Int. J. Nonlinear Anal. Appl. 13 (2022) 2, 1803–1810
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2021.24562.2769

Multikernel optimized beam forming using sparse representation for non-uniform linear array

Mamatha M. C.*, H. C. Sateesh Kumar

Department of Electronics and Communication Engg, Sapthagiri College of Engineering(Affiliated to Visvesvaraya Technological University, Belagavi) Bengaluru, India

(Communicated by Madjid Eshaghi Gordji)

Abstract

Antenna communication uses the sophisticated algorithms for Direction of Arrival estimation in the non uniform -linear array topology. Radar applications uses the mathematical models for Direction of Arrival estimation of the approaching signals. Recent developments in Basis pursuit solver algorithm have led to better beam forming techniques for Direction of Arrival algorithms. Sparse representation of signal helps in better signal analysis. This paper examines how to compute the direction of arrival of non-uniform linear array using sparse computation. A comparison between traditional techniques and sparse representation to estimate the direction of arrival is also studied. A novel method is proposed based on basis pursuit denoising (BPDN) to estimate the Direction of arrival. Simulation results are verified with the formulation developed for direction of arrival. A advanced manifold matrix is developed using the cumulative basis vector as the building element of the manifold matrix. MATLAB based simulation is developed with the advanced basis vector based manifold matrix to get the Direction of Arrival from multiple sources.

Keywords: Sparse representation, Direction of arrival, Non-uniform linear array, Multi-kernel learning, Beam forming
2020 MSC: 65K10, 78M50

1 Introduction

Estimating the Direction of arrival of signal from an array of linear arrangement of antenna is accomplished through to variety of methods. One of the most popular method is the basis pursuit method. In this approach the incoming signal is decomposed into a superposition of dictionary elements. There are several constrains that are inherent in this scenario such as speed and sparsity. Speed is dependent on the representation order i.e. $O(n)$ or $O(nlog(n))$. Sparsity must be as sparse as possible in the representation. Several methods that have been proposed by previous researcher is discussed below. Mathematics involved in the sparse representation implementation is applied in radar applications . Authors discussed a modified method utilizing the multi invariance property of the signal to calculate the direction of arrival of the signal is discussed in [1].This algorithm is a Modified version of Estimation of signal parameters via rotational invariance techniques (ESPRIT). The spatial distribution of the matrix is obtained by measuring the signal frequency and temporal variation from the uniform linear array. Estimating the DOA of a chirp signal using the MI-ESPRIT helps in faster detection and reducing the computational burden. Experimental results of DOA estimation

*Corresponding author
Email addresses: mamatharesearch2021@gmail.com (Mamatha M. C.), hcsateesh@gmail.com (H. C. Sateesh Kumar)

Received: September 2021 *Accepted:* December 2021

using support vector regression (SVR) method shows higher performance compared to both MUSIC and ESPRIT algorithms as outlined in [2].Improved accuracy and minimal computational time is one of the advantages of using SVR modeled on Laplacian distribution.[3] developed a sparse virtual aperture using the atomic norm minimization technique to estimate the direction of arrival. However, computation of virtual aperture without discretizing from co-prime array and coprime samplers increases the computation due to dense grid. Thus, simpler search free method, unitary ESPRIT, was chosen.Mohammed Abdul Hannan, 2016[4] concludes better accuracy of DOA estimation of multiple signals received from linear array using Multi Task Bayesian compression sensing method. The bandwidth of multiple signals extracted from the frequency spectral correlates to better detection of DOA.Underwater target detection using common DOA poses a challenge to the uniform linear array(ULA). A recent review of the literature on this subject found that "A" shaped antenna array has better performance with respect to lower SNR, Low beam width. Yue wang, 2017 [5] has proposed the use of minimum variance distortion less response (MVDR) algorithm on two ULA's distributed at specific angle. The result is great performance improvement in DOA estimation compared to traditional approach.In her analysis of joint DOA and direction of departure (DOD), Tingting Fan [6] has avoided Multidimensional spectrum peak searching algorithm by using ESPRIT-Root MUSIC algorithm. This is because the auto term and noise term is cancelled by using ambiguity algorithm, popular found in the literature on radar signals to estimate the DOA and DOD.Gengxin Ning [7] proposes the use of L shaped antenna array to compensate for the variation in wave velocity. An imprecise wave velocity affects the ability of the system to estimate the DOA. A modified MUSIC algorithm was developed by Genfxin Ning et al. to estimate the DOA.A reduced dimensional MUSIC algorithm was introduced in [8]. The algorithm reduces the computation complexity of MUSIC by reducing the two dimensional search space to single dimensional search space. Despite the reduced computation cost, the algorithm performs close to the 2d MUSIC algorithm.In his investigation into performance of MUSIC and ESPRIT algorithm, B. Vikas [9] shows that ESPRIT algorithm requires minimal computational time with respect to coherent signal. Coherent signal from the liner array is applied with spatial smoothing techniques to estimate the DOA.A method for joint DOD and DOA estimation using MIMO MUSIC algorithm was presented in [10]. Gibbs sampling was applied to the signal to aid in estimating the DOA and DOD. Markov Monte Carlo and MIMO MUSIC algorithm were combined together to estimate the joint DOD and DOA. Although the joint DOD and DOA estimation and DOA results were similar, the computational complexity was relatively higher.Estimation of DOA and DOD using electromagnetic vector sensor array for non-stationary signal was proposed in [11]. However, tracking of only two polarization parameter and one DOA was a limitation. This problem was solved by extending the geometry to a two dimensional rectangular array so as to measure both the azimuthal and elevation angles. This reduced the complexity inherent in 4th dimensional array to 2nd dimensional array, and thus the reduced computational complexity.Xuan Luong Tran in [14] proposed an 8 element ULA based DOA estimation to mitigate the problem of incoherence. DOA estimation fails for non-coherent sub arrays. One drawback of this approach is the increased complexity.A real time implementation of the DOA is computational expensive however Qianli wang in [15] proposes an implementation of alternating direction method of multipliers (ADMM) in place of Interior point method (IPM) for basis pursuit denoising algorithm. The advantage of this approach is the reduction of computation complexity, which is inherent in IPM. Also, ADMM splits the iterative procedure into smaller pieces which is advantageous for parallel processing.In [16] the authors analyzed the use of Artificial neural network (ANN) DOA . The network is trained on the incident values of elevation and azimuthal values. Linear vector quantization (LVQ) is implemented at the input stage to train the ANN. It was also overserved that there was decrease in the computation time if the training set was reduced.Liu Xiaozhi et al. in [17] discusses a new algorithm that incorporates weighted noise subspace method to deal with coherent signal. The received signal from the antenna is transformed into augmented matrix by cross covariance method. Using SVD, the augmented matrix's eigen values and eigen values are obtained. Consistency of the eigen vectors obtained from the SVD output is maintained using the weighted factor matrix and the noise subspace. The advantages of this method is that it works effectively for low SNR and lesser number of snapshots.In [18], the authors discuss about the implementation of mono pulse ratio (MR) based DOA in OFDM. For estimating DOA in OFDM was developed using the cyclic prefix (CP) for DOA. Experimental results of the method was also presented.Yuto Nakajima in [19] describes the use of SAGE algorithm for DOA estimation. The algorithm was shown to work with lesser number of snapshots and better estimation accuracy.Discrete azimuth in spatial grid is computed using orthogonal matching pursuit (OMP) was presented in [20]. This algorithm also called as Majorization minimization (MM) method is utilized to estimate the direction of arrival.The problem of estimating the DOA from multiple sources without bias is studied by Fang An in [21]. Traditional methods like MUSIC and ESPRIT are inadequate for single snapshot and correlated sources in MIMO. Fast iterative interpolation Beam (FIIB) algorithm was implemented, and various scenario of overlapped and non-overlapping virtual array was presented as well.Another method used for DOA in multiple sources is presented in [22]. The authors study the estimation of DOA using complex non negative matrix factorization (CNMF) on spatial covariance matrix. Sum of weighted DOA kernels are utilized to extract the difference in phase and the amplitude between sources for all possible directions.Electronically steerable parasitic array radiator (ESPAR) antenna implantation was proposed in the estimation of DOA by evaluating the received signal strength in [23]. Power pattern cross correlation is presented to find DOA. Doppler Coherent Angle -Of-Interest (AOI) detection along with DOA estimation is introduced on a 77GHz radar sensor was presented in [24]. Specific use case in automated vehicle navigation, environment mapping was also detailed.Histogram based DOA method was discussed in [25]. Recent study on the sparse representation based implementation of the sparse recovery is discussed in [27]. And discussion about the under-determined source identification implementation is discussed in recent publications [28]. By using weighted frequency components reduces the computational complexity and improves resolution. This method of DOA estimation finds uses in noisy or reverberated environment. The method of separation of unique signal from the cross spectrum was detailed.As a solution to the issues with the previous approaches as discussed, an optimized algorithm for non-uniform linear array is presented in this paper. The development and verification of this approach is as follows. First, the mathematical model of signal representation is outlined. A set of equation is derived using the Sparse Bayesian model. These equations are then used in Multikernel basis pursuit denoising model. Finally, the SNR and RMSE of NNSBL and Multikernel is tabulated using the simulation result.

2 Overview Of Beam Forming Algorithm

Omnidirectional antennas of uniform radiation pattern are placed at different distances in linear array arrangement. The distance is integral multiple of half wavelength. By improving the convergence, a consequence is that degree of freedom (DOF) increases. let the distance between the antenna d and M be the number of elements. The difference in co-array is given by.

$$\Omega = d_{m_1} - d_{m_2} \qquad m_1 = 0, 1, ., M-1; m_2 = 0, 1, ...M-1. \tag{2.1}$$

The DOF's increase with M antenna given by Ω. Assume that uncorrelated signals in the far field incident on the antenna, the narrow band source is given by

$$S_n(t), n = 1, 2 \tag{2.2}$$

Introducing noise into the channel, DOA estimation with spatially white Gaussian noise for M antenna is given as:

$$n_m(t), m = 0, 1, ..., \mathit{M\text{-}1}. \tag{2.3}$$

Consider simple signal model given as

$$x(t) = As(t) + n(t) \tag{2.4}$$

where $s(t)$ is a vector containing the transmitting source signal and is a $N \times 1$ vector of zero-mean spatially white sensor noise of variance ${\sigma_1}^2$ and the columns of the steering matrix $A(\theta)$ are the steering or array response vectors. The steering vector of all N sources are grouped in the manifold matrix A.

$$A = [a(\theta_1), \mathrm{a}(\theta_2), \cdots, \mathrm{a}(\theta_N)]. \tag{2.5}$$

The solution space is identified in the grid representation as $\theta = \{\theta_1, \theta_2, ..., \theta_N)\}$. This space spans the entire domain of the incident signals. The basis vector given in the manifold matrix A has the capability to improve the direction of arrival estimation as given in the algorithm given in [26].Grid generated by using the basis vector is utilized for better direction of arrival estimation in the radar communication applications.

$$\hat{Y} \sim C\mathcal{N}(\Phi w + \sigma_n^2 1_M, \tilde{R}_x). \tag{2.6}$$

All the direction in manifold matrix the grid is utilized in Φ and acts as the basis vector for matrix A.This inner product interpolation is the basis for the kernel.The technique discussed here is the Non-negative Sparse Bayesian learning (NNSBL) model using only positive real values. The incident signal is converter into gaussian distributed real values. The equation describing the NNSBL is given as the algorithm thus developed is relevant for both the uniform linear array and the sparse arrays. But this algorithm is better suited for the sparse array. The consideration that the incident signals must be uncorrelated makes it unsuitable for coherent signals. And for uniform array many algorithms are available that solves the DOA estimation. Since it is a sparse array, even if more sources are available compared to the number of antenna elements it can resolve and is particularly well suited for this under-determined condition. It will be discussed in the next section that the simulation results of this algorithm can handle partly correlated source but with degradation in performance. The NNSBL based DOA estimation algorithm uses the matrix Φ. It acts as the over-complete dictionary. This over-complete matrix is generated usually a Gaussian kernel. This kernel is advanced in the proposed algorithm to make it a multi kernel implementation. Using multi-kernel to finding manifold matrix using less time to achieve near to zero of the signals. Its helps in using this is kernel in Gaussian distribution for finding the manifold matrix.

$$k(x, x`) = \sum_{i=1}^{\infty} \Phi_i(x)\Phi_i(x`) = \Phi^T(x)\Phi(x`). \tag{2.7}$$

The manifold matrix thus created for the Sparse Bayesian Learning algorithm is developed using a combination of multiple basis vectors thus it is termed as Multi-kernel based Sparse Bayesian learning method.

3 Results and Discussions

The simulation of Multikernel based sparse learning algorithm in MATLAB was investigated. AWGN channel was used in the estimation of DOA, and comparison between traditional and proposed algorithm is also studied. The simulation parameter for the model developed is shown in table 1.

Details	**Configuration**
Number of Antennas	6
Antenna Array type	Non-uniform
Angle Range	$-\frac{\pi}{3}$ to $\frac{\pi}{3}$
Min to Max degrees	-40 to 40
Carrier frequency	280Hz
Propagation velocity	360
Interval of angle Searching	1
Angles of source signals	-54.8, -28.6 -9.2, 10.5 31.4, 56.7

Table 1: Table of Parameters considered for Proposed Algorithm

Manifold matrix which contains the basis vector of DOA is generated based on the number of source signals incident on the antenna array. The source signal incident on the antenna is shown fig. 2 and the channel noise is AWGN as shown in fig.3. Calculating the DOA from the manifold matrix is obtained by using Bayesian learning algorithm as discusses in previous section. The manifold matrix contains different azimuthal angles for different combination of kernel implementation. The resultant obtained from the multiplication of incident signal and manifold matrix is shown in figure 5. Proposed implementation exploits the stochastic nature of the manifold matrix by introducing different kernels that can better acquire basis pursuit while estimation DOA using the sparse learning algorithms. While NNSBL [26] is a fairly recent algorithm with better robustness, it is adopted in the proposed methodology. Multi-kernel manifold matrix is developed by combining the gaussian kernels iteratively to obtain the Multikernel manifold matrix. The SNR, mean and variance of the noise added to the signal is as given in the Table 2. The proposed algorithm is compared with the traditional and the recent algorithm to validate the performance of the method. The DOA estimation of the same setup with six incident source signals on six antennas for MUSIC algorithm is as given in Fig. 1.

Traditional MUSIC algorithm exhibits lesser sharpness in the estimated DOA beamforming. NNSBL algorithm DOA estimation as shown in Figure 2 shows a sharper DOA estimation compared to the MUSIC algorithm.

From the simulation it is observed that the proposed algorithm with the Multikernel based NNSBL algorithm exhibited sharper DOA estimation compared to the NNSBL algorithm. Figure 3 depicts the Multikernel NNSBL algorithm output. Performance validation of DOA estimation algorithms are carried out using the SNR vs Root Mean Square Error (RMSE) graph. The SNR is varied between -10 to 20db and for different possible decibel between this range the RMSE is estimated. SNR versus RMSE graph for NNSBL DOA estimation is as defined in Figure 4. The

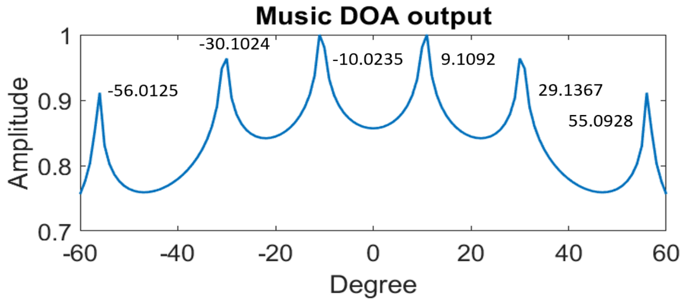

Figure 1: DOA estimation -MUSIC algorithm

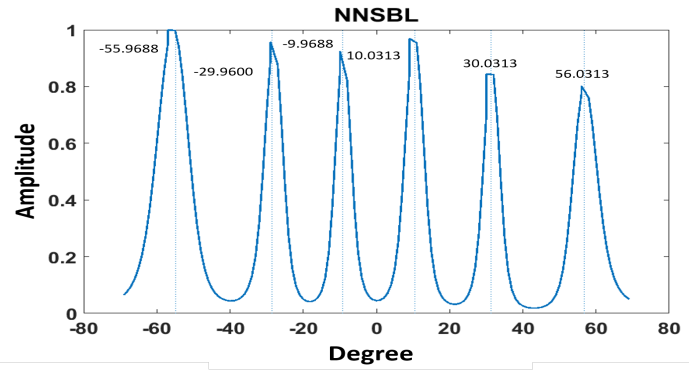

Figure 2: DOA estimation using NNSBL estimation

SNR vs RMSE graph for Multikernel is as given in Fig. 5. For validation of SNR-RMSE values obtained from the traditional MUSIC algorithm with the proposed algorithm the Table 2 is tabulated with the RMSE values for different values of SNR In Table 4, the RMSE values for Multikernel NNSBL algorithm are the lowest for similar SNR of all the algorithms. Considering Different snapshot window from the signal the RMSE is found for a range is SNR. The results are obtained as follows. Figure 11 and 12 are the graphs plotted with different snapshots. The Table 3. Discussed the execution time of each algorithm that indicates the computational complexity of different algorithms.

The RMSE vs SNR comparison for the proposed algorithm with the traditional algorithm is as shown in Table 4. The proposed algorithm of Multi-Kernel NNSBL method performed better than the algorithms that is compared with. MUSIC, NNSBL and MKNNSBL is checked for the best RMSE values and found that MKNNSBL method was found to be dominating.

The proposed algorithm of Multi-Kernel NNSBL method performed better than the algorithms that is compared with. MUSIC, NNSBL[26] and MKNNSBL is checked for the best RMSE values and found that MKNNSBL method was found to be dominating.

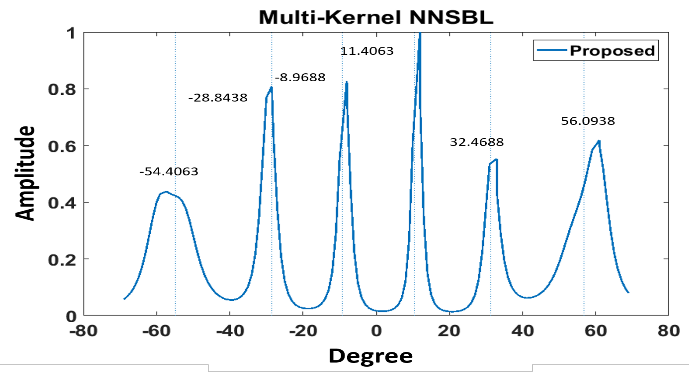

Figure 3: DOA estimation using Multikernel NNSBL estimation

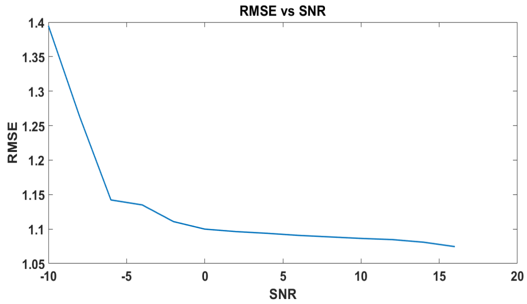

Figure 4: RMSE VS SNR NNSBL Graph

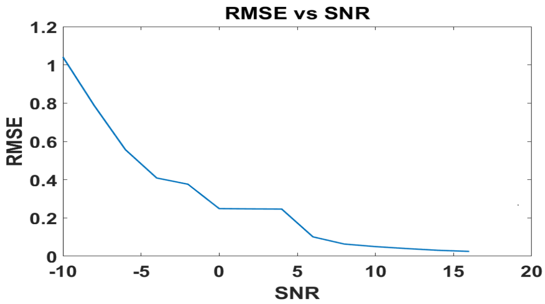

Figure 5: RMSE VS SNR Multikernel NNSBL Graph

Comparison of MUSIC vs NNSBL[26] vs MNNSBL		
SI. No.	Algorithm Type	Compilation Time
1	MUSIC	0.546342 seconds
2	NNSBL	0.448159 seconds
3	Multi-Kernel	0.391914 seconds

Table 2: Comparison Table of MUSIC vs NNSBL[26] vs MNNSBL

Comparison of MUSIC vs NNSBL[26] vs MNNSBL				
SI. No.	SNR	RMSE NNSBL	RMSE MK NNSBL	MUSIC
1	-10	1.3949	1.0410	1.0000
2	-8	1.2630	0.7875	0.7157
3	-6	1.1422	0.5566	0.7098
4	-4	1.1350	0.4088	0.4938
5	-2	1.0886	0.3761	0.4855
6	0	1.0963	0.2473	0.4013
7	2	1.0745	0.2464	0.3034
8	4	1.0864	0.2487	0.2618
9	6	1.1108	0.1012	0.2550
10	8	1.0938	0.0636	0.2516
11	10	1.0999	0.0501	0.2474
12	12	1.0908	0.0399	0.2425
13	14	1.0809	0.0308	0.2239
14	16	1.0847	0.0251	0.2122

Table 3: Comparison Table of MUSIC vs NNSBL[26] vs MNNSBL

4 Conclusion

This work presents the DOA estimation using Multikernel optimization beam forming for non -uniform array. Simulation results show that this algorithm has substantially improved the DOA estimation performance, compared to NNSBL. Novel basis vector generation for sparse learning is incorporated in this research. Theoretical aspects of the algorithm do not vary the basis vector generation but it is developed for proper implementation. However structural changes in the basis vector contribute to a performance variation in the sparse learning methodology. Multikernel basis vector generation increases the stochastic nature of the search algorithm. The stochastic nature of the search algorithm is utilized for the generation of basis vector using principle regularization. The important contribution of this paper is the proposed algorithm can leverage the regularization of sparse learning and greedy approach. The approach of the proposed algorithm is simulated and the results are tabulated. The results of this study suggest that there is a performance improvement in the estimation of direction of arrival.

References

[1] J. Lin, X. Ma, S. Yan and C Hao, *Time-frequency multi-invariance ESPRIT for DOA estimation*, IEEE Anten. Wireless Propag. Lett. **15** (2015), 770–773.

[2] C. Ashok and N. Venkateswaran, *Support vector regression based DOA estimation in heavy tailed noise environment*, Int. Conf. Wireless Commun. Signal Process. Network. (WiSPNET), Chennai, India, IEEE, 2016, pp. 99–102.

[3] J. Pan, C. Zhou, B. Liu and K. Jiang *Joint DOA and Doppler frequency estimation for coprime arrays and samplers based on continuous compressed sensing*, CIE Int. Conf. Radar (RADAR), Yangzhou, Jiangsu Province, China, IEEE, 2016, pp. 1–5.

[4] M.A. Hannan, N. Anselmi, G. Oliveri and P. Rocca, *Joint DoA and bandwidth estimation of unknown signals through single snapshot data and MT-BCS approach*, Int. Symp. Anten. Propag. USNC/URSI Nat. Radio Sci. Meet., San Diego, CA, USA, IEEE, 2017, pp. 2389–2390

[5] J. Shi, Q. Zhang and Y. Wang, *Wideband DOA estimation based on A-shaped array*, Int. Conf. Signal Process. Commun. Comput. (ICSPCC), Xiamen, China, IEEE, 2018, pp. 1–5.

[6] T. Fan, H. Jiang and J. Sun, *Ambiguity function-based ESPRIT-rootmusic algorithm for DOD-DOA estimation in MIMO radar*, Int. Conf. Radar Syst. (Radar), Belfast, IET, 2018, pp. 1–4

[7] G. Ning, B. Wang, C. Zhou and Y. Feng, *A velocity independent MUSIC algorithm for DOA estimation*, Int. Conf. Signal Process. Commun. Comput.(ICSPCC), Xiamen, China, IEEE, 2017, pp. 1-4.

[8] H. Zhao, M. Cai and H. Liu, *Two-dimensional DOA Estimation with reduced dimension MUSIC algorithm*, Int. Appl. Comput. Electromag. Soc. Symp. (ACES), Suzhou, China, IEEE, 2017, pp. 1–2.

[9] B.Vikas and D. Vakula, *Performance comparision of MUSIC and ESPRIT algorithms in presence of coherent signals for DoA estimation*, Int. Conf. Electron. Commun. Aerospace Technol. (ICECA), Coimbatore, India, IEEE, 2017, pp. 403–405.

[10] W. Shi, C. He, J. Huang and Q. Zhang, *Fast MUSIC algorithm for joint DOD-DOA estimation based on Gibbs sampling in MIMO array*, Int. Conf. Signal Process. Commun. Comput. (ICSPCC), Hong Kong, China, IEEE, 2016, pp. 1–4.

[11] Y. Chen, F. Wang, J. Wan and K. Xu, *DOA estimation for long-distance underwater acoustic sources based on signal self-nulling*, 2nd Adv. Inf. Technol. Electron. Autom. Control Conf. (IAEAC), Chongqing, China, IEEE, 2017, pp. 563–567

[12] X. Lan, W. Liu and H.Y.T. Ngan, *Joint 4-D DOA and polarization estimation based on linear tripole arrays*, Int. Conf. Digital Signal Process. (DSP), London, UK, IEEE, 2017, pp. 1–5

[13] A.-T. Nguyen, T. Matsubara and T. Kurokawa, *High-performance DOA estimation for coprime arrays with unknown number of sources*, Asia-Pacific Int. Symp. Electromag. Compat. (APEMC) , Seoul, Korea (South), IEEE, 2017, pp. 369–371.

[14] X.L. Tran, J. Veselý, F. Dvořák and T.B. Le, *SSubarray incoherency compensation for DOA estimation of 8-element antenna array*, Int. Conf. Military Technol. (ICMT), Brno, Czech Republic, IEEE, 2017, pp. 662–665.

[15] Q. Wang, Z. Zhao and Z. Chen, *Fast compressive sensing DOA estimation via ADMM solver*, Int. Conf. Inf. Autom. (ICIA), Macao, China, IEEE, 2017, pp. 53–57.

[16] A. Faye, J.D. Ndaw and A.S. Maiga, *Two-dimensional DOA estimation based on a single uniform linear array*, 25th Telecommun. Forum (TELFOR), Belgrade, Serbia, IEEE, 2017, pp. 1–4.

[17] L. Xiaozhi, S. Muye and Y. Yinghua, *An effective doa estimation method of coherent signals based on reconstruct weighted noise subspace*, 29th Chinese Control Decision Conf. (CCDC), Chongqing, China, IEEE, 2017, pp. 2218–2222.

[18] H. Kim, J. Kim, K.H. Lee and K.S. Kim, *DOA estimation in cyclic prefix OFDM systems in LOS mmWave channel using monopulse ratio*, Int. Conf. Inf. Commun. Technol. Convergence (ICTC), Jeju, Korea (South), IEEE, 2018, pp. 342–345.

[19] Y. Nakajima, N. Kikuma and K. Sakakibara, *Reduction effect of snapshots in DOA estimation using radio holography by SAGE algorithm*, Int. Workshop Electromagnetics: Appl. Student Innov. Compet., Nagoya, Japan, IEEE, 2018, pp. 1–1.

[20] X. Zhang, Y. Li, Y. Yuan, T. Jiang and Y. Yuan, *Low-complexity DOA estimation via OMP and majorization-minimization*, Asia-Pacific Conf. Antennas and Propagation (APCAP), Auckland, New Zealand, IEEE, 2018, pp. 18–19.

[21] F. An, H. Nosrati, E. Aboutanios and A. Hassanien, *Single-snapshot DOA estimation in MIMO radar using fast iterative interpolated beamforming*, 52nd Asilomar Conf. Signals, Syst. Comput. Pacific Grove, CA, USA, IEEE, 2018, pp. 924–928.

[22] J.J. Carabias-Orti1, P. Cabanas-Molero, P. Vera-Candeas and J. Nikunen, *Multi-source localization using a doa kernel based spatial covariance model and complex nonnegative matrix factorization*, 10th Sensor Array and Multichannel Signal Process. Workshop (SAM), Sheffield, UK, IEEE, 2018, pp. 440–444.

[23] M. Groth and L. Kulas, *Accurate PPCC-based doa estimation using multiple calibration planes for WSN nodes equipped with ESPAR antennas*, 15th Eur. Radar Conf. (EuRAD), Madrid, Spain, IEEE, 2018, 545—548.

[24] S. Choi, B. Kim, J. Kim, D. Kim and H. Cho, *Doppler coherent focusing DOA method for efficient radar map generation*, MTT-S Int. Conf. Microwaves Intell. Mobil.(ICMIM), Detroit, MI, USA, IEEE, 2019, pp.1–4.

[25] Y. Yao, T. N. Guo, Z. Chen and C. Fu, *A fast multi-source sound DOA estimator considering colored noise in circular array*, IEEE Sensors J. **19** (2019), 6914–6926.

[26] N. Hu, B. Sun, J. Wang, J. Dai and C. Chang, *Source localization for sparse array using nonnegative sparse Bayesian learning*, Signal Process. **127** (2016), 37–43.

[27] X. He and T. Liu, *Sparse matrix reconstruction based on sequential sparse recovery for multiple measurement mectors*, 6th Int. Conf. Comput. Commun. Syst., Chengdu, China, IEEE, 2021, pp. 480–483.

[28] H. Zhang, Y. Pang, S. Liang and Y. Hao, *Iterative deblending of off-the-grid simultaneous source data*, IEEE Access **9** (2021), 4923–4938.