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Fixed point theorems for modified generalized F-contraction and F-expansion of Wardowski kind via the notion of ψ-fixed point

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Abstract

In this manuscript, we introduce generalized $(\mathfrak{f}^*, \psi)$-contraction of kind (S) and use this concept to establish ψ-fixed point theorems in the frame of complete metric space. Secondly, we introduce new notion of generalized$(\mathfrak{f}^*, \psi)$ expansive mapping of kind (S) and utilize the same to prove some fixed point results for surjective mapping satisfying certain conditions. Our results improve the results of [8], [10] and [14] by omitting the continuity condition of $F \in \Im$ with the aid of ψ-fixed point. We also give an example which yields the main result. Also, many existing results in the frame of metric spaces are established.

Keywords: Generalized $(\mathfrak{f}^*, \psi)$-contraction, Generalized $(\mathfrak{f}^*, \psi)$-expansion, ψ-fixed point, Lower semi-continuous function
2020 MSC: 47H10, 54H25

1 Introduction and Preliminaries

In 1922, Banach established a useful result in fixed point theory regarding a contraction mapping, known as the Banach contraction principle. In 2012, Wardowski [17] gave a new contraction known as F-contraction and prove fixed point theorem concerning F-contractions. In this manner, Wardowski conclude the Banach contraction principle in a different way from the eminent results from the literature. Numerous generalities of F-contraction have been discussed by several prominant researchers. In 2012, Shahi et al. [16] recommend the view of (ξ, α) mapping of expansion and demonstrated a few aftereffects of fixed point for such assortment of functions in complete metric spaces. In 2013, Murthy and Prasad [13] generalize weak contraction by making combinations of $\sigma(\Omega, \mho)$. Later, Piri and Kumam [14] established Wardowski type fixed point theorems in complete metric spaces. Motivated by the perception of Dung and Hang [6], in 2016, Piri and Kumam [14] generalized the concept of generalized F-contraction and established some fixed point theorems for such kind of functions in complete metric spaces by addition of four terms $d(f^2x, x), d(f^2x, fx), d(f^2x, y), d(f^2x, fy)$. In 2017, Gornicki [8] established some results for F-expansion mapping in the context of metric and G-metric spaces. On the other hand one of the significant and imperative ideas, φ fixed point result, was presented by Jleli et al. [9] and conclude some results for partial metric spaces. They also established various $\varphi-$fixed point results for graphic and weak $(F, \varphi)-$contraction mappings in the edge of metric spaces. In 2019,

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Kumar and Arora [10] introduced new notions of generalized F-contractions of type (S) and type (M) and proved fixed point results of Wardowski kind utilising new notions. In 2020, Arora et al.[1] conferred common fixed point results for modified β-admissible contraction in the edge of metric space. Recently, Kumar et al. [12] explored the existence of fixed point with the aid of almost Z-contraction.

Wardowski [17] defined the F-contraction as follows.

Definition 1.1. [17] Let $\mathcal{Q}$ be a self-mapping in $(\mathcal{H},\sigma)$. Then, $\mathcal{Q}$ is named an F-contraction on $(\mathcal{H},\sigma)$ if we can find $F\in\Im$ with the goal that $\sigma(\mathcal{Q}\Omega,\mathcal{Q}\mho)>0\Rightarrow\gamma+F(\sigma(\mathcal{Q}\Omega,\mathcal{Q}\mho))\le F(\sigma(\mathcal{Q}\Omega,\mathcal{Q}\mho))$, for every $\Omega,\mho\in\mathcal{H}$, where $\Im$ be class of all mappings $F:(0,\infty)\to\mathcal{R}$ such that
(F1) F is strictly increasing function, that is, for all $\Omega,\mho\in(0,\infty)$, if $\Omega<\mho$, then $F(\Omega)<F(\mho)$.
(F2) For every arrangement $\{a_n\}$ of natural numbers, $\lim_{n\to\infty}\Omega_n=0$ in case $\lim_{n\to\infty}F(\Omega_n)=-\infty$.
(F3) There occur $q\in(0,1)$ in a manner that $\lim_{a\to 0^+}(\Omega^q F(\Omega))=0$.

Wardowski [17] gave some examples of $\Im$ as follows:
1.$F(\zeta)=\ln\zeta$.
2.$F(\zeta)=-\frac{1}{\zeta^{\frac{1}{2}}}$.
3.$F(\zeta)=\ln(\zeta)+\zeta$.
4.$F(\zeta)=\ln(\zeta^2+\zeta)$.

Remark 1.2. Let $F:\mathcal{R}_+\to\mathcal{R}$ be defined as $F=\ln(\beta)$, then $F\in\Im$. Now, F-contraction changes to a Banach contraction. Consequently, the Banach contractions are special case of F-contractions. There are F-contractions which are not Banach contractions (see[17]).

F-weak contraction was established by Wardowski and Dung in 2014 which is given as follows:

Definition 1.3. [18] Let $\mathcal{Q}$ be a self map in a metric space $(\mathcal{H},\sigma)$. Then, $\mathcal{Q}$ is named as F-weak contraction on $(\mathcal{H},\sigma)$ if there occur $F\in\Im$ and $\gamma>0$ with the goal that $\sigma(\mathcal{Q}\Omega,\mathcal{Q}\mho)>0$, then

$$\gamma+F\left(\sigma(\mathcal{Q}\Omega,\mathcal{Q}\mho)\right)\le F\left(\max\left\{\sigma(\Omega,\mho),\sigma(\mathcal{Q}\Omega,\Omega),\sigma(\mho,\mathcal{Q}\mho),\frac{\sigma(\Omega,\mathcal{Q}\mho)+\sigma(\mho,\mathcal{Q}\Omega)}{2}\right\}\right),$$

for every $\Omega,\mho\in\mathcal{H}$.

Theorem 1.4. [18] Let $\mathcal{Q}$ be a self map in a complete $(\mathcal{H},\sigma)$ and $\mathcal{Q}$ be an F-weak contraction. If F or $\mathcal{Q}$ is continuous, then $\mathcal{Q}$ has a fixed point ${\Theta_1}^*\in\mathcal{H}$ which is unique and the sequence $\{\mathcal{Q}^n\Omega\}$ tends to ${\Theta_1}^*$, where n varies from 1 to ∞.

Hang and Dung [6] investigated the concept of generalized F-contraction and proved useful fixed point results for such kind of functions.

Definition 1.5. [6] Let $\mathcal{Q}:\mathcal{H}\to\mathcal{H}$ be a self-mapping in $(\mathcal{H},\sigma)$. Then, $\mathcal{Q}$ is named as generalized F-contraction on $(\mathcal{H},\sigma)$ if we find $F\in\Im$ and $\delta>0$ with the goal that

$$\sigma(\mathcal{Q}\Omega,\mathcal{Q}\mho)>0\Rightarrow\delta+F(\sigma(\mathcal{Q}\Omega,\mathcal{Q}\mho))\le F(\max\{\sigma(\Omega,\mho),\sigma(\Omega,\mathcal{Q}\Omega),\sigma(\mho,\mathcal{Q}\mho),$$
$$\frac{\sigma(\Omega,\mathcal{Q}\mho)+\sigma(\mho,\mathcal{Q}\Omega)}{2},\frac{\sigma(\mathcal{Q}^2\Omega,\Omega)+\sigma(\mathcal{Q}^2\Omega,\mathcal{Q}\mho)}{2},\sigma(\mathcal{Q}^2\Omega,\mathcal{Q}\Omega),\sigma(\mathcal{Q}^2\Omega,\mho),\sigma(\mathcal{Q}^2\Omega,\mathcal{Q}\mho)\}),$$

for all $\Omega,\mho\in\mathcal{H}$.

Subsequently, Kumam and Piri[15] replace the $(F3)$ with $(F3^{'})$ in the definition of F-contraction given by Wardowski [17].

$(F3^{'})$: F is continuous on positive reals.

They gave the notation $\mathfrak{F}$ to denote the class of all maps $F:\mathcal{R}_+\to\mathcal{R}$ which fulfil $(F1),(F2)$ and $(F3^{'})$. Piri and Kumam also proved some useful fixed point results for $(\mathcal{H},\sigma)$. Now, the conditions $(F3)$ and $(F3^{'})$ are not associated with each other. For example, for $q\ge 1, F(\beta)=\frac{-1}{\beta^q}$, then F meet the conditions (F1) and (F2) but it does not fulfil(F3), while it fulfils the condition $(F3^{'})$. In view of this, it is significant to observe the sequel of Wardowski [17] with the functions $F\in\Im$ rather than F $\in\mathfrak{F}$. In 2017, Gornicki introduced the notion of F-expansion as follows:

Definition 1.6. [8] Let $\mathcal{Q}$ be a self-mapping in $(\mathcal{H},\sigma)$. Then, $\mathcal{Q}$ is named an F-expansion on $(\mathcal{H},\sigma)$ if we can find $F\in\Im$ with the goal that $\sigma(\mathcal{Q}\Omega,\mathcal{Q}\mho)>0\Rightarrow F(\sigma(\mathcal{Q}\Omega,\mathcal{Q}\mho))\geq F(\sigma(\mathcal{Q}\Omega,\mathcal{Q}\mho))+\gamma$, for every $\Omega,\mho\in\mathcal{H}$, where $\Im$ be class of all mappings $F:(0,\infty)\to\mathcal{R}$ such that
(F1) F is strictly increasing function, that is, for all $\Omega,\mho\in(0,\infty)$, if $\Omega<\mho$, then $F(\Omega)<F(\mho)$.
(F2) For every arrangement $\{a_n\}$ of natural numbers, $\lim_{n\to\infty}\Omega_n=0$ in case $\lim_{n\to\infty}F(\Omega_n)=-\infty$.
(F3) There occur $q\in(0,1)$ such that $\lim_{a\to0^+}(\Omega^qF(\Omega))=0$.

The aim of this paper is to introduce generalized $(\mathfrak{f}^*,\psi)$-contraction and generalized $(\mathfrak{f}^*,\psi)$-expansion mapping with the aid of ψ-fixed point. Also some ψ-fixed point results have been established by replacing the conditions $(F1)$, $(F3)$ and $(F3^{'})$ of [14] by a single condition (E).

Throughout this paper, we denote by $\mathcal{H}$, $\mathcal{R}$, $\mathcal{R}+$, $\mathbb{Z}$ and $\mathbb{Z}_+$ the nonempty set, the set of real numbers, the set of positive real numbers, the set of integers and the set of positive integers, respectively.

$$F_h=\{\Omega\in\mathcal{H}:h\Omega=\Omega\}\,\text{and}\,K_\psi=\{\Omega\in\mathcal{H}:\psi(\Omega)=0\}.$$

2 Main Results

Let $\mathcal{F}_\mathcal{E}$ be the class of all continuous functions $\mathfrak{f}^*:(0,\infty)\times[0,\infty)^2\to\mathcal{R}$, which gratify the accompanying condition:
(E) For all sequences $\{e_n\},\{f_n\}$ and $\{g_n\}\in\mathcal{R}_+$,

$$\lim_{n\to\infty}\mathfrak{f}^*(e_n,f_n,g_n)=-\infty\ \text{ if and only if }\ \lim_{n\to\infty}e_n^2+e_n+f_n+g_n=0.$$

Now, we introduce generalized $(\mathfrak{f}^*,\psi)$-contraction of kind (S).

Proposition 2.1. If $\{e_n\},\{f_n\}$ and $\{g_n\}\in\mathcal{R}_+$, then

$$\lim_{n\to\infty}e_n^2+e_n+f_n+g_n=0\Leftrightarrow\lim_{n\to\infty}e_n=0,\lim_{n\to\infty}f_n=0\ \text{ and }\ \lim_{n\to\infty}g_n=0.$$

Definition 2.2. Let $\psi:\mathcal{H}\to[0,\infty)$ be a map in $(\mathcal{H},\sigma)$. A function $h:\mathcal{H}\to\mathcal{H}$ is called generalized$(\mathfrak{f}^*,\psi)$-contraction of kind (S) if there occur $\mathfrak{f}^*\in\mathcal{F}_\mathcal{E}$ and $\chi>0$ in order that

$$h\Omega\neq h\mho\Rightarrow\chi+\mathfrak{f}^*(\sigma(h\Omega,h\mho),\psi(h\Omega),\psi(h\mho))\leq\mathfrak{f}^*(S(\Omega,\mho),\psi(\Omega),\psi(\mho)),\tag{2.1}$$

where

$$\begin{aligned}S(\Omega,\mho)=\max\{&\sigma(\Omega,\mho),\frac{\sigma(\Omega,h\mho)+\sigma(\mho,h\Omega)}{2},\frac{\sigma(h^2\Omega,\Omega)+\sigma(h^2\Omega,h\mho)}{2},\\&\sigma(h^2\Omega,h\Omega),\sigma(h^2\Omega,\mho),\sigma(h^2\Omega,h\mho)+\sigma(\Omega,h\Omega),\sigma(h\Omega,\mho)+\sigma(\mho,h\mho)\},\end{aligned}$$

$\forall\,\Omega,\mho\in\mathcal{H}$.

Lemma 2.3. Let $\psi:\mathcal{H}\to[0,\infty)$ and $h:\mathcal{H}\to\mathcal{H}$ be $(\mathfrak{f}^*,\psi)$-contraction in $(\mathcal{H},\sigma)$, where $\mathfrak{f}^*\in\mathcal{F}_\mathcal{E}$. If $\Omega_n\neq\Omega_{n+1},\forall n\in\mathbb{Z}_+$, then

(i) Ω_n is Cauchy;
(ii) $\lim_{n\to\infty}\sigma(\Omega_n,\mho_n)=\lim_{n\to\infty}\psi(\Omega_n)$.

Proof . Let $h\Omega_{n+1}=\Omega_{n+2}$ for every $n\in\mathbb{Z}_+$. Inserting $\Omega=\Omega_{n-1}$ and $\mho=\Omega_n$ in (2.1), we acquire

$$h\Omega_{n-1}\neq h\Omega_n\Rightarrow\chi+\mathfrak{f}^*(\sigma(h\Omega_{n-1},h\Omega_n),\psi(h\Omega_{n-1}),\psi(h\Omega_n))\leq\mathfrak{f}^*(S(\Omega_{n-1},\Omega_n),\psi(\Omega_{n-1}),\psi(\Omega_n)),\tag{2.2}$$

$\forall\,\Omega_{n-1},\Omega_n\in\mathcal{H}$, where

$$\begin{aligned}S(\Omega_{n-1},\Omega_n)&=\max\{\sigma(\Omega_{n-1},\Omega_n),\frac{\sigma(\Omega_{n-1},h\Omega_n)+\sigma(\Omega_n,h\Omega_{n-1})}{2},\frac{\sigma(h^2\Omega_{n-1},\Omega_{n-1})+\sigma(h^2\Omega_{n-1},h\Omega_n)}{2},\\&\quad\sigma(h^2\Omega_{n-1},h\Omega_{n-1}),\sigma(h^2\Omega_{n-1},\Omega_n),\sigma(h^2\Omega_{n-1},h\Omega_n)+\sigma(\Omega_{n-1},h\Omega_{n-1}),\sigma(h\Omega_{n-1},\Omega_n)+\sigma(\Omega_n,h\Omega_n)\},\\&=\max\{\sigma(\Omega_{n-1},\Omega_n),\frac{\sigma(\Omega_{n-1},\Omega_{n+1})+\sigma(\Omega_n,\Omega_n)}{2},\frac{\sigma(\Omega_{n+1},\Omega_{n-1})+\sigma(\Omega_{n+1},\Omega_{n+1})}{2},\\&\quad\sigma(\Omega_{n+1},\Omega_{n+1}),\sigma(\Omega_{n+1},\Omega_n),\sigma(\Omega_{n+1},\Omega_{n+1})+\sigma(\Omega_{n-1},\Omega_n),\sigma(\Omega_n,\Omega_n)+\sigma(\Omega_n,\Omega_{n+1})\},\\&=\max\{\sigma(\Omega_{n-1},\Omega_n),\sigma(\Omega_n,\Omega_{n+1})\}.\end{aligned}$$

If there occur $n \in \mathbb{Z}_+$ in order that $max\{\sigma(\Omega_{n-1}, \Omega_n), \sigma(\Omega_n, \Omega_{n+1})\} = \sigma(\Omega_n, \Omega_{n+1})$, subsequently (2.2) becomes $\chi + \mathfrak{f}^*(\sigma(h\Omega_{n-1}, h\Omega_n), \psi(h\Omega_{n-1}), \psi(h\Omega_n)) \leq \mathfrak{f}^*(\sigma(\Omega_n, \Omega_{n+1}), \psi(\Omega_{n-1}), \psi(\Omega_n))$. Thus,

$$\chi + \mathfrak{f}^*(\sigma(\Omega_n, \Omega_{n+1}), \psi(\Omega_n), \psi(\Omega_{n+1})) \leq \mathfrak{f}^*(\sigma(\Omega_n, \Omega_{n+1}), \psi(\Omega_{n-1}), \psi(\Omega_n)). \tag{2.3}$$

Since $\chi > 0$, we get a counter statement. Thus,

$$\max\{\sigma(\Omega_{n-1}, \Omega_n), \sigma(\Omega_n, \Omega_{n+1})\} = \sigma(\Omega_{n-1}, \Omega_n).$$

Now, (2.2) becomes

$$\begin{aligned}
\mathfrak{f}^*(\sigma(h\Omega_{n-1}, h\Omega_n), \psi(h\Omega_{n-1}), \psi(h\Omega_n)) &\leq \mathfrak{f}^*(\sigma(\Omega_{n-1}, \Omega_n), \psi(\Omega_{n-1}), \psi(\Omega_n)) - \chi \\
&\leq \mathfrak{f}^*(\sigma(\Omega_{n-2}, \Omega_{n-1}), \psi(\Omega_{n-2}), \psi(\Omega_{n-1})) - 2\chi \\
&\leq \mathfrak{f}^*(\sigma(\Omega_{n-3}, \Omega_{n-2}), \psi(\Omega_{n-3}), \psi(\Omega_{n-2})) - 3\chi \\
&\vdots \\
&\leq \mathfrak{f}^*(\sigma(\Omega_0, \Omega_1), \psi(\Omega_0), \psi(\Omega_1)) - n\chi.
\end{aligned} \tag{2.4}$$

Let $n \to \infty$ in (2.4), we acquire

$$\lim_{n\to\infty} \mathfrak{f}^*(\sigma(h\Omega_{n-1}, h\Omega_n), \psi(h\Omega_{n-1}), \psi(h\Omega_n)) = -\infty.$$

With the assistance of (E), we acquire

$$\lim_{n\to\infty} \sigma(\Omega_{n-1}, \Omega_n) = \lim_{n\to\infty} \psi(\Omega_n) = 0. \tag{2.5}$$

Imagine that $\{\Omega_n\}$ is not Cauchy in $\mathcal{H}$. Then, there exist $\delta > 0$ and subsequences $\{\Omega_{n_e}\}$ and $\{\Omega_{r_e}\}$ of $\{\Omega_n\}$ such that $\sigma(\Omega_{n_e}, \Omega_{r_e}) \geq \delta$ and $\sigma(\Omega_{n_e}, \Omega_{r_e-1}) < \delta$, for each $r_e > n_e > e$, where $e \in \mathbb{Z}_+$. Now,

$$\begin{aligned}
\delta &\leq \sigma(\Omega_{n_e}, \Omega_{r_e}) \\
&\leq \sigma(\Omega_{r_e}, \Omega_{r_e-1}) + \sigma(\Omega_{r_e-1}, \Omega_{n_e}) \\
&\leq \sigma(\Omega_{r_e}, \Omega_{r_e-1}) + \delta.
\end{aligned}$$

With the assistance of 2.5 and making $e \to \infty$, we acquire

$$\lim_{e\to\infty} \sigma(\Omega_{n_e}, \Omega_{r_e}) = \delta. \tag{2.6}$$

Further for each $e \geq n_1$, $\exists\, n_1 \in \mathbb{Z}_+$ ensuring that

$$\sigma(\Omega_{r_e}, \Omega_{r_e+1}) < \frac{\delta}{8} \text{ and } \sigma(\Omega_{n_e}, \Omega_{n_e+1}) < \frac{\delta}{8}. \tag{2.7}$$

Now, we exhibit that $\sigma(\Omega_{r_e+1}, \Omega_{n_e+1}) > 0$, for every $e \geq n_1$. Let us imagine $\exists\, g \geq n_1$ ensuring that

$$\sigma(\Omega_{r_g+1}, \Omega_{n_g+1}) = 0. \tag{2.8}$$

With the aid of (2.6), (2.7) and (2.8), we acquire

$$\begin{aligned}
\delta &\leq \sigma(\Omega_{r_g}, \Omega_{n_g}) \\
&\leq \sigma(\Omega_{r_g}, \Omega_{r_g+1}) + \sigma(\Omega_{r_g+1}, \Omega_{n_g}) \\
&\leq \sigma(\Omega_{r_g}, \Omega_{r_g+1}) + \sigma(\Omega_{r_g+1}, \Omega_{n_g+1}) + \sigma(\Omega_{n_g+1}, \Omega_{n_g}) \\
&< \frac{\delta}{8} + 0 + \frac{\delta}{8} = \frac{\delta}{4},
\end{aligned}$$

which is a counter statement. Consequently,

$$\sigma(\Omega_{r_e+1}, \Omega_{n_e+1}) > 0. \tag{2.9}$$

for every $e \geq n_1$. Further inserting $\Omega = \Omega_{r_e}$ and $\mho = \Omega_{n_e}$ in (2.1), we acquire

$$\chi + \mathfrak{f}^*(\sigma(h\Omega_{r_e}, h\Omega_{n_e}), \psi(h\Omega_{r_e}), \psi(h\Omega_{n_e})) \leq \mathfrak{f}^*(S(\Omega_{r_e}, \Omega_{n_e}), \psi(\Omega_{r_e}), \psi(\Omega_{n_e})),$$

where

$$\begin{aligned} S(\Omega_{r_e}, \Omega_{n_e}) &= max\{\sigma(\Omega_{r_e}, \Omega_{n_e}), \frac{\sigma(\Omega_{r_e}, h\Omega_{n_e}) + \sigma(\Omega_{n_e}, h\Omega_{r_e})}{2}, \frac{\sigma(h^2\Omega_{r_e}, \Omega_{r_e}) + \sigma(h^2\Omega_{r_e}, h\Omega_{n_e})}{2}, \\ &\sigma(h^2\Omega_{r_e}, h\Omega_{r_e}), \sigma(h^2\Omega_{r_e}, \Omega_{n_e}), \sigma(h^2\Omega_{r_e}, h\Omega_{n_e}) + \sigma(\Omega_{r_e}, h\Omega_{r_e}), \sigma(h\Omega_{r_e}, \Omega_{n_e}) + \sigma(\Omega_{n_e}, h\Omega_{n_e})\} \\ &= max\{\sigma(\Omega_{r_e}, \Omega_{n_e}), \frac{\sigma(\Omega_{r_e}, \Omega_{n_e+1}) + \sigma(\Omega_{n_e}, \Omega_{r_e+1})}{2}, \frac{\sigma(\Omega_{r_e+2}, \Omega_{r_e}) + \sigma(\Omega_{r_e+2}, \Omega_{n_e+1})}{2}, \\ &\sigma(\Omega_{r_e+2}, \Omega_{r_e+1}), \sigma(\Omega_{r_e+2}, \Omega_{n_e}), \sigma(\Omega_{r_e+2}, \Omega_{n_e+1}) + \sigma(\Omega_{r_e}, \Omega_{r_e+1}), \sigma(\Omega_{r_e+1}, \Omega_{n_e}) + \sigma(\Omega_{n_e}, \Omega_{n_e+1})\}. \end{aligned}$$

With the aid of (2.5), (2.6) and continuity property of $\mathfrak{f}^*$, we acquire

$$\chi + \mathfrak{f}^*(\delta, 0, 0) \leq \mathfrak{f}^*(\delta, 0, 0),$$

a counter statement, which indicates that $\{\Omega_n\}$ is a Cauchy sequence in $\mathcal{H}$. $\square$

Theorem 2.4. Let $h : \mathcal{H} \to \mathcal{H}$ be generalized $(\mathfrak{f}^*, \psi)$-contraction of kind (S) and $\psi : \mathcal{H} \to [0, \infty)$ be lower semi-continuous mapping having $F_h \subseteq K_\psi$ in complete $(\mathcal{H}, \sigma)$. Then, h possess a unique ψ-fixed point.

Proof . Let $\Omega_0 \in \mathcal{H}$ be any point. Inserting $h^n\Omega_0 = \Omega_{n+1}$, for every $n \in \mathbb{Z}_+$. If there occur $n \in \mathbb{Z}_+$ such that $h\Omega_n = \Omega_n$. Using given assumption of Theorem 2.4, we get that Ω_n is ψ-fixed point of h. Imagine that $\sigma(\Omega_n, h\Omega_n) > 0$, for every $n \in \mathbb{Z}_+$. With the assistance of Lemma 2.3, we get that $\{\Omega_n\}$ is a Cauchy sequence. But $(\mathcal{H}, \sigma)$ is complete, which indicates that there occur $\Omega \in \mathcal{H}$ such that

$$\lim_{n\to\infty} \Omega_n = \Omega. \tag{2.10}$$

With the assistance of Lemma 2.3 and lower semi-continuity property of ψ, we acquire

$$0 \leq \psi(\Omega) \leq \lim_{n\to\infty} inf \psi(\Omega_n) = 0,$$

which yields that

$$\psi(\Omega) = 0. \tag{2.11}$$

Let $\mathcal{D} = \{n \in \mathbb{Z}_+ : h\Omega = \Omega_n\}$. Now, two cases arise. When $\mathcal{D}$ is infinite set, then there exist a subsequence $\{\Omega_{n_e}\}$ of $\{\Omega_n\}$ having $h\Omega = \lim_{e\to\infty} \Omega_{n_e}$, which indicates that $h\Omega = \Omega$. Further, when $\mathcal{D}$ is finite set, then $\sigma(h\Omega, \Omega_n) > 0$, for infinte $n \in \mathbb{Z}_+$. Consequently, there exists a subsequence $\{\Omega_{n_e}\}$ of $\{\Omega_n\}$ such that $\sigma(h\Omega, \Omega_{n_e}) > 0$, for every $e \in \mathbb{Z}_+$. Since h is an generalized $(\mathfrak{f}^*, \psi)$-contraction of kind (S), we acquire

$$\chi + \mathfrak{f}^*(\Omega(h\Omega_{n_e}, h\Omega), \psi(h\Omega_{n_e}), \psi(h\Omega)) \leq \mathfrak{f}^*(S(\Omega_{n_e}, \Omega), \psi(\Omega_{n_e}), \psi(\Omega)),$$

where

$$\begin{aligned} S(\Omega_{n_e}, \Omega) &= max\{\sigma(\Omega_{n_e}, \Omega), \frac{\sigma(\Omega_{n_e}, h\Omega) + \sigma(\Omega, h\Omega_{n_e})}{2}, \frac{\sigma(h^2\Omega_{n_e}, \Omega_{n_e}) + \sigma(h^2\Omega_{n_e}, h\Omega)}{2}, \\ &\sigma(h^2\Omega_{n_e}, h\Omega_{n_e}), \sigma(h^2\Omega_{n_e}, \Omega), \sigma(h^2\Omega_{n_e}, h\Omega) + \sigma(\Omega_{n_e}, h\Omega_{n_e}), \sigma(h\Omega_{n_e}, \Omega) + \sigma(\Omega, h\Omega)\} \end{aligned}$$

With the aid of (2.10), (2.11), condition(E), continuity of $\mathfrak{f}^*$ and Lemma 2.3 as $e \to \infty$, we acquire

$$\chi + \mathfrak{f}^*(\sigma(\Omega, h\Omega), \psi(\Omega), \psi(h\Omega)) \leq \mathfrak{f}^*(\sigma(\Omega, h\Omega), \psi(\Omega), \psi(\Omega)),$$

which implies that

$$\chi + \mathfrak{f}^*(\sigma(\Omega, h\Omega), 0, 0) \leq \mathfrak{f}^*(\sigma(\Omega, h\Omega), 0, 0),$$

which is a counter statement because $\chi > 0$. Consequently,

$$\sigma(\Omega, h\Omega) = 0. \tag{2.12}$$

Equations (2.11) and (2.12) yields that Ω is ψ-fixed point of h. Now, we exhibit that ψ-fixed point of h is unique. Let us imagine that Ω_1, Ω_2 be distinct ψ-fixed points of h. Thus, $\sigma(h\Omega_1, h\Omega_2) = \sigma(\Omega_1, \Omega_2) > 0$. Now, inserting $\Omega = \Omega_1$ and $\mho = \Omega_2$ in (2.12), we acquire

$$\chi + \mathfrak{f}^*(\sigma(h\Omega_1, h\Omega_2), \psi(h\Omega_1), \psi(h\Omega_2)) \leq \mathfrak{f}^*(S(\Omega_1, \Omega_2), \psi(\Omega_1), \psi(\Omega_2)), \tag{2.13}$$

where

$$\begin{aligned} S(\Omega_1, \Omega_2) &= \max\left\{\sigma(\Omega_1, \Omega_2), \frac{\sigma(\Omega_1, h\Omega_2) + \sigma(\Omega_2, h\Omega_1)}{2}, \frac{\sigma(h^2\Omega_1, \Omega_1) + \sigma(h^2\Omega_1, h\Omega_2)}{2},\right. \\ &\left.\sigma(h^2\Omega_1, h\Omega_1), \sigma(h^2\Omega_1, \Omega_2), \sigma(h^2\Omega_1, h\Omega_2) + \sigma(\Omega_1, h\Omega_1), \sigma(h\Omega_1, \Omega_2) + \sigma(\Omega_2, h\Omega_2)\right\} \\ &= \max\left\{\sigma(\Omega_1, \Omega_2), \frac{\sigma(\Omega_1, \Omega_2) + \sigma(\Omega_2, \Omega_1)}{2}, \frac{\sigma(\Omega_1, \Omega_1) + \sigma(\Omega_1, \Omega_2)}{2},\right. \\ &\left.\sigma(\Omega_1, \Omega_1), \sigma(\Omega_1, \Omega_2), \sigma(\Omega_1, \Omega_2) + \sigma(\Omega_1, \Omega_1), \sigma(\Omega_1, \Omega_2) + \sigma(\Omega_2, \Omega_2)\right\} \\ &= \sigma(\Omega_1, \Omega_2). \end{aligned}$$

With the aid of (2.13), we acquire

$$\chi + \mathfrak{f}^*(\sigma(\Omega_1, \Omega_2), 0, 0) \leq \mathfrak{f}^*(\sigma(\Omega_1, \Omega_2), 0, 0),$$

which is a counter statement. Thus, $\Omega_1 = \Omega_2$, which indicates that h possess a unique ψ-fixed point in $\mathcal{H}$. □

Example 2.5. Consider $\mathcal{H} = [0, 6]$ associated with the usual metric σ. We define $h : \mathcal{H} \to \mathcal{H}$ by

$$h(\Omega) = \begin{cases} 0, & \text{if } 0 \leq \Omega < 5.5, \\ s\ln(\frac{\Omega}{9}), & 5.5 \leq \Omega < 6, \end{cases}$$

for all $\Omega \in \mathcal{H}$ and $s < 1$. Now, it is clear that h is not continuous at $\Omega = 5.5$. Let $\mathfrak{f}^* : (0, \infty) \times [0, \infty)^2 \to \mathbb{R}$ be defined as $\mathfrak{f}^*(e, f, g) = \ln(e + e^2 + f + g), \forall\, e, f, g \in [0, \infty)$ and $g \neq 0$. Let $\psi : \mathcal{H} \to [0, \infty)$ be defined as $\psi(\Omega) = \Omega$, for every $\Omega \in \mathcal{H}$. It is clear that ψ is lower semi-continuous and $\mathfrak{f}^* \in \mathcal{F}_{\mathcal{E}}$. Now, we assert that

$$\sigma(h\Omega, h\mho) + \psi(h\Omega) + \psi(h\mho) \leq e^{-\chi}(S(\Omega, \mho) + \psi(\Omega) + \psi(\mho)), \tag{2.14}$$

where

$$\begin{aligned} S(\Omega, \mho) = \max&\left\{\sigma(\Omega, \mho), \frac{\sigma(\Omega, h\mho) + \sigma(\mho, h\Omega)}{2}, \frac{\sigma(h^2\Omega, \Omega) + \sigma(h^2\Omega, h\mho)}{2},\right. \\ &\left.\sigma(h^2\Omega, h\Omega), \sigma(h^2\Omega, \mho), \sigma(h^2\Omega, h\mho) + \sigma(\Omega, h\Omega), \sigma(h\Omega, \mho) + \sigma(\mho, h\mho)\right\}, \end{aligned}$$

for all $\Omega, \mho \in \mathcal{H}$ and $h\mho \neq h\Omega$. Three cases arise:
Case 1: If $\Omega, \mho \in [0, 5.5)$, then (2.14) holds trivially.
Case 2: If $\Omega, \mho \in [5.5, 6]$, then

$$\begin{aligned} \sigma(h\Omega, h\mho) + \psi(h\Omega) + \psi(h\mho) &= 2\max\{h\Omega, h\mho\} \\ &\leq 2\max\{s\Omega, s\mho\} \\ &= s(2\max\{\Omega, \mho\}). \end{aligned} \tag{2.15}$$

Now,

$$\begin{aligned} S(\Omega, \mho) &= \max\left\{\sigma(\Omega, \mho), \frac{\sigma(\Omega, h\mho) + \sigma(\mho, h\Omega)}{2}, \frac{\sigma(h^2\Omega, \Omega) + \sigma(h^2\Omega, h\mho)}{2},\right. \\ &\left.\sigma(h^2\Omega, h\Omega), \sigma(h^2\Omega, \mho), \sigma(h^2\Omega, h\mho) + \sigma(\Omega, h\Omega), \sigma(h\Omega, \mho) + \sigma(\mho, h\mho)\right\} \\ &= 2\max\{\Omega, \mho\}. \end{aligned} \tag{2.16}$$

With the aid of (2.15) and (2.16), we acquire

$$\sigma(h\Omega, h\mho) + \psi(h\Omega) + \psi(h\mho) \leq s(\sigma(\Omega, \mho) + \psi(\Omega) + \psi(\mho)).$$

Case 3: If $\Omega \in [5.5, 6]$ and $\mho \in [0, 5.5)$, then

$$\begin{aligned}\sigma(h\Omega, h\mho) + \psi(h\Omega) + \psi(h\mho) &= 2\max\{h\Omega, h\mho\}\\ &= 2\max\{h\Omega, 0\}\\ &\leq 2\max\{s\Omega, 0\}\\ &= s(2\max\{\Omega, \mho\}). \qquad (2.17)\end{aligned}$$

Now,

$$\begin{aligned}S(\Omega, \mho) = \max\{\sigma(\Omega, \mho), \frac{\sigma(\Omega, 0) + \sigma(\mho, h\Omega)}{2}, \frac{\sigma(h^2\Omega, \Omega) + \sigma(h^2\Omega, 0)}{2},\\ \sigma(h^2\Omega, h\Omega), \sigma(h^2\Omega, \mho), \sigma(h^2\Omega, 0) + \sigma(\Omega, h\Omega), \sigma(h\Omega, \mho) + \sigma(\mho, 0)\} = 2\max\{\Omega, \mho\}. \qquad (2.18)\end{aligned}$$

With the aid of (2.17) and (2.18), we acquire

$$\sigma(h\Omega, h\mho) + \psi(h\Omega) + \psi(h\mho) \leq s(\sigma(\Omega, \mho) + \psi(\Omega) + \psi(\mho)).$$

Thus, h and ψ gratify all the conditions of Theorem 2.4 with $\chi = -\ln s > 0$. Consequently, h possess a unique ψ-fixed point, which is zero.

Corollary 2.6. [5] Let $(\mathcal{H}, \sigma)$ be a complete metric space and $h : \mathcal{H} \to \mathcal{H}$ be a map gratifying

$$\sigma(h\Omega, h\mho) \leq \lambda \max\left\{\sigma(\Omega, \mho), \frac{\sigma(\Omega, h\mho) + \sigma(\mho, h\Omega)}{2}, \frac{\sigma(\Omega, h\Omega) + \sigma(\mho, h\mho)}{2}\right\},$$

for some $\lambda \in (0, 1)$ and $\Omega, \mho \in \mathcal{H}$. Then, h possess a unique fixed point.

Proof . If $\mathfrak{f}^*(e, f, g) = \ln(e + f + g)$ and $\psi(\Omega) = 0$, for all $\Omega \in \mathcal{H}$ in Theorem 2.4, we get the result. $\square$

Corollary 2.7. (Banach Contraction Principle)[4] Let $(\mathcal{H}, \sigma)$ be a complete metric space and $h : \mathcal{H} \to \mathcal{H}$ be a map gratifying

$$\sigma(h\Omega, h\mho) \leq \lambda\sigma(\Omega, \mho),$$

for some $\lambda \in (0, 1)$ and $\Omega, \mho \in \mathcal{H}$. Then, h possess a unique fixed point.

Proof . If $\mathfrak{f}^*(e, f, g) = \ln(e + f + g)$ and $\psi(\Omega) = 0$, for all $\Omega \in \mathcal{H}$ in Theorem 2.4, we can deduce the result. $\square$

Next, we define new notion of generalized $(\mathfrak{f}^*, \psi)$-expansion of kind (S) and our second main result.

Definition 2.8. Let $\psi : \mathcal{H} \to [0, \infty)$ be a map in $(\mathcal{H}, \sigma)$. A function $h : \mathcal{H} \to \mathcal{H}$ is called generalized $(\mathfrak{f}^*, \psi)$-expansion of kind (S) if there occur $\mathfrak{f}^* \in \mathcal{F}_{\mathcal{E}}$ and $\chi > 0$ in order that

$$h\Omega \neq h\mho \Rightarrow \mathfrak{f}^*(\sigma(h\Omega, h\mho), \psi(h\Omega), \psi(h\mho)) \geq \mathfrak{f}^*(S(\Omega, \mho), \psi(\Omega), \psi(\mho)) + \chi, \qquad (2.19)$$

where

$$\begin{aligned}S(\Omega, \mho) = \max\{\sigma(\Omega, \mho), \frac{\sigma(\Omega, h\mho) + \sigma(\mho, h\Omega)}{2}, \frac{\sigma(h^2\Omega, \Omega) + \sigma(h^2\Omega, h\mho)}{2},\\ \sigma(h^2\Omega, h\Omega), \sigma(h^2\Omega, \mho), \sigma(h^2\Omega, h\mho) + \sigma(\Omega, h\Omega), \sigma(h\Omega, \mho) + \sigma(\mho, h\mho)\},\end{aligned}$$

for all $\Omega, \mho \in \mathcal{H}$.

Theorem 2.9. Let $h : \mathcal{H} \to \mathcal{H}$ be onto generalized $(\mathfrak{f}^*, \psi)$-expansion of kind (S) and $\psi : \mathcal{H} \to [0, \infty)$ be lower semi-continuous mapping having $F_{h^*} \subseteq K_\psi$ in complete $(\mathcal{H}, \sigma)$, where $h^* : \mathcal{H} \to \mathcal{H}$ in order that $h^* \circ h = I$, the identity function on $\mathcal{H}$. Then, h possess a unique ψ-fixed point.

Proof . Since h is surjective, so there exists a function $h^* : \mathcal{H} \to \mathcal{H}$ in order that $h^* \circ h = I$, the identity function on $\mathcal{H}$. Let $\Omega \neq \mho$, $h^*\Omega = \lambda$ and $h^*\mho = \mu$. Now, it is clear that $\lambda \neq \mu$, otherwise $h\lambda = h\mu$, which indicates that $\sigma(\Omega, \mho) = 0$, which is a counter statement. Since, h is generalized $(\mathfrak{f}^*, \psi)$-expansion of kind (S), we acquire

$$\mathfrak{f}^*(\sigma(h\lambda, h\mu), \psi(h\lambda), \psi(h\mu)) \geq \mathfrak{f}^*(S(\lambda, \mu), \psi(\lambda), \psi(\mu)) + \chi, \qquad (2.20)$$

where

$$S(\lambda,\mu)=\max\Big\{\sigma(\lambda,\mu),\frac{\sigma(\lambda,h\mu)+\sigma(\mu,h\lambda)}{2},\frac{\sigma(h^2\lambda,\lambda)+\sigma(h^2\lambda,h\mu)}{2},$$
$$\sigma(h^2\lambda,h\lambda),\sigma(h^2\lambda,\mu),\sigma(h^2\lambda,h\mu)+\sigma(\lambda,h\lambda),\sigma(h\lambda,\mu)+\sigma(\mu,h\mu)\Big\},$$

for all $\lambda,\mu\in\mathcal{H}$. Since, $h\lambda=\Omega$ and $h\mu=\mho$, the above inequality yields that

$$\chi+\mathfrak{f}^*(\sigma(h^*\Omega,h^*\mho),\psi(h^*\Omega),\psi(h^*\mho))\le\mathfrak{f}^*(S(\Omega,\mho),\psi(\Omega),\psi(\mho)),$$

where

$$S(\Omega,\mho)=\max\Big\{\sigma(\Omega,\mho),\frac{\sigma(\Omega,h\mho)+\sigma(\mho,h\Omega)}{2},\frac{\sigma(h^2\Omega,\Omega)+\sigma(h^2\Omega,h\mho)}{2},$$
$$\sigma(h^2\Omega,h\Omega),\sigma(h^2\Omega,\mho),\sigma(h^2\Omega,h\mho)+\sigma(\Omega,h\Omega),\sigma(h\Omega,\mho)+\sigma(\mho,h\mho)\Big\}$$

which indicates that h^* is $(\mathfrak{f}^*,\psi)$-contraction. Therefore, by mimicking the steps of Theorem 2.4, we get that $\psi-$fixed point of h^* exists and unique. Let Ω_1 be unique fixed point of h^*. Thus, $h^*\Omega_1=\Omega_1$ and $\psi(\Omega_1)=0$. Also, $h(\Omega_1)=h(h^*\Omega_1)=\Omega_1$ and $\psi(\Omega_1)=0$, consequently Ω_1 is also $\psi-$fixed point of h.

Now, we assert that h possess a unique $\psi-$fixed point. Let us imagine that Ω_1,Ω_2 be two $\psi-$fixed points of h such that $\Omega_1\neq\Omega_2$. Since h is $(\mathfrak{f}^*,\psi)$-expansive mapping, we acquire

$$\mathfrak{f}^*(\sigma(h\Omega_1,h\Omega_2),\psi(h\Omega_1),\psi(h\Omega_2))\ge\mathfrak{f}^*(S(\Omega_1,\Omega_2),\psi(\Omega_1),\psi(\Omega_2))+\chi,\tag{2.21}$$

where

$$S(\Omega_1,\Omega_2)=\max\Big\{\sigma(\Omega_1,\Omega_2),\frac{\sigma(\Omega_1,h\Omega_2)+\sigma(\Omega_2,h\Omega_1)}{2},\frac{\sigma(h^2\Omega_1,\Omega_1)+\sigma(h^2\Omega_1,h\Omega_2)}{2},$$
$$\sigma(h^2\Omega_1,h\Omega_1),\sigma(h^2\Omega_1,\Omega_2),\sigma(h^2\Omega_1,h\Omega_2)+\sigma(\Omega_1,h\Omega_1),\sigma(h\Omega_1,\Omega_2)+\sigma(\Omega_2,h\Omega_2)\Big\}$$
$$=\sigma(\Omega_1,\Omega_2).$$

With the aid of (2.21), we acquire

$$\mathfrak{f}^*(\sigma(\Omega_1,\Omega_2),\psi(\Omega_1),\psi(\Omega_2))=\mathfrak{f}^*(\sigma(h\Omega_1,h\Omega_2),\psi(h\Omega_1),\psi(h\Omega_2))\ge\mathfrak{f}^*(\sigma(\Omega_1,\Omega_2),\psi(\Omega_1),\psi(\Omega_2))+\chi,$$

which is a counterstatement. Consequently h possess a unique $\psi-$fixed point. □

Corollary 2.10. Let $h:\mathcal{H}\to\mathcal{H}$ be onto map and $\psi:\mathcal{H}\to[0,\infty)$ be lower semi-continuous mapping having $F_{h^*}\subseteq K_\psi$ in complete $(\mathcal{H},\sigma)$, where $h^*:\mathcal{H}\to\mathcal{H}$ in order that $h^*\circ h=I$, the identity function on $\mathcal{H}$. If there occur $\mathfrak{f}^*\in\mathcal{F}_\mathcal{E}$ and $\chi>0$ in order that:

$$h\Omega\neq h\mho\Rightarrow\mathfrak{f}^*(\sigma(h\Omega,h\mho),\psi(h\Omega),\psi(h\mho))\ge\mathfrak{f}^*(\sigma(\Omega,\mho),\psi(\Omega),\psi(\mho))+\chi,\tag{2.22}$$

for all $\Omega,\mho\in\mathcal{H}$. Then, h possess a unique ψ-fixed point.

Corollary 2.11. [19] Let $h:\mathcal{H}\to\mathcal{H}$ be an onto mapping defined on a complete metric space $(\mathcal{H},\sigma)$ gratifying the condition $\sigma(ha,hb)\ge c\sigma(a,b),\ \forall a,b\in\mathcal{H}$, where $c\ge1$. Then, h possess a unique fixed point in $\mathcal{H}$.

Proof . Inserting $\mathfrak{f}^*(e,f,g)=\ln(e+f+g),\ \forall\, e,f,g\in[0,\infty)$ and $\psi(\Omega)=0,\ \forall\,\Omega\in\mathcal{H}$ in Theorem 2.9, we get the following result. □

Corollary 2.12. [8] Let $h:\mathcal{H}\to\mathcal{H}$ be an onto mapping, $\mathfrak{f}^*\in\Im$ and $\chi>0$ in a complete metric space $(\mathcal{H},\sigma)$ gratifying the condition

$$h\Omega\neq h\mho\Rightarrow\mathfrak{f}^*(\sigma(h\Omega,h\mho))\ge\mathfrak{f}^*(S(\Omega,\mho))+\chi,$$

where $\Im$ be family of all functions $F:(0,\infty)\to\mathcal{R}$ such that
(F1) F is strictly increasing, that is, for all $a,b\in(0,\infty)$, if $a<b$,then $F(a)<F(b)$.
(F2) For each sequence a_n of positive numbers, $\lim_{n\to\infty}a_n=0$ if and only if $\lim_{n\to\infty}F(a_n)=-\infty$.
(F3) There exists $k\in(0,1)$ such that $\lim_{a\to0^+}(a^kF(a))=0$. Then, h possess a unique fixed point in $\mathcal{H}$.

Proof . Inserting $\psi(\Omega)=0$ in Theorem 2.9, we can deduce the result. □

3 Conclusion

In this paper, ψ-fixed point results are investigated with the aid of generalized $(\mathfrak{f}^*, \psi)$-contractive and expansive functions of kind (S) in the context of complete metric space. In this way, the relationship of the contractive and expansive functions of Wardowski kind with previous concept ψ-fixed point is investigated through indispensable theorems. Moreover, we established the results by substituting the continuity condition of $\mathfrak{f}^*$ by lower semi-continuity of ψ, for detail please see ([8], [10] and [14]). Additionally, an illustrative example and corollaries are provided to demonstrate the main results. Our results can be utilized to find solution of fractional non-linear differential and integral equations (see[7] and references therein).

4 Open Problems

In the main results section, we proved some ψ-fixed point theorems of Wardowski kind in complete metric space. On the other hand, now a days, many mathematicians have obtained the sequel in different spaces (see [2], [3], [11] and references therein). So, it is an open problem whether it is possible to prove our main results in the frame of partial, modular, b-metric and S-metric space. The future work is looking for generalized these results in the frame of partial and modular metric space with the assistance of ψ-fixed point.

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