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Finite-time boundedness of stochastic nonlinear reaction-diffusion systems with time delays and exogenous disturbances via boundary control

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Abstract

This paper investigates an finite-time boundedness of stochastic nonlinear reaction-diffusion systems (SNRDSs) with time delays and exogenous disturbances via boundary control. Both SNRDSs with and without exogenous disturbances are discussed. We design boundary controller is efforts to achieve the required dynamic behaviors of such SNRDSs. By utilizing Lyapunov-Krasovskii functional (LKF), Wirtinger's inequality, Gronwall inequality, and linear matrix inequalities (LMIs), sufficient conditions are derived to guarantee the finite-time bounded of proposed systems. Furthermore, the control gain matrices are defined for desired boundary controller. At last, two numerical examples are provided to demonstrate the efficacy and validity of obtained main results.

Keywords: Stochastic nonlinear systems; Reaction-diffusion terms; Time delays; Exogenous disturbances; Boundary control
2020 MSC: 93C10, 93E03, 93E15

1 Introduction

Recently, partial differential systems have been found to accurately describe a wide range of realistic systems, such as secure communication [32, 17], chemical reaction process [31], oncolytic M1-virotherapy model [10], virus transmission [18] and food web model [35]. An example is the flow of electrons in a non-uniform electromagnetic field. During the movement, the network structure and nonlinear dynamic behavior may change at the same time. Therefore, in the stochastic nonlinear systems [15], neural networks [36], complex-valued neural networks [30], complex dynamical networks [2], complex networks [9], and genetic regulatory networks [14], under study, considerable attention has been given to systems with partial derivatives. As we all known, diffusion effects are unavoidable in the completing this research of both biological and artificial neural networks since electrons flow in a non-uniform field of electromagnetic energy. As a result, the dynamic behaviors of reaction-diffusion systems (RDSs) are receiving more popularity in the study of research, both academically and experimentally [23]–[26].

Meanwhile, unknown disturbances, which can be termed stochastic, have an effect on the delayed real system behavior. In many fields of scientific and technical applications, stochastic processes have played an important role

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and gained a lot of attention over the past few decades. That is to tell, stochastic disturbances have a significant impact on the stability [43]–[8]. Unfortunately, some stochastic reaction-diffusion systems (SRDSs) are inherently unstable. Physically, designing a control function to maintain the stability of SRDSs while the original system is unstable. Due to its low cost, boundary control is a better control strategy. Recently, many authors are investigated the boundary control for RDSs [19]–[16]. Especially, in [42]–[41], the authors are constructed the suitable LKF to discussed the finite-time stability and boundedness of RDSs via boundary control. It is well known that, delays in practical systems cannot be avoided because they can cause systems instability or oscillations. As a result, delays have attracted much interest, and there are a lot of results in the literatures on the stability of RDSs with delays [33]–[4].

To best of our knowledge, few authors little attention for the stabilization and boundedness problems of SNRDSs with time delays via boundary control. Inspired by the preceding discussions, the aim of this paper is to investigate the finite-time boundedness of SNRDSs with time delays and exogenous disturbances using boundary control. The following are the main contributions of this paper: (i) A boundary controller was designed to finite-time boundedness the SNRDSs with time-delays and exogenous disturbances. (ii) By utilizing LKF, Wirtinger's inequality, and Gronwall inequality, sufficient conditions are derived to guarantee the finite-time bounded of SNRDSs. (iii) Our theoretical results reflects the effects of boundary control, reaction-diffusion terms and exogenous disturbances on the finite-time bounded. (iv) Sufficient criteria are presented in LMIs that can be verified by Matlab LMI toolbox.

Notations: $\mathbb{R}$ – set of all real numbers; $\mathbb{R}_+$ – set of all positive real numbers; $\mathbb{R}^n$ – n-dimensional Euclidean space; $\mathbb{R}^{m\times n}$ – $(m\times n)$-dimensional Euclidean space; $A<0$ – real symmetric negative definite matrix; $A>0$ – real symmetric positive definite matrix; B^T – transpose of the matrix B; $\lambda_{\min}(C)$ – minimum eigen value of the matrix C; $\lambda_{\max}(C)$ – maximum eigen value of the matrix C; $*$ – symmetric entries; $He\{B\}=(B+B^T)$; $\|\cdot\|$ – Euclidean norms; $\mathbb{E}(X)$ – mathematical expectation of X; $W^{1,2}([0,\Omega];\mathbb{R}^n)$ – Soblev n-dimensional space of continuous functions; $\int_0^1 \Im^T(x,t)\Im(x,t)dx = \|\Im(x,t)\|^2$.

2 System description and preliminaries

Consider the following SNRDSs with time delays and exogenous disturbances described by

$$\begin{aligned} d\Im(x,t) =& \Big[\mathcal{D}\frac{\partial^2\Im(x,t)}{\partial x^2} + \mathcal{A}\Im(x,t) + \mathcal{B}\Im(x,t-\varrho) + \mathfrak{f}(t,\Im(x,t)) + \mathfrak{g}(t,\Im(x,t-\varrho)) \\ &+ \varphi(x,t)\Big]dt + \sigma(t,\Im(x,t),\Im(x,t-\varrho))d\omega(t),\ x\in(0,1),\ t>0, \end{aligned} \tag{2.1}$$

with the following initial and boundary conditions as:

$$\Im(x,s) = \phi(x,s),\ x\in(0,1),\ s\in[-\varrho,0], \tag{2.2}$$

$$\frac{\partial\Im(x,t)}{\partial x}|_{x=0} = 0,\ \frac{\partial\Im(x,t)}{\partial x}|_{x=1} = u(t), \tag{2.3}$$

where $\Im(x,t)=(\Im_1(x,t),\Im_2(x,t),...,\Im_n(x,t))^T\in\mathbb{R}^n$ is a state vector; $t>0$ is a time variable; $x\in(0,1)$ is a space variable. $\phi(x,s)=(\phi_1(x,s),\phi_2(x,s),...,\phi_n(x,s))^T\in\mathbb{R}^n$ is a continuous initial functions. $u(t)=(u_1(t),u_2(t),...,u_n(t))^T\in\mathbb{R}^n$ is the boundary input control can be designed later. $\varphi(x,t)=(\varphi_1(x,t),\varphi_2(x,t),...,\varphi_n(x,t))^T\in\mathbb{R}^n$ is a exogenous disturbance and satisfying $\int_0^1\int_0^T \varphi^T(x,t)\varphi(x,t)dtdx \le d$ $(d\ge 0)$. $\mathcal{D}$ is a positive definite diffusion matrix. $\mathfrak{f},\mathfrak{g}:\mathbb{R}_+\times\mathbb{R}^n\to\mathbb{R}^n$ and $\sigma:\mathbb{R}_+\times\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^{n\times m}$ are the continuous nonlinear functions. $\omega(t)$ is a Brownian motion of m-dimensions. ϱ is a time delays. $\mathcal{A}$ and $\mathcal{B}$ are constant matrices with suitable dimensions.

Assumption $(\mathcal{H}_1)$**:** There exist a nonnegative constants α_1 and α_2 such that

$$\begin{aligned} (f(l)-f(m))^T(f(l)-f(m)) \le& \alpha_1(l-m)^T(l-m), \\ (g(l)-g(m))^T(g(l)-g(m)) \le& \alpha_2(l-m)^T(l-m),\quad \forall\, l,m\in\mathbb{R}^n. \end{aligned}$$

Assumption $(\mathcal{H}_2)$**:** There exist a nonnegative constants β_1 and β_2 such that

$$\begin{aligned} &trace[(\rho(l_1,l_2)-\rho(m_1,m_2))^T(\rho(l_1,l_2)-\rho(m_1,m_2))] \\ &\quad \le \beta_1(l_1-m_1)^T(l_1-m_1) + \beta_2(l_2-m_2)^T(l_2-m_2),\quad \forall\, l_1,l_2,m_1,m_2\in\mathbb{R}^n. \end{aligned}$$

Lemma 2.1. [43] The following matrix inequality applies to any real matrices M, N and one positive definite matrix R:

$$M^T N + N^T M \leq M^T R^{-1} M + N^T R N.$$

Lemma 2.2. [6] Let $\rho \in \mathbb{R}$ and $\kappa \in \mathbb{R}_+$ be a constants. If there is a function $x(\cdot)$ that meets the criteria,

$$x(t) \leq \rho + \int_b^t \kappa x(s) ds, \quad b \leq t \leq c,$$

then one must satisfying

$$x(t) \leq \rho e^{\kappa(t-b)}.$$

Lemma 2.3. [16] For a state vector $\Theta \in W^{1,2}([0, \Omega] : \mathbb{R}^n)$ with $\Theta(0) = 0$ or $\Theta(\Omega) = 0$ and a positive matrix M, we have

$$\int_0^\Omega \Theta^T(s) M \Theta(s) ds \leq \frac{4\Omega^2}{\pi^2} \int_0^\Omega \Big(\frac{d\Theta(s)}{ds}\Big)^T M \Big(\frac{d\Theta(s)}{ds}\Big) ds.$$

Lemma 2.4. [43] Let $\Gamma_1, \Gamma_2, \Gamma_3$ be given matrices such that $\Gamma_1^T = \Gamma_1$ and $\Gamma_2^T = \Gamma_2 > 0$, then

$$\Gamma_1 + \Gamma_3^T \Gamma_2^{-1} \Gamma_3 < 0 \Leftrightarrow \begin{bmatrix} \Gamma_1 & \Gamma_3^T \\ * & -\Gamma_2 \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -\Gamma_2 & \Gamma_3 \\ * & \Gamma_1 \end{bmatrix} < 0.$$

Definition 2.5. [15] Given three positive constants z_1, z_2 and $\mathcal{T}$ with $z_1 < z_2$, SNRDSs (2.1) without exogenous disturbance $\varphi(x,t)$ is said to be finite-time stable (FTS) with respect to $(z_1, z_2, \mathcal{T})$ if for any given initial conditions satisfying

$$\mathbb{E}\Big\{ \sup_{s\in[-\varrho,0]} ||\Im(x,s)||^2 \Big\} \leq z_1 \Rightarrow \mathbb{E}||\Im(x,t)||^2 < z_2, \ \forall\, t \in [0, \mathcal{T}].$$

Definition 2.6. [3] Given four positive constants $z_1, z_2, \mathcal{T}$ and d with $z_1 < z_2$, SNRDSs (2.1) with exogenous disturbance $\varphi(x,t)$ is said to be finite-time bounded (FTB) with respect to $(z_1, z_2, \mathcal{T}, d)$ if for any given initial conditions satisfying

$$\mathbb{E}\Big\{ \sup_{s\in[-\varrho,0]} ||\Im(x,s)||^2 \Big\} \leq z_1 \Rightarrow \mathbb{E}||\Im(x,t)||^2 < z_2, \ \forall\, t \in [0, \mathcal{T}].$$

3 Main results

In this section, we obtain the finite-time boundedness and stabilization results for SNRDSs via boundary control. The boundary controller is proposed by

$$u(t) = \mathcal{K} \int_0^1 \Im(x,t) dx. \tag{3.1}$$

where $\mathcal{K}$ is a control gain matrix will be designed later.

Theorem 3.1. Under Assumptions $(\mathcal{H}_1)$ and $(\mathcal{H}_2)$, the SNRDSs (2.1) is FTB with respect to $(z_1, z_2, \mathcal{T}, d)$ if there exist symmetric positive definite matrices $P, Q, R_q (q = 1, 2, 3)$ and a constant $\kappa > 0$ such that the following inequalities holds:

$$(i) \ \ \Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ * & \Pi_{22} & 0 \\ * & * & \Pi_{33} \end{bmatrix} < 0, \tag{3.2}$$

$$(ii) \ \ e^{\kappa \mathcal{T}} z_1 \Big[\lambda_{\max}(P) + \varrho e^{\kappa\varrho} \lambda_{\max}(Q) + \lambda_{\max}(R_3) d \Big] < z_2 \lambda_{\min}(P), \tag{3.3}$$

where

$$\Pi_{11} = He(PA + DPK) + Q + \alpha_1 R_1 + \lambda_{\max}(P)\beta_1 - \kappa P + PR_1^{-1}P + PR_2^{-1}P + PR_3^{-1}P,$$

$$\Pi_{12} = -K^T P^T \mathcal{D}^T, \ \Pi_{13} = PB, \ \Pi_{22} = -\frac{1}{2}\pi^2 P\mathcal{D}, \ \Pi_{33} = -Qe^{\kappa\varrho} + \alpha_2 R_2 + \lambda_{\max}(P)\beta_2.$$

Proof. Let us construct the following LKF candidate as:

$$V(t,\Im(x,t)) = \sum_{p=1}^{2} V_p(t,\Im(x,t)), \tag{3.4}$$

where

$$\begin{aligned} V_1(t,\Im(x,t)) &= \int_0^1 \Im^T(x,t)P\Im(x,t)dx, \\ V_2(t,\Im(x,t)) &= \int_0^1 \int_{t-\varrho}^{t} e^{\kappa(t-s)}\Im^T(x,s)Q\Im(x,s)dsdx. \end{aligned}$$

Calculating the derivative of $V(t,\Im(x,t))$ with the SNRDSs (1), we obtain that

$$\mathcal{L}V(t,\Im(x,t)) = \mathcal{L}V_1(t,\Im(x,t)) + \mathcal{L}V_2(t,\Im(x,t)). \tag{3.5}$$

Further, we have

$$\begin{aligned} \mathcal{L}V_1(t,\Im(x,t)) =& 2\int_0^1 \Im^T(x,t)P\Big[\mathcal{D}\frac{\partial^2\Im(x,t)}{\partial x^2} + \mathcal{A}\Im(x,t) + \mathcal{B}\Im(x,t-\varrho) + \mathfrak{f}(t,\Im(x,t)) \\ &+ \mathfrak{g}(t,\Im(x,t-\varrho)) + \varphi(x,t)\Big]dx + \int_0^1 trace\Big[\sigma^T(t)P\sigma(t)\Big]dx \\ &- \kappa\int_0^1 \Im^T(x,t)P\Im(x,t)dx + \kappa V_1(t,\Im(x,t)), \end{aligned} \tag{3.6}$$

where $\sigma(t) = \sigma(t,\Im(x,t),\Im(x,t-\varrho))$.

$$\begin{aligned} \mathcal{L}V_2(t,\Im(x,t)) =& \kappa\int_0^1\int_{t-\varrho}^{t} e^{\kappa(t-s)}\Im^T(x,s)Q\Im(x,s)dsdx + \int_0^1 \Im^T(x,t)Q\Im(x,t)dx \\ &- \int_0^1 e^{\kappa\varrho}\Im^T(x,t-\varrho)Q\Im(x,t-\varrho)dx \\ =& \kappa V_2(t,\Im(x,t)) + \int_0^1 \Im^T(x,t)Q\Im(x,t)dx \\ &- \int_0^1 e^{\kappa\varrho}\Im^T(x,t-\varrho)Q\Im(x,t-\varrho)dx. \end{aligned} \tag{3.7}$$

According to Lemma 2.1 and Assumption $(\mathcal{H}_1)$, we obtain

$$\begin{aligned} 2\Im^T(x,t)P\mathfrak{f}(t,\Im(x,t)) \leq& \Im^T(x,t)PR_1^{-1}P\Im(x,t) + \mathfrak{f}^T(t,\Im(x,t))R_1\mathfrak{f}(t,\Im(x,t)) \\ \leq& \Im^T(x,t)PR_1^{-1}P\Im(x,t) + \Im^T(x,t)\alpha_1R_1\Im(x,t), \end{aligned} \tag{3.8}$$

similarly

$$2\Im^T(x,t)P\mathfrak{g}(t,\Im(x,t-\varrho)) \leq \Im^T(x,t)PR_2^{-1}P\Im(x,t) + \Im^T(x,t-\varrho)\alpha_2R_2\Im(x,t-\varrho), \tag{3.9}$$

$$2\Im^T(x,t)P\varphi(x,t) \leq \Im^T(x,t)PR_3^{-1}P\Im(x,t) + \lambda_{\max}(R_3)\varphi^T(x,t)\varphi(x,t). \tag{3.10}$$

Based on Assumption $(\mathcal{H}_2)$, we have

$$trace\Big[\sigma^T(t)P\sigma(t)\Big] \leq \lambda_{\max}(P)\Big[\Im^T(x,t)\beta_1\Im(x,t) + \Im^T(x,t-\varrho)\beta_2\Im(x,t-\varrho)\Big]. \tag{3.11}$$

By using the Neumann boundary condition (2.3) and integration by parts, we obtain that

$$\begin{aligned} \int_0^1 \Im^T(x,t)\mathcal{D}\frac{\partial^2\Im(x,t)}{\partial x^2}dx =& \Big[\Im^T(x,t)\mathcal{D}\frac{\partial\Im(x,t)}{\partial x}\Big]_{x=0}^{x=1} - \int_0^1 \frac{\partial\Im^T(x,t)}{\partial x}\mathcal{D}\frac{\partial\Im(x,t)}{\partial x}dx \\ =& \int_0^1 \Im^T(1,t)\mathcal{D}\mathcal{K}\Im(x,t)dx - \int_0^1 \frac{\partial\Im^T(x,t)}{\partial x}\mathcal{D}\frac{\partial\Im(x,t)}{\partial x}dx. \end{aligned} \tag{3.12}$$

To obtain $\bar{\Im}(1,t)=0$, for create new state variable $\bar{\Im}(x,t)=\Im(x,t)-\Im(1,t)$ and satisfied the following conditions:

$$\frac{\partial \Im^T(x,t)}{\partial x}\mathcal{D}\frac{\partial \Im(x,t)}{\partial x}=\frac{\partial \bar{\Im}^T(x,t)}{\partial x}\mathcal{D}\frac{\partial \bar{\Im}(x,t)}{\partial x}. \tag{3.13}$$

Applying Lemma 2.3, we have

$$\begin{aligned}\int_0^1 \Im^T(x,t)\mathcal{D}\frac{\partial^2 \Im(x,t)}{\partial x^2}dx \leq& \int_0^1 \Im^T(1,t)\mathcal{D}\mathcal{K}\Im(x,t)dx-\frac{1}{4}\pi^2\int_0^1 \bar{\Im}^T(x,t)\mathcal{D}\bar{\Im}(x,t)dx\\ \leq& \int_0^1 \Im^T(x,t)\mathcal{D}\mathcal{K}\Im(x,t)dx-\int_0^1 \bar{\Im}^T(x,t)\mathcal{D}\mathcal{K}\Im(x,t)dx\\ &-\frac{1}{4}\pi^2\int_0^1 \bar{\Im}^T(x,t)\mathcal{D}\bar{\Im}(x,t)dx.\end{aligned} \tag{3.14}$$

Combining the inequalities (3.5)–(3.14), we have

$$\mathcal{L}V(t,\Im(x,t))\leq \int_0^1 \xi^T(x,t)\Pi\xi(x,t)dx+\lambda_{\max}(R_3)\int_0^1 \varphi^T(x,t)\varphi(x,t)dx+\kappa V(t,\Im(x,t)) \tag{3.15}$$

where

$$\xi(x,t)=\Big[\Im(x,t)\ \ \bar{\Im}(x,t)\ \ \Im(x,t-\varrho)\Big]^T.$$

Based on the condition (3.2), we have

$$\mathcal{L}V(t,\Im(x,t))\leq \lambda_{\max}(R_3)\int_0^1 \varphi^T(x,t)\varphi(x,t)dx+\kappa V(t,\Im(x,t)),\ \forall\ t\in[0,\mathcal{T}]. \tag{3.16}$$

Then, by using mathematical expectation, we have

$$\begin{aligned}\mathcal{D}^+\mathbb{E}V(t,\Im(x,t))=\mathbb{E}\mathcal{L}V(t,\Im(x,t))\leq&\lambda_{\max}(R_3)\mathbb{E}\Big\{\int_0^1 \varphi^T(x,t)\varphi(x,t)dx\Big\}\\ &+\kappa\mathbb{E}V(t,\Im(x,t)),\ \forall\ t\in[0,\mathcal{T}].\end{aligned} \tag{3.17}$$

Integrating from 0 to t and according to the Lemma 2.2, we have

$$\begin{aligned}\mathbb{E}V(t,\Im(x,t))\leq&\mathbb{E}V(0,\Im(x,0))+\lambda_{\max}(R_3)\mathbb{E}\Big\{\int_0^1\int_0^t \varphi^T(x,t)\varphi(x,t)dtdx\Big\}\\ &+\int_0^t \kappa\mathbb{E}V(t,\Im(x,t))\\ \leq&e^{\kappa t}\Big[\mathbb{E}V(0,\Im(x,0))+\lambda_{\max}(R_3)d\Big],\ \forall\ t\in[0,\mathcal{T}].\end{aligned} \tag{3.18}$$

From the definition of $V(t,\Im(x,t))$, we obtain that

$$\begin{aligned}\mathbb{E}V(0,\Im(x,0))=&\mathbb{E}\Big\{\int_0^1 \Im^T(x,0)P\Im(x,0)dx+\int_0^1\int_{-\varrho}^0 \Im^T(x,s)Q\Im(x,s)dsdx\Big\}\\ \leq&\Big[\lambda_{\max}(P)+\varrho e^{\kappa\varrho}\lambda_{\max}(Q)\Big]\mathbb{E}\Big\{\sup_{s\in[-\varrho,0]}\|\Im(x,s)\|^2\Big\}.\end{aligned} \tag{3.19}$$

Also, we have

$$\mathbb{E}V(t,\Im(x,t))\geq \lambda_{\min}(P)\mathbb{E}\Big\{\int_0^1 \Im^T(x,t)\Im(x,t)dx\Big\}=\lambda_{\min}(P)\mathbb{E}\|\Im(x,t)\|^2. \tag{3.20}$$

Combining the inequalities (3.18)–(3.20), we have

$$\begin{aligned}\mathbb{E}\|\Im(x,t)\|^2\leq&\frac{e^{\kappa t}}{\lambda_{\min}(P)}\Big[\lambda_{\max}(P)+\varrho e^{\kappa\varrho}\lambda_{\max}(Q)+\lambda_{\max}(R_3)d\Big]\\ &\times\mathbb{E}\Big\{\sup_{s\in[-\varrho,0]}\|\Im(x,s)\|^2\Big\},\ \forall\ t\in[0,\mathcal{T}].\end{aligned} \tag{3.21}$$

Considering inequality (3.3), when the following initial condition holds:

$$\mathbb{E}\Big\{\sup_{s\in[-\varrho,0]}\|\Im(x,s)\|^2\Big\} < z_1,$$

it implies immediately that $\mathbb{E}\|\Im(x,t)\|^2 < z_2,\ \forall\ t \in [0,\mathcal{T}]$. According to the Definition 2.6, the SNRDSs (2.1) is FTB with respect to $(z_1, z_2, \mathcal{T}, d)$. The proof is completed.

Remark 3.2. In SNRDSs (2.1), if the exogenous disturbances $\varphi(x,t) = 0$, then SNRDSs (2.1) is turns into the SNRDSs with time delays described by

$$\begin{aligned} d\Im(x,t) =& \Big[\mathcal{D}\frac{\partial^2\Im(x,t)}{\partial x^2} + \mathcal{A}\Im(x,t) + \mathcal{B}\Im(x,t-\varrho) + \mathfrak{f}(t,\Im(x,t)) + \mathfrak{g}(t,\Im(x,t-\varrho))\Big]dt \\ &+ \sigma(t,\Im(x,t),\Im(x,t-\varrho))d\omega(t),\ x \in (0,1),\ t > 0. \end{aligned} \tag{3.22}$$

The following corollary follows from Theorem 3.1.

Corollary 3.3. Under Assumptions $(\mathcal{H}_1)$ and $(\mathcal{H}_2)$, the SNRDSs (3.22) is FTS with respect to $(z_1, z_2, \mathcal{T})$ if there exist symmetric positive definite matrices $P, Q, R_q (q = 1, 2)$ and a constants $\kappa > 0$ such that the following inequalities holds:

$$(iii)\ \ \Sigma = \begin{bmatrix} \Sigma_{11} & \Pi_{12} & \Pi_{13} \\ * & \Pi_{22} & 0 \\ * & * & \Pi_{33} \end{bmatrix} < 0, \tag{3.23}$$

$$(iv)\ \ e^{\kappa\mathcal{T}} z_1\Big[\lambda_{\max}(P) + \varrho e^{\kappa\varrho}\lambda_{\max}(Q)\Big] < z_2\lambda_{\min}(P), \tag{3.24}$$

where

$$\Sigma_{11} = He(PA + DPK) + Q + \alpha_1 R_1 + \lambda_{\max}(P)\beta_1 - \kappa P + PR_1^{-1}P + PR_2^{-1}P,$$

and $\Pi_{12}, \Pi_{13}, \Pi_{22}, \Pi_{33}$ are defined in Theorem 3.1.

The following theorem states that the control gain matrix $\mathcal{K}$ for the boundary controller (3.1) can be designed to obtain the FTB of SNRDSs (2.1) with exogenous disturbances.

Theorem 3.4. Under Assumptions $(\mathcal{H}_1)$ and $(\mathcal{H}_2)$, the SNRDSs (2.1) is FTB with respect to $(z_1, z_2, \mathcal{T}, d)$ if there exist symmetric positive definite matrices $P, Q, R_q (q = 1, 2, 3)$, constant matrix L, and constant $\kappa > 0$ such that satisfied the following inequalities (3.3) and

$$(v)\ \ \hat{\Pi} = \begin{bmatrix} \Xi & \Phi \\ * & \Pi_{33} \end{bmatrix} < 0, \tag{3.25}$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & P & P & P & -L^T D^T \\ * & -R_1 & 0 & 0 & 0 \\ * & * & -R_2 & 0 & 0 \\ * & * & * & -R_3 & 0 \\ * & * & * & * & \Pi_{22} \end{bmatrix},\ \Phi = \begin{bmatrix} PB & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$
$$\Xi_{11} = He(PA + DL) + Q + \alpha_1 R_1 + \lambda_{\max}(P)\beta_1 - \kappa P,$$

and Π_{22}, Π_{33} are defined in Theorem 3.1. Moreover, the control gain matrix is designed by

$$(vi)\ \ \mathcal{K} = LP^{-1}. \tag{3.26}$$

Proof. The proof of Theorem 3.4 follows from Lemma 2.4 and Theorem 3.1.

The following corollary states that the control gain matrix $\mathcal{K}$ for the boundary controller (3.1) can be designed to obtain the FTS of SNRDSs (2.1) without exogenous disturbances.

Corollary 3.5. Under Assumptions $(\mathcal{H}_1)$ and $(\mathcal{H}_2)$, the SNRDSs (3.22) is FTS with respect to $(z_1, z_2, \mathcal{T})$ if there exist symmetric positive definite matrices $P, Q, R_q(q = 1, 2)$, constant matrix L and constant $\kappa > 0$ such that satisfied the following inequalities (3.24) and

$$(vii)\ \ \hat{\Sigma} = \begin{bmatrix} \Gamma & \Psi \\ * & \Pi_{33} \end{bmatrix} < 0, \tag{3.27}$$

where

$$\Gamma = \begin{bmatrix} \Xi_{11} & P & P & -L^T D^T \\ * & -R_1 & 0 & 0 \\ * & * & -R_2 & 0 \\ * & * & * & \Pi_{22} \end{bmatrix},\ \Psi = \begin{bmatrix} PB & 0 & 0 & 0 \end{bmatrix}^T,$$

and Π_{22}, Π_{33} are defined in Theorem 3.1 and Ξ_{11} is defined in Theorem 3.4. Moreover, the control gain matrix is designed by (3.26).

Remark 3.6. The obtained results in this paper extended with improve the results in [15]. In [15], the authors discussed the FTS of SRDSs with Markovian switching via boundary control. In this paper, we discussed the both FTS and FTB for SNRDSs with time delays and exogenous disturbances via boundary control.

Remark 3.7. In [15], the author investigated the finite-time stabilization for SRDSs with Markovian switching via boundary control. In [20], the author investigated the mean square finite-time boundary stabilization and H_∞ boundary control for SRDSs. In [40], the author investigated the finite-time boundary stabilization of RDSs. In [41], the author investigated the finite-time boundary control for delay RDSs. However, for the finite-time boundedness of SNRDSs with time delays and exogenous disturbances via boundary control, no exogenous disturbances related results can be found in previous works. To shorten the gap, this paper investigated the finite-time boundedness of SNRDSs with time delays and exogenous disturbances via boundary control.

Remark 3.8. In [19], the author discussed the exponential stabilization of RDSs via intermittent boundary control. In [40], the author discussed the finite-time boundary stabilization of reaction-diffusion systems. In [41], the author discussed the finite-time boundary control for delay RDSs. In [37], the author discussed the finite-time stability of impulsive RDSs with and without time delay. However, those authors have dealt with the stability and stabilization problems without stochastic perturbations. In fact, noise presented a fundamental issue in the transmission of information impacting all facets of the neuron systems operating within the neuron systems. It is worth noting that, stochastic perturbations is introduced into the systems, which may be more suitable to addressing a practical situations. Hence, it is also important to study the effects of stochastic perturbations existing in RDSs.

Remark 3.9. In Theorem 3.4, we established the FTB result of SNRDSs (2.1) with time delays and exogenous disturbances, it is expressed in terms of LMIs. It should be pointed out that the control gain matrix $\mathcal{K}$ is more flexible for the feasibility of (3.3) and (3.25) than those designed boundary controller in the literature [31, 13, 21, 22, 16, 20, 40, 41].

Remark 3.10. In SNRDSs (2.1), if the stochastic disturbances $\sigma(t) = 0$, then SNRDSs (2.1) is turns into the nonlinear reaction-diffusion systems (NRDSs) with time delays and exogenous disturbances described by

$$\begin{aligned}\frac{\partial \Im(x,t)}{\partial t} =& \mathcal{D}\frac{\partial^2 \Im(x,t)}{\partial x^2} + \mathcal{A}\Im(x,t) + \mathcal{B}\Im(x,t-\varrho) + \mathfrak{f}(t,\Im(x,t)) + \mathfrak{g}(t,\Im(x,t-\varrho)) \\ &+ \varphi(x,t)\ x \in (0,1),\ \ t > 0.\end{aligned} \tag{3.28}$$

Thus, Theorem 3.1 and Theorem 3.4 also apply to the NRDSs (3.28).

Remark 3.11. In SNRDSs (2.1), if $\mathcal{D} = 0$, then SNRDSs (2.1) is turns into the stochastic nonlinear systems (SNSs) with time delays and exogenous disturbances described by

$$\begin{aligned}d\Im(t) =& \Big[\mathcal{A}\Im(t) + \mathcal{B}\Im(t-\varrho) + \mathfrak{f}(t,\Im(t)) + \mathfrak{g}(t,\Im(t-\varrho)) + \varphi(t)\Big]dt \\ &+ \sigma(t,\Im(t),\Im(t-\varrho))d\omega(t),\ \ t > 0.\end{aligned} \tag{3.29}$$

Moreover, Theorem 3.1 and Theorem 3.4 also apply to the SNSs (3.29).

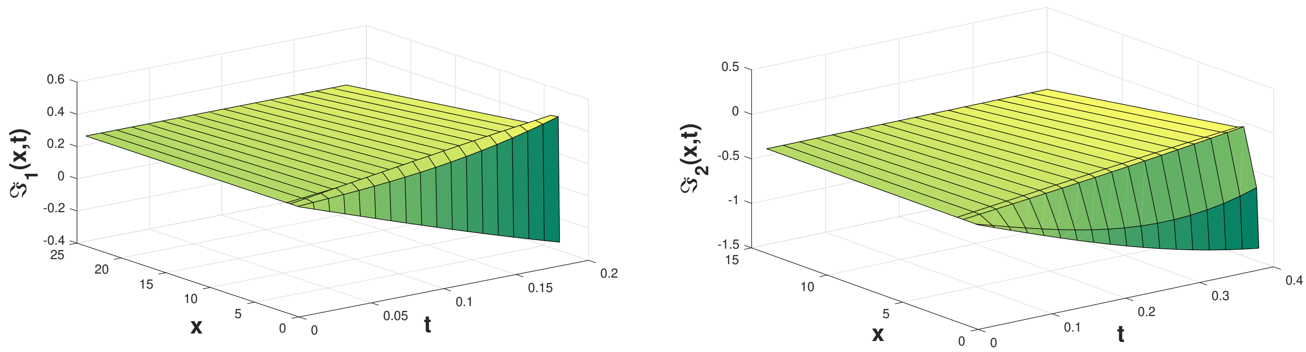

Figure 1: State trajectories of SNRDSs (33) without boundary control.

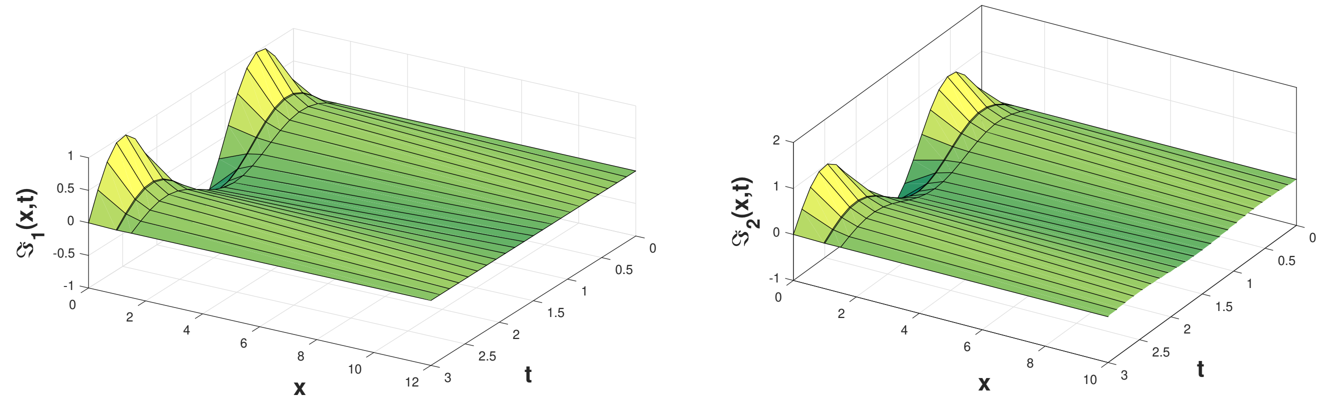

Figure 2: State trajectories of SNRDSs (33) with boundary control.

4 Numerical examples

In this section, two numerical examples are given to prove that our boundary controllers are effective.

Example 4.1. Consider the following SNRDSs with time delays and exogenous disturbances described by

$$\begin{aligned} d\Im(x,t) =& \Big[\mathcal{D}\frac{\partial^2\Im(x,t)}{\partial x^2} + \mathcal{A}\Im(x,t) + \mathcal{B}\Im(x,t-0.5) + \mathfrak{f}(t,\Im(x,t)) + \mathfrak{g}(t,\Im(x,t-0.5)) + \varphi(x,t)\Big]dt \\ &+ \sigma(t,\Im(x,t),\Im(x,t-0.5))d\omega(t),\ x \in (0,1),\ t > 0, \end{aligned} \tag{4.1}$$

where

$$\begin{aligned} &\mathcal{D} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},\ \mathcal{A} = \begin{bmatrix} 0.1 & -0.1 \\ 0.2 & -0.1 \end{bmatrix},\ \mathcal{B} = \begin{bmatrix} 0.2 & -1.2 \\ 0.5 & -1 \end{bmatrix}, \\ &\mathfrak{f}(t,\Im(x,t)) = 0.1(1+\sin(t))\Im(x,t), \\ &\mathfrak{g}(t,\Im(x,t-0.5)) = 0.1(1+\cos(t))\Im(x,t-0.5), \\ &\sigma(t,\Im(x,t),\Im(x,t-0.5)) = \begin{pmatrix} 0.55\Im_1(x,t) & 0.55\Im_2(x,t-0.5) \\ 0.55\Im_2(x,t) & 0.55\Im_1(x,t-0.5) \end{pmatrix}, \end{aligned}$$

Consider the initial conditions of the SNRDSs (4.1) as follows: $\phi_1(x,s) = 1.7\varpi(x)$, $\phi_2(x,s) = -2.1\varpi(x)$, $\varpi(x) = \cos([\pi(x-2)]/6)$. When $u(t) = 0$, the trajectories of states $\Im_p(x,t)(p = 1,2)$ are shown in Fig. 1. According to Fig. 1, the FTB of SNRDSs (4.1) cannot be realized if no boundary control input is present.

Next, we shall discuss the boundary control for FTB of SNRDSs (4.1). To boundedness the SNRDSs (4.1), we choose $\kappa = 2.1$, $z_1 = 1$, $z_2 = 4$, $\mathcal{T} = 10$, and $d = 0.5$. Solving the LMIs (3.3) and (3.25) in Theorem 3.4 by Matlab

LMI toolbox, we obtain the following feasible solutions:

$$P = 10^{-5} \times \begin{bmatrix} 0.3218 & -0.0116 \\ -0.0116 & 0.3699 \end{bmatrix}, \ Q = 10^{-6} \times \begin{bmatrix} 0.5111 & 0.0000 \\ 0.0000 & 0.5109 \end{bmatrix},$$
$$R_1 = 10^{-6} \times \begin{bmatrix} 0.3757 & -0.0031 \\ -0.0031 & 0.3897 \end{bmatrix}, \ R_2 = \begin{bmatrix} 56.8261 & -0.0200 \\ -0.0200 & 56.8625 \end{bmatrix},$$

$$R_3 = 10^{-4} \times \begin{bmatrix} -0.1580 & 0.0023 \\ 0.0023 & -0.1676 \end{bmatrix}, \ L = 10^{-3} \times \begin{bmatrix} -0.1433 & 0.0006 \\ 0.0006 & -0.1458 \end{bmatrix}.$$

Furthermore, the corresponding control gain matrix as follows:

$$\mathcal{K} = LP^{-1} = \begin{bmatrix} -44.5558 & -1.2373 \\ -1.2384 & -39.4604 \end{bmatrix}.$$

Therefore, based on Theorem 3.4, the SNRDSs (4.1) is FTB with respect to $(z_1, z_2, \mathcal{T}, d)$. The boundary controlled trajectories of states $\Im_p(x,t)(p = 1, 2)$ with the boundary controller (3.1) are shown in Fig. 2. In Fig. 2, the FTB of the SNRDSs (4.1) is realized, demonstrating the efficacy of Theorem 3.4 and the boundary controller (3.1).

Example 4.2. Consider the following SNRDSs with time delays described by

$$\begin{aligned} d\Im(x,t) =& \Big[\mathcal{D}\frac{\partial^2 \Im(x,t)}{\partial x^2} + \mathcal{A}\Im(x,t) + \mathcal{B}\Im(x,t-0.5) + \mathfrak{f}(t,\Im(x,t)) + \mathfrak{g}(t,\Im(x,t-0.5))\Big]dt \\ &+ \sigma(t,\Im(x,t),\Im(x,t-0.5))d\omega(t), \ x \in (0,1), \ t > 0, \end{aligned} \tag{4.2}$$

where

$$\begin{aligned} &\mathcal{D} = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}, \ \mathcal{A} = \begin{bmatrix} 1.1 & -1.1 \\ 2.2 & -2.1 \end{bmatrix}, \ \mathcal{B} = \begin{bmatrix} 2.2 & 2.2 \\ 1.5 & -1.6 \end{bmatrix}, \\ &\mathfrak{f}(t,\Im(x,t)) = 0.1(1+\sin(t))\Im(x,t), \\ &\mathfrak{g}(t,\Im(x,t-0.5)) = 0.1(1+\cos(t))\Im(x,t-0.5), \\ &\sigma(t,\Im(x,t),\Im(x,t-1)) = \begin{pmatrix} 0.55\Im_1(x,t) & 0.55\Im_2(x,t-0.5) \\ 0.55\Im_2(x,t) & 0.55\Im_1(x,t-0.5) \end{pmatrix}, \end{aligned}$$

To stabilize the SNRDSs (4.2), we choose $\kappa = 2.1$, $z_1 = 1$, $z_2 = 4$, and $\mathcal{T} = 10$. Solving the LMIs (3.24) and (3.27) in Corollary 3.5 by Matlab LMI toolbox, we obtain the following feasible solutions:

$$P = 10^{-5} \times \begin{bmatrix} -0.1614 & -0.0001 \\ -0.0001 & -0.1611 \end{bmatrix}, \ Q = 10^{-6} \times \begin{bmatrix} 0.1764 & 0.0001 \\ 0.0001 & 0.1760 \end{bmatrix},$$
$$R_1 = 10^{-3} \times \begin{bmatrix} 0.4105 & 0.0089 \\ 0.0089 & 0.3590 \end{bmatrix}, \ R_2 = \begin{bmatrix} 44.7942 & 0.0002 \\ 0.0002 & 44.7933 \end{bmatrix},$$
$$L = 10^{-4} \times \begin{bmatrix} -0.4142 & 0.0000 \\ 0.0000 & -0.4144 \end{bmatrix}.$$

Furthermore, the corresponding control gain matrix as follows:

$$\mathcal{K} = LP^{-1} = \begin{bmatrix} 25.6708 & -0.0100 \\ -0.0100 & 25.7287 \end{bmatrix}.$$

Therefore, based on Corollary 3.5, the SNRDSs (4.2) is FTS with respect to $(z_1, z_2, \mathcal{T})$.

5 Conclusion

In this paper, boundary controller design for finite-time boundedness of SNRDSs with time delays and exogenous disturbances are discussed. Both SNRDSs with and without exogenous disturbances are discussed. By construct a LKF and employing the Wirtinger's and Gronwall inequalities, sufficient conditions are derived to ensure that the FTB of SNRDSs. Moreover, the control gain matrices can be designed for boundary controller. At last, two numerical examples are provided to shown the our boundary controller are effective. Our future study will focus on the stability and stabilization problems for fractional-order SNRDSs using a boundary controller.

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