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Synchronization between two coupled fractional order neuron models using the optimized fuzzy logic controller in the presence of external disturbances

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Abstract

Appropriate modeling of the coupled neurons helps us understand neurons' natural functions. In this paper fuzzy logic controller has been designed to synchronize two coupled neuron models. The fractional-order neurons are based on the FitzHugh-Nagumo (FHN) model. An optimized fuzzy controller is designed to synchronize the behavior of two neurons with each other in the presence of external disturbances. This controller overcomes the disturbance. The simulation example shows the performance of the proposed method.

Keywords: Neroun model, Chaotic system, Fuzzy controller
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1 Introduction

Many fields of researchers studies have relation with chaos such as engineering and physics. Studies on the chaotic behavior have been attracted significant attention in the last decades. The main property of chaotic systems is the dependence of these systems to initial conditions. This property makes a high difference in the two same systems behavior with a little difference in the initial conditions [3, 24].

It was shown that nerve membranes have a nonlinear dynamic which has relation with action potentials. This nonlinear dynamic can generate chaotic behavior in the neurons.

For the first time, the mathematical model of nerve membranes is presented by A.L. Hodgkin and A.F. Huxley. This model has been obtained using electrophysiological experiments with squid giant atoms [8].

After that other studies in this field have been done. A model of action potentials of neurons was introduced by FitzHugh [13] and Nagumo [11]. This model is an extension of the Van der Pol equation. In this model, the Hodgkin-Huxley model of spike generation in giant squid axons is simplified. The considered Hodgkin-Huxley model is a two-dimensional model.

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Fractional calculus has a history more than 300 years. For many years these calculations have not many applications because they don't have analytical solutions. But nowadays, many analytical methods have been proposed [4]. Many differential equations have been modeled by fractional-order today [1, 5]. Fractional-order chaotic system is one of the most popular fractional order systems [14, 20]. One of the real world applications of these system is fractional-order neurons model [2].

Synchronization of coupled neurons according to the wide applications field has been done in many pieces of research. The most critical application of synchronization of coupled neurons is to understanding the behavior of neuron in the network [2].

Synchronization in chaotic systems is an old topic with different applications. Also, various methods have been introduced to this task [4, 7, 16, 17, 19].

Synchronization in the chaotic systems is happened with two systems which is named drive and response systems. Control signals are sending from response system to drive system. Then two system behaves like each others.

Different methods have been proposed to the control of chaotic systems. The well-known methods for chaos control is as OGY method, delay feedback, linear and nonlinear control, sliding mode control, fuzzy control, etc.

In the present paper, we use a fuzzy method for controlling the chaotic neural systems. For the first time, fuzzy set theory was introduced by Zadeh [23]. After that many other applications of fuzzy set theory have been introduced. One of the important applications of fuzzy theory is fuzzy control.

For chaos synchronizations, fuzzy systems have been used in some cases [18].

Bases of fuzzy controller is obtained from a conventional control method. Fuzzy control [4, 17] is consist of the theory of fuzzy sets and fuzzy logic [6, 22].

For the first time, fuzzy logic control (FLC) has been introduced near 40 years ago [15]. After that many improvements have been done in this field. Some of these improvements are as adaptive fuzzy controller to control and synchronization of chaotic systems [9, 10, 12, 21].

Tuning of FLC with trial and error is a traditional method in choosing the membership functions and rule base.

In this paper, we use particle swarm optimization method to determine the membership functions of the fuzzy logic controller. The cost function of this algorithm selected based on synchronization errors, then the results of optimized fuzzy control must have the least synchronization time which didn't notice in the past researches.

This paper is organized as follows. Section 2 describes the details the dynamics and properties of the FitzHugh-Nagumo chaotic neuron model. In section 3 basic concepts of the fuzzy logic controller is introduced. In section 4 synchronization method based on the fuzzy logic controller are proposed. Simulations of the proposed process are shown in section 5. Section 6 contains the main conclusions of this work.

2 Model of Neroun

2.1 Mathmatical model of fractional-order FitzHugh-Nagumo (FHN) model

Consider the system of differential equations:

$$D^{\alpha}x = x - \frac{x^3}{3} - y \tag{2.1}$$

$$D^{\beta}y = x + a \tag{2.2}$$

FHN model is as the above equations. In these equations x shows the membrane potential and y shows the recovery variable. Also a is a control parameter. In this paper we consider the parameters of systems as $\delta = 0.1$ and $a = 0.98$.

2.2 Stability analysis of FitzHugh-Nagumo (FHN) model

This system has one equilibrium:

$$x^* = -a \tag{2.3}$$

$$y^* = \frac{a^3}{3} - a \tag{2.4}$$

The type of the equilibrium point is an unstable focus, and around this point, the system acts chaotically. The phase portrait of this system is shown in Fig 1.

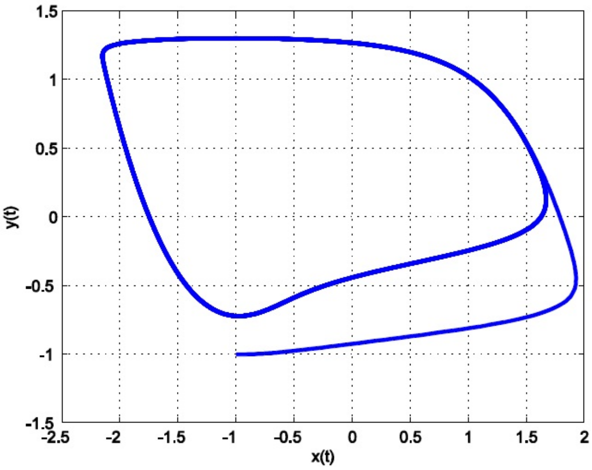

Figure 1: Phase portirate of FHN model

3 Synchronization Method

We consider two drive-response system with different initial conditions.so the drive system's equation is written as:

$$D^{\alpha} x_d = x_d - \frac{x_d^3}{3} - y_d + w_1 \tag{3.1}$$

$$\dot{y}_d = x_d + a + w_2 \tag{3.2}$$

Which has the initial condition as $\binom{-1}{-1}$ and w_1 and w_2 are external disturbances. The response system's equations are written as:

$$D^{\alpha} x_r = x_r - \frac{x_2^3}{3} - y_r \tag{3.3}$$

$$D^{\beta} y_r = x_r + a \tag{3.4}$$

Which has the initial condition as $\binom{0}{0}$. To control the chaotic model of neurons, we assume a control signal in the second state of the response system model. So this model is rewritten as:

$$D^{\alpha} x_r = x_r - \frac{x_r^3}{3} - y_r \tag{3.5}$$

$$D^{\beta} y_r = x_r + a + u \tag{3.6}$$

Here u is the control signal which is added to this system to control the chaos behavior. We calculate the difference between two systems states as below:

$$e_1 = x_r - x_d \tag{3.7}$$

$$e_2 = y_r - y_d \tag{3.8}$$

These equations demonstrate the synchronization error.

4 Fuzzy Logic Control

Fuzzy sets are defined using descriptive variables. Each of this variables have a membership function which shows the similarity of input and that variable. Fuzzy system uses these membership functions and rules to reason about the input.

The control signal based on the fuzzy logic controller is derived according to the following fuzzy rules.

if e is F_1^i and $\dot{e}$ is F_2^i then u is G^i

Where $F_1 \& F_2 \& G$ are membership functions for the i^{th} rule. Also, $F_1 \& F_2 \& G \subseteq [NB, NS, ZO, PS, PB]$ and notations are negative big, negative small, zero, positive small and positive big, respectively. The proposed rule-base table based on error and fractional derivative of error as the inputs of the fuzzy controller is according to Table 1.

Table 1: Rule base of fuzzy logic controller

$\dot{e}$ \ e	**NB**	**NS**	**ZO**	**PS**	**PB**
NB	NB	NB	NB	NS	ZO
NS	NB	NB	NS	ZO	PB
ZO	NB	NS	ZO	PB	PB
PS	NS	ZO	PB	PB	PB
PB	ZO	PB	PB	PB	PB

4.1 Particle swarm optimization

Particle swarm optimization (PSO) is an optimization method which is inspired by birds' social behavior. For the first time this algorithm is introduced by Dr. Eberhart and Dr. Kennedy in 1995 [6].

Structure of this algorithm is that the flying birds in the sky always are in the position that the best bird is in the front of all.

A function is optimized using this algorithm which is named cost function. The algorithm generates number of solutions which is named particle. And cost of each particle is calculated using the algorithm and the particle which has the minimum cost is selected as the best particle.

In each iteration position and velocity of these particle is updated. The best particle in each iteration is named personal best which is shown as pbest and the best particle of all of the iteration is called global best which is shown is gbest.

Particles update relations are as the follows:

$$v[i+1] = v[i] + c1 * rand() * (pbest[i] - present[i]) + c2 * rand() * (gbest[i] - present[i]) \tag{4.1}$$

$$present[i+1] = persent[i] + v[i+1] \tag{4.2}$$

where v is the particle velocity. $rand()$ generates a random number in $(0,1).c1, c2$ are constant coefficients.

Block diagram of this algorithm is shown at the Fig 2.

For optimizing the fuzzy logic controller, we use the adaptive particle swarm optimization algorithm. In this case, particles of PSO algorithm are considered as an interval of membership functions. According to the proposed method, we consider 30 particles for five membership functions of 2 inputs and one output of the system. The cost function of the system is defined according to the synchronization error of two states at each time step as the follows.

$$f = \int_0^T (e_1^2(t) + e_2^2(t))dt \tag{4.3}$$

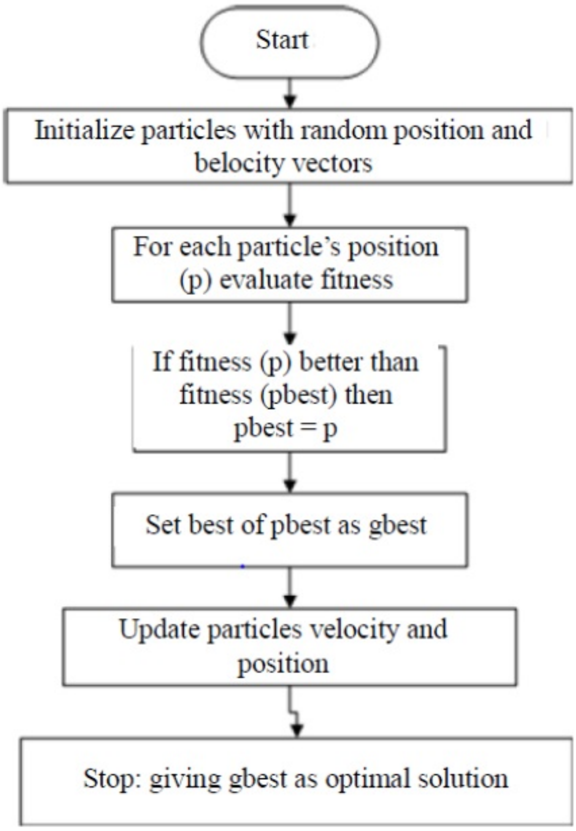

Figure 2: Folwchart of PSO algorithm

5 Simulations

The initial conditions for two systems are considered as $\binom{0}{0}$ and $\binom{-1}{-1}$ in the simulations. External disturbances are considered as $0.1\sin(10t)$ on two states. 100 iterations for the PSO algorithm is selected. Synchronization errors for the simple and optimized method are shown in Fig. 3, 4 and 5.

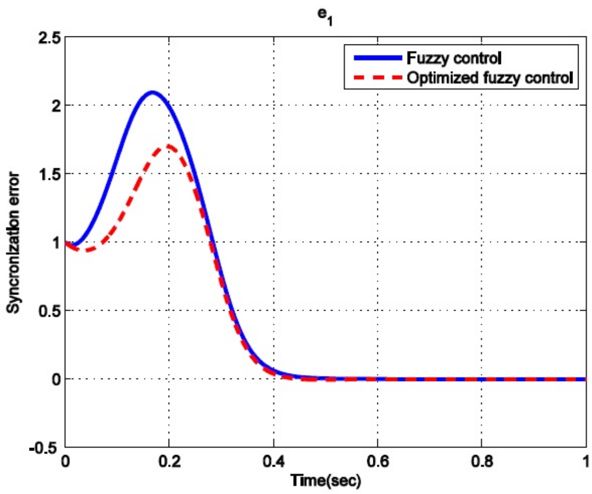

Figure 3: Synchronization error of first state

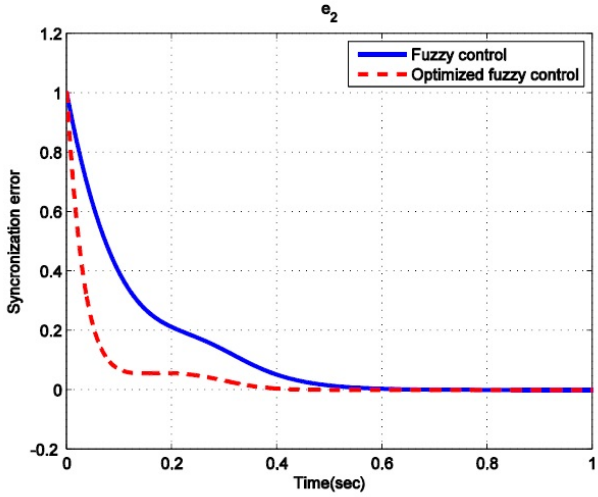

Figure 4: Synchronization error of second state

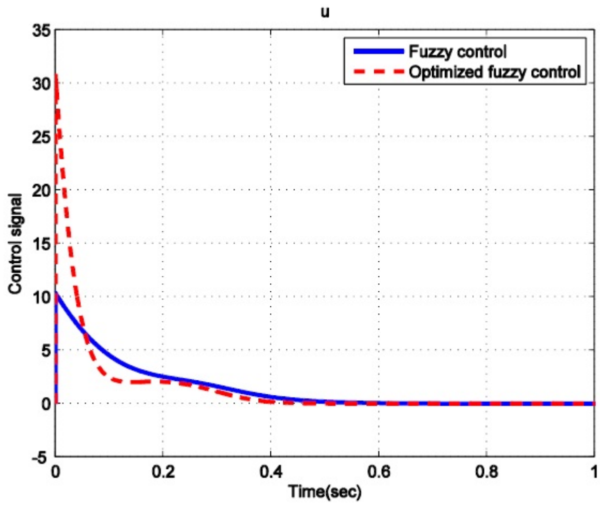

Figure 5: Control signals in two methods

6 Conclusion

In this paper, the chaotic model of the neuron was discussed. Synchronization problem of coupling neurons was introduced based on the fuzzy logic controller. Using the particle swarm optimization method, the designed controller was improved. The cost function was considered using synchronization errors. Optimization of the controller was shown on the synchronization times. Finally, it was shown fuzzy logic could be an appropriate tool to synchronize the behavior of two different chaotic system, which can be optimized by evolutionary algorithms.

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