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The disc structures of A4-graph for particular untwisted groups

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Abstract

Let t be an elements of order 3 in a finite simple group G. Let $\mathrm{X} = t^{\mathrm{G}}$ be a conjugacy class of t in G. The A4-graph, represented as $A_4(\mathrm{G}, \mathrm{X})$, is a simple graph has X as a vertex set and two vertices $x, y \in \mathrm{X}$, joined by edge whenever $\mathrm{x} \neq \mathrm{y}$ and $xy^{-1} = yx^{-1}$. In this paper, we investigate the discs structure and determine the clique number, girth and diameter of $A_4(\mathrm{G}, \mathrm{X})$ when G is isomorphic to one of the untwisted groups $\mathrm{G}_2(2)', \mathrm{G}_2(3)$ or $\mathrm{G}_2(4)$.

Keywords: Finite simple groups, A4-graph, connectivity, cliques
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1 Introduction

Graph theory and group theory are two separate fields of mathematical study, and each has its own set of standards for manipulating diverse features, concerns, and problems that appear to be detach from one another. Graph theory, on the other hand, are the ideal way to deal with a variety of problems involving algebraic structures. Many of the studies performed in this area see for example [2, 6, 7]. During investigating the algebraic characteristics of finite simple groups, the involution elements are significant. In finite simple groups, however, the elements of order 3 are equally significant. For instance, the Frobenius groups formed by two elements of order 3 are given with a comprehensive analysis in [12]. Furthermore, Maksimenko and Mamontov [8] showed that a group constructed by a conjugacy class of order 3 elements in which every pair yields an isomorphic subgroup to Z_3, A_4, $\mathrm{A}_5, \mathrm{SL}_2(3)$, or $\mathrm{SL}_2(4)$ have local finiteness. Further studies in this context can be found in [11] and [5]. Let G be a finite simple group and X is a G-conjugacy class for elements of order 3. The A4-graph, denoted by $A_4(\mathrm{G}, \mathrm{X})$, is a simple undirected graph, such that X being the vertex set and two vertices $x, y \in X$ are joined by an edge if and only $x \neq y$, where $xy^{-1} = yx^{-1}$. Aubad [1] was the first to present the A4-graph and its features, as well as the structure of $A_4(\mathrm{G}, \mathrm{X})$ when $\mathrm{G}^3\mathrm{D}_4(2)$ was investigated. One thing worth noting regarding $A_4(\mathrm{G}, \mathrm{X})$ is that the alternating group A_4 generated by two linked vertices. The paper aim to study the discs structure of the A4graph when G is one of untwisted groups: $\mathrm{G}_2(2)', \mathrm{G}_2(3)$ and $\mathrm{G}_2(4)$.

The study also involves calculate the diameter, the clique number and the girth of the A4-graph. From now on, we suppose that G be one of aforementioned group, and let $t \in G$ has order 3 and $X = t^G$ is a G-conjugacy class. One

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can see that the action by conjugation of G on X, generated A4-graph automorphisms and this action is transitive on graphs vertices.

Let we establish random element x in X and i $\in \mathbb{Z}^{+}$. Then the t^{th} disc of x, denoted by $\Delta_i(x)$ describe as the set of vertices in A4-graph, each has distance from x of size 1 (This distance function is denoted by d(,) when using the ordinary distance function for graphs). Also for the centralizer of x in G we put Gx (= $\mathrm{C_G}(x)$). We should note that he graph discs breakup into a union of particular Gx-orbits (as G acts by conjugation on X). As a consequence, we will identify the Gx-orbits of X in order to examine the characteristics of the A4-graph. Finally, for the identities of the G-conjugacy classes, we shall rely mainly on Atlas [4].

2 General Properties

This section discusses general features of the A4-graph of a finite simple group. Aubad [1] provides the following important work on the A4-graph for finite groups. The following findings show a common features for the A4-graph.

Lemma 2.1. [1] The A4-graph is simple, undirected, regular graph. Furthermore, two connected vertices generated the group A_4.

The next result associated to the disc structures of the A4-graph and its relation with the $C_G(t)$-orbits.

Lemma 2.2. Consider the A_4 (G, X) such that X = t^G (t be an element of order 3 in finite simple group G). Then the graph discs $\Delta_i(t)$ breakup into a union of particular $C_G(t)$-orbits. Moreover, for each vertex $y \in \Delta_1(t)$, we have the order of the element ty is 3 in G. Start by taking an advantage of the following list:

$$\mathrm{X_C} = \{y \in \mathrm{X} \mid ty \in X\}.$$

The set C is a random G-conjugacy class. We aim to look at the nonempty set X_C. This because such a set separated into union of particular $C_G(t)$-orbits of the class X. Knowing the way of the set X_C breaking into $C_G(t)$ - orbits is quite valuable for identifying whether $\Delta_i(t)$ have vertices in the set X_C. The set X_C length determination can also assist with class structure constants. Class structure constants depend on the size of the following set.

$$\{(s_1, s_2) \in C_1 \times C_2 \mid s_1 s_2 = s\}.$$

The sets C_1, C_2, C are G-conjugacy classes and $s \in$. The complex character table of G published in the Atlas will now be used to locate these constants, that have become available electronically in standard computer algebra package libraries of the Gap system [10].

Now if we let C_1 to be C and C_2, C_2 are equal to X, also let and $s = t$. Thus we have

$$|X_C| = \frac{|G|}{|C_G(t)|\,|C_G(w)|} \sum_{i=1}^{s} \frac{{}_i(w)_i(t)_i(t)}{{}_i(1)}$$

such that w is a representative of the class C and χ_i, where $i = 1, \ldots, s$ are the complex irreducible characters of the finite simple group G.

3 Graph Structures

Let G be one of the following groups $\mathrm{G_2(2)'}, \mathrm{G_2(3)}$ or $\mathrm{G_2(4)}$. Assume that $t \in G$ be an element of order 3. Set $X = t^G$ is a G-conjugacy class of t. The analyzing of $A_4(\mathrm{G}, \mathrm{X})$ structure can be investigated by computing the $\mathrm{C_G}(t)$-orbits as $\mathrm{C_G}(t)$ acting by conjugation on X. These orbits break up the discs of the graph. Thus, the goal is to determine such orbits for each class of elements of order 3 in G. The strategy we are adopting mainly focuses on the computational approach represented by the GAP. In addition, Online Atlas [9] has a very important role in distributing classes on $C_G(t)$ - orbits and provide a representation for each group. In the tables that come, exponential notation is being used to illustrate the multiple of a size. As previously stated, we use class names from the Atlas. We condense the letter component of the class name even more after we're going to merge these classes, and their characters are in alphabetical order to get things easier. For instance, $13AB$ is shortcut to $13A \cup 13B$ in Table 3.3. Moreover, The entry $\{9^2, 9^5, 9^4\}$ in Table **3.1** in the class name 7AB ensures X_{7AB} is a combine of $11 C_G(t)$-orbits with equal size 9, where 2 orbits in $\Delta_2(t)$, 5 in $\Delta_3(t)$ and 4 in $\Delta_4(t)$.

3.1 $A_4\left(\mathbf{G}_2(2)', \mathbf{X}\right)$

The group $\mathrm{G}_2(2)'$ has two classes of order 3 , and $A_4\left(\mathrm{G}_2(2)', \mathrm{X}\right)$ for each class describe as follows:
i. $A_4\left(\mathbf{G}_2(2)', 3 \mathrm{~A}\right)$
We note that for t a fixed element in 3 A, the $\mathrm{C}_G(t)$ $\left(\left(\mathrm{C}_3 \times \mathrm{C}_3\right): \mathrm{C}_3\right): \mathrm{C}_4$ with size of the class is 56 . The number of the $C_G(t)$ - orbits are 4 and split into 4 different classes which are $\mathrm{X}_{1 \mathrm{~A}}, \mathrm{X}_{3 \mathrm{~A}}, \mathrm{X}_{4\mathrm{C}}, \mathrm{X}_{6 \mathrm{~A}}$ and the size of the first two orbits 2 and the next two orbits 27 respectively. The clique number of the graph equal to 1 . Therefore, the A4-graph in this case is totally disconnected.
ii. $A_4\left(\mathrm{G}_2(2)', 3 \mathrm{~B}\right)$
Let t be a random element in the class 3 B. Then $C_G(t)$ $C_3 \times C_3$ and $|3B| = 672$. There are $84 C_G(t)$ - orbits separate into the following X_C classes. In the next table information about the set X_C the discs belong to are given:

Table 1: Discs Structure for the graph $A_4\left(\mathbf{G}_2(2)', 3 \mathrm{~B}\right)$

Class Name	$\Delta_1(t)$	$\Delta_2(t)$	$\Delta_3(t)$	$\Delta_4(t)$	$\Delta_5(t)$
1A				1	
2A			1		
3A					1^2
3B	9		$3^2, 9^4$	9^2	$1^2, 3^4, 9^2$
4AB				9	
4C		9^2	9^2	9^2	
6A			9^4	9^4	
7AB		9^2	9^5	9^4	
8AB				9^8	
12AB			9^2	9^2	

We can see immediately form the above table that $A_4\left(\mathrm{G}_2(2)', 3 \mathrm{~B}\right)$ is connected with diameter 5.

3.2 $A_4\left(\mathbf{G}_2(3), \mathbf{X}\right)$

In this simple group there are five classes of elements of order 3 . The A4-graph for such classes are given with details in the following:
i. $A_4\left(\mathrm{G}_2(\mathbf{3}), 3 \mathrm{~A}\right)$
The class 3A has size 728 and $C_G(t)$ $\left(\left(C_3 \times C_3 \times\left(\left(C_3 \times C_3\right): C_3\right)\right): Q_8\right): C_3$. The number of $C_G(t)$-orbits equal to 6 and the size of the class X_C are given in the next table:

Table 3.2. The $A_4\left(\mathrm{G}_2(3), 3 \mathrm{~A}\right)$ is totally disconnected this because the clique number of the graph is equal to 1.

Table 2: Description of the sets X_{C} in $A_4\left(\mathrm{G}_2(3), 3 \mathrm{~A}\right)$

Class Name	Number of Orbits	Size of the Orbits
1A	1	1
3A	2	25 including t
3D	1	216
4B	1	243
6A	1	243

ii. $A4\left(G_2(3), 3B\right)$
The graph $A_4\left(\mathrm{G}_2(3), 3 \mathrm{~B}\right) \cong A_4\left(\mathrm{G}_2(3), 3 \mathrm{~A}\right)$ then the information about both graphs are identical.
iii. $A_4\left(\mathrm{G}_2(3), 3\mathrm{C}\right)$
In the graph we consider the case when $t \Theta C$ and $X = t^G$. the size of X is 5824 and $C_G(t)$? $\left(\left(C_3 \times\left(\left(C_3 \times C_3\right): C_3\right)\right): C_3\right): C_3$. In the next table full details on how the set X_C break into the $48 C_G(t)$-orbits are provided:

We should note that the clique number of $A_4\left(\mathrm{G}_2(3), 3\mathrm{C}\right)$ equal 1 and hence the A4-graph is totally disconnected.
iv. $A_4\left(\mathrm{G}_2(3), 3\mathrm{D}\right)$
In this case we consider the $A_4\left(\mathrm{G}_2(3), 3\mathrm{D}\right)$ when $t \Theta \mathrm{D}$. The size of the class 3D is 26208 and $C_G(t)$ isomorphic to $C_3 \times C_3 \times\left(\left(C_3 \times C_3\right): C_2\right)$. The number of $C_G(t)$-orbits of the A4graph are 220. In the following table allocation of the sets X_{C} as they break the $\mathrm{C}_G(\mathrm{t})-$ orbits and which $A4-graph$ disc belong to are given below: From the above table the diameter of the $A_4\left(\mathrm{G}_2(3), 3\mathrm{D}\right)$ equal to 4 and thus, the graph is connected.

Table 3: Description of the sets X_C in $A_4(G_2(3), 3C)$

Class Name	Number of Orbits	Orbits discerption	Size of the Orbits
1A	1	1	
3AB	4	$1, 3^2, 27$	34
3C	5	$1, 3^4$	13
3D	8	$9^4, 27^4$	144
3E	6	$9^4, 27^2$	90
4AB	2	$243^2,$	486
6D	4	243^4	972
7A	1	729	729
9A	5	$81^4, 729$	1053
9BC	2	$81^2,$	162
13AB	1	729	729

Table 4: Discs Structure for the graph $A_4(\mathbf{G}_2(3), 3D)$

Class Name	$\Delta_1(t)$	$\Delta_2(t)$	$\Delta_3(t)$	$\Delta_4(t)$
1A				1
2A			81	
3AB			6	2
3C				$2^2, 6^4$
3D	81		$2^2, 6^4, 162^4$	$2^2, 6^2$
3E				6^6
4AB			$54^2, 162^3$	27
6AB		162	$54^2, 162^3$	
6C		$27^2, 162^2$		
6D			$54^4, 162^8$	
7A		162^8	162^{28}	
8AB		162^6	162^{12}	162^2
9A			$18^4, 162^8$	18^2
9BC			$18^4, 162^4$	18^2
12AB			162^6	54^2
13AB		162^2	162^7	

v. $A_4(G_2(\mathbf{3}), 3E)$
The graphs $A_4(G_2(3), 3D)$ and $A_4(G_2(3), 3E)$ are isomorphic. Therefore they have the same structure.

3.3 $A_4(\mathbf{G_2(4), X})$

In the group $G_2(4)$ there are two classes for elements of order 3 . The structure of the $A_4(G_2(4), X)$ explains as follows:
i. $A_4(\mathbf{G}_2(\mathbf{4}), \mathbf{3A})$
The class 3 A has size 4160 and for $t \in A$, we have $C_G(t) \cong NG(2,4)$. Furthermore, there 10 $C_G(t)$-orbits in the graph $A_4(G_2(4), 3\ A)$. The manner of the XC breaking into $C_G(t)$-orbits are listed in the next table and determine which discs be the possession of X_C. The above table shows the dimeter of $A_4(G_2(4), 3\ A)$ equal to 3 and this means the

Table 5: Discs Structure for the graph $A_4(\mathbf{G}_2(4), 3\ A)$

Class Name	$\Delta_1(t)$	$\Delta_2(t)$	$\Delta_3(t)$
1A			1
2B		622	
3A	622		
4A			945
5CD		1008	
6A		945	

graph is connected.
ii. $A_4\left(\mathrm{G}_2(\mathbf{4}), \mathbf{3B}\right)$
Let $t\Theta B$ then $C_G(t) \cong SL(3,4)$ and $|3B| = 1397760$. The $A4$-graph is very interesting in this case because the number of $C_G(t)$-orbits is quite big, indeed is equal to 8552 . Full information about the discs structure of $A_4\left(\mathrm{G}_2(4), 3\text{ B}\right)$ illustrate in the following table: From the above table we can see that $A_4\left(\mathrm{G}_2(4), 3\text{ B}\right)$ is connected with diameter 3.

Table 6: Discs Structure for the graph $A_4\left(\mathbf{G}_2(4), 3B\right)$

	D1	D2	D3
1A			1
2A		$3^2, 15^6$	15^5
2B		180^4	60^2
3A			$20, 60^2$
3B	$3^2, 15^{11}, 60^2, 180^4$	$15^{24}, 60^{12}, 180^{14}$	$20, 60^{117}, 180^8$
4A		$15^8, 60$	$60^{15}, 180^2$
4B		$15^{16}, 60^2, 180^2$	60^{30}
4C		180^4	180^{14}
5AB		$3, 15^{15}, 60^3, 180^4$	$60^{55}, 180^6$
5CD		180^3	$60, 180^{23}$
6A		$60^{14}, 180^{18}$	$60^{10}, 180^{40}$
6B		$60^2, 180^{228}$	180^{454}
7A		$60^{18}, 180^{118}$	$60^{69}, 180^{210}$
8A		180^{114}	180^{142}
8B		180^{60}	180^{164}
10AB		180^{108}	180^{276}
10CD		180^{140}	180^{244}
12A		$60^{22}, 180^{86}$	$60^{26}, 180^{82}$
12BC		$60^{18}, 180^{38}$	$60^{30}, 180^{114}$
13AB		180^{158}	180^{399}
15AB		$60^{37}, 180^{160}$	$60^{110}, 180^{328}$
15CD		$60^1, 180^{154}$	180^{340}
21AB		$60^{18}, 180^{110}$	$60^{69}, 180^{218}$

4 Main Results

We offer some results relating to the A4-graph of the aforementioned group in this part, which may be deduced from the preceding sections.

In the next theorem, the structure and diameters of the $A_4(\mathrm{G}, \mathrm{X})$ discs are stated with full details:

Theorem 4.1. Let G be a particular untwisted groups describe in below. Then the A4graph of G has the following features:

1- The graph $A_4\left(\mathrm{G}_2(2)', 3\text{ A}\right)$ is totally disconnected. Furthermore, for $t \in 3$ A the $\mathrm{C}_G(t)$ separating into 4 orbits and disperse among the set X_C such that $C \in \{1A, 3A, 4C, 6A\}$.

2- The graph $A_4\left(\mathrm{G}_2(2)', 3\text{ B}\right)$ is connected with diameter 5 , such that for $t \in 3$ B we have $|\Delta_1(t)| = 9, |\Delta_2(t)| = 54, |\Delta_3(t)| = 231, |\Delta_4(t)| = 343$ and $|\Delta_5(t)| = 34$.Furtherore, $C_G(t)$ separating into 4 orbits and disperse among the set X_C such that $C \in \{1A, 2A, 3AB, 4ABC, 6A, 7AB, 8AB, 12AB\}$.

3- The graphs $A_4\left(\mathrm{G}_2(3), 3\text{ A}\right)$ and $A_4\left(\mathrm{G}_2(3), 3\text{ B}\right)$ are isomorphic totally disconnected graphs. Furthermore, for $t \in 3A$ the $C_G(t)$ separating into 6 orbits and disperse among the set X_C such that $C \in \{1A, 3A, 3D, 4B, 6A\}$.

4- The graph $A_4\left(\mathrm{G}_2(3), 3C\right)$ is totally disconnected. Furthermore, for $t \in 3C$ the $C_G(t)$ separating into 48 orbits and disperse among the set X_C such that $C \in \{1A, 3ABCDE,\ 4AB, 6D, 7A, 9ABC, 13AB\}$.

5- The graphs $A_4\left(\mathrm{G}_2(3), 3\mathrm{D}\right)$ and $A_4\left(\mathrm{G}_2(3), 3\mathrm{E}\right)$ are isomorphic connected graphs with diameter 4. such that for $t \in 3D$ we have $|\Delta_1(t)| = 81, |\Delta_2(t)| = 4914, |\Delta_3(t)| = 20101$, $|\Delta_4(t)| = 1111$. Furthermore, for $t \in 3D$ the $C_G(t)$ separating into 220 orbits and disperse among the set X_C such that

$$C \in \{1A, 2A, 3ABCDE, 4AB, 6ABCD, 7A, 8AB, 9ABC, 12AB, 13AB\}.$$

6- The graph $A_4\left(\mathrm{G}_2(4), 3\text{ A}\right)$ is connected with diameter 3 , such that for $t \in 3$ A we have $|\Delta_1(t)| = 126, |\Delta_2(t)| = 3087, |\Delta_3(t)| = 946$.Furtherore, $C_G(t)$ separating into 10 orbits and disperse among the set X_C such that $C \in \{1A, 2B, 3A, 4A, 5CD, 6A\}$.

7- The graph $A_4\left(\mathrm{G}_2(4), 3\text{ B}\right)$ is connected with diameter 3 , such that for $t \in 3$ A we have $|\Delta_1(t)| = 1011, |\Delta_2(t)| = 446412, |\Delta_3(t)| = 950336$. Furthermore, $C_G(t)$ separating into 8552 orbits and disperse among the set X_c such that $C \in 1A, 2AB, 3AB, 4ABC, 5ABCD, 6AB, 7A,\ 8AB10ABCD, 12ABC, 13AB, 15ABCD, 21AB\}$.

Proof . The proof of the above theorem follow form graphs structure details which provided in previous section. It should be noted that the general results given in Section 2 are used to secure this information.

For the connected A4-graphs in the Theorem 4.1, the next result determine the girth and the clique number of the graph. The girth and the clique number can be calculated by using the gap YAGS [3]. □

Theorem 4.2. Suppose that G is untwisted groups isomorphic to $G_2(2)', G_2(3)$ or $G_2(4)$.
Then for connected A4-graph we have the following:

1- For the graph $A_4\left(\mathrm{G}_2(2)', 3\text{ B}\right)$ the girth is 3 and the clique number is 4.

2- For the isomorphic graphs $A_4\left(\mathrm{G}_2(3), 3\mathrm{D}\right)$ and $A_4\left(\mathrm{G}_2(3), 3\mathrm{E}\right)$ the girth is 3 and the clique number is 4 .

3- For the graph $A_4\left(\mathrm{G}_2(4), 3\text{ A}\right)$ the girth is 3 and the clique number is 4 .

4- For the graph $A_4\left(\mathrm{G}_2(4), 3\text{ B}\right)$ the girth is 3 and the clique number is 64.

Proof . The proof follow from Thereon 4.1 and the computational calculations can be done by using YAGS.

In the next results the relation between the alternating group A_4 and the $A4$-graph are given for our interesting groups: □

Corollary 4.3. Assume that G is untwisted groups isomorphic to one of the following groups $G_2(2)', G_2(3)$ or $G_2(4)$. Then

1- There no subgroup isomorphic to A_4 can be generated by any two random elements in the following classes 3 A in $\mathrm{G}_2(2)'$, 3ABC in $\mathrm{G}_2(3)$.

2- Let $t \in G$ be an elements of order 3 and $X = t^G$. Then the number of subgroups in G isomorphic to A_4 that can generated by t and random element in X are 9 if $X = 3B$ in $\mathrm{G}_2(2)'$, 81 if $X = 3D$ or $3E$ in $G_2(3)$, 126 if $X = 3A$ or 1011 if $X = 3B$ in $G_2(4)$

Proof . The proof follow immediately from Theorem 4.1 and Lemma 2.1. □

5 Conclusions

The structure of $A4$-graph for the groups $\mathrm{G}_2(2)', \mathrm{G}_2(3)$ or $\mathrm{G}_2(4)$ have been investigated with full details. We utilize the A4-graph to determine the number of subgroups isomorphism to the alternating group $A4$ inside the above groups. Such subgroups generated by elements of order 3 and random elements in the class of this element.

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