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Applying a suitable approximate-simulation technique of an epidemic model with random parameters

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Abstract

In this research, a suitable numerical simulation method is used to solve a non-linear system that contains multi-variables and multi-parameters with absent real data. The solution to such type of system needs a long time with some difficulty. Mean Latin Hypercube_Runge Kutta (MLH_RK4) propused method solve such system that has random parameters easily and fast. In addition, it is the appropriate method for solving the change in the values of the system coefficients with time. The mentioned system has been given realistic results with MLH_RK4 that has been applied to the epidemic model. The COVID-19 model from 2020 in Iraq is the application under the research. The comparison study between the numerical results with the proposed numerical simulation results is shown in tables, and more clear graphically. The COVID-19 pandemic in our study will vanish in the next few years, according to the behavior of the epidemic for all its stages mentioned in our study. The proposed method can lessen the number of iterations for the used numerical method, and the number of repetitions of the used simulation technique. As well as it is a faster technique in the generation of parameters that appears as random variables using the Latin Hypercube sampling technique. The MLH_RK4 method has been confirmed to be reliable, and effective to solve linear and nonlinear problems. The proposed method can predict the behavior of phases of the epidemic in the future of some epidemiological models.

Keywords: System of Ordinary Differential Equations, Epidemic Model, COVID-19 Model, Runge-Kutta Numerical Scheme, Numerical Simulation Methods
2020 MSC: 34E18, 34C60

1 Introduction

The coronavirus epidemic of type COVID-19 is a one of SARS-Cov-2 family. Because COVID-19 is a fast-spreading epidemic due to contact between humans, as well as breathing, which helps to increase the speed of the virus's spread globally, it is considered the biggest problem in our world at the moment. At this time, epidemiology modeling is an important tool for analyzing infectious diseases for individuals. The contribution of epidemiology modeling in understanding the dynamics of the virus, giving the best predictions for transmission and how to spread the virus quickly all over the world, [3, 5, 6, 7, 16].

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To solve nonlinear differential equations analytically or numerically if they have fixed coefficients, many researchers discussed this problem, like; the non-linear Korteweg-de Vries equations were solved analytically by reliable, fast, efficient Semi-Analytical Iterative Method, [1]. The nonlinear heat equation was solved numerically using the explicit finite difference method, [20].

Different types of epidemic models are analyzed to expected the behavior of epidemic spread. The first kind, social epidemics like obesity mathematical model that was been solved numerically using different ways of RK that are RK4, RK4, RK45, and RK78 methods, [13]. The alcohol consumption model too, as well, analyzed the behavior of the alcohol consumption and smoking habit epidemic analytically and numerically, [18, 19]. The second kind, the diseases epidemics such as the influenza epidemic model that analyzed the behavior of the epidemic numerically, using Runge-Kutta numerical method of order 4 and 45, [10]. As well, the behavior of the SIR epidemic model is solved analytically using Tamimi and Ansari iterative method with Laplace transform, [4]. The study concerned with COVID-19 modeling has recently increased to know whether the epidemic will increase or decrease in the future, since 2020. As well, the stability of the COVID-19 model was discussed by [15]. Euler's and Runge-Kutta methods of second-order and fourth-order were applied to the COVID-19 model Turkey and Iraq and were analyzed by [2]. Furthermore, the SIR COVID-19 model had been estimated the reproductive number in Kurdistan Region in Iraq, and the mode had been solved RK4 numerically and analyzed, [14]. As well, a new SEIVR COVID-19 model was modeled with Vaccination Campaign, as well, solved numerically by Runge-Kutta numerical scheme, [23].

On the other hand, some authors solved the epidemic models using numerical simulation methods; a new technique from Latin Hypercube random sampling with finite difference numerical method was created for the first time to solve the cocaine abuse model, [12]. After that, the second new technique that consists of Monte Carlo simulation mixed with finite difference numerical method was established to solve the obesity mathematical model, [9]. Then, the third numerical simulation was provided which contains Monte Carlo simulation and Runge-Kutta numerical scheme, solve smoking habit and alcohol consumption models, [17]. Recently, the fourth numerical simulation that mixed Latin Hypercube random sampling with Runge-Kutta numerical scheme was created to analyze the influenza epidemic model, [11]. In the present work, the last numerical simulation technique is applied to the COVID-19 model of SEIVR type with Vaccination Campaign.

The virus began to appear in Iraq at the end of February 2022, when the vaccine was not available in the first year. After that, the vaccine appeared to help reduce the spread of the virus. An increase in the number of those who received the vaccine was noted at the end of 2021 and the beginning of 2022, due to campaigns to raise awareness of the need to take the vaccine.

Some difficulty is encountered when solving nonlinear systems with many variables and multiple parameters. Sometimes, it is not easy to get the exact solution to it, so, maybe resorted to the analytical and numerical methods when the coefficients of the system have fixed values. In our study, these coefficients of the system are estimated and not real data, changing with time, and they can be considered as random variables with certain distributions. In order to solve this system, which has coefficients that are described as random variables, neither the analytical nor the numerical methods fits them, but another method called the approximate simulation methods to give a solution with every change of the random variables of the coefficients, and these are the methods that have been used in this study.

The present research is classified as follows: Section 2 deals with a mathematical model which is the COVID-19 model accompanied by vaccination campaign, Section 3 is solved the COVID-19 model using numerical and approximate-simulation methods. Section 4, gives some results that are offered tabular and graphically following the discussion of the given results. Finally, in Section 5, the conclusion of this study is provided.

2 COVID-19 Mathematical Model

The COVID-19 pandemic model including the Vaccination Campaign is one of natural phenomenon. The COVID-19 model based on SEIVR method with the vaccination campaign in Iraq is done. The COVID-19 model can be represented as a nonlinear system for the first order from five of differential equations, the model is formed (1), [23].

The population of COVID-19 pandemic model is divided into five individuals that are named compartments, $S(t), E(t), I(t), V(t)$ and $R(t)$, so this model is made (SEIVR), [23]. The first stage; Susceptible population is the healthy individuals and with the symbol (S), the second stage; Exposed population; the infected individuals in the incubation period who do not show symptoms of disease and denoted by (E), for the third stage; Infected population is the infected individuals and the given rise to infection after incubation interval and symbolized by (I), the fourth stage; Vaccinated population are the individuals who take the vaccinated and symbolized by (V), the fifth stage;

Recovered population are the individuals who recover which are coded by (R). The derivatives of all compartments of coronavirus model are continuous at $t \geq 0$. The solutions of the model are non-negativity, the existence and uniqueness can be viewed in [9]. The system of COVID-19 under study is offered in (2.1), [23]:

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - (\alpha E + m + \mu)S, \\
\frac{dE}{dt} &= \alpha SE + pVE + (fI + c + \mu)E, \\
\frac{dI}{dt} &= fEI - (z + \mu + \sigma)I, \\
\frac{dV}{dt} &= mS - (pE + \mu)V, \\
\frac{dR}{dt} &= zI + cE - \mu R,
\end{aligned} \tag{2.1}$$

where Table 1. has description of the parameters of COVID-19 model.

Table 1: Description parameters of COVID-19 model with their values [23]

Parameters	**Description**	**Value of Parameters**	**Source**
Λ	Mobilization rate of Coronavirus	50	[8]
α	Rate of transition from susceptible persons to exposed persons	0.002	estimated
m	Proportion of vaccinated susceptible persons	0.5	estimated
f	Rate at which exposed people become infected	0.008	estimated
p	Rate for vaccinated persons who are exposed to disease	0.08	estimated
z	Recovery rate of infected persons	0.012	estimated
μ	Rate of natural death	0.009	estimated
c	Recovery rate of exposed persons	0.05	estimated
σ	Mortality related to the disease	0.25	estimated

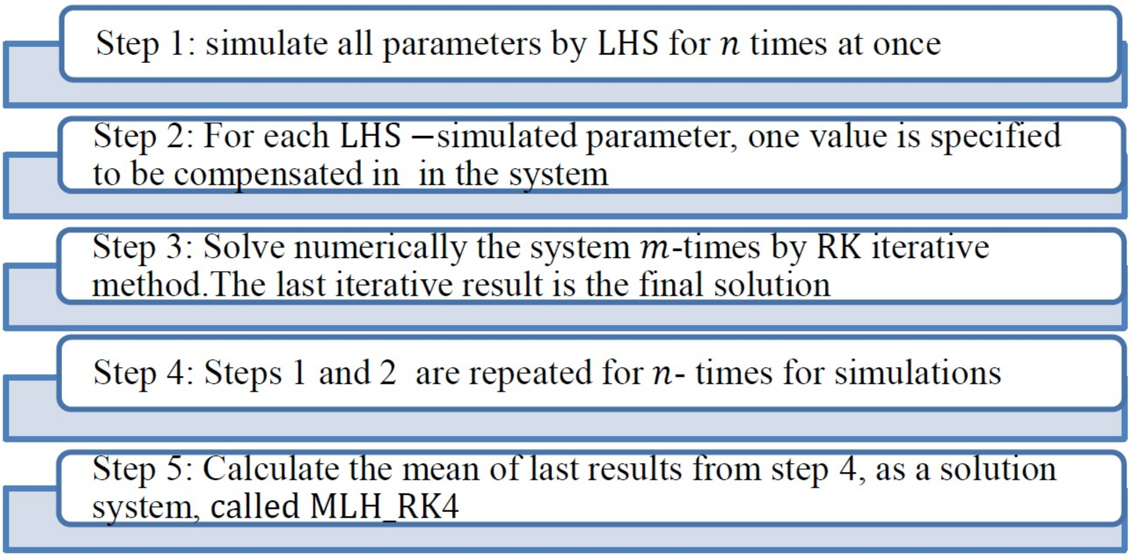
3 Solving Methods for COVID-19 Model

An Approximate-Simulation technique is a process to find the approximate solutions at specified points. The COVID-19 epidemic system in Iraq since 2020 with initial values can be solved in the present work using RK of the fourth-order numerical method using MATLAB code. As well, approximate-simulation methods had been utilized to solve the problem under study.

Different techniques of approximate-simulation methods. In the current study, the approximate-simulation method consists of Latin Hypercube random sampling as a simulation technique to simulate the system parameters, integrate Runge-Kutta numerical method to solve the system under study numerically. The obtained results consider as approximate-simulation results. This technique is called Mean Latin Hypercube_Runge Kutta method with code MLH_RK4. For more information about the process, see [11]. The MLH_RK4 results compare with another technique that is named Mean Monte Carlo_Runge Kutta method for coding MMC_RK4. MMC_RK4 is a process merge Monte Carlo simulation and Runge-Kutta numerical scheme, for more information about the process, see [17]. A flow chart to MLH_RK can be shown in Figure 1.

4 Results and Discussion

The World Health Organization website is the source of initial values of (1) are taken from the Iraq data, [21, 22] that is as follows: $S_0 = 500$, $E_0 = 46$, $I_0 = 23$, $V_0 = 0$, and $R_0 = 12$. Numerical and approximate-simulation results for COVID-19 model for 100 repetitions during 10 years in Iraq in 2020 are discussed and analyzed in this section. Table 2 gives the results of each of the epidemic stages $S(t), E(t), I(t), V(t)$ and $R(t)$ for RK_4, MLH_RK4 and MMC_RK4 methods. Because of the difficulty to find the exact solution of the nonlinear system under study, the RK_4 numerical



Step 1: simulate all parameters by LHS for n times at once

Step 2: For each LHS −simulated parameter, one value is specified to be compensated in in the system

Step 3: Solve numerically the system m-times by RK iterative method.The last iterative result is the final solution

Step 4: Steps 1 and 2 are repeated for n- times for simulations

Step 5: Calculate the mean of last results from step 4, as a solution system, called MLH_RK4

Figure 1: MLH_RK4 Procedur

results are represented as the exact solutions for this system for the purpose of comparison. It is noticeable that MLH_RK4 results more converge to RK_4 than MMC_RK4 with step size 0.08 in a month and 0.02 in a weak for all stages $S(t), E(t), I(t)$, and $R(t)$, in Table 2. Therefore, MLH_RK4 method is better than the MMC_RK4 method for solving such model under study.

These methods are distinguished from the analytical and numerical methods in that they give an estimated solution period specified by the least and largest value of the solution with an error rate of 5%, due to the randomness in simulating the coefficients of the system using one of the simulation methods. Table 3 displays the predicted interval Prediction intervals (5th percentile, 95th percentile) of approximate-simulation results with 100 repetition for COVID-19 model in 10 years from 2020 to 2030.

Figure 2 is to describe the behavior of epidemic stages $S(t), E(t), I(t), V(t)$ and $R(t)$ for RK_4 numerical and MLH_RK4 approximate-simulation results (100 repetition) for COVID-19 model in 10 years from 2020 when the step size h is 0.02 weakly in 10 years; from the beginning of breakout the virus 2020 to 2030.

Figure 2 has also shown the convergence of the RK4 numerical curve with the curve of the MLH_RK4 approximate-simulation, which indicates that the MLH_RK4 method is an efficient, convergent, and reliable method for solving such a model that features a nonlinear system with multi variables and multi randomness parameters.

It is noted that the $S(t)$ population number has decreased during the first two years until 2022 and then stabilize at one level, which indicates that the epidemic will not increase during the coming years until 2030. On the other hand, the rise of $E(t)$ population number has been increasing dramatically since 2020, reaching a peak in 2021, then starting to decline in 2022 and continuing to decline until 2024, then stabilizing in the year 2025 to 2030. As well, the number of people infected with the epidemic $I(t)$ is witnessing a noticeable increase since 2020, reaching a peak in 2022, then starting to decline rapidly from 2022 to 2024, and then gradually declining to 2030. It is noticeable that the vaccine for Covid-19 virus appeared at the end of 2021, therefore, the behavior of the vaccine curve before the middle of 2021 should be ignore and $V(t)$ has gradually increased to 2024 then begins to decline gradually until the end of the period under study. The number of people recovering $R(t)$ begins to rise gradually from the year 2020 until the end of the study period 2030.

All the results of the present study agree with the previous studies, for all stages of the Coronavirus epidemic. The MATLAB program of version R2017a has supported to provide the results of the COVID-19 pandemic model at the present study. In addition that, the curves have been drawn using the Magic plot program.

5 Conclusion

At present work, the model is convenient for the COVID-19 epidemic in Iraq in all its stages, and the observation data for Iraq as initial values are applied, so the obtained results match the prevalence of the virus in Iraq. The

Table 2: Numerical and approximate-simulation results 100 repetition for COVID-19 model in 10 years.

Population	Step Size (h)	RK4 10 Years	MLH_RK4 10 Years	MMC_RK4 10 Years
S	0.02 (weakly)	89.3593198158165	90.3354157846183	90.367603785434
	0.08 (monthly)	89.3593209518042	90.3776496156687	90.4117926880117
E	0.02 (weakly)	28.4319382456529	28.8921956740334	29.0624962936395
	0.08 (monthly)	28.4319252521013	28.815335511224	28.9874001863723
I	0.02 (weakly)	210.285219546379	212.317837900718	207.309737541351
	0.08 (monthly)	210.285313647745	213.044366254235	207.973433246921
V	0.02 (weakly)	19.9586546831238	20.4943866314048	20.1742527383927
	0.08 (monthly)	19.9586625404366	20.5566920641549	20.2426402810523
R	0.02 (weakly)	68.2557582669928	68.6369007961858	67.6098406070031
	0.08 (monthly)	68.2557993715537	68.4873728033566	67.4588995937681

Table 3: Predicted interval of approximate-simulation results with 100 repetition for COVID-19 model in 10 years.

Step Size (h)	Percentile Interval_S	Percentile Interval_E	Percentile Interval_I	Percentile Interval_V	Percentile Interval_R
0.02	68.4004504125162 114.828614889628	21.2884591742642 38.0085253596076	160.160865814436 262.933222748996	13.5261537819094 30.3257373065232	59.5965362784733 80.1186016993077
0.08	68.4311717378914 114.874415885433	21.2682058799006 37.8526057369374	160.862152549479 263.628416416727	13.6018774298088 30.3482444936138	59.4681031519441 79.9640688179357

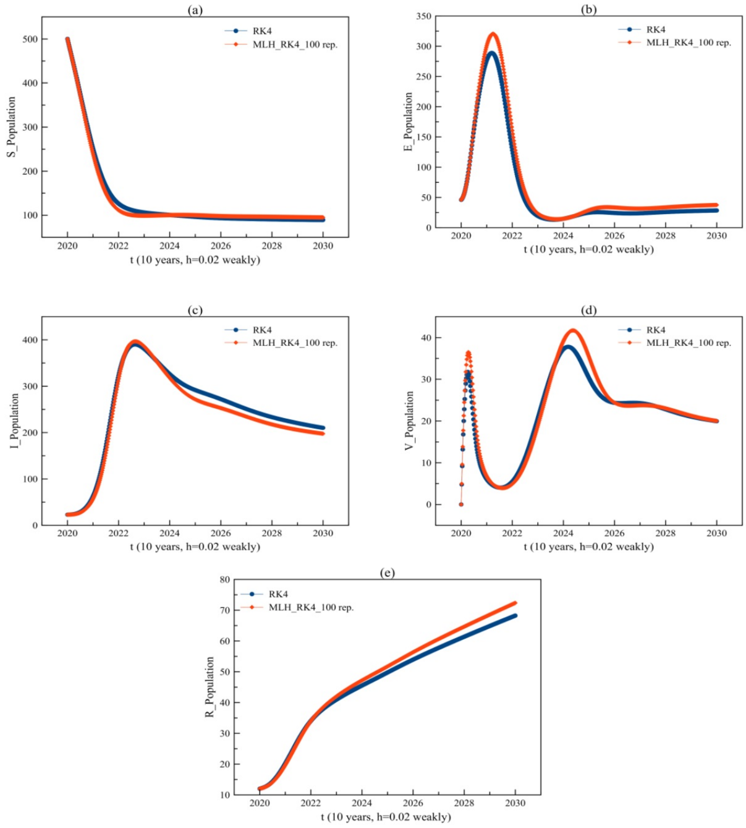


Figure 2: Numerical and approximate-simulation results (100 repetition) for COVID-19 model in 10 years from 2020.

approximate-simulation MLH_RK4 proposed method has helped us to analyze the spread of the COVID-19 epidemic model in Iraq.

From the results obtained, it has been found that the number of people exposed to infection gradually decreases during the first two years and then stabilizes in the third year of the virus's spread. On the contrary, the number of

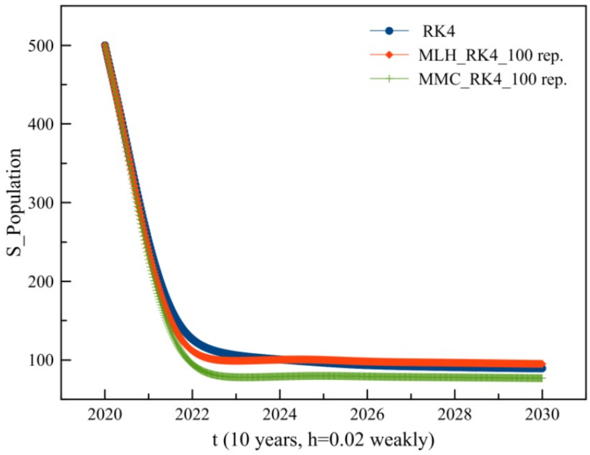


Figure 3: Comparison RK_4 numerical, MMC_RK_4 and MLH_RK_4 approximate-simulation results for S with 100 repetition for COVID-19 model in 10 years from 2020.

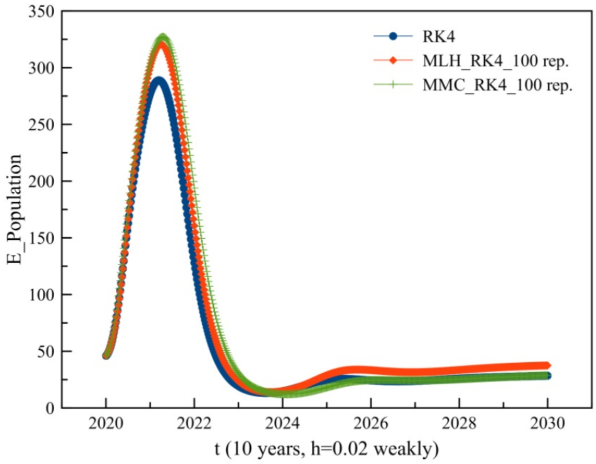


Figure 4: Comparison RK_4 numerical, MMC_RK_4 and MLH_RK_4 approximate-simulation results for E with100 repetition for COVID-19 model in 10 years from 2020.

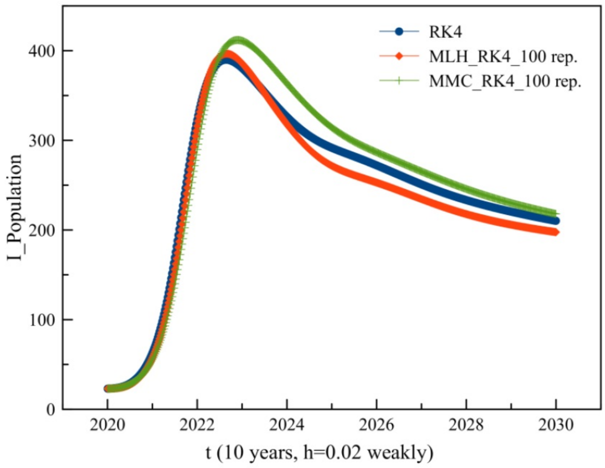


Figure 5: Comparison RK_4 numerical, MMC_RK_4 and MLH_RK_4 approximate-simulation results for I with100 repetition for COVID-19 model in 10 years from 2020.

people exposed to infection and infection during the first three years is increased. The number of vaccinated people has gradually raised from the beginning of the vaccination campaign in 2021 until 2025 then has being declined until

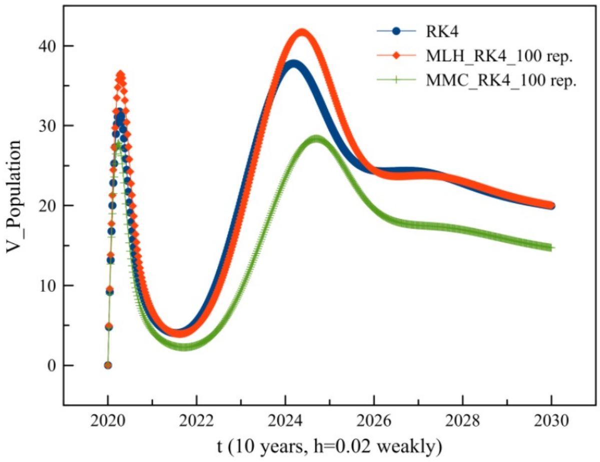


Figure 6: Comparison RK_4 numerical, MMC_RK_4 and MLH_RK_4 approximate-simulation results for V with100 repetition for COVID-19 model in 10 years from 2020.

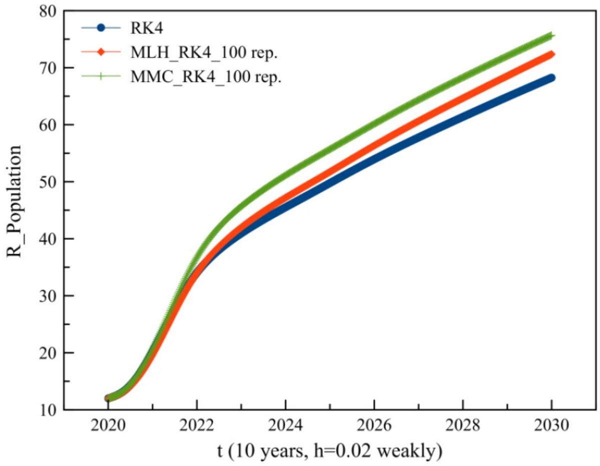


Figure 7: Comparison RK_4 numerical, MMC_RK_4 and MLH_RK_4 approximate-simulation results for R with100 repetition for COVID-19 model in 10 years from 2020.

2030. From the beginning of the virus, the number of recovered population has been gradually increasing until 2030. The previous results obtained indicate that the virus has not increased after 3 years and the epidemic will vanish in the future.

The advantage of the proposed method MLH_RK4 is in reducing the number of RK4 iterations for the solution of this model. MLH_RK4 proposed method is distinguished from the analytical and numerical methods in that its give an estimated solution interval (5th percentile, 95th percentile) of approximate-simulation results.

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