	19.5	13	11	18.5	13	20	19
14	18	19.5	14	11	7	14	18	19
15	9	19.5	15	3	14	15	20	19
16	16	15	16	20	20	16	8	9
17	15	19	17	14.5	19	17	4	5
18	15	17	18	6	8	18	6	7
19	13.5	18.5	19	16	4	19	8	10
20	12	18.5	20	16.5	9	20	12	15
21	13	17	21	12.5	7	21	17	16
22	12	18	22	11.5	5	22	4	2
23	12	15	23	11.5	19	23	15	16
24	11	18.5	24	13	17.5	24	7	9
25	11	19.5	25	15	19	25	11	14
26	9	16.5	26	7.5	17.5	26	20	18
27	9	19.5	27	14	14	27	7	8
28	7	19	28	14.5	18	28	19	20
29	8	19.5	29	11	17.5	29	18	17
30	8	19	30	15.5	17.5	30	20	19
31	11	17	31	20	20			
			32	12.5	-			

Table 3: The pre-test and the post-test scores of students with the retention after three weeks.

Dick and Carey			Merrill			Control		
student	Pre-test	Post-test	student	Pre-test	Post-test	student	Pre-test	Post-test
1	20	18	1	20	18	1	20	18
2	20	11	2	20	11	2	20	11
3	19	15	3	19	15	3	19	15
4	15	11	4	15	11	4	15	11
5	19.5	18	5	19.5	18	5	19.5	18
6	19.5	18	6	19.5	18	6	19.5	18
7	18.5	16	7	18.5	16	7	18.5	16
8	15	10	8	15	10	8	15	10
9	16.5	15	9	16.5	15	9	16.5	15
10	18.5	17	10	18.5	17	10	18.5	17
11	18.5	16	11	18.5	16	11	18.5	16
12	19	19	12	19	19	12	19	19

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Dick and Carey			Merrill			Control		
student	Pre-test	Post-test	student	Pre-test	Post-test	student	Pre-test	Post-test
13	19.5	19	13	19.5	19	13	19.5	19
14	19.5	19	14	19.5	19	14	19.5	19
15	19.5	17	15	19.5	17	15	19.5	17
16	15	11	16	15	11	16	15	11
17	19	17	17	19	17	17	19	17
18	17	15	18	17	15	18	17	15
19	18.5	15	19	18.5	15	19	18.5	15
20	18.5	13	20	18.5	13	20	18.5	13
21	17	14	21	17	14	21	17	14
22	18	13	22	18	13	22	18	13
23	15	11	23	15	11	23	15	11
24	18.5	14	24	18.5	14	24	18.5	14
25	19.5	13	25	19.5	13	25	19.5	13
26	16.5	10	26	16.5	10	26	16.5	10
27	19.5	16	27	19.5	16	27	19.5	16
28	19	17	28	19	17	28	19	17
29	19.5	18	29	19.5	18	29	19.5	18
30	19	19	30	19	19	30	19	19
31	17	15	31	17	15			
			32	20	18			

Table 4: Descriptive information of Merrill and Control groups.

Characteristics	N	Sig.
Experimental group pretest	32	0.2
Experimental group post	32	0.16
Control group pretest	30	0.064
Control group post test	30	0.073

At the first, we report the results of the Merrill group compared with the control group. Regarding the initial value of P, the null hypothesis cannot be rejected for the control and Merrill group at the level of 95% satisfaction. This means that the data are normal and independent samples.

Null hypothesis: The students' mean scores in two groups of Merrill and the control group are equal.

Alternative hypothesis: The students' mean scores in two groups of Merrill and the control group are not equal. The results are shown in Table 5.

Table 5: Independent Samples Test.

Variable: Performance Scores Before Teaching	Levene's Test for Equality of Variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean difference	Std. Error difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variances assumed	4.825	0.032	-1.10	59	0.272	-1.48	1.33	-4.15	1.19
Equal variances not assumed			-1.09	52.98	0.278	-1.48	1.35	-4.19	1.2

According to the results of Levine's exam ($0.032 < 0.05$) the presumption of equality of variances is rejected and the second row of Table 5 is used. In this row, the obtained measure ($0.278 > 0.05$) and the null hypothesis is accepted. It means that the students' mean scores in the two groups of Merrill and the control group are not significantly different. Moreover, they are equal in terms of educational level. Then, the difference between the mean score of the post-test performance of Merrill and control groups are considered. For this purpose, the presumption exam is considered.

Null hypothesis: the mean score of students in two groups of control and Merrill are the same after the formal teaching.

Alternative hypothesis: the mean score of students in two groups of control and Merrill are not the same after the formal teaching. The results are shown in Table 6.

Table 6: Independent Samples Test.

Variable: Performance Scores Before Teaching	Levene's Test for Equality of Variances		t-test for Equality of Means					95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean difference	Std. Error difference	Lower	Upper
Equal variances assumed	0.039	0.312	0.53	60	0.59	0.68	1.29	-1.89	3.26
Equal variances not assumed			0.53	57	0.59	0.68	1.29	-1.90	3.27

According to Table 6, the highest point and the lowest one are not positive. The difference between the mean score of the two populations is not significant and the mean scores are not the same. So two populations are not significantly different. The analysis of variance (ANOVA) is used to consider the effectiveness of Merrill's educational design pattern on the sixth level elementary students for remembering the fraction concept in Mathematics. The pre-test scores of the fraction concept and the post-test scores of this topic with the scores of the retention test after three weeks in the Merrill group are shown in Table 7.

The null hypothesis: The mean scores of the pre-test, post-test, and the retention scores of the Merrill group in the fraction concept are equal.

Alternative hypothesis: The mean scores of the pre-test, post-test, and the retention scores of the Merrill group in the fraction concept are not equal.

Table 7: The results of ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	159.98	2	79.93	3.74	0.027
Within Groups	1984.97	93	21.34		
Total	2144.95	95			

According to Table 7, since $(0.027 < 0.05)$, the null hypothesis is rejected. This means that there is a significant difference between the mean score of the groups. This test cannot show the difference between the mean score of groups. Therefore, a post hoc test is used and the results are reported in Table 8.

Table 8: The results of the post hoc test for Merrill and Control groups

group	Group comparison	Mean Difference	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Pretest Merrill	Posttest Merrill	-2.71	1.1	0.055	-5.48	0.04
	Retention Merrill	-2.46	1.1	0.089	-5.23	0.29
Posttest Merrill	Pretest Merrill	2.71	1.1	0.055	-0.042	5.48
	Retention Merrill	0.25	1.1	0.975	-2.51	3.01
Retention Merrill	Pretest Merrill	2.46	1.1	0.089	-0.29	5.23
	Posttest Merrill	-0.25	1.1	0.975	-3.01	2.51

According to Table 8, there is no significant difference between the groups. This means that the mean scores of the retention test and the post-test do not change in the presence of the retention after three weeks. Therefore, the Merrill method is effective in reminding the fraction concept.

In the following, we investigate the effectiveness of Dick and Carey's educational design pattern on the sixth level elementary students for remembering the fraction concept in Mathematics.

At the first, we will explain the difference between the mean scores of the pre-test performance in both control and Dick and Carey groups in the fraction concept. For this purpose, a T-test is used for two independent samples (mean equality test) and the mean equality test of two independent populations is described.

Null hypothesis: The students' mean scores in both Dick and Carey and control groups are the same before teaching.

Alternative hypothesis: The students' mean scores in both Dick and Carey and control groups are not the same before teaching.

Table 9: T-test with two independent samples for Dick and Carey group in the fraction concept

Variable: Performance Scores Before Teaching	Levene's Test for Equality of Variances		t-test for Equality of Means					95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean difference	Std. Error difference	Lower	Upper
Equal variances assumed	12.42	0.001	-0.51	59	0.612	-0.63	1.24	-3.11	1.85
Equal variances not assumed			-0.49	45.4	0.621	-0.63	1.27	-3.19	1.92

According to the results of Levine's exam $0.001 < 0.05$, the presumption of equality of variances is rejected and the second row of Table 9 is used. In this row, according to the calculated p-value $(0.621 > 0.05)$, the null hypothesis is accepted. This means that the students' mean scores in both Dick and Carey and control groups are the same and also they are the same in terms of the educational level. Now, we will explain the difference between the mean scores of the post-test performance in both control and Dick and Carey groups in the fraction concept. For this purpose, a T-test is used for two independent samples (mean equality test) and the mean equality test of two independent populations is described.

Null hypothesis: After teaching, the students' mean scores in both Dick and Carey and control groups are the same.

Alternative hypothesis: After teaching, the students' mean scores in both Dick and Carey and control groups are not the same.

According to the results of Levine's exam $0.000 < 0.05$, the presumption of variance equality is rejected and the second row of the table 10 is used. In this row, according to the calculated p-value $(0.04 < 0.05)$, the null hypothesis is rejected. This means that, after teaching, the students' mean scores in both Dick and Carey and control groups are not the same. Due to the positive upper and lower bounds, the mean difference between two populations is greater than zero, and the mean of the first population, i.e. the post-test scores of Dick and Carey group in the fraction concept is greater than the second group, i.e. control group.

4.2 The results of the DEA model

In this section, we apply the suggested DEA model (3.1) to evaluate the students of each group. For this purpose, the students of each group are considered as the decision-making units (from now on, we use the terms "DMU" or "unit" for each student), the pre-test score of the group's students is as the first input (x_1), the pre-test score of the group's students with the retention after three weeks is as the second input (x_2). Also, the post-test score of the group's students is as the first output (y_1), the post-test score of the group's students with the retention after three weeks is like the second output (y_2).

At the first, we use model (3.1) for assessing the students of the Dick and Carey group. In this group, there are 31 DMUs with two inputs (x_1, x_2), located on the second columns of Table 2 and Table 3, respectively. Also, there are two outputs (y_1, y_2), located on the third columns of Table 2 and Table 3, respectively. The results of model (3.1)

Table 10: T-test with two independent samples for Dick and Carey group in the fraction concept

Variable: Performance Scores Before Teaching	Levene's Test for Equality of Variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean difference	Std. Error difference	95% Confidence Interval of the Difference Lower	Upper
Equal variances assumed	31.27	0.000	3.12	59	0.03	3.16	1.01	1.13	5.18
Equal variances not assumed			3.07	33.76	0.04	3.16	1.02	1.07	5.24

for this group are summarized in Table 11. The second column of Table 11 shows the efficiency score of the Dick and Carey group's students. As we see in this table, the students 1, 2, 4, 5, 7, 9, 10, 12, 13, 16, 18, 23, 26, 28 and 31 are the efficient students. The third and the fourth columns of Table 11 show the output slack variables of DMUs. It should be noted that the output slack variables of the efficient DMUs are equal to zero. Columns 5 and 6 of Table 11 show the input target of DMUs and columns 7 and 8 of this table show the output target of units. It is clear that the efficient DMUs and their target units are the same.

Table 11: The results of model (3.1) for Dick and Carey group.

student	z_0^*	s_1^{+*}	s_2^{+*}	$\hat{x}_{1o}$	$\hat{x}_{2o}$	$\hat{y}_{1o}$	$\hat{y}_{2o}$
1	0.964	0.00	2.00	16.00	20.00	20.00	20.00
2	0.786	0.00	9.00	16.00	20.00	20.00	20.00
3	0.918	0.00	4.00	16.50	19.00	19.00	19.00
4	1.000	0.00	0.00	16.50	15.00	15.00	11.00
5	0.973	0.00	1.50	17.00	19.50	19.50	19.50
6	0.973	0.00	1.50	18.00	19.50	19.50	19.50
7	0.950	0.00	2.50	18.00	18.50	18.50	18.50
8	0.928	0.00	2.33	16.00	15.00	15.00	12.33
9	0.968	0.00	1.50	13.50	16.50	16.50	16.50
10	0.971	0.00	1.50	18.50	18.50	18.50	18.50
11	0.950	0.00	2.50	16.50	18.50	18.50	18.50
12	1.000	0.00	0.00	19.00	19.00	19.00	19.00
13	0.991	0.00	0.50	18.50	19.50	19.50	19.50
14	0.991	0.00	0.50	18.00	19.50	19.50	19.50
15	0.953	0.00	2.50	9.00	19.50	19.50	19.50
16	0.961	0.00	1.33	16.00	15.00	15.00	12.33
17	0.962	0.00	2.00	15.00	19.00	19.00	19.00
18	0.957	0.00	2.00	15.00	17.00	17.00	17.00
19	0.928	0.00	3.50	13.50	18.50	18.50	18.50
20	0.876	0.00	5.50	12.00	18.50	18.50	18.50
21	0.933	0.00	3.00	13.00	17.00	17.00	17.00
22	0.886	0.00	5.00	12.00	18.00	18.00	18.00
23	0.892	0.00	4.00	12.00	15.00	15.00	15.00
24	0.903	0.00	4.50	11.00	18.50	18.50	18.50
25	0.857	0.00	6.50	11.00	19.50	19.50	19.50
26	0.822	0.00	6.50	9.00	16.50	16.50	16.50
27	0.932	0.00	3.50	9.00	19.50	19.50	19.50
28	1.000	0.00	0.00	7.00	19.00	19.00	17.00
29	0.980	0.00	1.08	8.00	19.50	19.50	19.08
30	1.000	0.00	0.00	8.00	19.00	19.00	19.00
31	0.957	0.00	2.00	11.00	17.00	17.00	17.00

After that, model (3.1) is solved to evaluate the students of the Merrill group. In this group, there are 32 DMUs with two inputs (x_1, x_2), located on the fifth columns of Table 2 and Table 3, respectively. Also, there are two outputs (y_1, y_2), located on the sixth columns of Tables 2 and 3, respectively. The results of model (3.1) for this group are reported in Table 12. The second column of Table 12 shows the efficiency score of the Merrill group's students. As we see in this table, students 1, 6, 11, 14, 15 and 30 are efficient students. The third and the fourth columns of Table 12 show the output slack variables of DMUs. It should be noted that the output slack variables of the efficient DMUs are equal to zero. Columns 5 and 6 of Table 12 report the input target of DMUs and columns 7 and 8 of this table show the output target of units. It is clear that the efficient DMUs and their target units are the same.

Similarly, model (3.1) should be solved to assess the students of the Control group. In this group, there are 30 DMUs with two inputs (x_1, x_2), located on the eighth columns of Table 2 and Table 3, respectively. Also, there are two outputs (y_1, y_2), located on the ninth columns of Table 2 and Table 3, respectively. The results of model (3.1) for this group are reported in Table 13. The second column of Table 13 shows the efficiency score of the Control group's students. As we see in this table, students 1, 6, 11, 14, 15 and 30 are efficient students. The third and the fourth columns of Table 13 show the output slack variables of DMUs. It should be noted that the output slack variables of the efficient DMUs are equal to zero. Columns 5 and 6 of Table 13 report the input target of DMUs and columns 7 and 8 of this table show the output target of units. It is clear that the efficient DMUs and their target units are the same.

Therefore, using the suggested technique, in addition to evaluating the performance of each student after applying the educational designs Dick and Carey, Merrill and control, we can introduce a target unit for the student under evaluation to improve his / her performance by following the target unit. This is the main advantage of applying the DEA technique for assessing the students after using the educational designs.

5 Conclusion

This study evaluated the effectiveness of some educational design patterns on the sixth level elementary students for remembering the fraction concept in Mathematics. For this purpose, we implemented three teaching design patterns, Dick and Carey, Merill and control on three groups of the sixth level elementary students. The educational design pattern was assessed by two methods: DEA and statistical techniques. We suggested a DEA model evaluate the efficiency of students in each group when the educational design pattern was implemented on the students. In addition to evaluating the performance of each student in Dick and Carey, Merrill and control groups, a target unit for the student under evaluation could be introduced to improve students' performance by applying the DEA method. This is the main advantage of the DEA technique for assessing the students after using the educational designs.

Table 12: The results of model (3.1) for Merrill group.

student	z_0^*	s_1^{+*}	s_2^{+*}	$\hat{x}_{1o}$	$\hat{x}_{2o}$	$\hat{y}_{1o}$	$\hat{y}_{2o}$
1	0.957	0.00	2.17	16.50	16.50	20.00	18.17
2	0.934	1.00	1.44	9.50	9.50	20.00	10.44
3	0.855	5.00	2.63	16.00	16.00	20.00	17.63
4	0.905	2.00	3.25	17.50	17.50	20.00	19.25
5	0.962	1.00	0.85	12.50	12.50	20.00	13.85
6	1.000	0.00	0.00	9.00	9.00	16.00	10.00
7	0.911	4.00	0.44	9.50	9.50	20.00	10.44
8	0.948	2.00	1.00	19.00	19.00	20.00	20.00
9	0.978	0.00	1.09	15.50	15.50	20.00	17.09
10	0.948	1.00	2.00	18.50	18.50	20.00	20.00
11	1.000	0.00	0.00	0.50	0.50	16.00	16.00
12	0.900	2.00	3.09	15.50	15.50	20.00	17.09
13	0.940	1.50	1.21	11.00	11.00	20.00	12.21
14	0.615	13.00	0.21	11.00	11.00	20.00	12.21
15	0.991	0.40	0.00	3.00	3.00	14.40	3.00
16	1.000	0.00	0.00	20.00	20.00	20.00	20.00
17	0.913	1.00	3.01	14.50	14.50	20.00	16.01
18	0.630	12.00	1.32	6.00	6.00	20.00	6.32

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student	z_0^*	s_1^{+*}	s_2^{+*}	$\hat{x}_{1o}$	$\hat{x}_{2o}$	$\hat{y}_{1o}$	$\hat{y}_{2o}$
19	0.418	16.00	2.63	16.00	16.00	20.00	17.63
20	0.664	11.00	4.17	16.50	16.50	20.00	18.17
21	0.169	13.00	12.85	12.50	12.50	20.00	13.85
22	0.487	15.00	1.77	11.50	11.50	20.00	12.77
23	0.901	1.00	2.77	11.50	11.50	20.00	12.77
24	0.869	2.50	3.39	13.00	13.00	20.00	14.39
25	0.927	1.00	2.55	15.00	15.00	20.00	16.55
26	0.798	2.50	3.09	7.50	7.50	20.00	8.09
27	0.807	6.00	3.47	14.00	14.00	20.00	15.47
28	0.871	2.00	4.01	14.50	14.50	20.00	16.01
29	0.922	2.50	1.21	11.00	11.00	20.00	12.21
30	0.934	2.50	1.09	15.50	15.50	20.00	17.09
31	1.000	0.00	0.00	20.00	20.00	20.00	20.00
32	0.000	1.69	0.00	12.50	12.50	1.69	11.00

Table 13: The results of model (3.1) for Control group.

student	z_0^*	s_1^{+*}	s_2^{+*}	$\hat{x}_{1o}$	$\hat{x}_{2o}$	$\hat{y}_{1o}$	$\hat{y}_{2o}$
student	z_0^*	s_1^{+*}	s_2^{+*}	$\hat{x}_{1o}$	$\hat{x}_{2o}$	$\hat{y}_{1o}$	$\hat{y}_{2o}$
1	0.976	0.00	1.09	17.00	18.00	18.00	16.09
2	0.970	0.00	1.50	20.00	19.00	19.00	17.50
3	0.866	0.00	5.09	17.00	18.00	18.00	16.09
4	0.947	0.00	2.50	20.00	19.00	19.00	17.50
5	1.000	0.00	0.00	19.00	20.00	20.00	18.00
6	1.000	0.00	0.00	20.00	20.00	20.00	19.00
7	0.962	0.00	2.00	20.00	20.00	20.00	19.00
8	1.000	0.00	0.00	20.00	20.00	20.00	19.00
9	0.890	0.00	3.70	16.00	15.00	15.00	13.70
10	0.948	0.00	2.13	16.00	17.00	17.00	15.13
11	0.529	0.00	2.67	4.00	5.00	5.00	3.67
12	0.976	0.00	1.09	17.00	18.00	18.00	16.09
13	0.923	0.00	3.50	20.00	19.00	19.00	17.50
14	0.906	0.00	4.04	18.00	19.00	19.00	17.04
15	0.867	0.00	5.50	20.00	19.00	19.00	17.50
16	0.667	0.00	4.49	8.00	9.00	9.00	7.49
17	0.529	0.00	2.67	4.00	5.00	5.00	3.67
18	0.626	0.00	3.58	6.00	7.00	7.00	5.58
19	0.529	0.00	5.33	8.00	10.00	10.00	7.33
20	1.000	0.00	0.00	12.00	15.00	15.00	11.00
21	0.959	0.00	1.65	17.00	16.00	16.00	14.65
22	1.000	0.00	0.00	4.00	2.00	2.00	13.00
23	0.971	0.00	1.18	15.00	16.00	16.00	14.18
24	1.000	0.00	0.00	7.00	9.00	9.00	2.00
25	1.000	0.00	0.00	11.00	14.00	14.00	10.00
26	1.000	0.00	0.00	20.00	18.00	18.00	16.00
27	0.352	0.00	5.53	7.00	8.00	8.00	6.53
28	1.000	0.00	0.00	19.00	20.00	20.00	18.00
29	0.938	0.00	2.60	18.00	17.00	17.00	15.60
30	0.923	0.00	3.50	20.00	19.00	19.00	17.50

Also, we applied a T-test for two independent samples (mean equality test) and the mean equality test of two independent populations was described for assessing the effectiveness of each educational design pattern. Based on the findings of this research, there was no significant positive effect of the Merrill method on the performance of the students in learning the fraction concept, but it was effective on their retention after three weeks. In other words, since Merrill's theory is a cognitive one and all cognitive theories focus on the psychological processes of the learners and take a minor look at environmental factors for students' learning, it can be concluded that this teaching method has led to the students retention. On the other hand, the Merrill method was a positive effect on the learning of students for the fraction concept. A possible extension of this research would be to use some different DEA models or some different statistical tests for evaluating the effectiveness of the educational design pattern.

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