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A new subclass of bi-univalent functions associated with q-Chebyshev Polynomial

D. Kavitha[a,*], K. Dhanalakshmi[b], P. Manikandan[c]

[a]Department of Mathematics, SRM Institute of Science and Technology, Ramapuram, Chennai, 605108, Tamilnadu, India

[b]PG & Research Department of Mathematics, Theivanai Ammal College for Women (Autonomous), Villupuram, 605602, Tamilnadu, India

[c]Department of Mathematics, Anjalai Ammal Mahalingam Engineering College, Kovilvenni, 614403, Tamilnadu, India

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Abstract

In this article, using the concept of q-analogue, we define a new class of analytic functions associated with Chebyshev polynomial of second kind. Then with the help of symmetric q-Chebyshev polynomial, we introduce and estimating first two Maclaurin coefficients for new subclasses of analytic functions. Also as application of the results, we estimate the relevant connection to the famous classical Fekete-Szegö inequality belonging to the class $\tilde{S}^q_\Sigma(\lambda, \gamma, s)$.

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1 Introduction

The function class is represented by $\mathcal{A}$, which has the accompanying portrayal

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \qquad (z \in \mathbb{D}) \tag{1.1}$$

in the open disc $\mathbb{D} = \{z : z \in \mathbb{C} : |z| < 1\}$ with the normalization conditions

$$f(0) = 0 = f'(0) - 1.$$

Further, let $\mathcal{S}$ denote the class of functions in $\mathcal{A}$ which are also univalent in $\mathbb{D}$. Every univalent function f has an inverse f^{-1} satisfying $f^{-1}(f(z)) = z$, and

$$f^{-1}(f(w)) = w, (|w| < r_0(f), r_0(f) \geq 1/4)$$

where,

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \dots. \tag{1.2}$$

*Corresponding author
Email addresses: kavithad5@srmist.edu.in (D. Kavitha), ksdhanalakshmi@gmail.com (K. Dhanalakshmi), pmanikandan2007@gmail.com (P. Manikandan)

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A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{D}$ if both f and f^{-1} are univalent in $\mathbb{D}$. Let Σ denote the class of bi-univalent functions in $\mathbb{D}$ given by (1.1).

Let the functions f and g be analytic in $\mathbb{D}$. Then we say that f is subordinate to g, if there exits a Schwarz function $\omega(z)$, analytic in $\mathbb{D}$ with $\omega(0) = 0$ and $|\omega(z)| < 1$ in $\mathbb{D}$, such that $f(z) = g(\omega(z)), z \in \mathbb{D}$. We denote this subordination by $f \prec g$.

The problem of finding the sharp bounds for the non-linear functional $|a_3 - \mu a_2^2|$ of any compact family of functions is popularly known as the Fekete-Szegö problem. Several known authors at different time have applied the classical Fekete-Szegö inequality for various classes to obtain sharp bounds.

q-calculus is a fundamental instrument for understanding an extensive variety of scientific capabilities, and its applications in arithmetic and related fields have started interest among researchers. In light of the importance of q-calculus in mathematics and different disciplines, an enormous number of analysts have chipped away at q-calculus and concentrated on its various applications. A recent survey-cum-expository review paper of Srivatsava [9] made the researchers and academicians to working on q calculus in geometric function theory. In 2012, Johann Cigler [5] originally presented and considered the q-Chebyshev polynomials. Later Altinkaya and Yalcin [1] concentrated on the Chebyshev polynomial extensions to give assessments to the underlying coefficients of certain subclasses of bi-univalent capabilities characterized by the symmetric q-derivative operator. In 2018, Altinkaya computed the initial coefficients of a new class $\mathcal{L}(\alpha, t)$. Chebyshev polynomials are utilized to research a subclass of univalent in [2].In recent times the Komatu Integral operator was used to explore an original subclass of univalent functions as well as Chebyshev polynomials in [3]. one can see the recent contribution of Chebyshev polynomials in univalent function theory in [6], [7] and [10].

2 Preliminaries

For $q \in (0, 1)$, the q-derivative of the function f can be defined as

$$D_q f(z) = (z(1-q))^{-1}[f(z) - f(zq)] \tag{2.1}$$

where,

$$D_q f(0) - f^{'}(0) = 0$$

and

$$D_q^2 f(z) = D_q(D_q f); \quad z \neq 0, q \neq 0.$$

The new subclasses of symmetric q-derivative operator has been introduced and studied based on the idea of q-derivative operator. That is,

$$\tilde{D}_q f(z) = \begin{cases} [f(qz) - f(q^{-1}z)](q - q^{-1})^{-1} z^{-1} & (z \neq 0) \\ f^{'}(0) & (z = 0). \end{cases} \tag{2.2}$$

Definition 2.1. [5] q-Chebyshev polynomial of second kind defined as,

$$U_k(s, t, q) = \mathcal{F}_{k+1}(s, -1, t, q)(-q, q)_k = \sum_{l=0}^{(k/2)} q^{l^2} \begin{bmatrix} k-l \\ l \end{bmatrix} (1 + q^{l+1}) \dots (1 + q^{k-l}) t^l s^{k-2l} \tag{2.3}$$

which satisfies

$$U_k(s, t, q) = (1 + q^k) s U_{k-1}(s, t, q) + q^{k-1} t U_{k-2}(s, t, q). \tag{2.4}$$

with initial values

$$U_0(s, t, q) = 1 \quad and \quad U_1(s, t, q) = (1 + q)s.$$

and we can see that $U_k(s, -1, 1) = U_k(s)$ is the classical Chebyshev polynomial of second kind.

Inspired by the recent research of q-Chebyshev polynomials we can define the following:

Definition 2.2. Let $-1 < s < 1$ and $M(s,t,q,z)$ be the class of function defined as:

$$M(s,t,q,z) = 1 + \sum_{m=1}^{\infty} U_m(s,t,q)z^m \quad (z \in \mathbb{D}).$$
$$= 1 + U_1(s,t,q)z + U_2(s,t,q)z^2 + U_3(s,t,q)z^3 + \dots.$$

From (2.4),

$$U_1(s,t,q) = s(1+q)$$
$$U_2(s,t,q) = s^2(1+q)(1+q^2) + qt$$
$$U_3(s,t,q) = s^3(1+q)(1+q^2)(1+q^3) + sq(1+q)(1+q^2)t$$

Employ the concept of symmetric q-derivative operator discussed by Brahim et al.(2013) for $f(z)$ and is defined as follows:

Definition 2.3.

$$\tilde{D}_q z^k = [\tilde{k}]_q z^{k-1}$$
$$\tilde{D}_q f(z) = 1 + \sum_{k=2}^{\infty} [\tilde{k}]_q b_k z^{k-1}$$

where $[\tilde{k}]_q = \frac{q^k - q^{-k}}{q - q^{-1}}$.

Where,

$$\tilde{D}_q f(z) = D_{q^2} f(q^{-1}z)$$

represents the relation between D_q and $\tilde{D}_q$.

If there exist a inverse function of symmetric q-derivative operator, then

$$\tilde{D}_q f^{-1}(\omega) = 1 - [\tilde{2}]_q b_2\omega + [\tilde{3}]_q \left(2b_2^2 - b_3\right)\omega^2 - [\tilde{4}]_q \left(5b_2^2 - 5b_2b_3 + b_4\right)\omega^3 + \cdots.$$

Using the theory of subordination and the concept of symmetric q-derivative operator, we now define the new subclasses of bi-univalent function.

Definition 2.4. Let $\frac{1}{2} < s < 1, 0 \le \lambda \le 1, \gamma \in \mathbb{C}\backslash\{0\}, 0 < q < 1$ and $f \in \Sigma$ be in class $\tilde{\mathcal{S}}_{\Sigma}^{q}(\lambda, \gamma, s)$, if the following subordination holds:

$$1 + \frac{1}{\gamma}\left(\frac{z\tilde{D}_q f(z)}{(1-\lambda)z + \lambda f(z)} - 1\right) \prec M(s,t,q,z) \quad (z \in \mathbb{D}) \tag{2.5}$$

and

$$1 + \frac{1}{\gamma}\left(\frac{\omega\tilde{D}_q f^{-1}(\omega)}{(1-\lambda)\omega + \lambda f^{-1}(\omega)} - 1\right) \prec M(s,t,q,\omega) \quad (\omega \in \mathbb{D}). \tag{2.6}$$

With the suitable choice of parameter λ, we can obtain the next special classes:

Definition 2.5. For $\lambda = 1$, $\gamma \in \mathbb{C}\backslash\{0\}$ and $f \in \Sigma$ is said to be in class $\tilde{\mathcal{S}}_{\Sigma}^{q}(1, \gamma, s)$ if the following conditions are satisfied:

$$1 + \frac{1}{\gamma}\left(\frac{z\tilde{D}_q f(z)}{f(z)} - 1\right) \prec M(s,t,q,z) \quad (z \in \mathbb{D}) \tag{2.7}$$

and

$$1 + \frac{1}{\gamma}\left(\frac{\omega\tilde{D}_q f^{-1}(\omega)}{f^{-1}(\omega)} - 1\right) \prec M(s,t,q,\omega) \quad (\omega \in \mathbb{D}). \tag{2.8}$$

Definition 2.6. For $\lambda = 0$, $\gamma \in \mathbb{C}\backslash\{0\}$ and $f \in \Sigma$ is said to be in class $\tilde{\mathcal{S}}^q_\Sigma(\gamma, s)$ if the following conditions are satisfied:

$$1 + \frac{1}{\gamma}\left(\tilde{D}_q f(z) - 1\right) \prec M(s,t,q,z) \quad (z \in \mathbb{D}) \tag{2.9}$$

and

$$1 + \frac{1}{\gamma}\left(\tilde{D}_q f^{-1}(\omega) - 1\right) \prec M(s,t,q,\omega) \quad (\omega \in \mathbb{D}). \tag{2.10}$$

Definition 2.7. [4] For $\lambda = 0$, $\gamma = 1$ and $f \in \Sigma$ is said to be in class $\tilde{\mathcal{S}}^q_\Sigma(s)$ if the following conditions are satisfied:

$$\left(\tilde{D}_q f(z)\right) \prec M(s,t,q,z) \quad (z \in \mathbb{D}) \tag{2.11}$$

and

$$\left(\tilde{D}_q f^{-1}(\omega) - 1\right) \prec M(s,t,q,\omega) \quad (\omega \in \mathbb{D}). \tag{2.12}$$

Lemma 2.8. [8] If a function p $\in$ P is given by

$$p(z) = 1 + p_1 z + p_2 z^2 + \ldots \qquad (z \in \mathbb{D})$$

then $|p_k| \leq 2$, $k \in N$ where P is the family of all functions analytic in $\mathbb{D}$ for which $p(0) = 1$ and $\mathrm{Re}(p(z)) > 0$.

The primary purpose of this study is to investigate the properties of univalent functions associated with symmetric q-Chebyshev polynomials. By seeing the literature survey, there are only few authors have studied about q-Chebyshev polynomials and symmetric q-Chebyshev polynomials. Using this concept, author concentrated on finding initial coefficient estimates and the results of Fekete Szegö problem for the subclass $\tilde{S}^q_\Sigma(\lambda, \gamma, s)$.

3 Initial Coefficient Estimation

Theorem 3.1. Let $f \in \Sigma$ be in the class $\tilde{\mathcal{S}}^q_\Sigma(\lambda, \gamma, s)$. Then

$$|a_2| \leq \frac{|\gamma|(1+q)s\sqrt{(1+q)s}}{\sqrt{\left|\left[\gamma(\lambda^2 - (1+\tilde{[2]}_q)\lambda + \tilde{[3]}_q)(1+q) - (\tilde{[2]}_q - \lambda)^2(1+q^2)\right](1+q)s^2 - (\tilde{[2]}_q - \lambda)^2(qt - (1+q)s)\right|}}$$

and

$$|a_3| \leq \frac{(1+q)s|\gamma|}{(\tilde{[3]}_q - \lambda)} + \frac{(1+q)^2 s^2 |\gamma^2|}{(\tilde{[2]}_q - \lambda)^2}.$$

Proof . Let $f \in \tilde{S}^q_\Sigma(\lambda, \gamma, s)$ and there exists a holomorphic functions ω and ϖ. Then from (3.6) and (2.12),

$$1 + \frac{1}{\gamma}\left(\frac{z\tilde{D}_q f(z)}{(1-\lambda)z + \lambda f(z)} - 1\right) = M(s,t,q,\omega(z)) \tag{3.1}$$

and

$$1 + \frac{1}{\gamma}\left(\frac{\tilde{D}_q f^{-1}(\omega)}{(1-\lambda)\omega + \lambda f^{-1}(\omega)} - 1\right) = M(s,t,q,\varpi(\omega)). \tag{3.2}$$

If the functions $g, h \in \mathbb{D}$, then

$$\begin{aligned} g(z) &= \frac{1+\omega(z)}{1-\omega(z)} \\ &= 1 + g_1 z + g_2 z^2 + g_3 z^3 + \cdots \end{aligned} \tag{3.3}$$

$$\omega(z) = \frac{g(z) - 1}{g(z) + 1} \qquad (z \in \mathbb{D}). \tag{3.4}$$

$$\begin{aligned} h(\omega) &= \frac{1+\varpi(\omega)}{1-\varpi(\omega)} \\ &= 1 + h_1\omega + h_2\omega^2 + h_3\omega^3 + \cdots \end{aligned} \tag{3.5}$$

$$\varpi(\omega) = \frac{h(\omega)-1}{h(\omega)+1} \qquad (\omega \in \mathbb{D}). \tag{3.6}$$

From (3.4) and (3.6),

$$M(s,t,q,\omega(z)) = 1 + \frac{U_1(s,t,q)}{2} g_1 z + \left(\frac{U_1(s,t,q)}{2} \left(g_2 - \frac{g_1^2}{2} \right) + \frac{U_2(s,t,q)}{4} g_1^2 \right) z^2 + \cdots \tag{3.7}$$

and

$$M(s,t,q,\varpi(\omega)) = 1 + \frac{U_1(s,t,q)}{2} h_1 \omega + \left(\frac{U_1(s,t,q)}{2} \left(h_2 - \frac{h_1^2}{2} \right) + \frac{U_2(s,t,q)}{4} h_1^2 \right) \omega^2 + \cdots . \tag{3.8}$$

Also,

$$\begin{aligned} 1 + \frac{1}{\gamma} \left(\frac{z\tilde{D}_q f(z)}{(1-\lambda)z + \lambda f(z)} - 1 \right) &= 1 + \frac{1}{\gamma}([\tilde{2}]_q - \lambda) a_2 z \\ &+ \frac{1}{\gamma} \left(([\tilde{3}]_q - \lambda) a_3 - ([\tilde{2}]_q \lambda - \lambda^2) a_2^2 \right) z^2 + \cdots \end{aligned} \tag{3.9}$$

and

$$\begin{aligned} 1 + \frac{1}{\gamma} \left(\frac{\omega\tilde{D}_q g(\omega)}{(1-\lambda)\omega + \lambda g(\omega)} - 1 \right) &= 1 + \frac{1}{\gamma} \left(\lambda - [\tilde{2}]_q \right) a_2 \omega \\ &- \frac{1}{\gamma} \left((\lambda - [\tilde{3}]_q)(2a_2^2 - a_3) - (\lambda^2 - \lambda[\tilde{2}]_q) a_2^2 \right) \omega^2 + \cdots \end{aligned} \tag{3.10}$$

Comparing the like coefficients, we get

$$\frac{1}{\gamma} \left([\tilde{2}]_q - \lambda \right) a_2 = \frac{U_1(s,t,q)}{2} g_1, \tag{3.11}$$

$$\frac{1}{\gamma} \left[([\tilde{3}]_q - \lambda) a_3 - ([\tilde{2}]_q \lambda - \lambda^2) a_2^2 \right] = \frac{U_1(s,t,q)}{2} \left(g_2 - \frac{g_1^2}{2} \right) + \frac{U_2(s,t,q)}{4} g_1^2. \tag{3.12}$$

Similarly,

$$\frac{1}{\gamma} \left[\lambda - [\tilde{2}]_q \right] a_2 = \frac{U_1(s,t,q)}{2} h_1, \tag{3.13}$$

$$-\frac{1}{\gamma} \left[(\lambda - [\tilde{3}]_q)(2a_2^2 - a_3) - (\lambda^2 - \lambda[\tilde{2}]_q) a_2^2 \right] = \frac{U_1(s,t,q)}{2} \left(h_2 - \frac{h_1^2}{2} \right) + \frac{U_2(s,t,q)}{4} h_1^2. \tag{3.14}$$

From (3.11) and (3.13)

$$g_1 = -h_1 \Rightarrow g_1^2 = h_1^2 \tag{3.15}$$

Squaring and adding of (3.11) and (3.13),

$$a_2^2 = \frac{\gamma^2 U_1^2(s,t,q)(g_1^2 + h_1^2)}{8([\tilde{2}]_q - \lambda)^2}. \tag{3.16}$$

Now, adding (3.12) and (3.14) and substitute (3.15), we get

$$\frac{1}{\gamma} \left[(-2\lambda[\tilde{2}]_q + 2\lambda^2) a_2^2 + ([\tilde{3}]_q - \lambda) 2a_2^2 \right] = \frac{U_1(s,t,q)}{2}(g_2 + h_2) - \frac{U_1(s,t,q)}{2} g_1^2 + \frac{U_2(s,t,q)}{2} g_1^2. \tag{3.17}$$

By substituting (3.15) in (3.16)

$$g_1^2 = \frac{4(\tilde{[2]}_q - \lambda)^2 a_2^2}{\gamma^2 U_1^2(s,t,q)}. \tag{3.18}$$

Make use of (3.18) in (3.17),

$$a_2^2 = \frac{\gamma^2 U_1^3(s,t,q)(g_2 + h_2)}{4[\gamma(\lambda^2 - (1 + \tilde{[2]}_q)\lambda + \tilde{[3]}_q)U_1^2(s,t,q) - (\tilde{[2]}_q - \lambda)^2(U_2(s,t,q) - U_1(s,t,q))]}. \tag{3.19}$$

Applying Lemma 2.8 and the results defined in Defintion 2.2, we get the desired estimates of $|a_2|$.

Subtracting (3.14) from (3.12),

$$a_3 = \frac{U_1(s,t,q)(g_2 - h_2)\gamma}{4(\tilde{[3]}_q - \lambda)} + a_2^2. \tag{3.20}$$

From (3.16),

$$a_3 = \frac{U_1(s,t,q)(g_2 - h_2)\gamma}{4(\tilde{[3]}_q - \lambda)} + \frac{U_1^2(s,t,q)g_1^2\gamma^2}{4(\tilde{[2]}_q - \lambda)^2}. \tag{3.21}$$

Hence from Lemma 2.8

$$|a_3| \leq \frac{(1+q)s|\gamma|}{(\tilde{[3]}_q - \lambda)} + \frac{(1+q)^2 s^2 |\gamma^2|}{(\tilde{[2]}_q - \lambda)^2}. \tag{3.22}$$

4 Fekete Szegö Inequality

The classical Fekete Szegö functional $|a_3 - \nu a_2^2|$ has the great history in the geometric function theory for the univalent normalized function (1.1). In this section, authors concentrated on the Fekete Szegö inequality for the novel subclass $\tilde{S}_\Sigma^q(\lambda, \gamma, s)$ using symmetric q-Chebyshev polynomials.

Theorem 4.1. From (3.20) and (3.21),

$$\begin{aligned} a_3 - \nu a_2^2 &= \frac{U_1(s,t,q)(g_2 - h_2)\gamma}{4(\tilde{[3]}_q - \lambda)} \\ &\quad + (1-\nu)\frac{U_1^3(s,t,q)(g_2 + h_2)\gamma^2}{4\left[\gamma(\lambda^2 - (1 + \tilde{[2]}_q)\lambda + \tilde{[3]}_q)U_1^2(s,t,q) - (\tilde{[2]}_q - \lambda)^2(U_2(s,t,q) - U_1(s,t,q))\right]} \\ &= U_1(s,t,q)\left[\left(\mathcal{J}(\nu) + \frac{\gamma}{4(\tilde{[3]}_q - \lambda)}\right)g_2 + \left(\mathcal{J}(\nu) - \frac{\gamma}{4(\tilde{[3]}_q - \lambda)}\right)h_2\right], \end{aligned}$$

where

$$\mathcal{J}(\nu) = \frac{(1-\nu)U_1^2(s,t,q)\gamma^2}{4\left[\gamma(\lambda^2 - (1 + \tilde{[2]}_q)\lambda + \tilde{[3]}_q)U_1^2(s,t,q) - (\tilde{[2]}_q - \lambda)^2(U_2(s,t,q) - U_1(s,t,q))\right]}.$$

Hence,

$$|a_3 - \nu a_2^2| \leq \begin{cases} \frac{(1+q)s|\gamma|}{(\tilde{[3]}_q - \lambda)} & \text{if} \quad |\nu - 1| \leq \frac{\vartheta_q(q^{-1},s,t)}{(\tilde{[3]}_q - \lambda)(1+q^2)s^2} \\ \\ \frac{(1+q)^3 s^3 |\delta - 1||\gamma|^3}{\vartheta_q(q^{-1},s,t)} & \text{if} \quad |\nu - 1| \geq \frac{\vartheta_q(q^{-1},s,t)}{(\tilde{[3]}_q - \lambda)(1+q^2)s^2} \end{cases}$$

where

$$|\vartheta_q(q^{-1},s,t)| = |(1+q)s^2\left[\gamma(\lambda^2 - (1 + \tilde{[2]}_q)\lambda + \tilde{[3]}_q)(1+q) - (\tilde{[2]}_q - \lambda)^2(1+q^2)\right] - (qt - (1+q)s)(\tilde{[2]}_q - \lambda)^2|.$$

Corollary 4.2. Let $f \in \Sigma$ be in the class $\tilde{\mathcal{S}}^q_\Sigma(1, \gamma, s)$. Then

$$|a_2| \leq \frac{|\gamma|(1+q)s\sqrt{(1+q)s}}{\sqrt{\left|\left[\gamma([\tilde{3}]_q - [\tilde{2}]_q)(1+q) - ([\tilde{2}]_q - 1)^2(1+q^2)\right](1+q)s^2 - ([\tilde{2}]_q - 1)^2(qt - (1+q)s)\right|}}$$

and

$$|a_3| \leq \frac{(1+q)s|\gamma|}{([\tilde{3}]_q - 1)} + \frac{(1+q)^2 s^2 |\gamma^2|}{([\tilde{2}]_q - 1)^2}.$$

Also,

$$|a_3 - \nu a_2^2| \leq \begin{cases} \frac{(1+q)s|\gamma|}{([\tilde{3}]_q - 1)} & \text{if} \quad |\nu - 1| \leq \frac{\vartheta_q(q^{-1}, s, t)}{([\tilde{3}]_q - 1)(1+q^2)s^2} \\ \\ \frac{(1+q)^3 s^3 |\nu - 1||\gamma|^3}{\vartheta_q(q^{-1}, s, t)} & \text{if} \quad |\nu - 1| \geq \frac{\vartheta_q(q^{-1}, s, t)}{([\tilde{3}]_q - 1)(1+q^2)s^2} \end{cases}$$

where

$$|\vartheta_q(q^{-1}, s, t)| = |(1+q)s^2\left[\gamma([\tilde{3}]_q - [\tilde{2}]_q)(1+q) - ([\tilde{2}]_q - 1)^2(1+q^2)\right] - (qt - (1+q)s)([\tilde{2}]_q - 1)^2|.$$

Corollary 4.3. Let $f \in \Sigma$ be in the class $\tilde{\mathcal{S}}^q_\Sigma(\gamma, s)$. Then

$$|a_2| \leq \frac{|\gamma|(1+q)s\sqrt{(1+q)s}}{\sqrt{\left|\left[\gamma([\tilde{3}]_q(1+q) - [\tilde{2}]_q^2)(1+q^2)\right](1+q)s^2 - [\tilde{2}]_q^2(qt - (1+q)s)\right|}}$$

and

$$|a_3| \leq \frac{(1+q)s|\gamma|}{[\tilde{3}]_q} + \frac{(1+q)^2 s^2 |\gamma^2|}{[\tilde{2}]_q^2}.$$

Also,

$$|a_3 - \nu a_2^2| \leq \begin{cases} \frac{(1+q)s|\gamma|}{[\tilde{3}]_q} & \text{if} \quad |\nu - 1| \leq \frac{\vartheta_q(q^{-1}, s, t)}{[\tilde{3}]_q(1+q^2)s^2} \\ \\ \frac{(1+q)^3 s^3 |\nu - 1||\gamma|^3}{\vartheta_q(q^{-1}, s, t)} & \text{if} \quad |\nu - 1| \geq \frac{\vartheta_q(q^{-1}, s, t)}{[\tilde{3}]_q(1+q^2)s^2} \end{cases}$$

where

$$|\vartheta_q(q^{-1}, s, t)| = |(1+q)s^2\left[\gamma[\tilde{3}]_q^2(1+q) - [\tilde{2}]_q^2(1+q^2)\right] - (qt - (1+q)s)[\tilde{2}]_q^2|.$$

Remark 4.4. If $\lambda = 0$ and $\gamma = 1$, then the Theorem 3.1 and Theorem 4.1 are reduced to the results obtained by Bilal Khan et al.[4].

5 Conclusion

In this paper, we have introduced the new subclass of bi-univalent function associated with symmetric q-Chebyshev polynomial. Author have discussed the initial coefficients using the concept of subordination. Finally, as an application of this result, we have obtained the sharp estimates of classical Fekete-Szegö problem.

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