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Stability and superstability of n-Jordan $*$-homomorphisms in Fréchet locally C^*-algebras

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Abstract

Using fixed point methods, we prove the Hyers-Ulam stability and the superstability of n-Jordan $*$-homomorphisms in Fréchet locally C^*-algebras for the generalized Jensen-type functional equation

$$rf\left(\frac{a+b}{r}\right)+rf\left(\frac{a-b}{r}\right)=2f(a),$$

where r is a fixed real number greater than 1.

Keywords: n-Jordan $*$-homomorphism, Fréchet locally C^*-algebra, Fréchet algebra, fixed point methods, Hyers-Ulam stability
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1 Introduction and preliminaries

The stability of functional equations was first introduced by Ulam [37] in 1940. Assuming G_1 and G_2 to be Banach spaces, Hyers [17] gave a partial solution to *Ulam's problem* for the case of approximate additive mappings. Aoki [2] generalized Hyers' theorem for approximately additive mappings. In 1978, Rassias [35] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences. Rassias' influential paper [35] played a key role in the development of what we call Hyers-Ulam-Rassias stability of functional equations.

Theorem 1.1. [35] Let $f : E \to E'$ be a mapping from a normed vector space E into a Banach space E' subject to the inequality

$$\|f(a+b)-f(a)-f(b)\| \le \epsilon(\|a\|^p+\|b\|^p), \tag{1.1}$$

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for all $a, b \in E$, where $\epsilon > 0$ and $p < 1$ are constants. Then, there exists a unique additive mapping $T : E \to E'$ such that

$$\|f(a) - T(a)\| \leq \frac{2\epsilon}{2 - 2^p}\|a\|^p, \tag{1.2}$$

for all $a \in E$. If $p < 0$, then (1.1) holds for all $a, b \neq 0$, and (1.2) holds for $a \neq 0$. Also, if the function $t \mapsto f(ta)$ from $\mathbb{R}$ into E' is continuous for each fixed $a \in X$, then T is linear.

Rassias' theorem was generalized by Forti [13] and Gavruta [14], who permitted the Cauchy difference to be arbitrarily unbounded. Some results on the stability of single variable functional equations and nonlinear iterative equations can be found in [1, 38]. Isac and Rassias [22] were the first to apply the stability theory of functional equations to new fixed point theorems and their applications.

The concept of n-Jordan homomorphism in complex algebras was introduced by Eshaghi Gordji et al. [7]. Also, see [8, 9, 15, 30]. Jamalzadeh et al. [23] introduced the Hyers-Ulam stability and the superstability of n-Jordan $*$-derivations in Fréchet locally C^*-algebras.

During the last few decades, several stability problems of functional equations have been investigated by many mathematicians. See [5, 10, 11, 12, 18, 19, 21, 24, 25, 26, 27, 29, 31, 32, 33].

The remainder of this section is devoted to some preliminaries which will be needed in what follows.

Definition 1.2. Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a *generalized metric* on X whenever the following hold.

(1) Given $x, y \in X$, $d(x, y) = 0$ if and only if $x = y$.
(2) For all $x, y \in X$, $d(x, y) = d(y, x)$.
(3) For all x, y and z in X, $d(x, z) \leq d(x, y) + d(y, z)$.

Next, we recall a fundamental result of fixed point theory.

Theorem 1.3. ([3, 6]) Let (X, d) be a complete generalized metric space, and $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then for each given element x of X, either

$$d(J^n x, J^{n+1} x) = \infty$$

for all nonnegative integers n, or there exists a positive integer n_0 such that

(1) $d(J^n x, J^{n+1} x) < \infty$, for all $n \geq n_0$;
(2) the sequence $\{J^n x\}$ converges to a fixed point y^* of J;
(3) y^* is the unique fixed point of J in the set $Y = \{y \in X \mid d(J^{n_0} x, y) < \infty\}$;
(4) $d(y, y^*) \leq \frac{1}{1-L} d(y, Jy)$, for all $y \in Y$.

In this paper, we assume that n is an integer greater than 1.

Definition 1.4. ([16]) Let A and B be complex algebras. A $\mathbb{C}$-linear mapping $h : A \to B$ is called an n-Jordan homomorphism if

$$h(a^n) = h(a)^n$$

for all $a \in A$.

Definition 1.5. Let A and B be C^*-algebras. An n-Jordan homomorphism $h : A \to B$ is said to be an n-Jordan $*$-homomorphism if

$$h(a^*) = h(a)^*$$

for all $a \in A$.

Definition 1.6. A topological vector space X is said to be a Fréchet space if

(1) it is complete as a uniform space,

(2) it is locally convex, and
(3) its topology can be induced by a translation invariant metric, that is, a metric $d : X \times X \to \mathbb{R}$ such that $d(x, y) = d(x + a, y + a)$ for all a, x and y in X.

For more detailed definitions of such terminologies, we refer the reader to [10]. Note that a ternary algebra is called a ternary Fréchet algebra whenever it is a Fréchet space with a metric d.

A Fréchet algebra, named after Maurice Fréchet, is a special topological algebra whose topology can be induced by a translation invariant metric. Trivially, every Banach algebra is a Fréchet algebra, as the norm induces a translation invariant metric with respect to which the space is complete.

A locally C^*-algebra is a complete Hausdorff complex $*$-algebra A whose topology is determined by its continuous C^*-seminorms, in the sense that a net $\{a_i\}_{i\in I}$ converges to 0 if and only if the net $\{p(a_i)\}_{i\in I}$ converges to 0, for each continuous C^*-seminorm p on A. See [20, 34]. The set of all continuous C^*-seminorms on A is denoted by $S(A)$. A Fréchet locally C^*-algebra is a locally C^*-algebra whose topology is determined by a countable family of C^*-seminorms. Clearly, any C^*-algebra is a Fréchet locally C^*-algebra.

Given locally C^*-algebras A and B, a morphism of locally C^*-algebras from A to B is a continuous $*$-morphism φ from A to B. An isomorphism of locally C^*-algebras from A to B is a bijective mapping $\varphi : A \to B$ such that φ and φ^{-1} are morphisms of locally C^*-algebras.

Hilbert modules over locally C^*-algebras generalize Hilbert C^*-modules by allowing the inner products to take values in locally C^*-algebras rather than in C^*-algebras.

In this paper, using fixed point methods, we prove the Hyers-Ulam stability and the superstability of n-Jordan $*$-homomorphisms in Fréchet locally C^*-algebras for the following generalized Jensen-type functional equation

$$rf\left(\frac{a+b}{r}\right) + rf\left(\frac{a-b}{r}\right) = 2f(a).$$

2 Stability of n-Jordan $*$-homomorphisms

Lemma 2.1. ([28]) Let A and B be linear spaces, and $f : A \to B$ be an additive mapping such that $f(\mu a) = \mu f(a)$ for all $a \in A$ and all $\mu \in T^1 := \{\lambda \in \mathbb{C} : |\lambda| = 1\}$. Then, the mapping $f : A \to B$ is $\mathbb{C}$-linear.

Theorem 2.1. Let A and B be Fréchet locally C^*-algebras, and $f : A \to B$ be a mapping for which a function $\varphi : A \times A \to [0, \infty)$ exists such that

$$r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a) \le \varphi(a, b), \tag{2.1}$$

$$\|f(a^n) - f(a)^n\| \le \varphi(a, b) \tag{2.2}$$

and

$$\|f(a^*) - f(a)^*\| \le \varphi(a, b), \tag{2.3}$$

for all $\mu \in T^1$ and all $a, b \in A$. If there exists $L < 1$ such that $\varphi(a, b) \le rL\varphi(\frac{a}{r}, \frac{b}{r})$ for all $a, b \in A$, then there exists a unique n-Jordan $*$-homomorphism $h : A \to B$ such that

$$\|f(a) - h(a)\| \le \frac{L}{1-L}\varphi(a, 0), \tag{2.4}$$

for all $a \in A$.

Proof . It follows from $\varphi(a, b) \le rL\varphi(\frac{a}{r}, \frac{b}{r})$ that

$$\lim_{j\to\infty} r^j \varphi(r^j a, r^j b) \le \lim_{j\to\infty} r^j \frac{L^j}{r^j}\varphi(a, b) = 0, \tag{2.5}$$

for all $a, b \in A$. Letting $\mu = 1$ and $b = 0$ in (2.1), we get

$$\left\| rf\left(\frac{a}{r}\right) - f(a) \right\| \le \varphi(a, 0) \tag{2.6}$$

for all $a \in A$. Hence,

$$\left\| \frac{1}{r} f(ra) - f(a) \right\| \le \frac{1}{r}\varphi(ra, 0) \le L\varphi(a, 0) \tag{2.7}$$

for all $a \in A$. Define a generalized metric on the set $X = \{g|\ g : A \to B\}$ by

$$d(h, g) = \inf\{C \in \mathbb{R}^+ : \|g(a) - h(a)\| \le C\varphi(a, 0), \quad \forall a \in A\}.$$

It is easy to show that (X, d) is complete. Now, we define the linear mapping $J : X \to X$ by

$$J(h)(a) = \frac{1}{r} h(ra),$$

for all $a \in A$. By [4, Theorem 3.1],

$$d(J(g), J(h)) \le Ld(g, h)$$

for all $g, h \in X$. It follows from (2.7) that

$$d(f, J(f)) \le L.$$

By Theorem 1.1, J has a unique fixed point in the set $X_1 = \{h \in X : d(f, g) < \infty\}$. Let h be the fixed point of J. Then, h is the unique mapping for which

$$h(ra) = rh(a)$$

holds for all $a \in A$. On the other hand,

$$\lim_{k\to\infty} d(J^k(f), h) = 0.$$

So,

$$\lim_{k\to\infty} \frac{1}{r^k} f(r^k a) = h(a) \tag{2.8}$$

for all $a \in A$. It follows from $d(f, g) \le \frac{1}{1-L} d(f, J(f))$ that

$$d(f, h) \le \frac{L}{1 - L}.$$

This implies (2.4). Also, it follows from (2.1), (2.5) and (2.8) that

$$\begin{aligned} \left\| rh(\frac{a+b}{r}) + rh(\frac{a-b}{r}) - 2h(a) \right\| &= \lim_{k\to\infty} \frac{1}{r^k} \|f(r^{k-1}(a + b)) + f(r^{k-1}(a - b)) - f(r^k a)\| \\ &\le \lim_{k\to\infty} \frac{1}{r^k} \varphi(r^k a, r^k b) = 0, \end{aligned}$$

for all $a \in A$. Hence,

$$rh\left(\frac{a+b}{r}\right) + rh\left(\frac{a-b}{r}\right) = 2h(a)$$

for all $a, b \in A$. Letting $s = \frac{a+b}{r}$ and $t = \frac{a-b}{r}$ in the above equation, we get

$$h(s) + h(t) = \frac{2}{r} h\left(\frac{r(s + t)}{2}\right) \tag{2.9}$$

for all $s, t \in A$. If we let $t = 0$ in (2.9), we get $\frac{r}{2}h(s) = h(\frac{r}{2}s)$. So, $\frac{r}{2}h(t) = h(\frac{r}{2}t)$. Therefore, H is Cauchy additive. Letting $b = a$ in (2.1), we obtain

$$\left\| r\mu f\left(\frac{2a}{r}\right) - 2f(\mu a) \right\| \le \varphi(a, a)$$

for all $a \in A$. This implies that

$$\begin{aligned} \|h(2\mu a) - 2\mu h(a)\| &= \lim_{k\to\infty} \frac{1}{r^k}\|f(2\mu r^k a) - 2\mu f(r^k a)\| \\ &\le \lim_{k\to\infty} \frac{1}{r^k}\varphi(r^k a, r^k a) = 0, \end{aligned}$$

for all $\mu \in T$ and all $a \in A$. By Lemma 2.1, the mapping $h : A \to B$ is $\mathbb{C}$-linear. It follows from (2.2) that

$$\begin{aligned} \|h(a^n) - (h(a)^n)\| &= \lim_{k\to\infty} \left\| \frac{1}{r^{nk}} h(a^n) - \frac{1}{r^{nk}}(h(a))^n \right\| \\ &\le \lim_{k\to\infty} \frac{1}{r^{nk}}\varphi(r^k a, r^k a) \\ &\le \lim_{k\to\infty} \frac{1}{r^{k}}\varphi(r^k a, r^k a) \\ &= 0, \end{aligned}$$

for all $a \in A$. Having (2.3) in mind, we find that

$$\begin{aligned} \|h(a^*) - (h(a)^*)\| &= \lim_{k} \left\| \frac{1}{r^{nk}} h(a^*) - \frac{1}{r^{nk}}(h(a))^* \right\| \\ &\le \lim_{k\to\infty} \frac{1}{r^{nk}}\varphi(r^k a, r^k a) \\ &\le \lim_{k\to\infty} \frac{1}{r^{k}}\varphi(r^k a, r^k a) \\ &= 0, \end{aligned}$$

for all $a \in A$. Thus, $h : A \to B$ is an n-Jordan $*$-homomorphism satisfying (2.4), as desired. □

Now, we prove the Hyers-Ulam stability for n-Jordan $*$-homomorphisms in Fréchet locally C^*- algebras.

Corollary 2.2. Let $p \in (0, 1)$ and $\theta \in [0, \infty)$ be real numbers. Suppose $f : A \to B$ satisfies

$$\left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a) \right\| \le \theta(\|a\|^p + \|b\|^p),$$

$$\|f(a^n) - f(a)^n\| \le 2\theta\|a\|^p$$

and

$$\|f(a^*) - f(a)^*\| \le 2\theta\|a\|^p,$$

for all $\mu \in T$ and $a, b \in A$. Then, there exists a unique n-Jordan $*$-homomorphism $h : A \to B$ such that

$$\|f(a) - h(a)\| \le \frac{2^p\theta}{2 - 2^p},$$

for all $a \in A$.

Proof . Let $\varphi(a, b) := \theta(\|a\|^p + \|b\|^p)$, for all $a, b \in A$, and $L = 2^{p-1}$ in Theorem 2.1 to obtain the desired result. □

3 Superstability of n-Jordan $*$-homomorphisms

In this section, we prove the superstability of n-Jordan $*$-homomorphisms on Fréchet locally C^*-algebras for the generalized Jensen-type functional equation. We need the following lemma to establish our main results.

Lemma 3.1. Suppose that A and B are Fréchet locally C^*-algebras. Let $\theta \geq 0$ and, p and q be real numbers with $q > 0$ and $p + q \neq 1$. If $f : A \to B$ satisfies $f(0) = 0$ and

$$\left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a)\right\| \leq \theta \|a\|^p \|b\|^q \tag{3.1}$$

for all $\mu \in T$ and all $a, b \in A$, then f is $\mathbb{C}$-linear.

Proof . Letting $b = 0$ in (3.1) we obtain

$$f(ra) = rf(a), f(\mu a) = \mu f(a), \tag{3.2}$$

for all $\mu \in T$ and all $a \in A$. Hence, it follows from (3.1) and (3.2) that

$$\|f(a+b) + f(a-b) - 2f(a)\| \leq \theta \|a\|^p \|b\|^q, \tag{3.3}$$

for all $a, b \in A$ with $a \neq 0$. Since $f(r^n a) = r^n f(a)$ for all $a \in A$ and all integers n, we get

$$\|f(a+b) + f(a-b) - 2f(a)\| \leq \theta \left(\frac{r^{p+q}}{r}\right)^n \|a\|^p \|b\|^q,$$

for all integers n and all $a, b \in A$ with $a \neq 0$. Thus,

$$f(a+b) + f(a-b) = 2f(a)$$

for all $a, b \in A$ with $a \neq 0$. Since f is odd, the last equality holds for all $a, b \in A$. Hence, f is Cauchy additive and we conclude that f is $\mathbb{C}$-linear by Lemma 2.1. □

Now, we prove the superstability of n-Jordan $*$-homomorphisms in Fréchet locally C^*-algebras.

Corollary 3.1. Consider $p, s \in \mathbb{R}$ and $\theta, q \in (0, \infty)$ with $p+q \neq 1, s \neq 2$. Let A and B be Fréchet locally C^*-algebras. Suppose that $f : A \to B$ satisfies $f(0) = 0$,

$$\left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a)\right\| \leq \theta \|a\|^p \|b\|^q$$

and

$$\|f(a^n) - f(a)^n\| \leq \theta \|a\|^s,$$

for all $\mu \in T$ and all $a, b \in A$. Then, f is an n-Jordan $*$-homomorphism.

Proof . By Lemma 3.1, f is $\mathbb{C}$-linear. Hence,

$$\|f(a)^n - (f(a))^n\| \leq \theta k^{s-n} \|a\|^s$$

for all integers k and all $a \neq 0$. Therefore, $f(a^n) = f(a)^n$, and the desired result follows. □

Corollary 3.2. Consider $p \in \mathbb{R}$ and $\theta, q \in (0, 1)$ with $p + q \neq 1, 2$. Suppose that A and B are Fréchet locally C^*-algebras. If $f : A \to B$ satisfies $f(0) = 0$ and

$$\max\{\|f(a^*) - (f(a))^*\|, \|f(a^n) - (f(a))^n\|, \left\| r\mu f\left(\frac{a+b}{r}\right) + r\mu f\left(\frac{a-b}{r}\right) - 2f(\mu a)\right\| \} \leq \theta \|a\|^p \|b\|^q$$

for all $\mu \in T$ and all $a, b \in A$, then f is an n-Jordan $*$-homomorphism.

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