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Radiation effect on MHD copper suspended nanofluid flow through a stenosed artery with temperature-dependent viscosity

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Abstract

In the present paper, the effects of radiation, variable viscosity, and the inclination of the artery on copper nanofluid through composite stenosis with chemical reaction are discussed. The viscosity of blood is varied with temperature as represented in the Reynolds viscosity model. The coupled nonlinear equations of the nanofluid model are simplified by considering the mild stenosis case. The governing equations are solved numerically by applying the Finite Difference Method. The effects of the physical parameters on the velocity, temperature, and concentration along the radial axis have been studied and are physically interpreted for medical applications. The effect of shear stress along the increasing height of stenosis has been explained with the help of graphs. The proposed work will be beneficial to clinicians, hematologists, and biomedical engineers because they serve as useful approximations, which are capable of throwing some light toward the understanding of the genesis of pathological states, like arteriosclerosis as well as the mechanism of gaseous exchanges that take place within arteries and capillaries.

Keywords: Radiation effect, Variable Viscosity, MHD, Inclined Artery, Nano Particles
2020 MSC: 78A40 , 76W05, 82D80

1 Introduction

Arterial blood flow has important aspects due to engineering as well as medical application points of view. In particular, the circulatory connected issues are the major causes of health problems and deaths in the present world. Blood is a biomagnetic fluid which consists of red blood cells, white blood cells, platelets and plasma.The red blood cells, white blood cells and platelets are suspended in aqueous plasm. Plasma is an aqueous composition of lipoproteins, nutrients and clotting factors [45]. The hemoglobin present in the red blood cells helps in the transport of oxygen[2]. Blood helps in the supply of oxygen and nutrients to the cells and also carries away wastes from the cells. Blood is conducted through blood vessels called arteries and veins. Blood can be considered as both Newtonian and non Newtonian fluid depending on the diameter of the arteries[13]. Some of the basic studies dealing with different

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models of Newtonian and non-Newtonian fluid are given in Shukla and Rahman [42], Naz et al. [27], Tripathi et al.[46]. Blood flow in the arteries is affected by the deposition of fatty substances inside the lumen called stenosis. Arteriosclerosis occurs in the blood vessels due to the accumulation of lipids in the arterial wall or pathological changes in the tissue structure [43]. When a blood clot forms in an artery, blocking the blood flow to the heart muscle or the brain, a heart attack or stroke can happen. Hence, the study of blood flow through a stenosed artery plays an important role in understanding cardiovascular diseases. The study of bio-fluids under the presence of magnetic field with viscus dissipation finds its applications in various fields like innovative drug targeting, surgical operations etc. Also, various surgical operations require significant control of the fluid flow and heat transfer of biological fluids. The presence of electromagnetic fields during such operations can have impacts on the human circulation system. It has also been reported that application of a magnetic field is useful for nerve regeneration[24], bone grafts [41] and fracture healing [15]. In magnetic resonance imaging (MRI), spectral separation of scan images can be increased by applying magnetic strength upto 8 T. MHD unsteady flow and heat transfer over a flat plate with Navier slip and Newtonian heating has been investigated by Makinde [20]. Heat and mass transfer in magneto-biofluid flow through different physical conditions has been discussed by Sharma at el. [34, 37]. A numerical investigation of unsteady flow of blood through arteries under stenotic condition has been discussed by Majee and Shit [3]. Blood viscosity is an important property to be analysed as it has huge impact on blood flow. The mechanical properties of red blood cells also affect the viscosity. The major determinants of blood viscosity are packed cell volume and plasma viscosity[17]. The red blood cells are highly deformable and hence aid in the flow of blood under bulk flow conditions and microcirculation[4]. In a biological system, temperature variation even for 2 °C or 3 °C has remarkable significance. For an increase of temperature by 1 °C, the time of treatment process reduces to half for a specific biological result like the reduction to one-third of cancer cells in a tumor [16, 18].An unsteady MHD mixed convection for a heat generating fluid with and without thermal radiation and chemical reaction is analysed by Sharma et al. [40, 31]. Makinde et al. [22] analyzed the heat transfer and MHD effects on the peristaltic flow of Walters-B fluid in a compliant wall channel with thermal radiation and heat generation. Recently, radiation effect on MHD blood flow through a tapered porous stenosed artery with thermal and mass diffusion has been discussed by Sharma et al. [30]. Nanofluids are the fluids of nanometer-sized particles of metals, carbides, oxides, nitrides, or nanotubes. Nowadays, nanofluids, among researchers, are considered an active area of research. Nanofluids have their huge applications in heat transfer, like microelectronics, pharmaceutical processes, fuel cells, and hybrid-powered engines, domestic refrigerator, nuclear reactor coolant, and grinding and space technology et. In fact, nanofluids are a suspension of nanosized solid particles in a base fluid. The nanofluids have high thermal conductivity as compared to the base fluid so, nanofluids basically increase heat transfer rate. The nanoparticles used in the nanofluid are metals, oxides, carbides, nitrites and nanotubes [26] . The commonly used base fluids are water, ethylene glycol, oil, biofluids and polymer solutions [44]. Nanofluids exhibit enhanced thermophysical properties such as thermal conductivity, thermal diffusivity, viscosity and convective heat transfer [10].

Nanofluids are used in the treatment of arterial stenosis. Hence the analysis of blood flow in the presence of nanofluid plays an important role in the treatment process. All the above studies are carried out for the fluids having constant viscosity through the flow regime. However, it is known that the physical property may change significantly with viscosity. To accurately predict the flow behavior, it is necessary to take the variation of viscosity. The effects of hematocrit and temperature on cerebral blood flow velocity can be related either to their effect on blood viscosity, on cerebral blood flow adaptation to cerebral metabolic rate for oxygen, or both. Changes in hematocrit can change blood viscosity. Blood viscosity increases when blood temperature decreases. Blood viscosity increases by 50 % to 300% as temperature is decreased from 37°C to 22°C [7, 21]. Often, even lower blood temperatures, in the range of 8°C–12°C, are typically encountered when performing deliberate deep hypothermia for cardiac or thoracic aortic operations requiring temporary circulatory arrest [8, 14]. Recently Sharma et al. [38] studied the effects of thermal and mass diffusion in biomagnetic fluid of blood flow through a tapered porous stenosed artery. Thermal radiation effect in blood flow is an important topic of research, because it has got major applications in biomedical engineering and several medical treatment methods. Thermal radiation is one of the methods applied by medical practitioners to ease cardiovascular diseases. Different aspects of therapeutic procedure of hyperthermia were discussed/studied by several researchers (Abe and Hiraoka [1], Molls [23], Chaudhary et al. [6] and Sharma et al [35]). Combined heat and mass transfer problems with chemical reaction are of prominence in many processes and have received a significant amount of attention in recent years. In processes such as evaporation at the surface of water body, energy transfer in wet cooling tower, drying and the flow in a desert cooler, heat and mass transfer occur simultaneously. Mass transfer with chemical reaction has special significance in chemical and hydrometallurgical industries. Das et al. [9] considered the effects of first order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Makinde [19] discussed the chemically reacting hydromagnetic unsteady flow of a radiating fluid past a vertical plate with constant heat flux. Recently, the effects of chemical reaction on a magneto- fluid flow from a radiative surface with variable permeability was discussed by Sharma et al. [39]. Recently, Sharma et al. [36] discussed the inclined arterial blood flow with magnetic field.

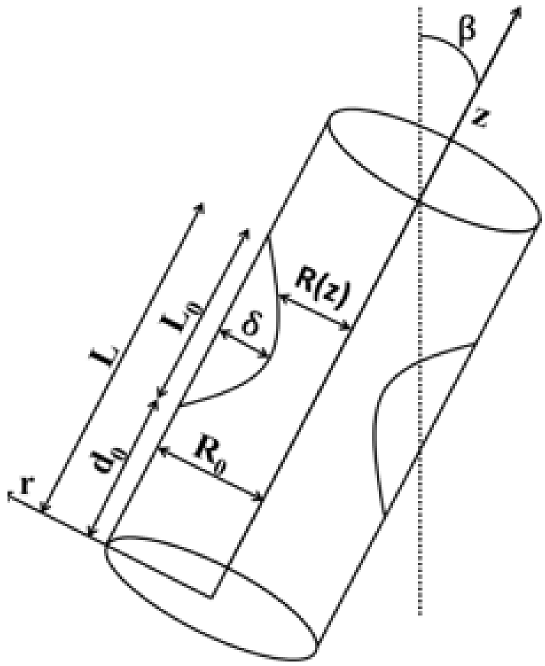

Figure 1: Geometry of the stenosed artery

In all the above referred to research, horizontal or vertical blood carrying vessels were considered. but, it's far widely known that many ducts in physiological structures are not horizontal or vertical but have some inclination to the axis. However, the available work still lacks the investigation of the arterial blood flow characteristics through an inclined artery with variable viscosity. Hence, this paper deals with the copper nanoparticles for blood with water as base fluid. The dependence of blood viscosity on temperature is taken into consideration. The viscosity of blood varies exponentially with temperature according to Reynolds viscosity model. Under constant pressure gradient and assuming the flow to be steady, the velocity, concentration and temperature profiles are plotted. The effects of Brinkmann number, viscosity parameter, thermal radiation, magnetic field parameter and chemical reaction are considered. These results are interpreted graphically in order to highlight the effects of various parameters on blood flow under pathological state.

2 Formulation of Problem

Considered an axisymmetric incompressible nanofluid flow of blood flow through a composite stenosed circular tartery of finite length L as shown in Fig.1. Let (r, θ, z) be the coordinates of a material point in the cylindrical polar coordinate system where z-axis is taken along the axis of artery, while r and θ are along the radial and circumferential directions, respectively.

The geometry of arterial wall with composite stenosis is described by Joshi and Srivastava [5] as:

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \frac{2\delta}{R_0 L_0}(z-d) & d < z \leq d + \frac{L_0}{2}, \\ 1 - \frac{\delta}{2R_0}(1 + \cos\frac{2\pi}{L_0}(z - d - \frac{L_0}{2})) & d + \frac{L_0}{2} < z \leq d + L_0, \\ 1 & otherwise. \end{cases} \tag{2.1}$$

where R(z) is the radius of the artery in the obstructed region while R_0 is the radius of normal artery. L_0, d, δ are the length, location and height of the stenosis respectively. β is the angle of inclination of the artery from vertical axis. The governing equations for incompressible nano fluid can be written as,

$$\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial (rv)}{\partial r} = 0. \tag{2.2}$$

$$\rho_f \left(u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r} \right) = -\frac{\partial p}{\partial r} - \left[\frac{1}{r}\frac{\partial (r\tau_{rr})}{\partial r} + \frac{\partial \tau_{zr}}{\partial z} - \frac{\tau_{\theta\theta}}{r} \right]. \tag{2.3}$$

$$\rho_f \left(u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} \right) = -\frac{\partial p}{\partial z} - \left[\frac{1}{r}\frac{\partial (r\tau_{rz})}{\partial r} + \frac{\partial \tau_{zz}}{\partial z} \right] + \rho_f g\cos(\beta)\alpha_T(T-T_0) + \rho_f g\cos(\beta)\alpha_C(C-C_0) - \sigma B_0^2 u. \tag{2.4}$$

$$(\rho_{C_p})_f \left(v\frac{\partial T}{\partial r} + u\frac{\partial T}{\partial z} \right) = \kappa_T \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right] - \frac{1}{r}\frac{\partial (rq_r)}{\partial r} + \mu(T)\left(\frac{\partial u}{\partial r}\right)^2. \tag{2.5}$$

$$v\frac{\partial C}{\partial r} + u\frac{\partial C}{\partial z} = D_m \left[\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right] - kC. \tag{2.6}$$

Following the Rosseland approximation with the radiative heat flux q_r is modeled as:

$$q_r = -\frac{4\sigma_s}{3k_e}\frac{\partial T^4}{\partial r}. \tag{2.7}$$

Where σ_s is the Stefan-Boltzmann constant and k_e is absorption coefficient. The differences in temperature within the flow are assumed such that T^4 can be expressed as a linear combination of the temperature, therefore T^4 is expanded using Taylor series about T_0 as:

$$T^4 = 4T_0^3 T - 3T_0^4. \tag{2.8}$$

The corresponding boundary conditions are the symmetry at the centerline and no-slip at the walls:

$$\begin{cases} \frac{\partial u}{\partial r} = 0, \frac{\partial T}{\partial r} = 0, \frac{\partial C}{\partial r} = 0 & \text{at } r = 0 \\ u = 0, T = T_0, C = C_0 & \text{at } r = \frac{R(z)}{R_0}. \end{cases} \tag{2.9}$$

We non-dimensionalize the governing equations and boundary conditions by introducing the following non-dimensional parameters:

$$\begin{gathered}
\bar{r} = \frac{r}{R_0}, \quad \bar{p} = \frac{R_0^2}{UL_0\mu_f}, \quad \bar{z} = \frac{z}{L_0}, \quad \mu(\theta) = \frac{\mu(T)}{\mu_f}, \quad M^2 = \frac{\sigma B_0^2 R_0^2}{\mu_f}, \quad \bar{R} = \frac{R}{R_0}, \theta = \frac{T-T_0}{T_0}, \quad \alpha = \frac{k}{(\rho C_p)_f} \\
\bar{u} = \frac{u}{U}, \quad Ec = \frac{U^2}{C_{pf}T_0}, \quad Br = EcPr, \quad \sigma = \frac{C-C_0}{C_0}, \quad S_c = \frac{\mu_f}{\rho_f D_m}, \quad Gr = \frac{g\alpha_T R_0^2 T_0 \rho_f}{U\mu_f}, \quad Pr = \frac{\mu_f C_{pf}}{k} \\
R = \frac{k_e k_T}{4\sigma_s T_0^3}, \quad \bar{v} = \frac{L_0}{\delta U}v, \quad S_r = \frac{D_m K_T \rho T_0}{\mu T_m C_0}, \quad Cr = \frac{g\alpha_C R_0^2 C_0 \rho_f}{U\mu_f}, \quad \bar{\delta} = \frac{\delta}{R_0}, \quad K = \frac{kR_0^2\rho_f}{\mu_f}, \quad \bar{d} = \frac{d}{L_0}
\end{gathered} \tag{2.10}$$

Making use of these non-dimensional variables and applying the additional conditions $\epsilon = \frac{R_0}{L_0} = o(1)$ for the case of mild stenosis ($\frac{\delta}{R_0} << 1$), [25] to the equations (2.2)-(2.6), the non-dimensional governing equations after dropping the dashes can be written as:

$$\frac{\partial p}{\partial r} = 0, \tag{2.11}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\mu(\theta)\frac{\partial u}{\partial r}) + Gr\cos\beta\theta + Cr\cos\beta\sigma - M^2 u, \tag{2.12}$$

$$\left(1 + \frac{4}{3R}\right)\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right)\right) + Br\mu(\theta)\left(\frac{\partial u}{\partial r}\right)^2 = 0, \tag{2.13}$$

$$\frac{1}{S_c}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\sigma}{\partial r}\right)\right)-K\sigma=0, \tag{2.14}$$

In the present problem we have considered the Reynold's model of viscosity which is defined as follows:

$$\mu(\theta)=e^{-\omega\theta}. \tag{2.15}$$

The non-dimensional boundary conditions on velocity, temperature and concentration are:

$$\begin{cases}\frac{\partial u}{\partial r}=0, \frac{\partial\theta}{\partial r}=0, \frac{\partial\sigma}{\partial r}=0 & \text{at } r=0\\ u=0,\theta=0,\sigma=0 & \text{at } r=R(z).\end{cases} \tag{2.16}$$

And the geometry of the arterial wall in non dimensional form will be:

$$\frac{R(z)}{R_0}=\begin{cases}1-2\delta(z-d) & d<z\leq d+\frac{1}{2},\\ 1-\frac{\delta}{2}(1+\cos 2\pi(z-d-\frac{1}{2}) & d+\frac{1}{2}<z\leq d+1,\\ 1 & otherwise.\end{cases} \tag{2.17}$$

3 Numerical Solution

The above non-linear dimensionless partial differential equations with boundary conditions have been solved numerically by applying explicit finite difference method. The discretization for first order derivatives terms are based on the first order forward difference scheme and for second order terms are based on central difference scheme. To obtain the difference equations, the region of the blood flow is divided into a grid or mesh lines. Solution of these difference schemes are obtained at the intersection of these mesh lines called nodes. Axis is discretized by incrementing j and radial direction by incrementingi. The finite difference equations at every internal nodal point on a particular n-level constitute a tri diagonal system of equations. These equations are solved using Thomas Algorithm[12]. The finite difference equations are:

$$\left(\frac{\partial p}{\partial z}\right)_{i,j}=\frac{u_{i,j}}{r_{i,j}}\frac{u_{i+1,j}-u_{i,j}}{\Delta r}+u_{i,j}\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{(\Delta r)^2}+\left(\frac{u_{i+1,j}-u_{i,j}}{\Delta r}\right)^2-M^2u_{i,j}+Gr\cos\beta\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta r}+Cr\cos\beta\frac{\sigma_{i+1,j}-\sigma_{i,j}}{\Delta r}, \tag{3.1}$$

$$(1+\frac{4}{3R})\left(\frac{1}{r_{i,j}}\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta r}+\frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta r)^2}\right)+E_cPr(u_{i,j})\left(\frac{u_{i+1,j}-u_{i,j}}{\Delta r}\right)^2. \tag{3.2}$$

$$\frac{1}{Sc}\left(\frac{1}{r_{i,j}}\frac{\sigma_{i+1,j}-\sigma_{i,j}}{\Delta r}+\frac{\sigma_{i+1,j}-2\sigma_{i,j}+\sigma_{i-1,j}}{(\Delta r)^2}\right)-K\sigma_{i,j}=0, \tag{3.3}$$

The appropriate mesh size for the above calculation is $\Delta r=0.025$.The procedure was carried out iteratively till the error was less than 10^{-6}. In order to access the accuracy of our results, the computation is carried out for slightly changed values of Δr. Negligible changed is observed in the values of u,θ, and σ and also after each cycle of iteration the convergence checking is performed i.e. $|f^{n+1}-f^n|<10^{-6}$ is satisfied at all points.

4 Results and Discussion

In this section, for getting the physical insight into the problem, graphs have been plotted for velocity, temperature, concentration and shear stress with different quantities of interest. Following figures have been plotted using the default parameter values which are given as: $M=0.2, Gr=0.5, Cr=0.3, Br=0.3, Sc=0.5, R=2, Pr=0.7, K=1.5, \beta=30^\circ, \omega=0, d=0.25, \delta=0.5$.

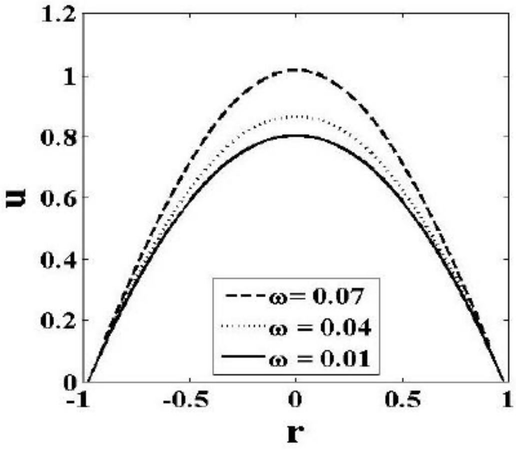

Figure 2: Velocity profile against radial axis for varying ω

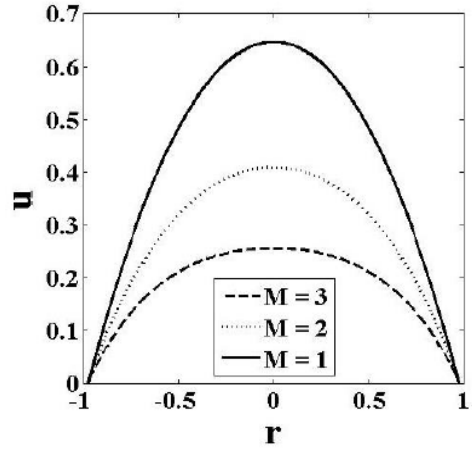

Figure 3: Velocity profile against radial axis for varying M

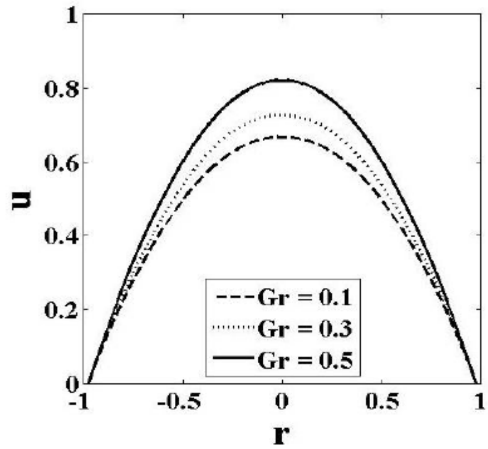

Figure 4: Velocity profile against radial axis for varying Gr

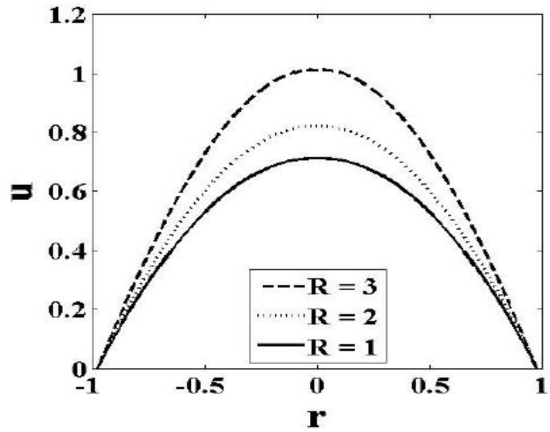

Figure 5: Velocity profile against radial axis for varying R

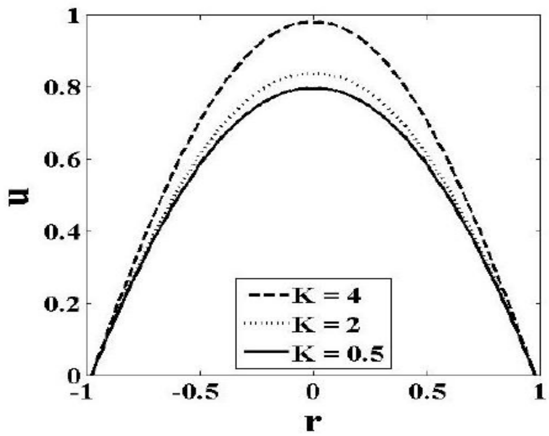

Figure 6: Velocity profile against radial axis for varying K

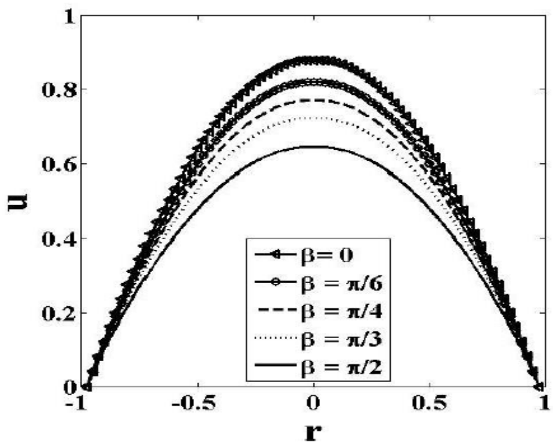

Figure 7: Velocity profile against radial axis for varying β

In Fig.2, it is observed that as values of the viscosity parameter ω increase from 0.01 to 0.07, velocity profile increases, respectively. For the particular value of the viscosity parameter velocity profile attains its highest value at the center of the artery and it starts decreasing towards the wall of the artery. The effect of Hartmann number (M) on blood flow is depicted in Fig.3. It is clear from the figure that the velocity profile of the blood flow decreases with increasing Hartmann number M by following the parabolic convex upward profile. This happens due to the Lorentz force which acts as a retarding force on blood flow. This force has the tendency to slow down the motion of the fluid in the boundary layer. As blood contains hemoglobin in its content which are oxides of iron so the velocity of blood flow is highly influenced by the presence of a magnetic field. The present result is very much useful for medical persons during the time of surgical process as result shows that the blood can be regulated or controlled at the desired level under the influence of an external magnetic field [37]. Fig.4, displays the variations of the velocity profile of blood flow for different values of the Grashof number Gr. It is observed from the figure that velocity profile of the blood flow increases with increasing values of Gr and this is due to increased Boussinesq source terms [29]. It is noted from Fig.5 that the velocity profile of the fluid increases as values of the radiation parameter increases. It is clear from the figure that for a particular value of the radiation parameter, following the parabolic profile the velocity of fluid attains maximum value at the center of the artery and slowly decreases towards the wall of the artery. It is clear that, as the thermal radiation parameter increases, the nano-fluid velocity distributions across the boundary layer increases. The increase in radiation parameter means the release of heat energy from the flow region and so the fluid velocity increases as the thermal boundary layer thickness become thinner. Fig.6 reveals the effect of chemical reaction parameter on velocity profile. The figure shows that fluid velocity increases as values of the chemical reaction increase.

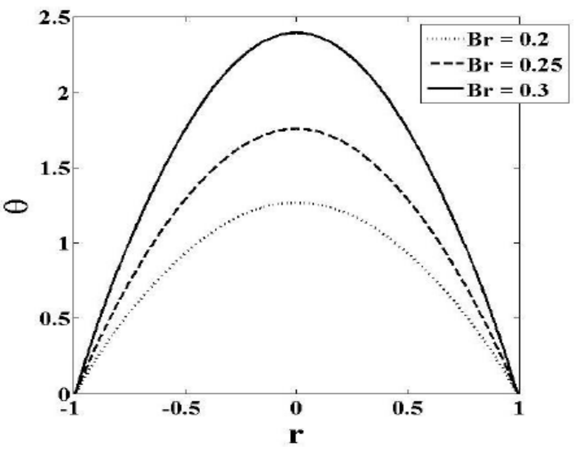

Figure 8: Velocity profile against radial axis for varying Br

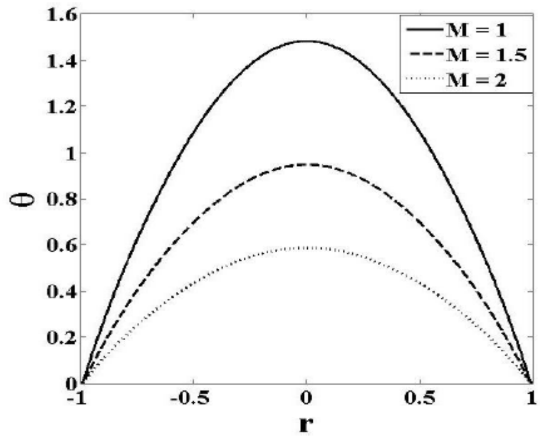

Figure 9: Velocity profile against radial axis for varying M

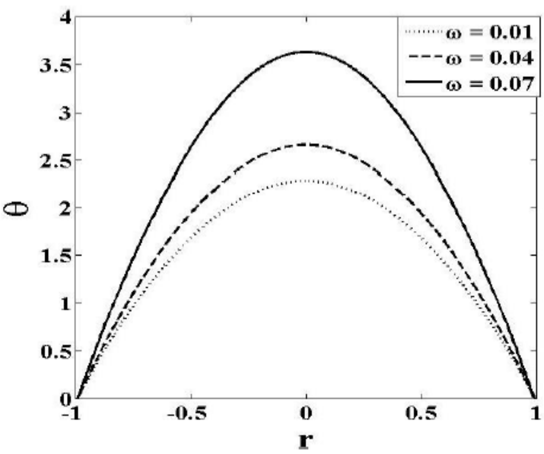

Figure 10: Velocity profile against radial axis for varying ω

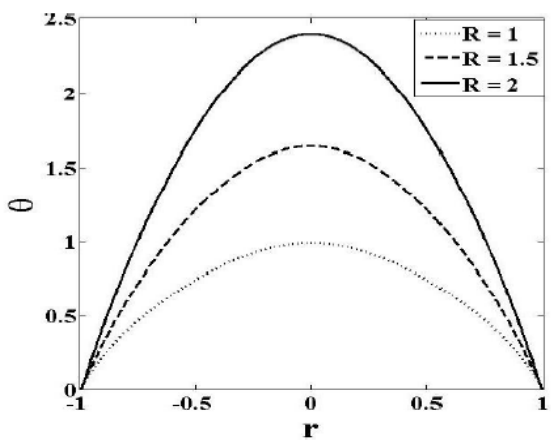

Figure 11: Velocity profile against radial axis for varying R

Fig.7 reveals that as values of the inclined angle made by the artery from the vertical axis increase from 0 to $\pi/2$, nano-fluid velocity decreases, respectively. Fig.8 shows the effect of Brinkman number on the temperature profile of the nanofluid. Brinkman Number is the ratio of heat produced by viscous dissipation to heat produced by molecular transport. It is noted that as values of the Brinkman number increase, the temperature profile of the nanofluid also increases which implies that viscous dissipation effect becomes more dominant as compared to the thermal diffusion. Fig.9 displays the variations of the temperature profile for different values of the magnetic field parameter. From the figure, it is observed that as values of the magnetic field parameter increase, temperature profile of the nanofluid decreases respectively. As the high intensity of an external magnetic force slows down the flow. In result of this, the kinetic energy of the flow decreases which directly affects the temperature profile of the blood flow. Fig.10 and Fig.11 depicts the variations of the temperature profile of the blood flow for different values of the viscosity parameter ω and radiation parameter R. It is clear from these figures that as values of the viscosity and radiation parameter increase, the temperature profile of the blood flow decreases. Temperature profile shows this behavior with radiation parameter due to Buyoncy force [47, 48].

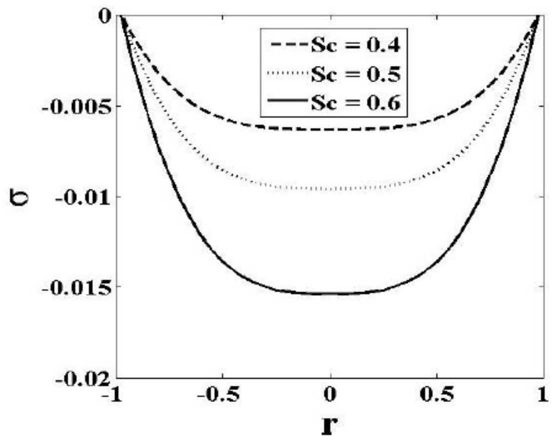

Figure 12: Concentration profile against radial axis for varying Sc

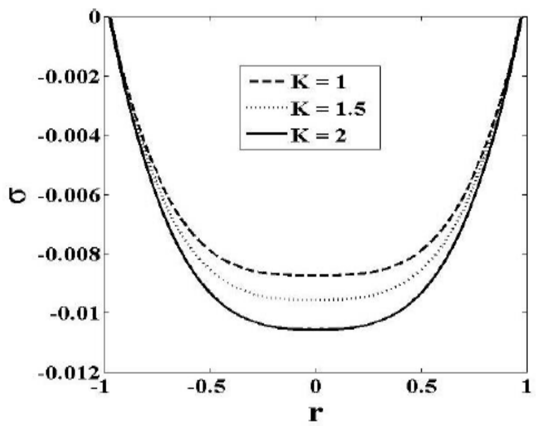

Figure 13: Concentration profile against radial axis for varying K

Concentration profiles for different values of Schmidt number and chemical reaction parameter are shown in Fig.12 and Fig.13, respectively. The Schmidt number defines the relative effectiveness of momentum and mass transport by diffusion in the velocity and concentration fields. It is noticed that the concentration of the nano-fluid decreases as values of the Schmidt number increase. Concentration decreases with increase in Sc due to the increase in momentum diffusivity. This causes the concentration of buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the momentum and concentration boundary layers thickens. Further, it is observed in Fig.13 that increasing the values of chemical reaction parameter, decreases the concentration profile of the nano-fluid. This is due to the fact that the chemical reaction parameter suppresses the concentration of the fluid due to a fall in the values of chemical molecular diffusivity of the species concentration. Thus the species concentration experiences retarding effect and reduce the total mass transfer of inside the fluid [28, 33].

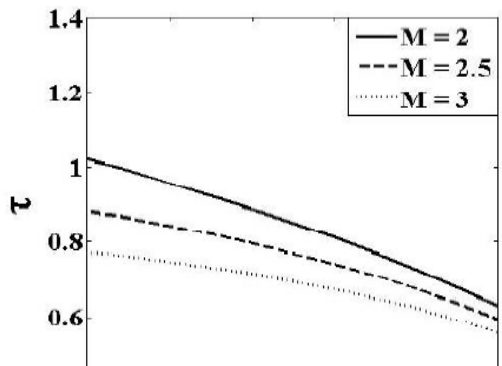

Figure 14: Concentration profile against radial axis for varying Sc

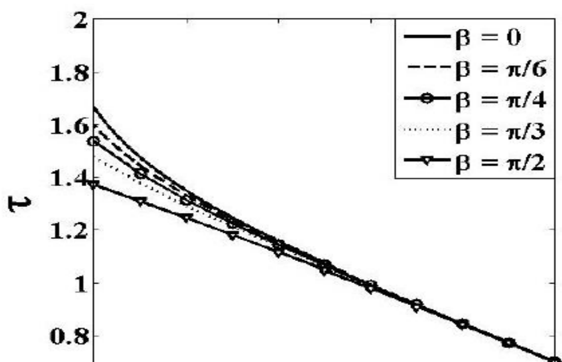

Figure 15: Concentration profile against radial axis for varying K

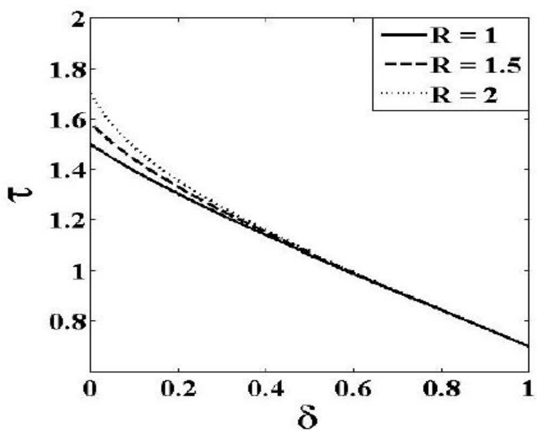

Figure 16: Concentration profile against radial axis for varying Sc

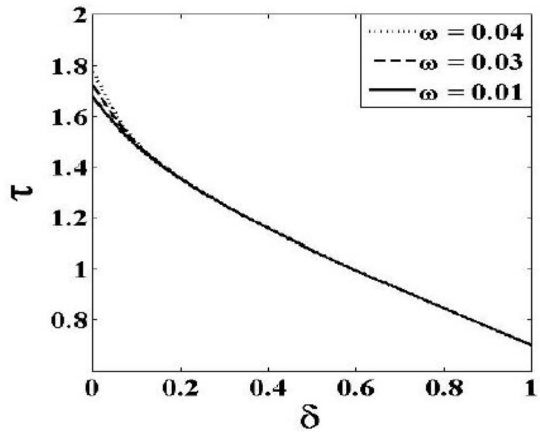

Figure 17: Concentration profile against radial axis for varying K

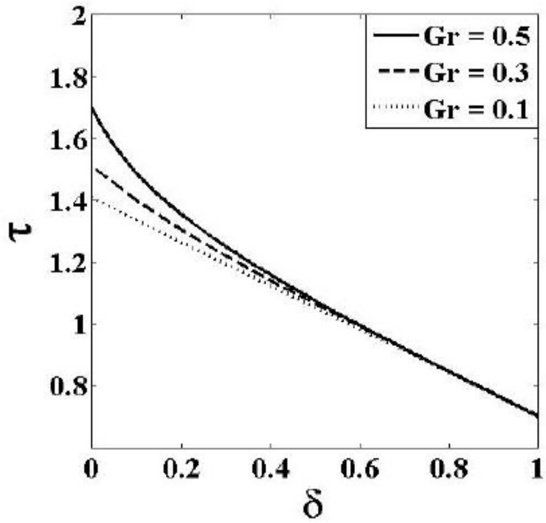

Figure 18: Concentration profile against radial axis for varying Sc

Shear stress profiles for different values of the magnetic field parameter, different inclination angle of the artery, viscosity parameter, Grashof number and Radiation parameter have been displayed in Fig.14-Fig.18, respectively. From Fig.14 and Fig.15 it is noticed that shear stress at stenosis throat decreases, as values of the magnetic field parameter (M) and the inclined angle made by the artery increases. These figures display that for a particular value of magnetic field parameter and the inclined angle of the artery shear stress attains its maximum value for the case of mild stenosis. Fig.16-Fig.17, reveal that shear stress at stenosis throat increases as values of the radiation and viscosity parameter increase. As increasing the values of radiation dosage and viscosity parameter indirectly increase the blood velocity and hence the wall shear stress also increases [11]. Fig.18 reveals the effect of Grashof on shear stress. It is noted that shear stress increases with increasing Grashof number. An increase in the Grashof number would directly increase the velocity via the increased Boussinesq source terms and hence also the magnitude of the slope of the velocity at the wall.

5 Conclusion

The present study deals with the copper nanoparticles for blood with water as base fluid. The dependence of blood viscosity on temperature is taken into consideration. The viscosity of blood varies exponentially with temperature according to Reynolds viscosity model. The model has been solved using finite difference numerical methods. The numerical simulations have shown that:

- It is observed that velocity field rises with increasing Br, Gr, K, R, ω whereas it decreases with increasing β, and M.
- When we increase chemical reaction rate constant K concentration of copper nanoparticle in fluid decreases.
- It is also observed that concentration profile has an opposite behavior as compared with the temperature profile.
- It is noted that increase in Br, M, R and ω increases the temperature whereas increase in β, decreases the temperature.
- It is seen that Shear stress is experienced by fluid for variation of M at all stenosis heights whereas for all other parameters the shear stress experienced by fluid with variation of parameters is almost the at larger stenosis heights. Some variation could be seen at small stenosis heights.

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