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Differential subordination and superordination for a q-derivative operator connected with the q-exponential function

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Abstract

In this article, we define a q-derivative operator of univalent functions associated with the q-exponential function. Moreover, we introduce differential subordination and differential superordination for the subordination class defined by this operator. Sandwich-type theorems of several known results also are derived by applying these results.

Keywords: q-exponential function, q-derivative Operator, best dominant, best subordination, and sandwich-type theorems
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1 Introduction

Let $\mathcal{S}$ be the class of analytic and univalent functions $f(z)$ in the open unit disc $\mathcal{O}=\{z:|z|<1\}$ with their normalized form

$$f(z)=z+\sum_{j=2}^{\infty}a_jz^j. \tag{1.1}$$

Let $\mathcal{H}[a,l]$ be the subclass of the functions $f\in\mathcal{S}$ defined as

$$\mathcal{H}[a,l]=\left\{f:f(z)=a+a_lz^l+a_{l+1}z^{l+1}+\dots\right\}.$$

Next, let $f(z)$ and $h(z)$ be two analytic functions in $\mathcal{O}$, the function $f(z)$ is called subordinate to $h(z)$, or $h(z)$ is superordinate to $f(z)$, denoted by $f(z)\prec h(z)$ and $h(z)\prec f(z)$, respectively, if there is a Schwarz function φ with $\varphi(z)=0$, $|\varphi(z)|<1$ and $f(z)=h(\varphi(z))$. In addition, we get the following equivalence if the function h is univalent in $\mathcal{O}$

$$f(z)\prec h(z)\Longrightarrow f(0)=h(0)\text{ and }f(\mathcal{O})\subset h(\mathcal{O})\ (z\in\mathcal{O}).$$

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If f and F be two functions in $\mathcal{S}$, the convolution (*or Hadamard product*) denoted by $f * F$ defined by

$$(f * F)(z) := z + \sum_{j=2}^{\infty} a_j d_j z^j, \ z \in \mathcal{O},$$

where f is defined (1.1) and $F(z) = z + \sum_{j=2}^{\infty} d_j z^j$, $z \in \mathcal{O}$.
Assume that κ and $\hbar$ in $\mathcal{O}$ are two analytic functions

$$\Lambda(r, e, \ell; z) : \mathbb{C}^3 \times \mathcal{O} \to \mathbb{C}.$$

If κ and $\Lambda\left(\kappa(z), z\kappa'(z), z^2\kappa''(z); z\right)$ are univalent functions and κ satisfies the second-order superordination in $\mathcal{O}$.

$$\hbar(z) \prec \Lambda\left(\kappa(z), z\kappa'(z), z^2\kappa''(z); z\right). \tag{1.2}$$

Then κ is said to be a solution to the differential superordination (1.2). The analytic function γ is called a subordinate of (1.2), if $\gamma \prec \kappa$ for every the function κ satisfying (1.2). A univalent subordinate ω that satisfies $\gamma \prec \omega$ for all subordinate γ of (1.2) (see, [26], [27]).

Miller and Mocanu [26] discovered sufficient conditions on the functions $\gamma, \hbar$ and Λ to prove the following:

$$\hbar(z) \prec \Lambda\left(\kappa(z), z\kappa'(z), z^2\kappa''(z); z\right) \Longrightarrow \gamma \prec \kappa. \tag{1.3}$$

In the same methods above, Bulboacă ([7, 8]), defined general families of first-order differential superordinations and superordination-preserving integral operators. Furthermore, using Bulboacă's [8] results, Ali et al.[1] found sufficient conditions for normalized analytic functions f to fulfill

$$\gamma_1(z) \prec \frac{zf'(z)}{f(z)} \prec \gamma_2(z),$$

where γ_1 and γ_2 are univalent functions in $\mathcal{O}$ with $\gamma_1(0) = 1$ and $\gamma_2(0) = 1$.
After that, many researchers have been interested in studying the properties of subordination and superordination (see, [2],[3], [6], [12], [15], [17], [24], [28], [29], [34], [33], [35], [36]).
Many reserchers have interested on the topic of the q-calculus (or q-analysis). The reason for focusing on studying the q-calculus is due to its wide applications in the field of mathematical, quantum physics, and operator theory. Jackson ([19, 20]) pioneered the introduced of q-derivative and q-integral. More recently, Kanas and Raducanu [21] (also see, [9], [10], [13], [14], [16], [18], [31], [25], [22], [23], [32]) investigated certain classes of functions that are analytic in $\mathcal{O}$ using fractional q-calculus operators.
In this article, we present a q-derivative operator $\mathcal{Q}_{\vartheta,\sigma,q}^{\xi,m} f(z)$ associated with the q-exponential function. Furthermore, we introduce differential subordination and superordination results related to this operator. Also, we obtain some applications of the results of sandwich-type results.

Definition 1.1. [20] (i) Let $0 < q < 1$, the q-factorial denoted by $[j]_q!$, is defined by:

$$[j]_q! = \begin{cases} [j]_q [j-1]_q \dots [2]_q [1]_q, & \text{if} \quad j = 1, 2, 3, \dots, \\ 1, & \text{if} \quad j = 0, \end{cases}$$

where

$$[j]_q := \frac{1 - q^j}{1 - q}, \quad \text{and} \quad [0]_q := 0.$$

(ii) The q-derivative operator with $0 < q < 1$, is defined by

$$D_q f(z) := \frac{f(qz) - f(z)}{(q-1)z}, \ z \neq 0.$$

If we let the q-exponential function e_q defined by the power series expansion (see, [25])

$$e_q(z) := \sum_{j=0}^{\infty} \frac{z^j}{[j]_q!}, \ (z \in \mathcal{O}). \tag{1.4}$$

As a result

$$e(z) := lim_{q\to 1^-} e_q(z) = \sum_{j=0}^{\infty} \frac{z^j}{j!}.$$

Then e_q is a unique function that satisfies the condition

$$\frac{D_q e(z)}{D_q z} = \sum_{j=0}^{\infty} \frac{d_q z^j}{[j]_q!} = \sum_{j=1}^{\infty} \frac{[j]_q z^{j-1}}{[j]_q!} = \sum_{j=1}^{\infty} \frac{z^{j-1}}{[j-1]_q!} = \sum_{j=0}^{\infty} \frac{z^j}{[j]_q!} = e_q(z),\ (z \in \mathcal{O}).$$

Now, for $\xi > 0, \vartheta \geq 0, \sigma > 0, \sigma \neq \vartheta,$ and $m \in \mathbb{N}_0$, we introduce a q-derivative operator $\mathcal{F}^{\xi,m}_{\vartheta,\sigma,q} f(z) : \mathcal{S} \to \mathcal{S}$ as below

$$\mathcal{F}^{\xi,0}_{\vartheta,\sigma,q} f(z) = f(z), \tag{1.5}$$

$$\mathcal{F}^{\xi,1}_{\vartheta,\sigma,q} f(z) = (1 - \xi(\sigma - \vartheta))f(z) + \xi(\sigma - \vartheta)z d_q(f(z)) \tag{1.6}$$

$$\vdots$$

$$\mathcal{F}^{\xi,m}_{\vartheta,\sigma,q} f(z) = \mathcal{F}^{\xi,1}_{\vartheta,\sigma,q} \left(\mathcal{F}^{\xi,m-1}_{\vartheta,\sigma,q} f(z)\right). \tag{1.7}$$

Then from the functions (1.1) and (1.7), we have

$$\mathcal{F}^{\xi,m}_{\vartheta,\sigma,q} f(z) = z + \sum_{j=2}^{\infty} (1 + \xi(\sigma - \vartheta)([j]_q - 1))^m a_j z^j. \tag{1.8}$$

Remark 1.2. We note the following special cases of the operator $\mathcal{F}^{\xi,m}_{\vartheta,\sigma,q} f(z)$ previously obtained by several authors:

1. Taking $q \to 1$, we obtain differential operator introduced by Darus and Ibrahim [11].
2. When $\vartheta = 0$ and $\xi = 1$, we get q-Al-Oboudi operator defined by Aouf et al. [5].
3. Let $q \to 1, \vartheta = 0$ and $\xi = 1$, this operator defined by Al-Oboudi [4].
4. If $\vartheta = 0, \sigma = 1,$ and $\xi = 1$, we have q-Salagean operator presented by Govindaraj and Sivasubramanian [16].
5. Let $q \to 1, \vartheta = 0, \sigma = 1,$ and $\xi = 1$, we obtain Salagean operator presented by Sălăgean [30].

Definition 1.3. For the function $f \in \mathcal{S}$, we define a new q-derivative operator $\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q} f(z) : \mathcal{S} \to \mathcal{S}$ as below

$$\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q} f(z) = \mathcal{F}^{\xi,m}_{\vartheta,\sigma,q} f(z) * e_q(z).$$

From the above definition, it follows that

$$\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q} f(z) = z + \sum_{j=2}^{\infty} \psi(q) a_j z^j,\ (z \in \mathcal{O}) \tag{1.9}$$

where $\psi(q) = \frac{(1+\xi(\sigma-\vartheta)([j]_q-1))^m}{[j]_q!}$.
From (1.9), we show that the following relationship

$$\xi(\sigma - \vartheta)z(\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q} f(z))' = \mathcal{Q}^{\xi,m+1}_{\vartheta,\sigma,q} f(z) - (1 - \xi(\sigma - \vartheta))\, \mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q} f(z). \tag{1.10}$$

The main purpose of this article is to find sufficient conditions for certain normalized analytic functions $f(z)$ to satisfy

$$\gamma_1(z) \prec \{\frac{\mathcal{Q}^{\rho,m+1}_{\vartheta,\sigma,q} f(z)}{z}\} \prec \gamma_2(z)$$

and

$$\gamma_1(z) \prec \{\frac{\mathcal{Q}^{\rho,m+1}_{\vartheta,\sigma,q} f(z)}{\mathcal{Q}^{\rho,m}_{\vartheta,\sigma,q} f(z)}\} \prec \gamma_2(z),$$

where γ_1 and γ_2 are univalent functions in $\mathcal{O}$ with $\gamma_1(0) = 1$ and $\gamma_2(0) = 1$.

2 Main Lemmas

Firstly, to obtain our results we need the following lemmas

Definition 2.1. [26, 27] Let Q is the set of all analytic and injective functions f on $\overline{\mathcal{O}}\diagdown E(f)$, where

$$E(f) = \left\{ \eta \in \partial\mathcal{O} : \lim_{z\to\eta} f(z) = \infty \right\},$$

such that $f'(\eta) \neq 0$ for $\eta \in \partial\mathcal{O}\diagdown E(f)$.

Lemma 2.2. [26] In the unit disk $\mathcal{O}$, let κ be a univalent function. In a domain D containing $\kappa(\mathcal{O})$, let θ and φ be analytic with $\varphi(u) \neq 0$ when $u \in \kappa(\mathcal{O})$. Set $Q(z) = z\kappa'(z)\varphi(\kappa(z))$ and $\hbar(z) = \theta(\kappa(z)) + Q(z)$. Assume that
(1) $Q(z)$ is a starlike univalent in $\mathcal{O}$.
(2) $\mathcal{R}\left\{\frac{z\hbar'(z)}{Q(z)}\right\} > 0$ for $z \in \mathcal{O}$. If μ is analytic in $\mathcal{O}$, with $\gamma(0) = \kappa(0)$, $\gamma(\mathcal{O}) \subset D$ and

$$\theta(\gamma(z)) + z\gamma'(z)\varphi(\gamma(z)) \prec \theta(\kappa(z)) + z\kappa'(z)\varphi(\kappa(z)), \tag{2.1}$$

then $\gamma \prec \kappa$ and κ is the best dominant of (2.1).

Lemma 2.3. [26] Let κ be convex univalent function in $\mathcal{O}$. In a domain D containing $\kappa(\mathcal{O})$, let θ and φ be analytic with $\varphi(u) \neq 0$ when $u \in \kappa(\mathcal{O})$. Let $Q(z) = z\kappa'(z)\varphi(\kappa(z))$. Assume that
(1) $\mathcal{R}\left\{\frac{\theta'(\kappa(z))}{\varphi(\kappa(z))}\right\} > 0$ for $z \in \mathcal{O}$.
(2) $Q(z)$ is a starlike univalent in $\mathcal{O}$.
If $\gamma \in \mathcal{H}[\kappa(0), 1] \cap Q$, with $\gamma(\mathcal{O}) \subset D$, $\theta(\gamma(z)) + z\gamma'(z)\varphi(\gamma(z))$ is a univalent in $\mathcal{O}$ and

$$\theta(\kappa(z)) + z\kappa'(z)\varphi(\kappa(z)) \prec \theta(\gamma(z)) + z\gamma'(z)\varphi(\gamma(z)), \tag{2.2}$$

then $\kappa \prec \gamma$ and κ is the best subordinate of (2.2).

Lemma 2.4. [27] In the unit disk $\mathcal{O}$, let κ be a convex univalent function, $\zeta \in \mathbb{C}$, and $\mathcal{R}(\zeta) > 0$.
If $\gamma \in \mathcal{H}[\kappa(0), 1] \cap Q$ and $\gamma(z) + \rho z\gamma'(z)$ is univalent in $\mathcal{O}$, we get

$$\kappa(z) + \rho z\kappa'(z) \prec \gamma(z) + \rho z\gamma'(z), \tag{2.3}$$

then, $\kappa \prec \gamma$ and κ is the best subordinate of (2.3).

Lemma 2.5. [26] Let κ be a convex univalent function in $\mathcal{O}$, $\zeta \in \mathbb{C}$, and $\rho \in \mathbb{C}\backslash\{0\}$ with

$$\mathcal{R}\left(1 + \frac{z\kappa''(z)}{\kappa'(z)}\right) > max\left\{0; -\mathcal{R}\left(\frac{\zeta}{\rho}\right)\right\}.$$

If γ is analytic in $\mathcal{O}$, we have

$$\zeta\gamma(z) + \rho z\gamma'(z) \prec \zeta\kappa(z) + \rho z\kappa'(z). \tag{2.4}$$

Then $\gamma(z) \prec \kappa(z)$, and κ is the best dominant of (2.4).

Theorem 2.6. In the unit disk $\mathcal{O}$, let κ be a convex univalent function with $\kappa(0) = 1$, $\tau \in \mathbb{C}\backslash\{0\}$, $\mu > 0$, and κ satisfies the following condition

$$\mathcal{R}\left(1 + \frac{z\kappa''(z)}{\kappa'(z)}\right) > max\left\{0; -\mathcal{R}\left(\frac{\mu}{\tau\xi(\sigma - \vartheta)}\right)\right\}. \tag{2.5}$$

If $f \in \mathcal{S}$ satisfies the subordination condition below

$$\begin{aligned} &\tau\left[\frac{\mathcal{Q}^{\rho,m+1}_{\vartheta,\sigma,q}f(z)}{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)} - 1\right]\left[\frac{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)}{z}\right]^{\mu} + (1-\tau)\left[\frac{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)}{z}\right]^{\mu} \\ &\prec \kappa(z) + \frac{\tau\xi(\sigma - \vartheta)}{\mu} z\kappa'(z). \end{aligned} \tag{2.6}$$

Then

$$\left[\frac{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)}{z}\right]^{\mu} \prec \kappa(z), \tag{2.7}$$

and $\kappa(z)$ is the best dominant of (2.6).

Proof . Assume that

$$\gamma(z) = \left[\frac{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)}{z}\right]^{\mu}. \tag{2.8}$$

Since κ is univalent in $\mathcal{O}$ and $\kappa(0) = 1$.
Now, by differentiation logarithmically with respect to z, we get

$$\frac{z\gamma'(z)}{\gamma(z)} = \mu\left[\frac{z(\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z))'}{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)} - 1\right] = \frac{\mu}{\xi(\sigma-\vartheta)}\left[\frac{\mathcal{Q}^{\xi,m+1}_{\vartheta,\sigma,q}f(z)}{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)} - 1\right].$$

Then

$$\begin{aligned}\frac{\xi(\sigma-\vartheta)z\gamma'(z)}{\mu} &= \left[\frac{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)}{z}\right]^{\mu}\left[\frac{\mathcal{Q}^{\xi,m+1}_{\vartheta,\sigma,q}f(z)}{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)} - 1\right] = \tau\left[\frac{\mathcal{Q}^{\xi,m+1}_{\vartheta,\sigma,q}f(z)}{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)} - 1\right]\left[\frac{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)}{z}\right]^{\mu} \\ &+ (1-\tau)\left[\frac{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)}{z}\right]^{\mu} = \gamma(z) + \frac{\tau\xi(\sigma-\vartheta)}{\mu}z\gamma'(z).\end{aligned}$$

From (2.6), it follows that

$$\gamma(z) + \frac{\tau\xi(\sigma-\vartheta)}{\mu}z\gamma'(z) \prec \kappa(z) + \frac{\tau\xi(\sigma-\vartheta)}{\mu}z\kappa'(z).$$

By Lemma 2.5, with $\rho = 1$ and $\zeta = \frac{\tau\xi(\sigma-\vartheta)}{\mu}$, we get (2.7). □

Corollary 2.7. In the unit disk $\mathcal{O}$, let κ be a convex univalent function with $\kappa(0) = 1$, $\tau \in \mathbb{C}\backslash\{0\}$, and $\mu > 0$, and κ satisfies the following condition

$$\mathcal{R}\left(1 + \frac{z\kappa'(z)}{\kappa'(z)}\right) > max\left\{0; -\mathcal{R}\left(\frac{\mu}{\tau}\right)\right\}.$$

If $f \in \mathcal{S}$ fulfill the following condition

$$\begin{aligned}\frac{\tau}{\xi(\sigma-\vartheta)}\left[\frac{\mathcal{Q}^{\xi,m+1}_{\vartheta,\sigma,q}f(z)}{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)} - 1\right]\left[\frac{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)}{z}\right]^{\mu} + \frac{(1-\tau)}{\xi(\sigma-\vartheta)}\left[\frac{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)}{z}\right]^{\mu} \\ \prec \kappa(z) + \frac{\tau}{\mu}z\kappa'(z).\end{aligned} \tag{2.9}$$

Then

$$\left[\frac{\mathcal{Q}^{\xi,m}_{\vartheta,\sigma,q}f(z)}{z}\right]^{\mu} \prec \kappa(z),$$

and κ is the best dominant of (2.9).

Theorem 2.8. In the unit disk $\mathcal{O}$, let κ be a convex univalent function with $\kappa(0) = 1$, $\kappa_s \in \mathbb{C}\backslash\{0\}$ $(s = 1, 2, 3, 4)$, $\mu > 0$, and κ satisfies the following condition

$$\mathcal{R}\left(1 + \frac{\kappa_2}{\kappa_4}\kappa(z) + \frac{2\kappa_3}{\kappa_4}(\kappa(z))^2 - \frac{z\kappa'(z)}{\kappa(z)} + \frac{z\kappa''(z)}{\kappa'(z)}\right) > 0. \tag{2.10}$$

Consider $\frac{z\kappa'(z)}{\kappa(z)}$ to be univalent starlike in $\mathcal{O}$. If $f \in \mathcal{S}$ satisfies the subordination condition below

$$\psi^{\xi,m}_{\sigma,\vartheta}(\kappa_1, \kappa_2, \kappa_3, \kappa_4, \mu; z) \prec \kappa_1 + \kappa_2\kappa(z) + \kappa_3(\kappa(z))^2 + \kappa_4\frac{z\kappa'(z)}{\kappa(z)}, \tag{2.11}$$

where

$$\psi_{\sigma,\vartheta}^{\xi,m}(\kappa_1,\kappa_2,\kappa_3,\kappa_4,\mu;z)=\kappa_1+\kappa_2\left[\frac{\mathcal{Q}_{\vartheta,\sigma,q}^{\xi,m}f(z)}{z}\right]^{\mu}+\kappa_3\left[\frac{\mathcal{Q}_{\vartheta,\sigma,q}^{\xi,m}f(z)}{z}\right]^{2\mu} +\kappa_4\mu\left[\frac{\mathcal{Q}_{\vartheta,\sigma,q}^{\xi,m+1}f(z)}{\mathcal{Q}_{\vartheta,\sigma,q}^{\xi,m}f(z)}-1\right]. \tag{2.12}$$

Hence

$$\left[\frac{\mathcal{Q}_{\vartheta,\sigma,q}^{\xi,m}f(z)}{z}\right]^{\mu}\prec\kappa(z), \tag{2.13}$$

and κ is the best dominant in (2.11).

Proof . Taking

$$\gamma(z)=\left[\frac{\mathcal{Q}_{\vartheta,\sigma,q}^{\xi,m}f(z)}{z}\right]^{\mu}. \tag{2.14}$$

Now, by computing logarithmic differentiation with respect to z, we have

$$\frac{z\gamma'(z)}{\gamma(z)}=\mu\left[\frac{z(\mathcal{Q}_{\vartheta,\sigma,q}^{\xi,m}f(z))'}{\mathcal{Q}_{\vartheta,\sigma,q}^{\xi,m}f(z)}-1\right]=\frac{\mu}{\vartheta}\left[\frac{\mathcal{Q}_{\vartheta,\sigma,q}^{\xi,m+1}f(z)}{\mathcal{Q}_{\vartheta,\sigma,q}^{\xi,m}f(z)}-1\right].$$

By assuming $\theta(\omega)=\kappa_3\omega^2+\kappa_2\omega+\kappa_1$ and $Q(\omega)=\frac{\kappa_4}{\omega}$, it is clear θ is an analytic in $\mathbb{C}$, $\omega\in\mathbb{C}\backslash\{0\}$, φ is also analytic in $\mathbb{C}\backslash\{0\}$ with $\varphi(\omega)\neq 0$, we obtain

$$\mathrm{Q(z)}=z\kappa'(z)\varphi(\kappa(z))=\kappa_4\frac{\mathrm{z}\kappa'(z)}{\kappa(z)}$$

and

$$\hbar(z)=\theta(\kappa(z))+Q(z)=\kappa_1+\kappa_2\kappa(z)+\kappa_3(\kappa(z))^2+\kappa_4\frac{z\kappa'(z)}{\kappa(z)}.$$

Since $Q(z)$ is an analytic function, we have

$$\hbar'(z)=\kappa_2+\kappa'(z)+2\kappa_3\kappa(z)\kappa^{'}(z)+\kappa_4\frac{(\kappa'(z)+z\kappa'(z))\kappa(z)-z(\kappa'(z))^2}{(\kappa(z))^2},$$

then

$$\frac{z\hbar'(z)}{Q(z)}=1+\frac{\kappa_2}{\kappa_4}\kappa(z)+\frac{2\kappa_3}{\kappa_4}(\kappa(z))^2-\frac{z\kappa'(z)}{\kappa(z)}+\frac{z\kappa''(z)}{\kappa'(z)},$$

and

$$\mathcal{R}(\frac{z\hbar'(z)}{Q(z)})=\mathcal{R}(1+\frac{\kappa_2}{\kappa_4}\kappa(z)+\frac{2\kappa_3}{\kappa_4}(\kappa(z))^2-\frac{z\kappa'(z)}{\kappa(z)}+\frac{z\kappa'(z)}{\kappa'(z)})>0,$$

$$\kappa_1+\kappa_2\gamma(z)+\kappa_3(\gamma(z))^2+\kappa_4\frac{z\gamma'(z)}{\gamma(z)}=\psi_{\sigma,\vartheta}^{\xi,m}(\kappa_1,\kappa_2,\kappa_3,\kappa_4,\mu;z).$$

Using (2.11), therefore

$$\kappa_1+\kappa_2\gamma(z)+\kappa_3(\gamma(z))^2+\kappa_4\frac{z\gamma'(z)}{\gamma(z)}\prec\kappa_1+\kappa_2\kappa(z)+\kappa_3(\kappa(z))^2+\kappa_4\frac{z\kappa'(z)}{\kappa(z)}.$$

By Lemma 2.3, we get $\gamma(z)\prec\kappa(z)$. □

Corollary 2.9. In the unit disk $\mathcal{O}$, let the condition (2.10) be satisfied, $\kappa_s\in\mathbb{C}\backslash\{0\}$ $(s=1,2,3,4)$, and $\mu>0$, if

$$\psi_{\sigma,\vartheta}^{\xi,m}(\kappa_1,\kappa_2,\kappa_3,\kappa_4,\mu;z)\prec\kappa_1+\kappa_2\frac{(\mathcal{V}-\mathcal{W})z}{(1+\mathcal{V}z)(1+\mathcal{W}z)}+\kappa_3\left(\frac{1+\mathcal{V}z}{1+\mathcal{W}z}\right)^2+\kappa_4\frac{1+\mathcal{V}z}{1+\mathcal{W}z},$$

with $-1\leq\mathcal{W}<\mathcal{V}\leq 1$ and $\psi_{\sigma,\vartheta}^{\xi,m}(\kappa_1,\kappa_2,\kappa_3,\kappa_4,\mu;z)$ defined in (2.12), thus we get

$$\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f(z)}{z}\right]^{\mu}\prec\frac{1+\mathcal{V}z}{1+\mathcal{W}z}, \tag{2.15}$$

and $\kappa(z)=\frac{1+\mathcal{V}z}{1+\mathcal{W}z}$ is the best dominant.

Corollary 2.10. In the unit disk $\mathcal{O}$, if the condition (2.11) is satisfied, $\varkappa_s \in \mathbb{C}\backslash\{0\}$ $(s = 1, 2, 3, 4)$, $\mu > 0$, and $0 < \alpha \leq 1$, if

$$\psi_{\sigma,\vartheta}^{\xi,m}\left(\kappa_1, \kappa_2, \kappa_3, \kappa_4, \mu; z\right) \prec \kappa_1 + \kappa_2\left(\frac{1+z}{1-z}\right)^{\alpha} + \kappa_3\left(\frac{1+z}{1-z}\right)^{2\alpha} + \kappa_4\frac{2\alpha z}{1-z^2},$$

where $\psi_{\sigma,\vartheta}^{\xi,m}\left(\kappa_1, \kappa_2, \kappa_3, \kappa_4, \mu; z\right)$ defined in (2.12), thus we get

$$\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f\left(z\right)}{z}\right]^{\mu} \prec \left(\frac{1+z}{1-z}\right)^{\alpha}, \tag{2.16}$$

and $\kappa\left(z\right) = \left(\frac{1+z}{1-z}\right)^{\alpha}$ is the best dominant.

Theorem 2.11. In the unit disk $\mathcal{O}$, let κ be a convex univalent function with $\kappa\left(0\right) = 1$, $\kappa_s \in \mathbb{C}\backslash\{0\}$ $(s = 1, 2, 3, 4, 5)$, $\mu > 0$, and κ satisfies the following condition

$$\mathcal{R}\left(1 + \frac{\kappa_2}{\kappa_5}\kappa(z) + \frac{2\kappa_3}{\kappa_5}(\kappa\left(z\right))^2 + \frac{3\kappa_4}{\kappa_5}(\kappa\left(z\right))^3 - \frac{z\kappa'(z)}{\kappa(z)} + \frac{z\kappa''(z)}{\kappa'(z)}\right) > 0, \tag{2.17}$$

where

$$\begin{aligned}\psi_{\sigma,\vartheta}^{\xi,m}\left(\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \mu; z\right) = \kappa_1 + \kappa_2\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f\left(z\right)}{z}\right]^{\mu} + \kappa_3\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f\left(z\right)}{z}\right]^{2\mu} \\ + \kappa_4\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f\left(z\right)}{z}\right]^{3\mu} + \kappa_5\mu\left[\frac{\mathcal{Q}_{\varpi(q)}^{\vartheta,m+1} f\left(z\right)}{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f\left(z\right)} - 1\right].\end{aligned} \tag{2.18}$$

If the function κ satisfies the following subordination condition

$$\psi_{\sigma,\vartheta}^{\xi,m}\left(\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \mu; z\right) \prec \kappa_1 + \kappa_2\kappa(z) + \kappa_3(\kappa\left(z\right))^2 + \kappa_4(\kappa\left(z\right))^3 + \kappa_5\frac{z\kappa'\left(z\right)}{\kappa\left(z\right)}. \tag{2.19}$$

Thus, we have

$$\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f\left(z\right)}{z}\right]^{\mu} \prec \kappa\left(z\right), \tag{2.20}$$

with κ is the best dominant of (2.19).

In the similar way of the Theorem 2.8, we can get the result.

Theorem 2.12. In the unit disk $\mathcal{O}$, let κ be a convex univalent function with $\kappa\left(0\right) = 1$, $\tau \in \mathbb{C}\backslash\{0\}$, and $\mu > 0$. If $f \in \mathcal{S}$ satisfies the conditions below

$$\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f\left(z\right)}{z}\right]^{\mu} \in \mathcal{H}\left[\kappa\left(0\right), 1\right] \cap Q.$$

If

$$\begin{aligned}\kappa\left(z\right) + \frac{\tau\xi(\sigma-\vartheta)}{\mu} z\kappa'\left(z\right) \prec \tau\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m+1} f\left(z\right)}{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f\left(z\right)} - 1\right]\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f(z)}{z}\right]^{\mu} \\ + (1-\tau)\left[\frac{\mathcal{Q}_{\varpi(q)}^{\vartheta,m} f\left(z\right)}{z}\right]^{\mu}.\end{aligned} \tag{2.21}$$

So that $\tau\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m+1} f(z)}{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f(z)} - 1\right]\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f(z)}{z}\right]^{\mu} + (1-\tau)\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f(z)}{z}\right]^{\mu}$ be a univalent in $\mathcal{O}$. Then

$$\kappa\left(z\right) \prec \left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m} f\left(z\right)}{z}\right]^{\mu}, \tag{2.22}$$

and κ is the best subordinate in (2.21).

Proof . Taking

$$\gamma\left(z\right)=\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f(z)}{z}\right]^{\mu}. \tag{2.23}$$

Using calculating logarithmic differentiation with respect to z, we have

$$\frac{z\gamma'\left(z\right)}{\gamma\left(z\right)}=\mu\left[\frac{z(\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right))'}{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}-1\right]=\frac{\mu}{\xi(\sigma-\vartheta)}\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m+1}f\left(z\right)}{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}-1\right].$$

Then

$$\begin{aligned}\frac{\xi(\sigma-\vartheta)z\gamma'\left(z\right)}{\mu}&=\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}{z}\right]^{\mu}\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m+1}f\left(z\right)}{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}-1\right]=\\ \tau\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m+1}f\left(z\right)}{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}-1\right]\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}{z}\right]^{\mu}&+(1-\tau)\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}{z}\right]^{\mu}\\ &=\gamma\left(z\right)+\frac{\tau\xi(\sigma-\vartheta)}{\mu}z\gamma'\left(z\right).\end{aligned} \tag{2.24}$$

By (2.23) and (2.24), we obtain

$$\kappa\left(z\right)+\frac{\tau\xi(\sigma-\vartheta)}{\mu}z\kappa'\left(z\right)\prec\gamma\left(z\right)+\frac{\tau\xi(\sigma-\vartheta)}{\mu}z\gamma'\left(z\right).$$

We will get the result using Lemma 2.3, with $\zeta=1$ and $\rho=\frac{\tau\xi(\sigma-\vartheta)}{\mu}$. □

Theorem 2.13. In the unit disk $\mathcal{O}$, let κ be a convex univalent function with $\kappa\left(0\right)=1$, $\kappa_s\in\mathbb{C}\backslash\{0\}$ $(s=1,2,3,4)$, $\mu>0$, and κ satisfies the following condition

$$\mathcal{R}\left(\frac{\kappa_2}{\kappa_4}\kappa(z)+\frac{2\kappa_3}{\kappa_4}(\kappa(z))^2\right)>0. \tag{2.25}$$

Assume that $\frac{z\kappa'(z)}{\kappa(z)}$ is univalent starlike in $\mathcal{O}$. If $f\in\mathcal{S}$ fulfill the subordination conditions below

$$\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}{z}\right]^{\mu}\in\mathcal{H}\left[\kappa(0),1\right]\cap Q,$$

and

$$\kappa_1+\kappa_2\kappa(z)+\kappa_3(\kappa\left(z\right))^2+\kappa_4\frac{z\kappa'\left(z\right)}{\kappa\left(z\right)}\prec\psi_{\sigma,\vartheta}^{\xi,m}\left(\kappa_1,\kappa_2,\kappa_3,\kappa_4,\mu;z\right), \tag{2.26}$$

where

$$\begin{aligned}\psi_{\sigma,\vartheta}^{\xi,m}\left(\kappa_1,\kappa_2,\kappa_3,\kappa_4,\mu;z\right)=\kappa_1+\kappa_2\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}{z}\right]^{\mu}+\kappa_3\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}{z}\right]^{2\mu}\\ +\kappa_4\mu\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m+1}f\left(z\right)}{\mathcal{Q}_{\sigma,\vartheta}^{\xi,m}f\left(z\right)}-1\right].\end{aligned} \tag{2.27}$$

Hence

$$\kappa\left(z\right)\prec\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}{z}\right]^{\mu}, \tag{2.28}$$

and κ is the best subordinate in (2.26).

Proof . By taking

$$\gamma\left(z\right)=\left[\frac{\mathcal{Q}_{\sigma,\vartheta,q}^{\xi,m}f\left(z\right)}{z}\right]^{\mu}. \tag{2.29}$$

Furthermore, using calculating logarithmic differentiation with respect to z, we have

$$\frac{z\gamma'(z)}{\gamma(z)} = \mu\left[\frac{z\left(\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)\right)'}{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)} - 1\right] = \frac{\mu}{\xi(\sigma-\vartheta)}\left[\frac{\mathcal{Q}^{\xi,m+1}_{\sigma,\vartheta,q}f(z)}{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)} - 1\right].$$

By assuming $\theta(\omega) = \kappa_3\omega^2 + \kappa_2\omega + \kappa_1$ and $Q(\omega) = \frac{\kappa_4}{\omega}$. It is clear θ is analytic in $\mathbb{C}$, $\omega \in \mathbb{C}\backslash\{0\}$, φ is also analytic in $\mathbb{C}\backslash\{0\}$ with $\varphi(\omega) \neq 0$, we obtain

$$\frac{v'(\kappa(z))}{\phi(\kappa(z))} = \frac{\kappa'(z)[\kappa_2 + 2\mu\kappa(z)]\kappa(z)}{\kappa_4}$$

it is obvious that $Q(\omega)$ is starlike,

$$\mathcal{R}\left(\frac{v'(\kappa(z))}{\phi(\kappa(z))}\right) = \mathcal{R}\left(\frac{\kappa_2}{\kappa_4}\kappa(z) + \frac{2\kappa_3}{\kappa_4}(\kappa(z))^2\right) > 0.$$

Using (2.25), therefore

$$\kappa_1 + \kappa_2\kappa(z) + \kappa_3(\kappa(z))^2 + \kappa_4\frac{z\kappa'(z)}{\kappa(z)} \prec \kappa_1 + \kappa_2\gamma(z) + \kappa_3(\gamma(z))^2 + \kappa_4\frac{z\gamma'(z)}{\gamma(z)}.$$

By Lemma 2.3 we get $\kappa(z) \prec \gamma(z)$. □

Theorem 2.14. In the unit disk $\mathcal{O}$, let κ be a convex univalent function with $\kappa(0) = 1$, $\kappa_s \in \mathbb{C}\backslash\{0\}$ $(s = 1, ..., 5)$, $\mu > 0$, and κ satisfies the following condition

$$\mathcal{R}\left(\frac{\kappa_2}{\kappa_4}\kappa(z) + \frac{2\kappa_3}{\varkappa_4}(\kappa(z))^2 + \frac{3\kappa_4}{\kappa_5}(\kappa(z))^3\right) > 0. \tag{2.30}$$

Assume that $\frac{z\kappa'(z)}{\kappa(z)}$ is univalent starlike in $\mathcal{O}$. If $f \in \mathcal{S}$ and the following condition holds

$$\left[\frac{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)}{z}\right]^{\mu} \in \mathcal{H}[\kappa(0), 1] \cap Q$$

and

$$\kappa_1 + \kappa_2\kappa(z) + \kappa_3(\kappa(z))^2 + \kappa_4(\kappa(z))^3 + \kappa_5\frac{z\kappa'(z)}{\kappa(z)} \prec \psi^{\xi,m}_{\sigma,\vartheta,q}(\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \mu; z) \tag{2.31}$$

where $\psi^{\vartheta,m}_{\varpi(q)}(\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \mu; z)$ defined by (2.12).
Hence

$$\kappa(z) \prec \left[\frac{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)}{z}\right]^{\mu}, \tag{2.32}$$

and κ is the best subordinate in (2.30).

Proof . The proof is the similar method of Theorem 2.13. □

3 Results of Sandwich Theorem

Theorem 3.1. In the unit disk $\mathcal{O}$, let κ be a convex univalent function with $\kappa_1(0) = \kappa_2(0) = 1$, $\tau \in \mathbb{C}\backslash\{0\}$, and $\mu > 0$. If $f \in \mathcal{S}$ satisfies the condition below

$$\left[\frac{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)}{z}\right]^{\mu} \in \mathcal{H}[1, 1] \cap Q.$$

If

$$\begin{aligned} \kappa_1(z) + \frac{\tau\xi(\sigma-\vartheta)}{\mu}z{\kappa_1}'(z) \prec (\tau+\varsigma)\left[\frac{\mathcal{Q}^{\xi,m+1}_{\sigma,\vartheta,q}f(z)}{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)} - 1\right]\left[\frac{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)}{z}\right]^{\mu} \\ + (1-\tau)\left[\frac{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)}{z}\right]^{\mu} \prec \kappa_1(z) + \frac{\tau\xi(\sigma-\vartheta)}{\mu}z{\kappa_1}'(z). \end{aligned} \tag{3.1}$$

So that $\tau\left[\frac{\mathcal{Q}^{\xi,m+1}_{\sigma,\vartheta,q}f(z)}{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)}-1\right]\left[\frac{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)}{z}\right]^{\mu}+(1-\tau)\left[\frac{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)}{z}\right]^{\mu}$ be univalent in $\mathcal{O}$.
Then

$$\kappa_1(z)\prec\left[\frac{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta}f(z)}{z}\right]^{\mu}\prec\kappa_2(z), \tag{3.2}$$

and $\kappa_1(z)$ and $\kappa_2(z)$ are the best subordinate and best dominant in (3.1).

Theorem 3.2. In the unit disk $\mathcal{O}$, let κ be a convex univalent function with $\kappa_1(0)=\kappa_2(0)=1$, $\kappa_s\in\mathbb{C}\backslash\{0\}$ $(s=1,..,4)$, $\mu>0$, and κ satisfies the following condition

$$\mathcal{R}\left(\frac{\kappa_2}{\kappa_4}\kappa(z)+\frac{2\kappa_3}{\kappa_4}(\kappa(z))^2\right)>0. \tag{3.3}$$

Assume that $\frac{z\kappa'(z)}{\kappa(z)}$ is univalent starlike in $\mathcal{O}$. If $f\in\mathcal{S}$ and the following subordination condition holds

$$\left[\frac{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)}{z}\right]^{\mu}\in\mathcal{H}[1,1]\cap Q,$$

and

$$\begin{aligned}\kappa_1+\kappa_2\kappa_1(z)+\kappa_3(\kappa_1(z))^2+\kappa_4\frac{z\kappa'_1(z)}{\kappa_1(z)}&\prec\psi^{\xi,m}_{\sigma,\vartheta}(\kappa_1,\kappa_2,\kappa_3,\kappa_4,\mu;z)\\&\prec\kappa_1+\kappa_2\kappa_2(z)+\kappa_3(\kappa_2(z))^2+\kappa_4\frac{z\kappa'_2(z)}{\kappa_2(z)}\end{aligned} \tag{3.4}$$

where $\psi^{\xi,m}_{\sigma,\vartheta}(\kappa_1,\kappa_2,\kappa_3,\kappa_4,\mu;z)$ defined in (2.12).
Hence

$$\kappa_1(z)\prec\left[\frac{\mathcal{Q}^{\xi,m}_{\sigma,\vartheta,q}f(z)}{z}\right]^{\mu}\prec\kappa_2(z),$$

and $\kappa_1(z)$ and $\kappa_2(z)$ are the best subordinate and best dominant in (3.4).

Remark 3.3. Theorem 2.11, and Corollaries 2.9 and 2.10 can be study by using the sandwich theorem.

4 Conclusion

Given the importance of q-calculus in many branches of mathematics and quantum physics, many studies focused on introducing new concepts by using q-calculus. Therefore, we have presented a new q-derivative operator connected with the q-exponential function $e_q(z)$. Moreover, we studied the best dominant and the best subordinate results. After that, we investigated subordination results in sandwich theorems.
Hence we obtained some new properties of this operator by using the subordination and superordination properties. Other subclasses of analytic functions can also be introduced using this operator, and further investigations into the geometric properties like (distortion theorems, coefficient estimates, neighborhoods and starlikeness radii, closure theorems, and convexity or close-to-convexity of functions).

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