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Stability analysis of Swine flu epidemics model with awareness and fear

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Abstract

Many awareness programmes are suggested by health care agencies to reduce the adverse effects of swine flu infection on society. These awareness programmes help to create fear for behavioural changes, which may control the spread of various diseases. Several mathematical models have been studied by many researchers earlier. In this manuscript, we suggest an SEIR mathematical model see the impact of awareness and fear on swine flu infection. The bounded region has been carried out in which, disease-free equilibrium point and endemic equilibrium point exist. The basic reproduction number has been evaluated to determine the local and global stability conditions around equilibrium points. Sensitivity analysis of the basic reproduction number is done to find out the dominant parameters that have a significant impact on infection level. Moreover, suitable graphs are obtained and it is found that awareness is more effective than fear to reduce the risk of swine flu infection.

Keywords: Swine flu infection, Awareness and fear, Equilibrium Points, Basic Reproduction Number, Stability analysis and Sensitivity analysis
2020 MSC: 34D20, 34D23, 37C75, 49Q12, 90C31, 93C15, 93D05

1 Introduction

The social behavior of individuals and dynamics of disease are correlated to each other. Behavior of individual is one of the causes which makes susceptible infectious one. Behavioral changes help to shape epidemic size by improving the life style and contact places. The spread of several diseases can be controlled by behavioral changes. Promotion of awareness among group of people through media can create fear (threat) for behavioral changes which helps to limit the spread of infection [11]. Changes in knowledge, behavior and psychological responses can manage the anxiety in the community about the outbreak phase of swine flu infection [22, 23, 7, 3]. Mathematical model is one of the mean which provides the information about the population and disease dynamics at any moment of time with precautionary course for controlling the disease [20].

Several researchers have proposed and analyzed the ecological and epidemics models incorporating the effect of awareness and fear. I. Ghosh et al. (2018) proposed a SI epidemic model for HIV/AIDS with media and self-imposed psychological fear. They calculated basic reproduction number to suggest the stability conditions. They regulated the HIV case data sets for Uganda and Tanzania to the model and estimated some parameters. On comparing of

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graphs for awareness and self-imposed psychological fear, they observed that awareness is more effective to reduce the risk of acquiring HIV infection [4]. A. Sha et al. (2019) gave an eco-epidemiological model with disease in the prey population incorporating fear effect. They evaluated equilibrium points and their stability conditions. Their numerical results show the backward bifurcation, presence of oscillations and occurrence of chaos due to fear induced lower disease transmission in the prey population [21]. I. Papst (2015) proposed SIR epidemic model to see the effects of fear on transmission dynamics of infectious diseases. They calculated equilibrium points and basic reproduction number for providing the stability conditions. They studied the effects of fear on two important public health metrics (the epidemic length and peak prevalence) [15]. The ecological model based on three species food chain incorporating fear with stability and bifurcation analysis in 2018 [13] and three species food chain model with fear induced trophic cascade in 2019 [14] were studied by P. Panday et al. S. Pal et al. (2019) studied the impact of fear effect in a predator–prey model with Beddington–DeAngelis functional response. They observed that due to fear of predator the birth rate of prey population reduces and as the strength of fear increases then model system becomes stable. They also observed that the fear of predation risk can have both stabilizing and destabilizing effects [12]. K. Kundu et al. (2018) have given a discrete time prey predator model to see the effect of fear and observed that fear effect enhances stability in a predator-prey system [6]. A. Das et al. (2019) have proposed stochastic prey predator model with additional food for predator incorporating fear effect. They have also investigated the model with fear as well as without fear. Some new results have been carried out for different fear functions [2]. C. Maji (2021) has analyzed an SEIR model to study the effect of fear induced by media on transmission of epidemic COVID-19 [8]. M. A. Mamun et al. (2020) has done statistical analysis on Bangladesh population & observed that COVID 19 patients have risk factor of depression and suicidal ideation due to poor knowledge and greater fear of pandemic [9]. H. Purushwani et al. (2019) have given SIR model to see the effect of various awareness policy on swine flu infection [19]. H. Purushwani et al. (2021) have suggested SEIR mathematical model to observe the Impact of early treatment programs on Swine flu infection with optimal controls [18]. F. Bozurt et al. (2021) has examine an fractional order model of COVID-19 incorporating the effect of fear produced by media and social network in community during lockdown period [1]. Moreover, we have formed an SEIR epidemic model for swine flu epidemics with awareness and fear using system of non linear differential equations. Bounded region for the existence of solutions is derived. Disease free equilibrium point, endemic equilibrium point and basic reproduction number are evaluated. Using variational matrix and lyapunov's function, conditions for stability are carried out. Sensitivity analysis of basic reproduction number is performed to guess, most influence parameter that have impact on infection spread and control. Numerical simulation is done to support obtained results and theoretical results by relevant graphs. Conclusion and discursion is also provided.

2 Formulation of model

To formulate a SEIR mathematical model for swine flu infection with awareness and fear, we assume that the total population (N) is divided into five distinct subclasses namely; the susceptible unaware class (S_u), the susceptible aware class (S_a), Exposed (E) , Infected (I) and Recovered (R). The lack of awareness about treatment and hygienic condition in individuals may precede to the growth of infection level. Many awareness programmes are assisted by the health care agencies to create fear for behavioural changes so that the risk of infection can be reduced. Keeping the above-mentioned facts into consideration, the transmission dynamics of swine flu infection with awareness and fear can be described by system of nonlinear differential equations as follows:

$$\frac{dS_u}{dt} = \Delta - \alpha S_u + \varepsilon S_a - \frac{\beta_1 S_u I}{N} - dS_u \tag{2.1}$$

$$\frac{dS_a}{dt} = \alpha S_u - \varepsilon S_a - \frac{\beta_2 S_a I}{1 + k\left(S_a + E + I + R\right)} - dS_a + tE \tag{2.2}$$

$$\frac{dE}{dt} = \frac{\beta_1 S_u I}{N} + \frac{\beta_2 S_a I}{1 + k\left(S_a + E + I + R\right)} - \left(\xi + t + d\right) E \tag{2.3}$$

$$\frac{dI}{dt} = \xi E - \left(d + \sigma + \nu\right) I \tag{2.4}$$

$$\frac{dR}{dt} = \nu I - dR \tag{2.5}$$

with initial conditions

$$S_u(0) = S_{u0} > 0,\ S_a(0) = S_{a0} > 0,\ E(0) = E_0 > 0,\ I(0) = I_0 > 0,\ R(0) = R_0.$$

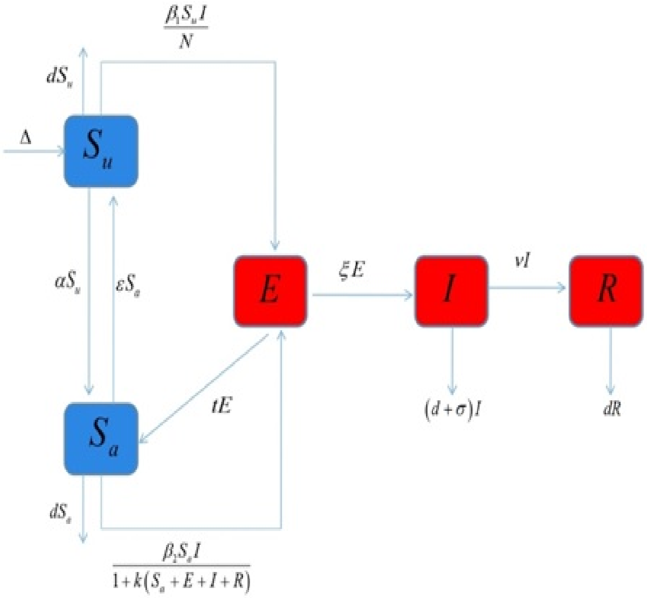

Figure 1: **Flow of disease**

The flow of disease in individuals easily can be demonstrated by Figure 1 given below;

In this model, the classes of individuals are connected as follows:

Suppose that constant recruitment rate of individuals is Π. As newly recruited individuals are not aware about infection severity. Awareness spreads among unaware individuals at a constant rate α by media coverage and reporting. The information loses the rate ε. Since latent period for swine flu infection is 2-3 days, so initially susceptible population joins the exposed class then after acquiring proper infection move to infected class. Transmission rate of infection in unaware and aware population are β_1 and β_2 respectively where $\beta_1 > \beta_2$. Suppose $\frac{\beta_1 S_u I}{N}$ and $\frac{\beta_2 S_a I}{1+k(S_a+E+I+R)}$ are portions of unaware population and aware population respectively joining exposed class where k is the fear intensity that helps to reduce the chance of getting infection. Due to this some exposed people rejoins aware susceptible class again at the temporary recovery rate t. Conversion rate from exposed to infection class is ξ. Infected population recovers at the rate ν. Mortality rates are d (natural death rate) and σ (disease induced death rate) of each population class. The list of variables and parameters is mentioned in Table 1.

3 Basic Properties of the model

3.1 Bounds of the solutions

Suppose $N(t) = S_u(t) + S_a(t) + E(t) + I(t) + R(t)$ is total population associated to epidemic.

Differentiating N(t) w.r.to t and using model system (2.1)-(2.5), we get

$$\frac{dN}{dt} = \Delta - d\left(S_u + S_a + E + I + R\right) - \sigma I$$

$$\frac{dN}{dt} \leq \Delta - d\left(S_u + S_a + E + I + R\right) = \Delta - dN$$

On solving, we get following result;

$$N \leq \frac{\Delta}{d}\left(1 - e^{-dt}\right) + N_0 e^{-dt}$$

$$\lim_{t\to\infty} N \leq \frac{\Delta}{d}$$

Thus, all solutions of model system (2.1)-(2.5) lie in the compact positively invariant region τ, where,

$$\tau = \left\{(S_u, S_a, E, I, R) : (S_u + S_a + E + I + R) \leq \frac{\Delta}{d}\right\}.$$

Table 1: **List of Variables and Parameters**

Symbols	**Variables and Parameters**	**Unit**
$S_u(t)$	Unaware susceptible population at time t.	
$S_a(t)$	Aware susceptible population at time t.	
$E(t)$	Exposed population at time t.	
$I(t)$	Infected population at time t.	
$R(t)$	Recovered population at time t.	
Π	Constant recruitment rate of unaware population.	Person/day
α	Communication rate of awareness.	Person/day
ε	Lose of information.	Person/day
β_1	Transmission rate of swine flu infection in unaware population.	Person/day
β_2	Transmission rate of swine flu infection in aware population.	Person/day
k	Fear intensity.	/Person
ξ	Conversion rate from exposed to infection class.	Person/day
t	Temporary recovery rate of exposed to susceptible.	/day
d	Natural death rate.	/day
σ	Disease induced death rate.	/ day
υ	Recovery rate.	/ day

3.2 Disease-free equilibrium point

The disease-free equilibrium point E_{df} of the designed model is given by

$$E_{df} = \left(\bar{S}_u, \bar{S}_a, 0, 0, 0\right) = \left(\frac{\Delta(\varepsilon + d)}{d(\alpha + \varepsilon + d)}, \frac{\Delta\alpha}{d(\alpha + \varepsilon + d)}, 0, 0, 0\right)$$

which always exist in region τ.

3.3 Basic Reproduction Number

The average number of secondary infections produced by one infected individual during the entire period of infectiousness is known as basic reproduction number. It is required for determining the status of the disease and also stability conditions of the model around equilibrium points. Mathematically, basic reproduction number is the dominating Eigen value of the next generation matrix FV^{-1}, where F is the growth rate of new infection in infected class and V is the outside moment of individuals from infected class by any mean.

$$F_1 = \frac{\beta_1 S_u I}{N} + \frac{\beta_2 S_a I}{1 + k(S_a + E + I + R)}, \qquad F_2 = 0$$

$$V_1 = (\xi + t + d)E, \qquad V_2 = -\xi E + (d + \sigma + \nu) I$$

This implies;

$$F = \begin{bmatrix} 0 & \frac{\beta_1 \bar{S}_u}{\bar{S}_u + \bar{S}_a} + \frac{\beta_2 \bar{S}_a}{1 + k\bar{S}_a} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{\beta_1(\varepsilon+d)}{(\alpha+\varepsilon+d)} + \frac{\beta_2\Delta\alpha}{d(\alpha+\varepsilon+d)+k\Delta\alpha} \\ 0 & 0 \end{bmatrix}$$

and

$$V = \begin{bmatrix} (\xi + t + d) & 0 \\ -\xi & (d + \sigma + \nu) \end{bmatrix} \Rightarrow V^{-1} = \frac{1}{(\xi+t+d)(d+\sigma+\nu)} \begin{bmatrix} (d + \sigma + \nu) & 0 \\ \xi & (\xi + t + d) \end{bmatrix}$$

Therefore,

$$FV^{-1} = \frac{1}{(\xi + t + d)(d + \sigma + \nu)} \begin{bmatrix} \xi\left(\frac{\beta_1(\varepsilon+d)}{(\alpha+\varepsilon+d)} + \frac{\beta_2\Delta\alpha}{d(\alpha+\varepsilon+d)+k\Delta\alpha}\right) & (\xi + t + d)\left(\frac{\beta_1(\varepsilon+d)}{(\alpha+\varepsilon+d)} + \frac{\beta_2\Delta\alpha}{d(\alpha+\varepsilon+d)+k\Delta\alpha}\right) \\ 0 & 0 \end{bmatrix}$$

Consequently,

$$R_0 = \frac{\xi}{(\xi + t + d)(d + \sigma + \nu)} \left(\frac{\beta_1(\varepsilon + d)}{(\alpha + \varepsilon + d)} + \frac{\beta_2\Delta\alpha}{d(\alpha + \varepsilon + d) + k\Delta\alpha}\right)$$

3.4 Endemic equilibrium point

The endemic equilibrium point $E_e\left(\overset{*}{S}_u, \overset{*}{S}_a, \overset{*}{E}, \overset{*}{I}, \overset{*}{R}\right)$ of proposed model is obtained by equating the equations of the model system to zero. where,

$$\overset{*}{S}_a = \frac{\left(\alpha + \frac{\beta_1 \overset{*}{I}}{\overset{*}{N}} + d\right)\overset{*}{S}_u - \Delta}{\varepsilon}$$

$$\overset{*}{E} = \frac{(d+\sigma+\nu)\left(\Delta - d\overset{*}{N}\right)}{\sigma\xi},$$

$$\overset{*}{I} = \frac{\Delta - d\overset{*}{N}}{\sigma},$$

$$\overset{*}{R} = \frac{\nu\left(\Delta - d\overset{*}{N}\right)}{d\sigma},$$

Further, $\overset{*}{S}_u$ can be obtained by following quadratic equation;

$$A_2 \overset{*}{S}_u{}^2 + A_1 \overset{*}{S}_u + A_0 = 0 \tag{3.1}$$

where,

$$A_0 = \frac{\Delta(\varepsilon+d)\left(1+k\overset{*}{N}\right)}{\varepsilon} + \frac{\left(\Delta - d\overset{*}{N}\right)}{\sigma}\left[\frac{t(d+\sigma+\nu)\left(1+k\overset{*}{N}\right)}{\xi} + \frac{\beta_2\Delta}{\varepsilon}\right],$$

$$A_1 = -\left[\frac{tk(d+\sigma+\nu)\left(\Delta - d\overset{*}{N}\right)}{\sigma\xi} + \frac{k\Delta(\varepsilon+d)}{\varepsilon} + \left(1+k\overset{*}{N}\right)\left(\frac{\beta_1\left(\Delta - d\overset{*}{N}\right)}{\sigma\overset{*}{N}} + d\right)\right.$$
$$\left. + \left(\frac{d\left(1+k\overset{*}{N}\right)}{\varepsilon} + \frac{\beta_2\left(\Delta - d\overset{*}{N}\right)}{\varepsilon\sigma}\right)\left(\alpha + \frac{\beta_1\left(\Delta - d\overset{*}{N}\right)}{\sigma\overset{*}{N}} + d\right)\right]$$

$$A_2 = \frac{dk(\alpha+\varepsilon+d)}{\varepsilon} + \frac{\beta_1 k(\varepsilon+d)\left(\Delta - d\overset{*}{N}\right)}{\varepsilon\sigma\overset{*}{N}}$$

Since $\Delta > d\overset{*}{N}$, hence under this condition one root of equation (3.1) and value of $\overset{*}{S}_a, \overset{*}{E}, \overset{*}{I}, \overset{*}{R}$ will be positive. Consequently when $\Delta > d\overset{*}{N}$ then endemic equilibrium point $E_e\left(\overset{*}{S}_u, \overset{*}{S}_a, \overset{*}{E}, \overset{*}{I}, \overset{*}{R}\right)$ will exist.

4 Stability Analysis

In this section, stability conditions are proposed by following theorems given below;

Theorem 4.1. The disease-free equilibrium point E_{df} of the model (2.1) to (2.5) is linearly stable when $R_0 < 1$ and unstable otherwise.

Proof . To obtain stability condition, we will find Jacobian matrix J_{df} around the disease free equilibrium point $E_{df} = \left(\bar{S}_u, \bar{S}_a, 0, 0, 0\right)$ as follows;

$$J_{df} = \begin{bmatrix} -(\alpha+d) & \varepsilon & 0 & -\frac{\beta_1 \bar{S}_u}{\bar{N}} & 0 \\ \alpha & -(\varepsilon+d) & t & -\frac{\beta_2 \bar{S}_a}{1+k\bar{S}_a} & 0 \\ 0 & 0 & -(\xi+t+d) & \frac{\beta_1 \bar{S}_u}{\bar{N}} + \frac{\beta_2 \bar{S}_a}{1+k\bar{S}_a} & 0 \\ 0 & 0 & \xi & -(d+\sigma+\nu) & 0 \\ 0 & 0 & 0 & \nu & -d \end{bmatrix}$$

Eigen equation of above matrix is given by;

$$(d+\lambda)^2(\alpha+\varepsilon+d+\lambda)\left(B_0\lambda^2+B_1\lambda+B_2\right)=0$$

where,
$B_0=\left(\bar{S_a}+\bar{S_u}\right)\left(1+k\bar{S_a}\right)>0,$

$B_1=((\xi+t+d)+(d+\sigma+\nu))\left(\bar{S_a}+\bar{S_u}\right)\left(1+k\bar{S_a}\right)>0,$

$$\begin{aligned}B_2&=\left(\bar{S_a}+\bar{S_u}\right)\left(1+k\bar{S_a}\right)(\xi+t+d)(d+\sigma+\nu)-\xi\beta_1\bar{S_u}\left(1+k\bar{S_a}\right)-\xi\beta_2\bar{S_a}\left(\bar{S_a}+\bar{S_u}\right)\\&=\left(\bar{S_a}+\bar{S_u}\right)\left(1+k\bar{S_a}\right)(\xi+t+d)(d+\sigma+\nu)(1-R_0)>0 \text{ when } R_0<1\end{aligned}$$

Hence, all the Eigen values of above equation will be negative when $R_0<1$. Consequently, disease-free equilibrium point E_{df} of the model (2.1) to (2.5) is linearly stable when $R_0<1$ and unstable otherwise. □

Theorem 4.2. The endemic equilibrium point $E_e\left(\overset{*}{S_u},\overset{*}{S_a},\overset{*}{E},\overset{*}{I},\overset{*}{R}\right)$ of the model system (2.1) to (2.5) is linearly stable when $R_0>1$ and unstable otherwise.

Proof . To obtain stability condition, we will find Jacobian matrix J_e around the endemic equilibrium point $E_e\left(\overset{*}{S_u},\overset{*}{S_a},\overset{*}{E},\overset{*}{I},\overset{*}{R}\right)$ as follows;

$$J_e=\begin{bmatrix}J_{11}&J_{12}&0&J_{14}&0\\J_{21}&J_{22}&J_{23}&J_{24}&J_{25}\\J_{31}&J_{32}&J_{33}&J_{34}&J_{35}\\0&0&J_{43}&J_{44}&0\\0&0&0&J_{54}&J_{55}\end{bmatrix}$$

where,
$J_{11}=-\left(\alpha+d+\frac{\beta_1\overset{*}{I}}{\overset{*}{N}}\right),\ J_{12}=\varepsilon,\ J_{14}=-\frac{\beta_1\overset{*}{S_u}}{\overset{*}{N}},\ \ J_{21}=\alpha,$

$J_{22}=-\left(\varepsilon+d+\frac{\beta_2\overset{*}{I}}{1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)}-\frac{k\beta_2\overset{*}{S_a}\overset{*}{I}}{\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)^2}\right),\ J_{23}=t+\frac{k\beta_2\overset{*}{S_a}\overset{*}{I}}{\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)^2},$

$J_{24}=-\left(\frac{\beta_2\overset{*}{S_a}}{1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)}-\frac{k\beta_2\overset{*}{S_a}\overset{*}{I}}{\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)^2}\right),\ J_{25}=\frac{k\beta_2\overset{*}{S_a}\overset{*}{I}}{\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)^2},J_{31}=\frac{\beta_1\overset{*}{I}}{\overset{*}{N}},$

$J_{32}=\frac{\beta_2\overset{*}{I}}{1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)}-\frac{k\beta_2\overset{*}{S_a}\overset{*}{I}}{\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)^2},\ J_{33}=-\left((\xi+t+d)+\frac{k\beta_2\overset{*}{S_a}\overset{*}{I}}{\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)^2}\right),$

$J_{34}=\frac{\beta_1\overset{*}{S_u}}{\overset{*}{N}}+\frac{\beta_2\overset{*}{S_a}}{1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)}-\frac{k\beta_2\overset{*}{S_a}\overset{*}{I}}{\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)^2},\ J_{35}=-\frac{k\beta_2\overset{*}{S_a}\overset{*}{I}}{\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)^2},\ \ J_{43}=\xi,$

$J_{44}=-(d+\sigma+\nu),\ \ J_{54}=\nu,\ \ J_{55}=-d$ Eigen equation of above matrix is given by;

$$D_0\lambda^5+D_1\lambda^4+D_2\lambda^3+D_3\lambda^2+D_4\lambda+D_5=0$$

where,
$D_0=-1,$
$D_1=J_{11}+J_{22}+J_{33}+J_{44}+J_{55},$
$D_2=J_{12}J_{21}+J_{23}J_{32}+J_{34}J_{43}-J_{11}(J_{22}+J_{33}+J_{44}+J_{55})-J_{22}(J_{33}+J_{44}+J_{55})-J_{33}(J_{44}+J_{55})$
$-J_{44}J_{55},$
$D_3=J_{12}J_{23}J_{31}+J_{11}J_{22}J_{33}+J_{14}J_{31}J_{43}+J_{24}J_{32}J_{43}+J_{11}J_{22}J_{44}+J_{11}J_{33}J_{44}+J_{22}J_{33}J_{44}+J_{35}J_{43}J_{54}$
$+J_{11}J_{22}J_{55}+J_{11}J_{33}J_{55}+J_{22}J_{33}J_{55}+J_{11}J_{44}J_{55}+J_{22}J_{44}J_{55}+J_{33}J_{44}J_{55}-(J_{11}+J_{44}+J_{55})J_{23}J_{32}$
$-(J_{33}+J_{44}+J_{55})J_{12}J_{21}-(J_{11}+J_{22}+J_{55})J_{34}J_{43},$
$D_4=J_{12}J_{24}J_{31}J_{43}+J_{14}J_{21}J_{32}J_{43}+J_{11}J_{23}J_{32}J_{44}+J_{12}J_{21}J_{33}J_{44}+J_{25}J_{32}J_{43}J_{54}+J_{11}J_{23}J_{32}J_{55}$

$+J_{12}J_{21}J_{33}J_{55} + J_{12}J_{21}J_{44}J_{55} + J_{23}J_{32}J_{44}J_{55} + J_{11}J_{22}J_{34}J_{43} + J_{11}J_{34}J_{43}J_{55} + J_{22}J_{34}J_{43}J_{55}$
$-J_{14}J_{22}J_{31}J_{43} - J_{11}J_{24}J_{32}J_{43} - J_{12}J_{23}J_{31}J_{44} - J_{11}J_{22}J_{33}J_{44} - J_{11}J_{35}J_{43}J_{54} - J_{22}J_{35}J_{43}J_{54}$
$-J_{12}J_{23}J_{31}J_{55} - J_{11}J_{22}J_{33}J_{55} - J_{14}J_{31}J_{43}J_{55} - J_{24}J_{32}J_{43}J_{55} - J_{11}J_{22}J_{44}J_{55} - J_{11}J_{33}J_{44}J_{55}$
$-J_{22}J_{33}J_{44}J_{55} - J_{12}J_{21}J_{34}J_{43}$,
$D_5 = J_{12}J_{25}J_{31}J_{43}J_{54} + J_{11}J_{22}J_{35}J_{43}J_{54} + J_{14}J_{22}J_{31}J_{43}J_{55} + J_{11}J_{24}J_{32}J_{43}J_{55} + J_{12}J_{23}J_{31}J_{44}J_{55}$
$+J_{11}J_{22}J_{33}J_{44}J_{55} + J_{12}J_{21}J_{34}J_{43}J_{55} - J_{11}J_{25}J_{32}J_{43}J_{54} - J_{12}J_{21}J_{35}J_{43}J_{54} - J_{12}J_{24}J_{31}J_{43}J_{55}$
$-J_{14}J_{21}J_{32}J_{43}J_{55} - J_{11}J_{23}J_{32}J_{44}J_{55} - J_{12}J_{21}J_{33}J_{44}J_{55} - J_{11}J_{22}J_{34}J_{43}J_{55}$

Here, all $D_i < 0;\ i = 0,1,2,3,4,5$, when $R_0 > 1$ Hence, all the Eigen values of above equation will be negative when $R_0 > 1$. Consequently, endemic equilibrium point E_e of the model (2.1) to (2.5) is linearly stable when $R_0 > 1$ and unstable otherwise. □

Theorem 4.3. The endemic equilibrium point $E_e\left(\overset{*}{S}_u, \overset{*}{S}_a, \overset{*}{E}, \overset{*}{I}, \overset{*}{R}\right)$ of model system (2.1) to (2.5) is globally stable under following condition, otherwise unstable.

$$\left(t + \frac{\beta_2\Delta\left(1+k\left(\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right) + k\beta_2 d\overset{*}{S}_a\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S}_a+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right) > \frac{\beta_1 d\overset{*}{S}_u\overset{*}{I}}{\Delta\overset{*}{N}}$$

Proof . Let a positive definite function U as follows;

$$U = \frac{1}{2}\left[\left(S_u - \overset{*}{S}_u\right)^2 + \left(S_a - \overset{*}{S}_a\right)^2 + \left(E - \overset{*}{E}\right)^2 + \left(I - \overset{*}{I}\right)^2 + \left(R - \overset{*}{R}\right)^2\right]$$

Differentiating U w.r.to t and using proposed model system, we have following;

$$\begin{aligned}
\frac{dU}{dt} &= \left(S_u - \overset{*}{S}_u\right)\left[\left(-(\alpha+d)\left(S_u - \overset{*}{S}_u\right) + \varepsilon\left(S_a - \overset{*}{S}_a\right) - \frac{\beta_1 S_u I}{N} + \frac{\beta_1\overset{*}{S}_u\overset{*}{I}}{\overset{*}{N}}\right)\right] \\
&+ \left(S_a - \overset{*}{S}_a\right)\left[\left(\alpha\left(S_u - \overset{*}{S}_u\right) - (\varepsilon+d)\left(S_a - \overset{*}{S}_a\right) - \frac{\beta_2 S_a I}{1+k(S_a+E+I+R)} + \frac{\beta_2\overset{*}{S}_a\overset{*}{I}}{1+k\left(\overset{*}{S}_a+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)} + t\left(E - \overset{*}{E}\right)\right)\right] \\
&+ \left(E - \overset{*}{E}\right)\left[\left(\frac{\beta_1 S_u I}{N} - \frac{\beta_1\overset{*}{S}_u\overset{*}{I}}{\overset{*}{N}} + \frac{\beta_2 S_a I}{1+k(S_a+E+I+R)} - \frac{\beta_2\overset{*}{S}_a\overset{*}{I}}{1+k\left(\overset{*}{S}_a+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)} - (\xi+t+d)\left(E - \overset{*}{E}\right)\right)\right] \\
&+ \left(I - \overset{*}{I}\right)\left[\xi\left(E - \overset{*}{E}\right) - (d+\sigma+\nu)\left(I - \overset{*}{I}\right)\right] + \left(R - \overset{*}{R}\right)\left[\nu\left(I - \overset{*}{I}\right) - d\left(R - \overset{*}{R}\right)\right]
\end{aligned}$$

This implies

$$\begin{aligned}
\frac{dU}{dt} &= -\left[\left(\alpha + d + \frac{\beta_1 I\left(\overset{*}{S}_a+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)}{N\overset{*}{N}}\right)\left(S_u - \overset{*}{S}_u\right)^2 - \left(\varepsilon + \alpha + \frac{\beta_1\overset{*}{S}_u\overset{*}{I}}{N\overset{*}{N}}\right)\left(S_u - \overset{*}{S}_u\right)\left(S_a - \overset{*}{S}_a\right)\right. \\
&+ \left(\varepsilon + d + \frac{\beta_2\left(1+k\left(\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)I}{(1+k(S_a+E+I+R))\left(1+k\left(\overset{*}{S}_a+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right)\left(S_a - \overset{*}{S}_a\right)^2 + (d+\sigma+\nu)\left(I - \overset{*}{I}\right)^2 \\
&- \left(t + \frac{\beta_2\Delta\left(1+k\left(\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)+k\beta_2 d\overset{*}{S}_a\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S}_a+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)} - \frac{\beta_1 d\overset{*}{S}_u\overset{*}{I}}{\Delta\overset{*}{N}}\right)\left(S_a - \overset{*}{S}_a\right)\left(E - \overset{*}{E}\right) \\
&+ \left((\xi+t+d) + \frac{k\beta_2\overset{*}{S}_a\overset{*}{I}}{(1+k(S_a+E+I+R))\left(1+k\left(\overset{*}{S}_a+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)} + \frac{\beta_1\overset{*}{S}_u\overset{*}{I}}{N\overset{*}{N}}\right)\left(E - \overset{*}{E}\right)^2 \\
&- \left(\frac{-\beta_1\overset{*}{S}_u\left(S_u+\overset{*}{S}_a+\overset{*}{E}+\overset{*}{R}\right)}{N\overset{*}{N}}\right)\left(S_u - \overset{*}{S}_u\right)\left(I - \overset{*}{I}\right) - \frac{\beta_1\overset{*}{S}_u\overset{*}{I}}{N\overset{*}{N}}\left(S_u - \overset{*}{S}_u\right)\left(R - \overset{*}{R}\right) \\
&- \frac{\left(\beta_1\overset{*}{S}_u\overset{*}{I}+\beta_1 I\left(\overset{*}{S}_a+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}{N\overset{*}{N}}\left(S_u - \overset{*}{S}_u\right)\left(E - \overset{*}{E}\right) + d\left(R - \overset{*}{R}\right)^2
\end{aligned}$$

$$-\left(\xi+\frac{\beta_1\overset{*}{S_u}\left(S_u+\overset{*}{S_a}+\overset{*}{E}+\overset{*}{R}\right)}{N\overset{*}{N}}+\frac{\beta_2\left(1+k\left(S_a+\overset{*}{E}+\overset{*}{R}\right)\right)\overset{*}{S_a}}{(1+k(S_a+E+I+R))\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right)\left(E-\overset{*}{E}\right)\left(I-\overset{*}{I}\right)$$
$$-\left(-\left(\frac{\beta_1\overset{*}{S_u}\overset{*}{I}}{N\overset{*}{N}}+\frac{k\beta_2\overset{*}{S_a}\overset{*}{I}}{(1+k(S_a+E+I+R))\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right)\right)\left(E-\overset{*}{E}\right)\left(R-\overset{*}{R}\right)$$
$$-\left(\frac{-\beta_2\left(1+k\left(S_a+\overset{*}{E}+\overset{*}{R}\right)\right)\overset{*}{S_a}}{(1+k(S_a+E+I+R))\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right)\left(S_a-\overset{*}{S_a}\right)\left(I-\overset{*}{I}\right)$$
$$-\left(\frac{k\beta_2\overset{*}{S_a}\overset{*}{I}}{(1+k(S_a+E+I+R))\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right)\left(S_a-\overset{*}{S_a}\right)\left(R-\overset{*}{R}\right)-\nu\left(I-\overset{*}{I}\right)\left(R-\overset{*}{R}\right)\Big]$$

Putting the conditions of bounded region, we have following

$$\frac{dU}{dt}\le-[b_{uu}\left(S_u-\overset{*}{S_u}\right)^2-b_{ua}\left(S_u-\overset{*}{S_u}\right)\left(S_a-\overset{*}{S_a}\right)+b_{aa}\left(S_a-\overset{*}{S_a}\right)^2-b_{ae}\left(S_a-\overset{*}{S_a}\right)\left(E-\overset{*}{E}\right)$$
$$+b_{ee}\left(E-\overset{*}{E}\right)^2+b_{ii}\left(I-\overset{*}{I}\right)^2+b_{rr}\left(R-\overset{*}{R}\right)^2-b_{ui}\left(S_u-\overset{*}{S_u}\right)\left(I-\overset{*}{I}\right)-b_{ue}\left(S_u-\overset{*}{S_u}\right)\left(E-\overset{*}{E}\right)$$
$$-b_{ur}\left(S_u-\overset{*}{S_u}\right)\left(R-\overset{*}{R}\right)-b_{ei}\left(E-\overset{*}{E}\right)\left(I-\overset{*}{I}\right)-b_{er}\left(E-\overset{*}{E}\right)\left(R-\overset{*}{R}\right)-b_{ai}\left(S_a-\overset{*}{S_a}\right)\left(I-\overset{*}{I}\right)$$
$$-b_{ar}\left(S_a-\overset{*}{S_a}\right)\left(R-\overset{*}{R}\right)-b_{ir}\left(I-\overset{*}{I}\right)\left(R-\overset{*}{R}\right)]\qquad,$$

On rearranging the terms, we have;

$$\frac{dU}{dt}\le-\left[\left\{\frac{b_{uu}}{4}\left(S_u-\overset{*}{S_u}\right)^2-b_{ua}\left(S_u-\overset{*}{S_u}\right)\left(S_a-\overset{*}{S_a}\right)+\frac{b_{aa}}{4}\left(S_a-\overset{*}{S_a}\right)^2\right\}\right.$$
$$+\left\{\frac{b_{aa}}{4}\left(S_a-\overset{*}{S_a}\right)^2-b_{ae}\left(S_a-\overset{*}{S_a}\right)\left(E-\overset{*}{E}\right)+\frac{b_{ee}}{4}\left(E-\overset{*}{E}\right)^2\right\}$$
$$+\left\{\frac{b_{uu}}{4}\left(S_u-\overset{*}{S_u}\right)^2-b_{ui}\left(S_u-\overset{*}{S_u}\right)\left(I-\overset{*}{I}\right)+\frac{b_{ii}}{4}\left(I-\overset{*}{I}\right)^2\right\}$$
$$+\left\{\frac{b_{uu}}{4}\left(S_u-\overset{*}{S_u}\right)^2-b_{ue}\left(S_u-\overset{*}{S_u}\right)\left(E-\overset{*}{E}\right)+\frac{b_{ee}}{4}\left(E-\overset{*}{E}\right)^2\right\}$$
$$+\left\{\frac{b_{uu}}{4}\left(S_u-\overset{*}{S_u}\right)^2-b_{ur}\left(S_u-\overset{*}{S_u}\right)\left(R-\overset{*}{R}\right)+\frac{b_{rr}}{4}\left(R-\overset{*}{R}\right)^2\right\}$$
$$+\left\{\frac{b_{ee}}{4}\left(E-\overset{*}{E}\right)^2-b_{ei}\left(E-\overset{*}{E}\right)\left(I-\overset{*}{I}\right)+\frac{b_{ii}}{4}\left(I-\overset{*}{I}\right)^2\right\}$$
$$+\left\{\frac{b_{ee}}{4}\left(E-\overset{*}{E}\right)^2-b_{er}\left(E-\overset{*}{E}\right)\left(R-\overset{*}{R}\right)+\frac{b_{rr}}{4}\left(R-\overset{*}{R}\right)^2\right\}$$
$$+\left\{\frac{b_{aa}}{4}\left(S_a-\overset{*}{S_a}\right)^2-b_{ai}\left(S_a-\overset{*}{S_a}\right)\left(I-\overset{*}{I}\right)+\frac{b_{ii}}{4}\left(I-\overset{*}{I}\right)^2\right\}$$
$$+\left\{\frac{b_{aa}}{4}\left(S_a-\overset{*}{S_a}\right)^2-b_{ar}\left(S_a-\overset{*}{S_a}\right)\left(R-\overset{*}{R}\right)+\frac{b_{rr}}{4}\left(R-\overset{*}{R}\right)^2\right\}$$
$$\left.+\left\{\frac{b_{ii}}{4}\left(I-\overset{*}{I}\right)^2-b_{ir}\left(I-\overset{*}{I}\right)\left(R-\overset{*}{R}\right)+\frac{b_{rr}}{4}\left(R-\overset{*}{R}\right)^2\right\}\right]$$

where,

$$b_{uu}=\left(\alpha+d+\frac{\beta_1\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)}{\overset{*}{N}}\right),b_{ua}=\left(\varepsilon+\alpha+\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}\right),b_{aa}=\left(\varepsilon+d+\frac{\beta_2\Delta\left(1+k\left(\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right),$$
$$b_{ae}=\left(t+\frac{\beta_2\Delta\left(1+k\left(\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)+k\beta_2 d\overset{*}{S_a}\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}-\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}\right),b_{ee}=\left((\xi+t+d)+\frac{k\beta_2 d\overset{*}{S_a}\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}+\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}\right),$$

$b_{ii}=(d+\sigma+\nu)\,,\ b_{rr}=d,\, b_{ui}=-\left(\frac{\beta_1 d\overset{*}{S_u}\left(S_u+\overset{*}{S_a}+\overset{*}{E}+\overset{*}{R}\right)}{\Delta\overset{*}{N}}\right),b_{ue}=\frac{\left(\beta_1 d\overset{*}{S_u}\overset{*}{I}+\beta_1\Delta\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}{\Delta\overset{*}{N}},$

$b_{ur}=\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}},\ b_{ei}=\left(\xi+\frac{\beta_1 d\overset{*}{S_u}\left((\Delta/d)+\overset{*}{S_a}+\overset{*}{E}+\overset{*}{R}\right)}{\Delta\overset{*}{N}}+\frac{\beta_2 d\left(1+k\left((\Delta/d)+\overset{*}{E}+\overset{*}{R}\right)\right)\overset{*}{S_a}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right),b_{ar}=\left(\frac{k\beta_2 d\overset{*}{S_a}\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right),$

$b_{er}=-\left(\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}+\frac{k\beta_2 d\overset{*}{S_a}\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right),\ b_{ai}=-\left(\frac{\beta_2 d\left(1+k\left((\Delta/d)+\overset{*}{E}+\overset{*}{R}\right)\right)\overset{*}{S_a}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right),b_{ir}=\nu$

Further, under following conditions $\frac{dU}{dt}$ will be negative definite;

$$\frac{1}{4}\left(\alpha+d+\frac{\beta_1\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)}{\overset{*}{N}}\right)\left(\varepsilon+d+\frac{\beta_2\Delta\left(1+k\left(\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right)>\left(\varepsilon+\alpha+\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}\right)^2,$$

$$\frac{1}{4}\left(\varepsilon+d+\frac{\beta_2\Delta\left(1+k\left(\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right)\left((\xi+t+d)+\frac{k\beta_2 d\overset{*}{S_a}\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}+\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}\right)$$
$$>\left(t+\frac{\beta_2\Delta\left(1+k\left(\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)+k\beta_2 d\overset{*}{S_a}\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}-\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}\right)^2,$$

$$\frac{1}{4}(d+\sigma+\nu)\left(\alpha+d+\frac{\beta_1\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)}{\overset{*}{N}}\right)>\left(\frac{\beta_1 d\overset{*}{S_u}\left(S_u+\overset{*}{S_a}+\overset{*}{E}+\overset{*}{R}\right)}{\Delta\overset{*}{N}}\right)^2,$$

$$\frac{1}{4}\left(\alpha+d+\frac{\beta_1\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)}{\overset{*}{N}}\right)\left((\xi+t+d)+\frac{k\beta_2 d\overset{*}{S_a}\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}+\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}\right)$$
$$>\left(\frac{\left(\beta_1 d\overset{*}{S_u}\overset{*}{I}+\beta_1\Delta\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}{\Delta\overset{*}{N}}\right)^2,$$

$$\frac{1}{4}d\left(\alpha+d+\frac{\beta_1\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)}{\overset{*}{N}}\right)>\left(\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}\right)^2,$$

$$\frac{1}{4}(d+\sigma+\nu)\left((\xi+t+d)+\frac{k\beta_2 d\overset{*}{S_a}\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}+\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}\right)$$
$$>\left(\xi+\frac{\beta_1 d\overset{*}{S_u}\left((\Delta/d)+\overset{*}{S_a}+\overset{*}{E}+\overset{*}{R}\right)}{\Delta\overset{*}{N}}+\frac{\beta_2 d\left(1+k\left((\Delta/d)+\overset{*}{E}+\overset{*}{R}\right)\right)\overset{*}{S_a}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right)^2,$$

$$\frac{1}{4}d\left((\xi+t+d)+\frac{k\beta_2 d\overset{*}{S_a}\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}+\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}\right)$$
$$>\left(\frac{\beta_1 d\overset{*}{S_u}\overset{*}{I}}{\Delta\overset{*}{N}}+\frac{k\beta_2 d\overset{*}{S_a}\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right)^2,$$

$$\frac{1}{4}(d+\sigma+\nu)\left(\varepsilon+d+\frac{\beta_2\Delta\left(1+k\left(\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}{(d+4\Delta k)\left(1+k\left(\overset{*}{S_a}+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right)$$

$$> \left(\frac{\beta_2 d \left(1 + k\left((\Delta/d) + \overset{*}{E} + \overset{*}{R}\right)\right) \overset{*}{S}_a}{(d+4\Delta k)\left(1+k\left(\overset{*}{S}_a + \overset{*}{E} + \overset{*}{I} + \overset{*}{R}\right)\right)} \right)^2,$$

$\frac{1}{4}d\left(\varepsilon + d + \frac{\beta_2\Delta\left(1+k\left(\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}{(d+4\Delta k)\left(1+k\left(\overset{*}{S}_a+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right) > \left(\frac{k\beta_2 d\overset{*}{S}_a\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S}_a+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right)^2,$
$\frac{d(d+\sigma+\nu)}{4} > \nu^2$. Hence, under following condition $\frac{dU}{dt}$ is negative definite;

$$\left(t + \frac{\beta_2\Delta\left(1+k\left(\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right) + k\beta_2 d\overset{*}{S}_a\overset{*}{I}}{(d+4\Delta k)\left(1+k\left(\overset{*}{S}_a+\overset{*}{E}+\overset{*}{I}+\overset{*}{R}\right)\right)}\right) > \frac{\beta_1 d\overset{*}{S}_u\overset{*}{I}}{\Delta\overset{*}{N}}$$

Consequently, using Lyapunov Stability theorem, under above mentioned condition the endemic equilibrium point $E_e\left(\overset{*}{S}_u, \overset{*}{S}_a, \overset{*}{E}, \overset{*}{I}, \overset{*}{R}\right)$ of model system (2.1) to (2.5) is globally stable, otherwise unstable. □

5 Numerical simulation

In this section, proposed model is simulated for the set of parameters values to observe the effect of awareness and fear on swine flu infection. Some numerical facts are verified for the existence and stability properties of the equilibrium points.
For the analysis of swine flu transmission with awareness and fear, we choose set-1 for values of parameters given below-

Δ=1.5 Person/day, β_1=0.035 Person/day, β_2=0.025 Person/day, α=0.4 Person/day, ε=0.3 Person/day, d=0.02 /day, σ=0.01 /day, υ=0.01 /day, k=0.8 / Person, t=0.4 /day, ξ=0.5 Person/day then for this set basic reproduction number R_0=0.6236 <1 and disease free equilibrium point is E_{df}(33.333, 41.666, 0, 0, 0) Figure 2 shows all species versus time plot, it is cleared that all the trajectories moves towards the disease free condition, which represents that the disease free equilibrium is locally asymptotically stable. Next, if we take same set expect k=0.3 /Person then for this set basic reproduction number R_0=1.2597 >1 and endemic equilibrium point is E_{e}(27.783, 34.179, 0.5015, 6.2683, 3.1341). In same Figure 2, It is clear that endemic equilibrium point is stable under these values of parameters. Figure 3 shows as fear intensity increases R_0 decreases and awareness has parabolic nature.

Figure 4 and Table 2 depict that for fixed value of k=0.3, as awareness increases then basic reproduction number and numbers of infectious decreases. Similarly, Figure 5 and Table 3 highlight that for fixed value of α=0.4, as fear increases then basic reproduction number and numbers of infectious decreases. In Figure 6, plot between time versus

Table 2: **Basic reproduction number and Equilibrium Point for various values of α**

α	**Basic reproduction number**	**Equilibrium Point**
0.4	1.2597	E_{e}(27.783, 34.179, 0.5015, 6.2683, 3.1341)
0.6	1.2254	E_{e}(21.844, 40.362, 0.492, 6.151, 3.076)
0.8	1.2018	E_{e}(18.069, 44.563, 0.476, 5.946, 2.973)

Table 3: **Basic reproduction number and Equilibrium Point for various values of k**

k	**Basic reproduction number**	**Equilibrium Point**
0.4	1.0125	E_{e}(33.009, 41.231, 0.029, 0.366, 0.183)
0.6	0.7557	E_{df}(33.333, 41.666, 0, 0, 0)
0.8	0.6236	E_{df}(33.333, 41.666, 0, 0, 0)

infected population is traced to compare the effect of awareness and fear on swine flu infection. It is observed that awareness minimizes infection cases speedily whereas fear minimizes infection cases slowly. See the scale of both graphs also. Further, sensitivity indices are obtained to guess the most dominant parameters that have significant impact on spread and control of swine flu infection, using normalized forward sensitivity index techniques. It can be observed

Table 4: **Sensitivity Indices for Basic Reproduction Number**

Parameters	**Sensitivity indices**
Δ	+0.0616
α	-0.0658
ε	+0.0617
β_1	+0.1678
β_2	+0.8322
k	-0.7706
ξ	+0.4565
t	-0.4348
d	-0.5793
σ	-0.2500
ν	-0.2500

from above table, that the recruitment rate of susceptible, loss of awareness, transmission rate of infection in unaware, aware population and conversion rate from exposed to infection class are those parameters that have significant positive association with basic reproduction number i.e. sensitivity indices for β_1 is +0.1678, indicates that 10% increase in β_1 would approximately increase R_0 by 1.678%. Similarly, communication rate of awareness, fear intensity, temporary recovery rate of exposed to susceptible, mortality rate (natural and disease induced) of individual, recovery rate are those parameters that have significant negative association with basic reproduction number i.e. sensitivity indices for α is -0.0658, indicates that 10% increase in α would approximately reduce R_0 by 0.658 %.

6 Conclusion

In this manuscript, we have analyzed a mathematical model consists of nonlinear ordinary differential equations for five different interacting populations to see the spread of swine flu infection with awareness and fear. Bounded feasible region has been obtained in which all the solutions of proposed model exist. The disease free equilibrium point, endemic equilibrium point and basic reproduction number are evaluated to provide the stability conditions. It has been derived that the disease free equilibrium point is linearly asymptotically stable if $R_0 < 1$ otherwise unstable (Theorem 4.1 and Figure 2) i.e. swine flu infection will wash out from the community. It has been also noticed that a unique endemic (positive) equilibrium point exists and stable if $R_0 > 1$ otherwise unstable (Theorem 4.2 and Figure 2) i.e. swine flu infection will persist from the community. It is also pointed about that awareness and fear may reduce the basic reproductive number or the infection severity (Figure 3). Further, the phase plane plots between infected and susceptible also indicate that awareness and fear help to minimize the number of infected population (Figure 4 , 5). Moreover, it is also found that awareness can reduce the infection cases more rapidly than the fear (Figure 6). Consequently, it can be concluded that awareness is more influence parameter than fear to control infection severity and protect community structure. From sensitivity indices, influence of parameter on disease spread can be observed and it can be decided that how the parameters should be handled by adopting suitable way to save the community from the risk of infection.

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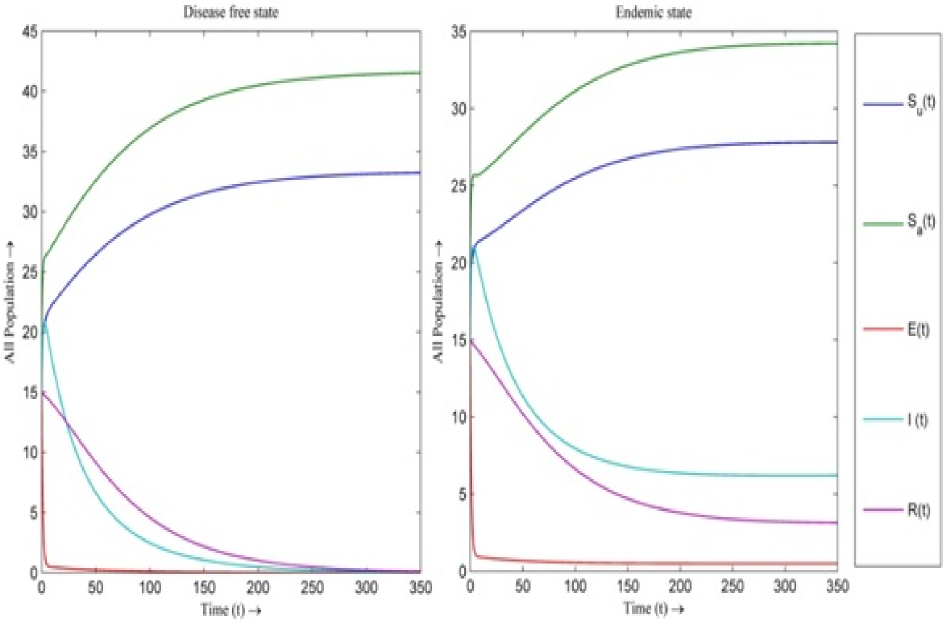

Figure 2: **Time versus All Species in Disease free and endemic state**

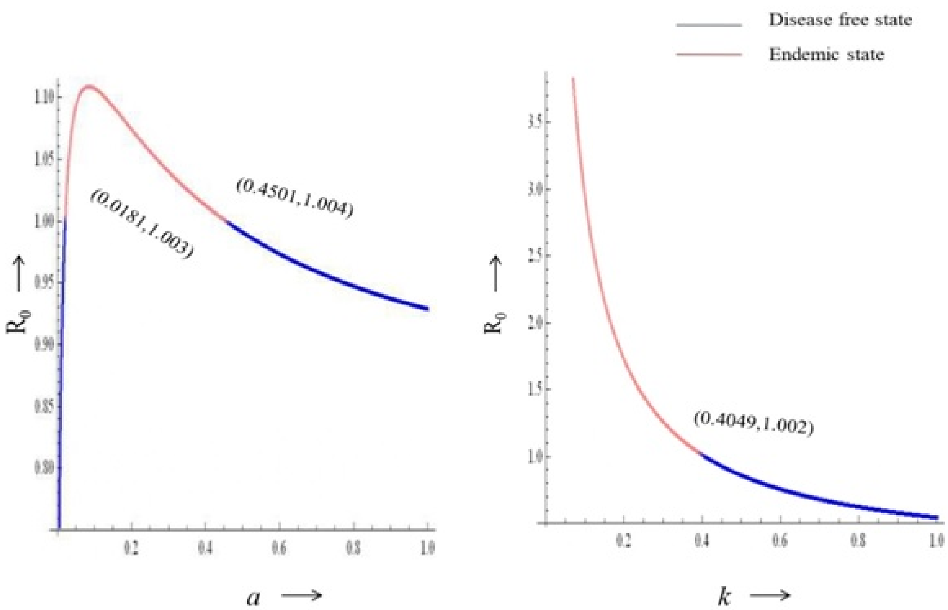

Figure 3: α **and** k **versus Basic reproduction number in Disease free and endemic state**

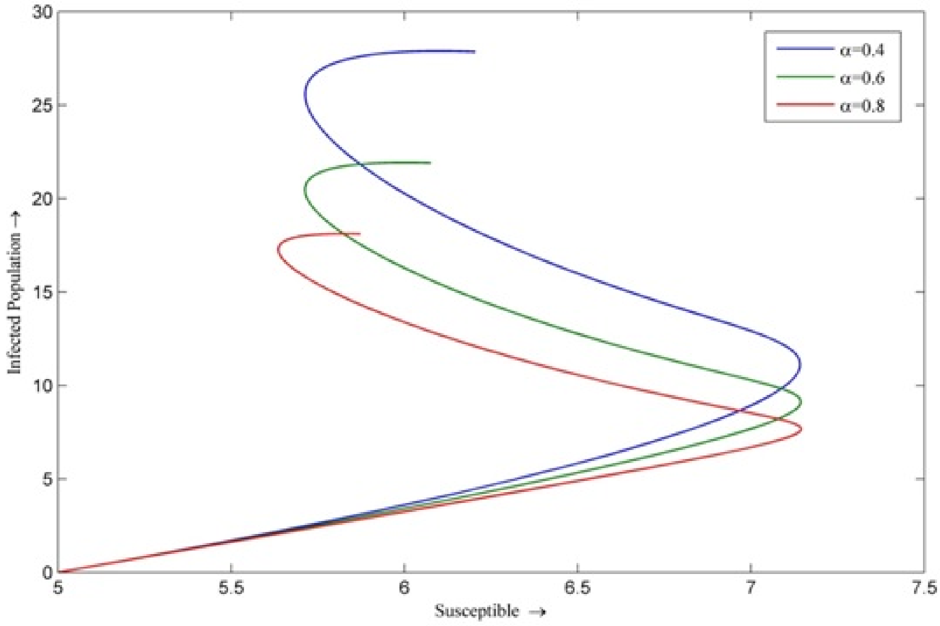

Figure 4: **Susceptible versus Infected population for various values of α**

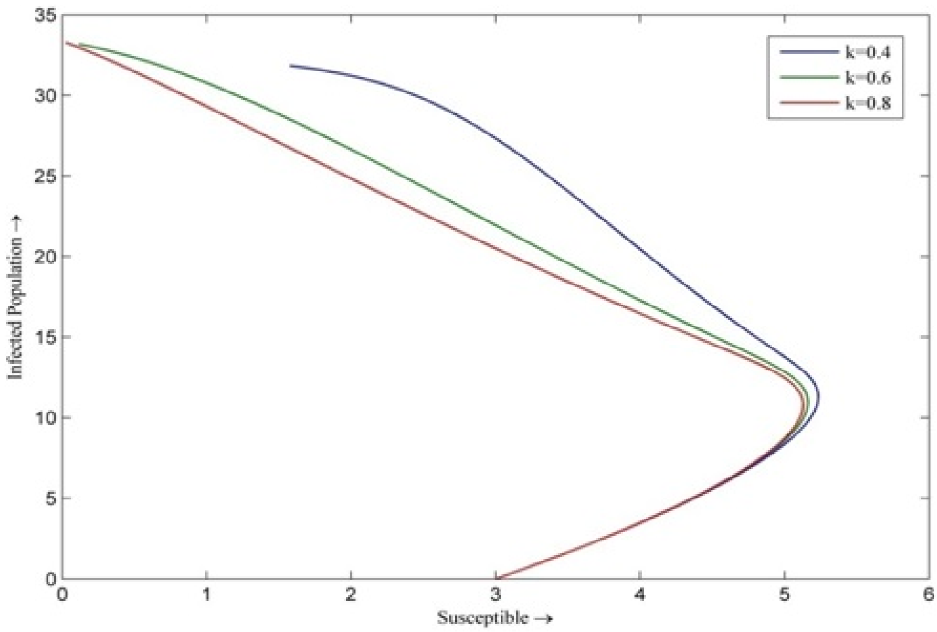

Figure 5: **Susceptible versus Infected population for various values of k**

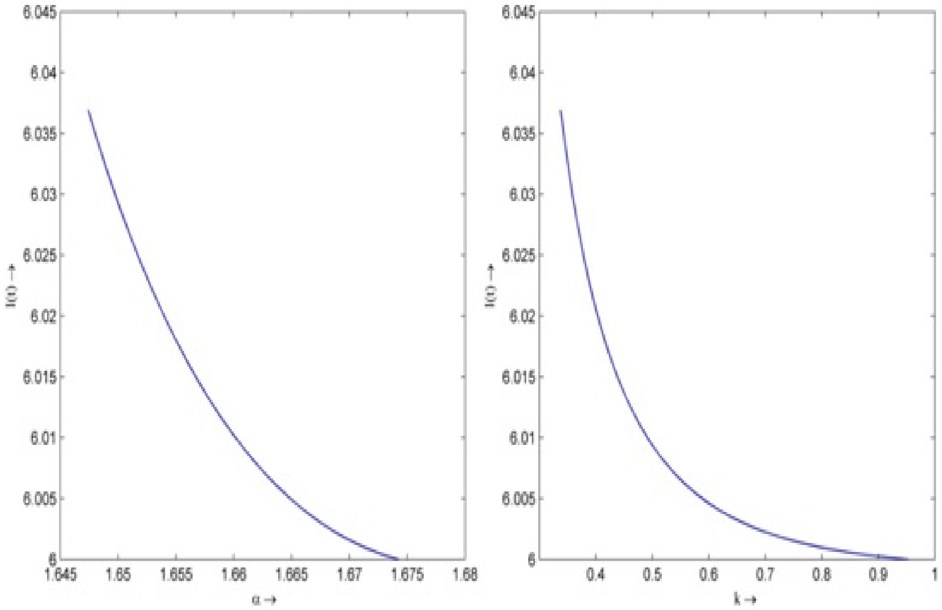

Figure 6: **α and k versus Infected population**

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