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Solving cubic objective function programming problem by modification simplex method

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Abstract

In this paper, a cubic objective programming problem (COPP)is defined, which is in the form of multiplying three linear functions. The simplex method is modified to solve a cubic objective programming problem. An algorithm for its solution is suggested. The algorithm of the usual simplex method is also reported. A vital application talks about how the developed algorithm can be utilized to unravel non-linear. The proposed technique can be illustrated with the constructed numerical examples and it showed steps by tables. The results of the study indicate that the new technique, modified simplex, gets the same result which is exactly similar to other methods such us (quadratic, linear simplex method).

Keywords: New Approach, Cubic Objective Programming Problem, Simplex Method.
2020 MSC: 90C70; 90C33, 90C29, 90C32

1 Introduction

Cubic objective programming problems (COPP), might be specified as a really critical point with respect to nonlinear programming. In expansion, direct programming is exceptionally vital for a few purposes counting (well-being care, generation and etc) arranging. More specifically, in mentioned applications of nonlinear programming, two given portions or functions could be maximised and minimised. The number of methods with provide examples clearly discussed [5] presented a specialization of the convex simplex method to cubic programming. [2] presented a method that's utilized to illuminate a set of nonlinear programming issues by simplex strategy. This technique also makes a difference to supply the arrangement of direct programming problems(Abdulrahim). Nonlinear optimization with financial applicative is been examined by [1]. Also, by utilizing the altered simplex approach and Wolfes strategy QFPP is illuminated by [6]. (sulaiman and Basiya K. Abdulrahim) used two methods to solve the problem one of them is Modified Simplex Method and the other is Feasible Direction Development. (Azara, Shrali and Shetty) presented Nonlinear Programming: Theory and Algorithms. 3rd. the cubic-quartic nonlinear Schrödinger and resonant nonlinear Schrödinger equation in parabolic law media are investigated to obtain the dark, singular, bright-singular combo and periodic soliton solutions by [4]. [8] are studied Soliton solutions of higher-order dispersive cubic-quintic nonlinear. To broaden this work, we considered a unique case issue in which the target capacities are QF (Quadratic partial) however contain direct limitations. The issue will settle by another adjusted simplex strategy. Likewise, the issue

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of the extraordinary case will be tackled by the simplex strategy after converting the target capacity to the pseudo partiality work. The two outcomes will be contrasted with test legitimacy. In order to extend this work, we have defined a COPP and suggested the algorithm solve cubic programming problem which is the objective function as the form multiplying of three linear equations; and proposed a new modification simplex method to find the solution.

2 Some Definition and theorems

2.1 linear programming (LP)

The general linear programming model with n decision variables and m constraints can be stated in the following form. Optimize (max or min) $Z = \sum_{i=1}^{n} C_i t_i$ subject to:

$$a_{11}t_1 + a_{12}t_2 + \ldots + a_{1n}t_n \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} b_1$$
$$a_{21}t_1 + a_{22}t_2 + \ldots + a_{2n}t_n \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} b_2$$
$$\ldots$$
$$a_{n1}t_1 + a_{n2}t_2 + \ldots + a_{nn}t_n \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} b_n, \quad t \geq 0.$$

where $c_1, c_2, \ldots, c_n$ represent the per unit profit (or cost) of decision variables $t_1, t_2, \ldots, t_n$ to the value of the objective function and a_{ij} where $i, j = 1, 2, \ldots, n$ represent the amount of resource consumed per unit of the decision variables. The b_i represents the total availability of the ith resource. Z represent the measure-of-performance which can be either profit, or cost or reverence etc.

2.2 Quadratic Programming

the optimization problems assume that form Max(Min), $Z = a + C^T t + t^T G t$ subject to:

$$At \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} b, \quad t \geq 0.$$

where $A = (a_{ij})_{m\times n}$ Matrix of coefficients, for all $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, $b = (b_1, b_2, \ldots, b_n)^T$, $t = (t_1, t_2, \ldots, t_n)^T$, $Ct = (C_1, C_2, \ldots, C_n)^T$ and $a = (g_{ij})_{n\times n}$ mentioned as a positive semi-definite organized four-sided matrix, also, the objective functions is quadratic and constraints are linear.

So, shown problem could be expressed as a QP problem. for more details, see Suleiman and Nawkhass.

2.3 Nonlinear programming problem

The general non-linear programming problem can be stated in the following form: Optimize (max or min) $Z = f(t_1, t_2, \ldots, t_n)$ subject to

$$g_i(t_1, t_2, \ldots, t_n) \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b_i, \quad t_j \geq 0$$

$i = 1, 2, \ldots, m$ and for all $j = 1, 2, \ldots, n$. where $f(t_1, t_2, \ldots, t_n)$ and $g_i(t_1, t_2, \ldots, t_n)$ are original esteemed function of n choice variables, and one of these expected to be non-linear. Several ways have been established for answering non-linear programming problems. In this article review, we will discuss the methods for solving quadratic programming problems, separable programming problems, geometric programming problems and stochastic programming problems.

Theorem: Fundamental Theorem of LP

The ideal value of the target function in a LP issue exists, at that point that esteem (known as the ideal arrangement) or (optimal solution) should happen at least one of the limit points of the practical area.

3 Mathematical form of COPP

The mathematical form of COPP cubic objective programming problem as follows: $\max Z = \sum_{i=1}^{n} C^T t^p$ subject to:

$$At \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b, \quad t \geq 0.$$

where c is n-dimensional column vector, $p = 1, 2, 3$, A is an $(m \times n)$ matrix and b is an m-dimensional column vector α, β and γ are scalars.

In this paper, the problem that has objective function from as multiplying three linear function is tried to be solved can be represented as follows

$$\max Z = (a_1 t_1 + a_2 t_2 + \alpha)(b_1 t_1 + b_2 t_2 + \beta)(c_1 t_1 + c_2 t_2 + \gamma)$$

Subject to:

$$At \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b, \quad t \geq 0.$$

A is an $m \times n$ matrix, all vectors are assumed to be column vectors unless transposed (T), where t is an n-dimensional column vector of decision variables, c is the n-dimensional column vector of constant, α, β and γ are scalars.

4 The algorithm of standard division technique to solve COPP (cubic objective programing problem)

Below algorithm shown to find the optimal average of maximum and minimum for the COPP as follows:

Step 1: through clarifying and appearing slack and manufactured factors standard shape of the issue can be composed to limitations, and stamp starting simplex table.

Step 2: compute the μ by through below equations $\mu = \min(\frac{VB}{t_j})$.

Step 3: compute the Δ_j by through below equations

$$\Delta_j = (Z_1 \Delta_{j_2} + Z_2 \Delta_{j_1}) + (Z_1 \Delta_{j_3} + Z_3 \Delta_{j_2}) + (Z_2 \Delta_{j_3} + Z_3 \Delta_{j_2}) + \mu \Delta_{j_1} \Delta_{j_2} \Delta_{j_3}$$

Then mark or write computed value in the beginning simplex table.

Step 4: get arrangement of to begin with objective issue through utilizing simplex way.

Step 5: check the reply for attainability in step 4, in case of being doable go to step 6, and in case not, double simplex strategy will be utilizing in order to remove in feasibility.

Step 6: the arrangement for optimality will be check in the event that all $\Delta_j \geq 0$ at that point go to step 7, something else back to step 4.

Step 7: dole out a title to ideal esteem of the greatest objective work Z_i say $\forall i = 1, 2, \ldots, r$ and allot a title to the ideal esteem of the most extreme objective work Z_i where $\forall i = r + 1, r + 2, \ldots, s$.

Step 8: include overall objective functions through repeat procedure from the step 4: for $i = 1, 2, \ldots, s$.

5 Solution for quadratic programming problem by modified simplex method

This portion oversees the course of action of the quadratic programming issue by the methodology accurately like simplex strategy in LP. This way can be successfully balanced to quick computational. They might apply this procedure in the event that the impediments of the issue are straight capacity:

$$\max, Z_1 (\text{or } \min, z) = (C^T t + \alpha)(C^T t + \beta) \tag{5.1}$$

Subject to $At \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} b, \quad t \geq 0.$

i) A is $m \times n$ matrix;

ii) t, c, d are $n \times 1$ column vectors;

iii) b is $m \times I$ column vector;

iv) α, β are scalars and prime (T) denoted then transpose of the vector.

Here it is assumed that $(Cmt + \alpha)(dmt + \beta)$ are positive all feasible solutions, the set of all feasible bounded and closed convex polyhedron.

Also, at least two distinct feasible solutions exist (Suleiman and Nawkhass). To apply the simplex process, first, Δ_{j_1} and Δ_{j_2} require to be discovered from the coefficients of the first and second linear vector of objective function respectively. By using below equation

$$\begin{aligned}
&\Delta_{ji} = C_{ij} - CB_i t_{ij}, \quad i = 1, 2, j = 1, 2, \dots, m+n \\
&Z_1 = CB_1 VB + \alpha \\
&Z_2 = C_{ij} VB + \beta \\
&Z = Z_1 \times Z_2
\end{aligned}$$

And calculate $M_j = \min[\frac{VB}{t_j}]$ for non-basis vector. In this methodology, the formula is explained or defined to discovery Δ_j from $\Delta_{j_1}, \Delta_{j_2}, Z_1, Z_2$ and μ_j following

$$\Delta_j = Z_1 \Delta_{j_2} + Z_2 \Delta_{j_1} + \mu_j \Delta_{j_2}.$$

After solve the (5.1), then solve the $\max z(\text{or } \min z) = Z, Z_3$ when

$$Z = (C_1^T t + \alpha)(C_2^T t + \beta), \quad Z_3 = (C_3^T t + \gamma).$$

Formulation of new method for solving a cubic objective function.

6 Construct Numerical example

Example 6.1. $\max, Z = (2t_1 + t_2)(t_1 - t_2)(3t_1 - 2t_2)$ Subjected to:

$$\begin{aligned}
&8t_1 + 6t_2 \leq 24 \\
&10t_1 + 5t_2 \leq 10 \\
&t_1, t_2 \geq 0.
\end{aligned}$$

Solution. 1) Solve by using linear simplex method
$\max Z_1 = (2t_1 + t_2)$ subjected to:

$$\begin{aligned}
&8t_1 + 6t_2 \leq 24 \\
&10t_1 + 5t_2 \leq 10 \\
&t_1, t_2 \geq 0.
\end{aligned}$$

The optimal solution is $t_1 = 1, t_2 = 0, s_1 = 16, s_2 = 0$ and $\max Z_1 = 2$, $\max Z_2 = (t_1 - t_2)$ subjected to:

$$\begin{aligned}
&8t_1 + 6t_2 \leq 24 \\
&10t_1 + 5t_2 \leq 10 \\
&t_1, t_2 \geq 0.
\end{aligned}$$

The optimal solution is $t_1 = 1, t_2 = 0, s_1 = 16, s_2 = 0$ and $\max Z_2 = 1$, $\max Z_2 = (3t_1 - 2t_2)$ subjected to:

$$\begin{aligned}
&8t_1 + 6t_2 \leq 24 \\
&10t_1 + 5t_2 \leq 10 \\
&t_1, t_2 \geq 0.
\end{aligned}$$

The optimal solution is $t_1 = 1, t_2 = 0, s_1 = 16, s_2 = 0$ and $\max Z_3 = 3$ then $\max Z = 2 \times 1 \times 3 = 6$.

2) Solve by using quadratic method
$\max Z_1 = (2t_1 + t_2)(t_1 - t_2)$ subjected to:

$$8t_1 + 6t_2 \leq 24$$
$$10t_1 + 5t_2 \leq 10$$
$$t_1, t_2 \geq 0.$$

The optimal solution is $t_1 = 2, t_2 = 0, s_1 = 16, s_2 = 0$ and $\max Z = 2$, $\max Z_2 = (t_1 - 2t_2)$ subjected to:

$$8t_1 + 6t_2 \leq 24$$
$$10t_1 + 5t_2 \leq 10$$
$$t_1, t_2 \geq 0.$$

By above The optimal solution (5.1) is $t_1 = 1, t_2 = 0, s_1 = 16, s_2 = 0$ and $\max Z_3 = 3$ then $\max Z = \max Z_1 \times \max Z_2 = 2 \times 3 = 6$.

3) Solving by using a new modified simplex method
$\max Z = (2t_1 + t_2)(t_1 - t_2)(3t_1 - 2t_2)$ subjected to:

$$8t_1 + 6t_2 \leq 24$$
$$10t_1 + 5t_2 \leq 10$$
$$t_1, t_2 \geq 0$$

$\max Z = (2t_1 + t_2)(t_1 - t_2)(3t_1 - 2t_2)$ subjected to:

$$8t_1 + 6t_2 + s_1 = 24$$
$$10t_1 + 5t_2 + s_2 = 10$$
$$t_1, t_2 \geq 0$$

Table 1: First table of modification simplex method for solving cubic objective function

				CB_1	2	1	0	0	
				CB_2	1	1	0	0	
				CB_3	3	-2	0	0	Min ratio
B_v	CB_1	CB_2	CB_3	t_B	t_1	t_2	s_1	s_2	
s_1	0	0	0	24	6	8	1	0	3
s_2	0	0	0	10	10	5	0	1	1
			$z_1 = 0$	Δ_{j_1}	-2	-1	0	0	
			$z_2 = 0$	Δ_{j_2}	-1	1	0	0	
			$z_3 = 0$	Δ_{j_3}	-3	2	0	0	
				μ	1	2	0	0	
				Δ_j	-6	-4	0	0	

The optimal solution is $t_1 = 1, t_2 = 0, s_1 = 16, s_2 = 0$ and $\max Z = 6$.

Example 6.2. $\max Z = t_1(t_1 - t_2)(t_1 + 1)$ subjected to:

$$t_1 + t_2 \leq 2$$
$$-t_1 + t_2 \leq 1$$
$$t_1, t_2 \geq 0.$$

Solution. 1) Solve by linear simplex method
$\max Z_1 = t_1$ subjected to:

$$t_1 + t_2 \leq 2$$
$$-t_1 + t_2 \leq 1$$
$$t_1, t_2 \geq 0.$$

The optimal solution is $t_1 = 2, t_2 = 0, s_2 = 3, s_1 = 0$ and $\max Z_1 = 2$, $\max Z_2 = (t_1 - t_2)$ subjected to:

$$t_1 + t_2 \leq 2$$
$$-t_1 + t_2 \leq 1$$
$$t_1, t_2 \geq 0.$$

The optimal solution is $t_1 = 2, t_2 = 0, s_2 = 3, s_1 = 0$ and $\max Z_2 = 2$, $\max Z_3 = (t_1 + 1)$ subjected to:

$$t_1 + t_2 \leq 2$$
$$-t_1 + t_2 \leq 1$$
$$t_1, t_2 \geq 0.$$

The optimal solution is $t_1 = 2, t_2 = 0, s_2 = 3, s_1 = 0$ and $\max Z_3 = 3$ then $\max Z = 2 \times 2 \times 3 = 12$.

2) Solve by quadratic method
$\max Z = t_1(t_1 - t_2)$ subjected to:

$$t_1 + t_2 \leq 2$$
$$-t_1 + t_2 \leq 1$$
$$t_1, t_2 \geq 0.$$

The optimal solution is $t_1 = 4, t_2 = 0, s_2 = 0, s_2 = 3$ and $\max Z = 4$, $\max Z_2 = (t_1 + 1)$ subjected to:

$$t_1 + t_2 \leq 2$$
$$-t_1 + t_2 \leq 1$$
$$t_1, t_2 \geq 0.$$

By above The optimal solution (5.1) is $t_1 = 1, t_2 = 0, s_1 = 0, s_2 = 3$ and $\max Z_2 = 3$ then $\max Z = \max Z_1 \times \max Z_2 = 4 \times 3 = 12$.

3) Solving by new modified simplex method
$\max Z = t_1(t_1 - t_2)(t_1 + 1)$ subjected to:

$$t_1 + t_2 \leq 2$$
$$-t_1 + t_2 \leq 1$$
$$t_1, t_2 \geq 0$$

Table 2: First table of modification simplex method for solving cubic objective function

				CB_1	1	0	0	0	
				CB_2	1	-1	0	0	
				CB_3	1	0	0	0	Min ratio
B_v	CB_1	CB_2	CB_3	t_B	t_1	t_2	s_1	s_2	
s_1	0	0	0	2	1	1	1	0	2
s_2	0	0	0	1	-1	1	0	1	–
			$z_1 = 0$	Δ_{j_1}	-1	0	0	0	
			$z_2 = 0$	Δ_{j_2}	-1	1	0	0	
			$z_3 = 0$	Δ_{j_3}	-1	0	0	0	
				μ	-1	1	2	1	
				Δ_j	-2	0	0	0	

The optimal solution is $t_1 = 2, t_2 = 0, s_1 = 0, s_2 = 3$ and $\max Z = 12$.

Table 3: Comparison between results of the numerical approaches

		Linear approach	Quadratic	New approach
Example(6.1)	Z_1 2 Z_2 1 Z_3 3	6	6	6
Example(6.2)	Z_1 2 Z_2 2 Z_3 3	12	12	12

In the above table, it is clear that the results obtained in examples. Which solved by modification simplex method is the same results which solved by other methods.

7 Conclusion

A new modification simplex method approach is proposed for solving the Cubic objective programming problem which is in the form of multiplying three linear functions. For treating the problem, the methods: Quadratic programming, linear simplex method and the cubic simplex method are used. An algorithm is suggested for characterizing the solution concept of the (COP) programming problem. Comparisons of these methods are based on the value of the objective function. After solving the numerical examples, we found that $\max Z$ obtained by a new technique is promising.

References

[1] M. Bartholomew-Biggs, *Nonlinear optimization with financial applications*, Springer Science and Business Media, 2006.

[2] K.A. Bhat and A. Ahmed,*Simplex method and non-linear programming*, Int. J. Comput. Sci. Math. **4** (2012), no. 3, 299–303.

[3] M.S. Bazaraa, H.D. Sherali and CM. Shetty, *Nonlinear programming: Theory and algorithms*, John Wiley & Sons, 2013.

[4] W. Gao, HF. Ismael, AM. Husien, H. Bulut and HM. Baskonus *Optical soliton solutions of the cubic-quartic nonlinear Schrödinger and resonant nonlinear Schrödinger equation with the parabolic law*, Appl. Sci. **10** (2020), no. 1, 219.

[5] C. Henin and J. Doutriaux, *A specialization of the convex simplex method to cubic programming*, Riv. Math. Sci. Econ. Soc. **3** (2013), no. 2, 61–72.

[6] N. Suleiman and M. Nawkhass, *Solving quadratic fractional programming problem*, Int. J. Appl. Math. Res. **2** (2013), no. 2, 303–309.

[7] N. Suleiman and M. Nawkhass, *Using standard division to solve multi-objective quadratic fractional progamming problem*, J. Zankoy Sulaimani **18** (2017), no. 3, 157–163.

[8] A.M. Sultan, D. Lu, M. Arshad, H.U. Rehman and M.S. Saleem, *Soliton solutions of higher order dispersive cubic-quintic nonlinear Schrödinger equation and its applications*, Chinese J. Phys. **67** (2020), 405–413.