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Symmetric fuzzy approach to solve complementary multi-objective linear fractional programming problem

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Abstract

In this paper, the property of fuzzy sets is used as approach to instrument for the construction and finding the value of "multi-objective linear fractional programming problem" (MOLFPP) and applied complementarity condition on such problems which is one of regulation of order problems which cover by fuzzy dealings. Introduce a technique to convert and solve the problem by symmetric fuzzy approach. Suggest an algorithm and show how fuzzy LFPP can be answered without rising the Arithmetic potency. Our technique is used to convert complementarity multi-objective linear fractional programming problem (CMOLFPP) to LP through symmetric fuzzy linear fractional problem (SFLFP). To demonstrate the efficacy of the suggested technique, a numerical example is presented. then compared the result with other techniques fuzzy linear fractional programming (FLFP) which are solved by using computer applications to test the algorithm of the above method to indicate that the results obtained by such approach are promising.

Keywords: Symmetric Fuzzy, Fuzzy Mathematical Programming, CMOLFPP, Complementary Condition, FLFP
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1 Introduction

Linear fractional programming (LFP) which is written as a fraction of dual-line functions, is optimum. The area is comparable to, fiscal and business projection, output planning, emporium and broadcasting Section, campus planning and scholar admittance, verdure care and infirmary planning, and so on. usually, appearance problems to make a verdict that improves section/ righteousness fraction, income/fee, stocking/selling, genuine fee/normal fee and so on. In the applied implementation, a classical includes several restrictions whose significance is assumed by specialists. although, together with specialists and decisional makers usually do non-recognize the cost of those strictures. [6] Ibaraki studied CP and defined a new case of optimum problems, known as CPP. In (1982) Gupta and Sharma had worked on a new algorithm for solving a QCPP with indefinite [4]. Das and Edalatpanah [3] studied a general form of fuzzy linear fractional programs with trapezoidal fuzzy numbers, [7] Pal, Moitra and Maulik present a goal programming (GP) process for "fuzzy multi-objective linear fractional programming (FMOLFP) problems". A GP model is developed for achieving the uppermost membership value of each of the fuzzy goals set for fractional objectives. To solve the problem effectively utilizing linear goal programming (LGP) methodology, the procedure of mutable modification on the down and top of variation mutable of the organism goals connected through the fuzzy goals of the classical is presented in the solution process. [5] Mamadameen research focuses on multi-objective fuzzy

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linear programming (MFLP) issues with ternary fuzzy numbers as factors in the goal functions. In addition, a modern method for converting MFLP issues into a single fuzzy linear programming problem is proposed. [8] Loganathan and Ganesan offered a method for addressing "completely fuzzy linear fractional programming problems" in which all limitations and mutable are represented by triangular fuzzy integers translate entirely ternary fuzzy numbers to parametric formula and the fractional PP to a "solo objective linear programming problem" in parameter formula finally find the best solution to the provided "completely fuzzy linear fractional programming" issue deprived of changing to its alike crisp line programming problem by using "novel fuzzy arithmetic" and "fuzzy ranking". [2] Bellman and Zadeh worked on Fuzzy objectives and restraints" are fuzzy groups in the interplanetary of replacements that can be stated accurately. The intersection of the provided goals and restrictions may thus be seen as a fuzzy choice. A maximizing choice is one in which the "membership function" of a fuzzy deduction achieves its highest value in the space of alternatives. [13] Zimmermann forced on Fuzzy sets are introduced as a novel instrument for formulating and solving systems and decision problems that involve fuzzy processes or fuzzy connections. Following a brief overview of fuzzy sets' underlying theory, the implications for systems theory and decision-making are discussed. After that, "fuzzy linear programming problems" are solved using fuzzy set theory. [1] Abo-Sinna and Baky purposes the involvement aimed at the definite fuzzy target of the choice creators goal functional on together stages as well as the relationship functional for vectorial of a fuzzy function of a choice changeable dominance by 1st-stage choice makers are industrialized primary in the classical design of an issue. [12] Veeramani and Sumathi resolve the process was projected to FLFPP when remuneration of the goal occupation, the resources and the scientific constants are three-sided fuzzy numerals. Now, the FLFP problem is changed for an alike specified "multi-objective linear fractional programming. Through utilising the "Fuzzy Mathematical programming" method changed MOLFPP is compact "single objective linear programming (LP) problem". [10] Sulaiman and Nawkhass, their work depends on to solve solving multi-objective quadratic fractional programming problems by an optimal average of maximin and minimax techniques. This work is prepared as follows: Section 2 defines a few of the fundamentals of fuzzy set and Fuzzy Algebra Operation. Section 3 addresses and defines some mathematical model. Section 4 addresses Symmetric Fuzzy Linear Fractional Programming (SFLFP) to describe the proposed method. An algorithm is obtainable in Section 5. In Section 6, to exemplify of the proposed process, an arithmetical sample is solved. Finally, Conclusions in Section 7.

2 Preliminaries Definition

Some fundamentals and algebra operations of fuzzy environment which utilized in this research are listed [5]. If X is a assemble of contraption define basically by x, posteriorly en a fuzzy set $\tilde{A}$ in X is a set of well-ordered sets: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))|x \in X\}$, $\mu_{\tilde{A}}(x)$ is titled by grade of membership of x in $\tilde{A}$.

The support of a fuzzy set $\tilde{A}$, $S(\tilde{A})$, is the crisp set of all $x \in X$ if $\mu_{\tilde{A}}(x) > 0$. The fragile group of fundamentals element that pertinence for a fuzzy set $\tilde{A}$ at the lower to the grade α is named by α-level set: $A_\alpha = \{x \in X|\mu_{\tilde{A}}(x) \geq \alpha\}$, $A'_\alpha = \{x \in X|\mu_{\tilde{A}}(x) > \alpha\}$ is define by "strong α-level set" or "strong α-cut".

The algebraic sum (likelihood sum) $\tilde{C} = \tilde{A} + \tilde{B}$ is illustrate as $\tilde{C} = \{(x, \mu_{\tilde{A}+\tilde{B}}(x))|x \in X\}$, where $\mu_{\tilde{A}+\tilde{B}}(x) = \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - \mu_{\tilde{A}(x)}, \mu_{\tilde{B}}(x)$.

Definition bounded sum: The bounded sum $\tilde{C} = \tilde{A} \oplus \tilde{B}$ is definite as

$$\tilde{C} = \{(x, \mu_{\tilde{A}\oplus\tilde{B}}(x))|x \in X\}, \quad \text{wherever} \quad \mu_{\tilde{A}\oplus\tilde{B}}(x) = \min\{1, \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x)\}.$$

A bounded difference $\tilde{C} = \tilde{A} \ominus \tilde{B}$ is definite as

$$\tilde{C} = \{(x, \mu_{\tilde{A}\ominus\tilde{B}}(x))|x \in X\} \quad \text{wherever} \quad \mu_{\tilde{A}\ominus\tilde{B}}(x) = \min\{1, \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x)\}.$$

3 Formulation of the mathematical model

3.1 Linear Fractional Programming problem (LFPP)

The linear fractional programming problem (LFPP) can expressed as follows [9]:

$$\begin{aligned} &\max, W = \frac{c_1'x + \beta}{c_2'x + \delta} = \frac{z_1}{z_2}, \\ &\text{s.to}: \quad Ax = b, x \geq 0 \end{aligned} \tag{3.1}$$

where (i) x, c_1 and c_2 are $n \times 1$ column vectors. (ii) A is $n \times m$ matrix. (iii) b is an $m \times 1$ vectors. (iv) The prime $\{'\}$ on the vectors c_1, and c_2 indicate as a transpose of vectors and (v) β, δ are comparatively numeric.

3.2 Complementary LFPP

Ibaraki [6] defined complementary programming problem and Gupta and Sharma [4] defined complementary quadratic programming problem (CQPP), CLFPP can be defined such as:

$$\begin{aligned} &\max, W = \frac{c_1'x+\beta}{c_2'x+\delta} \\ &\text{s.to :} \\ &Ax = b, x \geq 0, \quad uv = 0, \quad x,u,v \geq 0 \end{aligned} \tag{3.2}$$

Where x, u, v are n, m-dimensional and m-dimensional vectors of variables respectively, c_1, c_2 are n-dimensional and b is n-dimensional vector of constants; A is $p \times n$ matrix of constraints β, δ are constants.

3.3 Complementary Multi-Objective Linear Fractional Programming Formula (CMOLFPP)

The multi-objective functions that are the proportion of dual linear objective functions can call as CMOLFPP which can be define by:

$$\begin{aligned} &\max, W_1 = \frac{c_{11}'x+\beta_1}{c_{12}'x+\delta_1} = \frac{z_{11}}{z_{12}} \\ &\max, W_2 = \frac{c_{11}', x+\beta_2}{c_{22}'x+\delta_2} = \frac{z_{21}}{z_{22}} \\ &\vdots \\ &\max, W_r = \frac{c_{r1}'x+\beta_r}{c_{r2}'x+\delta_r} = \frac{z_{r1}}{z_{r2}} \\ &\min, W_{r+1} = \frac{c_{(r+1)1}'x+\beta_{r+1}}{c_{(r+1)2}'x+\delta_{r+1}} = \frac{z_{r1}}{z_{r2}} \\ &\vdots \\ &\min, W_n = \frac{c_n'x+\beta_n}{c_n'x+\delta_n} = \frac{z_{n1}}{z_{n2}} \\ &\text{s.to:} \\ &Ax = b, \quad uv = 0, \quad x,u,v \geq 0 \end{aligned} \tag{3.3}$$

here r is the amount of goals function which is maximized, n is the amount of goals functions which is max. and min. and $n-r$ is the amount of goals function which is min, other cyphers have the similar element as earlier stated in [11].

4 Symmetric Fuzzy Linear Fractional Programming (SFLFP)

Our technique depend on the adopted "fuzzy" version of formula (3.1) is: find x such that

$$\begin{aligned} &\frac{c_1'x+\beta}{c_2'x+\delta} \gtrsim \frac{z_1}{z_2} \\ &Ax \lesssim b, \quad x \geq 0 \end{aligned} \tag{4.1}$$

here c_1 and c_2 are the vector of coefficients of numerator and denominator respectively of the ratio of the goal function, b denoted as a vector of constraints, and A is the factor of matrix. The sign " $\gtrsim$ " indicate the fuzzified type of " $\geqq$ " and read out "basically greater than or commensurate to".

Note that (4.1) is wholly symmetric with observance to goal function and restraints, and we tried to do that be easy to understandable by exchange $\begin{pmatrix} c_1 \\ c_2 \\ A \end{pmatrix} = \beta$, $\begin{pmatrix} z_1 \\ z_2 \\ b \end{pmatrix} = d$, Then (4.1) converts to:

find x, were

$$\beta x \lesssim d, x \geqq 0 \tag{4.2}$$

Apiece of the $(m+2)$ rows of formula (4.2) ought to now be signified by a fuzzy set; grade of membership is $\mu_i(x)$ which define by a function $\mu : R^{m+2} \to [0,1]$ such that

$$\mu(\beta x) = \begin{cases} 0 & \text{if } \beta x \leqq d \text{ is highly contravene} \\ 1 & \text{if } \beta x \leqq d \text{ is convinced} \end{cases}$$

By using new version of function $\mu(\beta x)$ by suppose it to be in lines and the junction of the (fuzzy) restraints and the (fuzzy) objective function. Hence $\mu(\beta x) = \min \mu_i(\beta_i x), x \geq 0$ with

$$\mu_i(\beta_i x) = \begin{cases} 1 & \text{for } \beta x \leqq b_i \\ \frac{1-\beta_i x - b_i}{d_i} & \text{for } b_i < \beta_i x \leqq b_i + d_i, \quad i = 1, \cdots, m+2 \\ 0 & \text{for } b_i + d_i \leqq \beta_i x \end{cases} \tag{4.3}$$

Here d_i are individual elected coefficients of permissible contravene of the restraints and the goal function. $\mu_i(\beta_i x)$ is the grade of membership of the ith row of the linear suit βx. $\min \mu_i(\beta_i x)$ is fuzzy decision and

$$\underset{x \geq 0}{\text{maximum}}(\min_i \mu_i(\beta_i x))$$

the decision through the maximum grade of membership. By simplifying (4.3) by neglected the " 1 " (because does not have effect on the issue) we attain at the next problem:

$$\underset{x \geq 0}{\text{maximum}} \min_i (b_i^{'} - \beta_i x)^{'}) = \underset{x \geq 0}{\text{maximum}} \mu_D(x) \tag{4.4}$$

Therefore, this problem is valent to solving the following LP. Maximize λ s.to:

$$\lambda \leqq b_i - \beta_i x, i = 1, \cdots, m+2, \quad x \geq 0 \tag{4.5}$$

The optimum result to (4.5) is the optimal solution as well to (4.4) and as well to (4.1).

5 Algorithm to Solve CMOLFPP

To determine the value of optimal solution for the CMOLFPP in formula (3.3), a procedure is given as below:

Step1: Put ordered number to each goal function which are to be max, min. or together.

Step2: By resolve each goal function of formula (3.3) by SFLFP without complementary condition $x_1 x_2 = 0$.

Step3: Combined all objective functions together to create formula (4.5).

Step4: Optimize the combined objective function subject to the same constraint in (3.3) without complementary condition $x_1 x_2 = 0$.

Step5: Find the optimal solution by applying complementary condition $x_1 x_2 = 0$.

6 Numerical Example

A firm produce two types of products $\mathbb{A}$ and $\mathbb{B}$ with income about 5$ and about 3$ each item, individually. And, the value for every product of component of products $\mathbb{A}$ and $\mathbb{B}$ is about 5$ and about 2$ per units. It is assuming that a constant price of about 1$ is additional to the price function due to predictable period over the procedure of making. Assume the material required for industrial output $\mathbb{A}$ and $\mathbb{B}$ is around 3 items for pound and 5 items for pound for every unit, the stock for this factual is limited to for 15 pounds. Guy-periods per item of an output $\mathbb{A}$ about 5 h and output $\mathbb{B}$ is about 2 h for item for industrial but total Guy-periods available is about 10 h daily. find how many output $\mathbb{A}$ and $\mathbb{B}$ would be factory-made in ordered to exploit of an entire income. In this case, let x_1 and x_2 to be the quantity of items of $\mathbb{A}$ and $\mathbb{B}$ to be formed. Then the above issue canister be expressed as [12]:

$$\begin{aligned} &\max, W = \frac{\bar{5}x_1 + \bar{3}x_2}{\bar{5}x_1 + \bar{2}x_2 + \bar{1}} \\ &s.to: \\ &\bar{3}x_1 + \bar{5}x_2 \leq \bar{15} \\ &\bar{5}x_1 + \bar{2}x_2 \leq \bar{10} \\ &x_1 x_2 = 0, x_1, x_2 \geq 0 \end{aligned}$$

Let us assume that

$$\bar{5} = (3,5,7), \bar{3} = (2,3,4), \bar{5} = (4,5,6), \bar{2} = (1,2,3), \bar{1} = (0,1,2), \bar{3} = (2,3,4),$$

$$\bar{5} = (3,5,7), \bar{15} = (11,15,19), \bar{5} = (4,5,6), \bar{2} = (1,2,3), \bar{10} = (8,10,12).$$

Then the problem can be formed as

$$\begin{aligned}
&\max, W = \frac{(3,5,7)x_1 + (2,3,4)x_2}{(4,5,6)x_1 + (1,2,3)x_2 + (0,1,2)} \\
&s.to: \\
&(2,3,4)x_1 + (3,5,7)x_2 \leq (11,15,19) \\
&(4,5,6)x_1 + (1,2,3)x_2 \leq (8,10,12) \\
&x_1 x_2 = 0, x_1 x_2 \geq 0
\end{aligned}$$

directly above FLFP issue is equal to the next MOLFP problem

$$\begin{aligned}
&\max, W_1 = \frac{3x_1 + 2x_2}{6x_1 + 3x_2 + 2} \\
&\max, W_2 = \frac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1} \\
&\max, W_3 = \frac{7x_1 + 4x_2}{4x_1 + x_2} \\
&s.to: \\
&2x_1 + 3x_2 \leq 11 \\
&3x_1 + 5x_2 \leq 15 \\
&4x_1 + 7x_2 \leq 19 \\
&4x_1 + x_2 \leq 8 \\
&5x_1 + 2x_2 \leq 10 \\
&6x_1 + 3x_2 \leq 12 \\
&x_1 x_2 = 0, x_1, x_2 \geq 0.
\end{aligned} \tag{6.1}$$

Here, by using SFLFP of each of $\max, W_i = 1,2,3$ individually subject to the same constraint in (6.1) transformed to linear programming problem as following. Maximize λ

$$\begin{aligned}
&\lambda \leqq -4.43 + 1.87x_1 + 1.25x_2 \\
&\lambda \leqq -4.66 + 2x_1 + x_2 \\
&\lambda \leqq -4.26 + 1.92x_1 + 1.15x_2 \\
&\lambda \leqq -4.4 + 2x_1 + 0.8x_2 \\
&\lambda \leq -4.19 + 1.94x_1 + 1 - 1.11x_2 \\
&\lambda \leqq -4 + 2x_1 + 0.5x_2 \\
&\lambda \leqq 4.6 - 0.6x_1 - x_2
\end{aligned} \tag{6.2}$$

$$\begin{aligned}
&\lambda \leqq -4 + 2x_1 + 0.5x_2 \\
&\lambda \leqq 4.6 - 0.6x_1 - x_2 \\
&\lambda \leqq 6 - x_1 - 1.66x_2 \\
&\lambda \leqq 7.33 - 1.33x_1 - 2.33x_2 \\
&\lambda \leqq 2.6 - 0.8x_1 - 0.2x_2 \\
&\lambda \leqq 4.33 - 1.6x_1 - 0.6x_2 \\
&\lambda \leqq 5 - 2x_1 - x_2 \\
&x_1 x_2 = 0, \lambda, x_1 x_2 \geqq 0.
\end{aligned}$$

Solution: Solve the above linear programming problem without complementary condition $x_1x_2 = 0$ and answer of the main issue is: $x_1 = 0.34, x_2 = 2.74$.
therefore, we going to fulfilled the complementary state $(x_1x_2 = 0)$, since $x_1x_2 \neq 0$. The best solution is issue $x_1 = 0, x_2 = 2.74$. The value of each objective function one by one are $\max, W_1 = 0.5362$, $\max, W_2 = 1.2685$, $\max, W_3 = 4$ And the comparison with FLFP Method presented as following

Table 1: The result of problem solved by FLFP Method [12] and CMOLFPP

Method	(x_1, x_2)	$\max, W_1$	$\max, W_2$	$\max, W_3$
CMOLFPP	(0.2.74)	0.5362	1.2685	4
FLFP	(0.2.7)	0.53	1.26	4

7 Conclusion

The fuzzy set theory show that has a lot of applicable and auspicious for the field of system theory also for optimization or general decision making, however, in this study; CMOLFPP was solved by SFLFP methods and the comparison of these approaches are built on value of goal function. By convert CMOLFPP to single LPP using SFLFP technique. And from table 1 that clear it is obvious that the suggesting techniques donate suppleness to the maker to select his ideal answer. Furthermore, the process going be used for resolving LFPP, when the charge of the goal function, the incomes and the scientific coefficients remain Symmetric Fuzzy approach.

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