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Medical and color images compression using new wavelet transformation

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Abstract

In recent years, technology has developed very dramatically in all fields, such as imaging technology, which has become crucial in most medical fields. Currently, widely medical images are used to diagnose and treat various illnesses. This research describes a novel waveform developed from the principle of torsion in Haar with Cosine and Sine waves (CAS) for compressing color and medical images such as Magnetic Resonance Imaging (MRI) and computed tomography (CT). This research proposes a method that discusses medical image decomposition and color images using novel waveform analysis for image compression, and how wavelet is utilized for image restoration. The experiments demonstrate that our method is more accurate and compressive than the Haar wavelet Transform (Haar) when comparing numerical results of Compressed file size, CR (Compression Ratio), PSNR (Peak Signal Noise Ratio) and RMSE (Root Mean Square Error). Furthermore, the method achieves better image quality.

Keywords: Haar, Sine and Cosine waves, Image Compression, Compressed file size, CR, PSNR, and RMSE
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1 Introduction

In recent years, Image files have become one of the most common files to be utilized and shared. Nevertheless, image sizes are large, making them difficult to transmit and store [17]. Medical imaging and diagnostic tools have evolved rapidly, becoming more significant in clinical analysis and illness diagnosis. Interior human body organs such as the brain, lungs, and heart require medical photography to be captured [12]. Only when compression techniques maintain all relevant and significant image information is diagnosis possible in medical image compression applications [11]. The function of wavelets in image compression has grown in importance as the number of digital images has grown fast in recent years, resulting in high transmission and memory costs [9]. Digital images may be compressed by removing redundant information since they contain a significant degree of spatial, temporal, and spectral redundancy. Lossy compression yields greater compression rates, but precise data cannot be restored [1].

Typically, wavelet transform is implemented in medical images, which provides a suitable approach for the compression of biomedical images. For example, X-ray angiography (XA), magnetic resonance imaging (MRI), and others are

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widely utilized in medical diagnostics [5, 18]. As a result, one of the goals of this work is to look at the impact of using various wavelet types for lossless compression of medical and color images [11]. Furthermore, to picture distortion, the human visual system has poor sensitivity. Therefore, when these factors are taken into account, the compression potential increases dramatically, saving storage space and transmission time. This research uses a new wavelet formula to describe a medical image and color image compression technique. We can attain a higher peak signal-to-noise ratio (PSNR), a greater compression ratio, and a lower root means square error (RMSE) while reconstructing an image.

This research is organized as follows: Section 2 reviews previous works. The analysis wavelet is presented in Section 3. Image Quality metrics and Image compression are explained in Sections 4 and 5. The proposed method is presented in Section 6. Section 7 evaluates the performance of the method. Finally, Section 8 concludes the research.

2 Related Works

The suggested technique was created to compress the image. The goal is to create a computationally efficient and effective lossy image compression method employing wavelet techniques. We used the Haar Wavelet Transform in this work. Images were compressed using general-purpose compression applications, although the results were less than ideal. Some of the image's finer features can be sacrificed to conserve a bit more bandwidth or storage space. Sub-band coders have proven very effective in speech and picture reduction, and the discrete wavelet is essentially a sub-band–coding scheme. The findings achieved in terms of reconstructed image quality and retention of important image elements are encouraging [14].

In [8], the authors used the Discrete Haar Wavelet Transform (DWT) to offer the best approach for compressing medical images. Nowadays, medical image compression is beneficial and essential for image transfer between two sites or for data storage and simple data. At the transmitting point, to transfer images, they need to be compressed. Then, they must be decompressed at the destination to reconstruct the original image or a close representation of it. Singular Value Decomposition, Block Truncation Coding, Gaussian Pyramids, and Discrete Cosine Transform are just a few image compression algorithms studied. Discrete HAAR wavelet transform compression is an essential and efficient method for storing and transmitting medical pictures. The compression ratio in Percent and PSNR value between the original and compressed images was calculated, and this study summarized the findings.

The authors in [6] presented an effective methodology using Discrete Cosine Transform (DCT) with block processing to give an efficient picture compression solution for medical imaging. The DCT divides an image into several sections while maintaining picture quality. Additionally, the DCT transformation is the most often utilized Fourier transformation. Block processing is done to the converted image after successful transformation using the DCT technique. Instead of processing the entire image, block processing performs actions on parts of it. Lastly, the authors described a fast 3D medical image compression technique based on block processing with DCT. The suggested approach dramatically decreased compression size, improved the compression ratio and MSE, and increased the PSNR by 6

3 Analysis Wavelet

Recently, wavelets (Wavelet analysis) have gained popularity and have become effectively employed in various applications, such as image analysis, transient signal analysis, and other signal processing tasks. Therefore, the wavelet is a new numerical idea that allows you to express a function in terms of identifiable fundamental functions in space. In the early 1980s, wavelets were presented for the first time. It has piqued the curiosity of the mathematics community as well as members in a variety of areas where wavelets have shown promise. Many books on the subject have appeared due to this interest and a vast number of research publications [4]. Wavelets theory is a relatively young and developing instrument in applied mathematics. It has been used in many engineering fields, including waveform representation and segmentation, time-frequency analysis, and rapid algorithms for straightforward implementation.

Wavelets make it possible to express a wide range of functions and operators accurately. Furthermore, wavelets have a relationship with rapid numerical methods [3, 2]. A wavelet is a wavelike oscillation whose amplitude begins at 0, grows, and returns to 0. Moreover, wavelets are irregular in shape and compactly supported, making them suitable for evaluating non-stationary data. Wavelets are mathematical functions that divide data into a variety of subsets. After that, investigate each component with a resolution corresponding to its scale [7]. As shown in equation 3.1, the following set of continuous wavelets can be achieved when the expansion parameter a and the translation parameter b change continuously.

$$\Psi_{x,y}(u) = |x|^{\frac{-1}{2}} \Psi\left(\frac{u-y}{x}\right) \cdot \mathrm{x,y} \in \mathrm{P}, \mathrm{x} \neq 0 \tag{3.1}$$

If the parameters x and y are restricted to discrete values, which are

$$x = x_0^{-s},\ y\ =\ zy_0\ \ x_0^{-s}\ , x_0\ >\ 1,\ y_0 >\ 0$$

Where s and z are positive integers, Then we have the next family of separate wavelets as shown in equation 3.2:

$$\Psi_{s,z}(u) == |x_0|^{\frac{s}{2}}\, \Psi\left(x_0^s u - z y_0\right) \tag{3.2}$$

Where $\Psi_{s,z}$ (u) is a basis of wavelet for $L^2(P)$.

In particular, when $x_0 = 2, y_0 = 1$, the form $\Psi_{s,z}(u)$ is an orthonormal basis [1].

3.1 CAS Wavelet:

A wavelet CAS is denoted by $\psi_{v,w}(a)$ Wavelet CAS relies on four arguments which are $\psi_{v,w}(a) =\ \psi(s,\ v,\ w,\ a)$ have four arguments: $v\ =\ 0,\ 1,\ ...,\ 2^s -\ 1,\ s$ Any positive integer is considered to be. They are defined as follows on the interval [0, 1] and as shown in equations 3 and 4.

$$\Psi_{v,w}(\mathrm{a}) = \begin{cases} 2^{\frac{s}{2}} CAS_w\left(2^s a - v\right),\ \text{for}\ \frac{v}{2^s} \leq a \leq \frac{v+1}{2^s} \\ 0 \quad otherwise \end{cases} \tag{3.3}$$

With

$$CAS_w\,(a) =\ cos(2w\pi a)\ +\ sin(2w\pi a), \tag{3.4}$$

Where $w\ =\ -W,\ -\ (W\ -\ 1)\,, ...\ 0.,\ ...,(W\ -\ 1),\ W$ [15].

3.2 Haar Wavelet function

Haar transform is one of the earliest instances of what is currently known as a compact, dyadic, orthonormal wavelet transform. The Haar function is the oldest and simplest orthonormal wavelet with compact support, as it is an odd rectangular pulse pair. In many applications, such as image coding, edge extraction, and binary logic design, Haar functions appear to be particularly appealing. Several definitions and extensions of the Haar functions have been published and utilized. Haar wavelets have recently been used widely in signal processing studies in communications and physics and have shown to be a fantastic mathematical tool.

The Haar wavelet family for u ∈ [0, 1] is illustrated in equation 3.5.

$$h_c(u) = \begin{cases} 1\ \text{for}\ u \in \left[\frac{s}{w}, \frac{s+0.5}{w}\right] \\ -1\ \text{for}\ u \in \left[\frac{s+0.5}{w}, \frac{s+1}{w}\right] \\ 0 \quad elsewhere \end{cases} \tag{3.5}$$

Integer $w\ =\ 2^v\ \ (v\ =\ 0, 1, 2, ..., V)$ indicates the level of the wavelet; Maximal level of resolution is $V; s\ =\ 0, 1, 2, ..., w\ -1$ is the translation parameter. According the formula $c\ =\ w\ +\ s\ +\ 1$, the index c is calculated; in the case of minimal values. $w\ =\ 1,\ s\ =\ 0$ [5].

3.3 New wavelet

By convoluting Haar and CAS wavelets, a new waveform is generated. The CAS waves are given in trigonometric functions whose integral is Periodic and Constrained [10] (see equation 3.6), which we will apply in our proposed method to compress images and medical images.

Allow $\Psi_{v,w} = CAS$ wavelet, and $H_c =$ Haar wavelet since the convolution is commutative, we have:

$$W_{v,W}^{NEW}(\mathrm{a}) = \left(\Psi_{v,w} * H_c\right)(\mathrm{a}) = \int_0^a \Psi_{v,w}(\mathrm{u}) \cdot H_c(\mathrm{a} - \mathrm{u}) du \tag{3.6}$$

$$W_{v,w}^{NEW}(\mathrm{a}) = \begin{cases} \begin{cases} \frac{2^{\frac{-s}{2}-1}}{\pi w}\left[\cos\left(2w\pi\left(2^s a - v\right)\right) - \sin\left(2w\pi\left(2^s a - v\right)\right) + 2\sin(w\pi)\right. \\ \left. -2\cos(w\pi) + 1\right], \frac{v}{2^s} \le a \le \frac{v+1}{2^s} \end{cases} \\ 0, \quad otherwise \end{cases} \tag{3.7}$$

Where $\mathrm{w} \in \{-\mathrm{W}, -\mathrm{W}+1, \ldots, \mathrm{W}\}$

4 Image Quality Metric

Image quality is an essential requirement in image-based object recognition. Ground truth is necessary for an accurate image quality rating. For all things and their functionality, quality is a critical factor. Full reference measurements are used to evaluate image quality. Several image quality approaches are often used to measure and compare image quality [13], including the following:

4.1 Compression Ratio (CR)

The ratio of the size of the uncompressed image (the original image) to the size of the compressed image is used to calculate it [16].

$$Compression\ Ratio = \frac{Origianal\ image\ Size}{compressed\ image\ Size}$$

4.2 The RMSE

It's a metric for how good a compression method is and calculated as the difference between the original and reconstructed images after compression. Therefore, the lower the RMSE number, the better the algorithm's compression [13].

$$\mathrm{RMSE} = \frac{1}{V \times W} \sum_{c=0}^{V-1} \sum_{d=0}^{W-1} [A(c,d) - B(c,d)]^2$$

where V and W represent the width and height of the image. Whereas, A and B are the reconstructed and original images.

4.3 PSNR is an image quality metric

It determines the degree of similarity between the raw and processed images. This ratio is frequently used to compare the quality of an original and a compressed image. The better the quality of the compressed or reconstructed image, the higher the PSNR [13].

$$\mathrm{PSNR} = 10.\log_{10}\left[\frac{\max(l(a,b))^2}{\frac{1}{v_a \cdot v_b} \cdot \sum_0^{v_a-1} \sum_0^{v_b-1} [l(a,b) - u(a,b)]^2}\right]$$

5 Image Compression

A broad range of wavelet-based image compression schemes has been documented, such as basic entropy to more advanced approaches like vector quantization adaptive transform, tree encoding, coding, edge-based coding, and Huffman coding. There are two forms of compression: lossless compression and lossy compression. The lossless compression method compresses binary data, such as executables, documents, etc. It entails compressing data that, when compressed together, produces an exact replica of the original material. But, lossy compression gives a higher compression ratio than lossless compression. The compressed image produced by this method is not identical to the original image; some information is lost. Utilizes lossless compression of color and medical images, especially medical images, so that this compression does not lead to loss of image information, because it affect the doctor's diagnosis of the patient's condition. Finally, our method is based on the compression of functions with the help of Haar and CAS wavelet [7].

6 Methodology Suggestions

This research aims to design a compression method for medical and color images that produce a high-quality reconstruction image after decompressing by CAS and Haar wavelet. CAS and Haar wavelet combines the benefits of both while avoiding the downsides of each. Therefore, the following steps present are the wavelet transforms-based compression technique:

1. For compressing the medical and color image, the DWT of the image is formed by acquiring wavelet decomposition coefficients for the necessary levels using New Wavelet
2. For uncompressing the medical and color image, inverse New Wavelet is used to recreate the image.

7 Proposed Apply

The proposed approach can be implemented by performing the following steps:

Step 1: Insert the medical and color image into the MATLAB workspace.

Step 2: Divide the used image into 16 x 16 blocks.

Step 3: Seperate the red, green and blue components from this image.

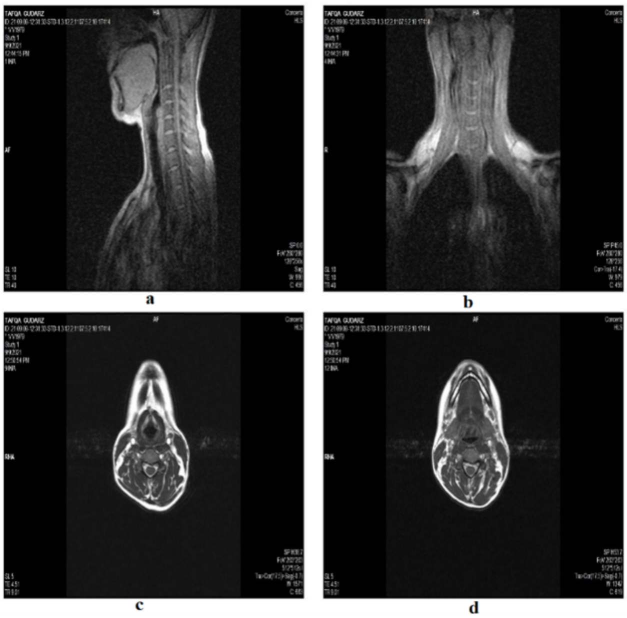
Step 4: Apply the new wavelet to components of the image used by implementing Equation 3.5 (see Figures 1 and 2).

Step 5: Reconstruct the approximation of the original image used by implementing the corresponding inverse of the new wavelet.

Step 6: Combine the red, green, and blue components of this image

Step 7: Calculate various compression ratio, RMSE, and PSNR values for the reconstructed images are the illustrated in Figures 3 and 5.

Step 8: Haar Transformation were applied, repeat the operation on the same used images, as shown in Figures 1 and 2. The compression ratios of the reconstructed images were then specified using the CR, RMSE, and PSNR values.



a b c d

Figure 1: Original Medical images used

Figure 2: Original Images Color used

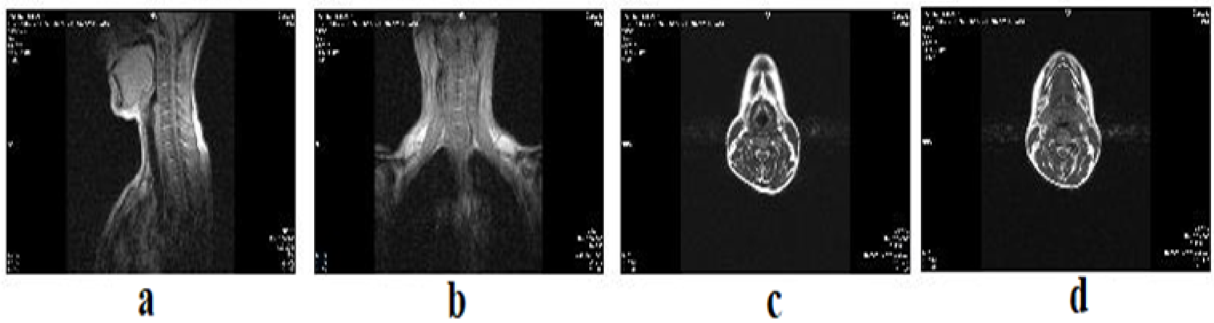


Figure 3: Medical Images Compressed by New Wavelet Transformation

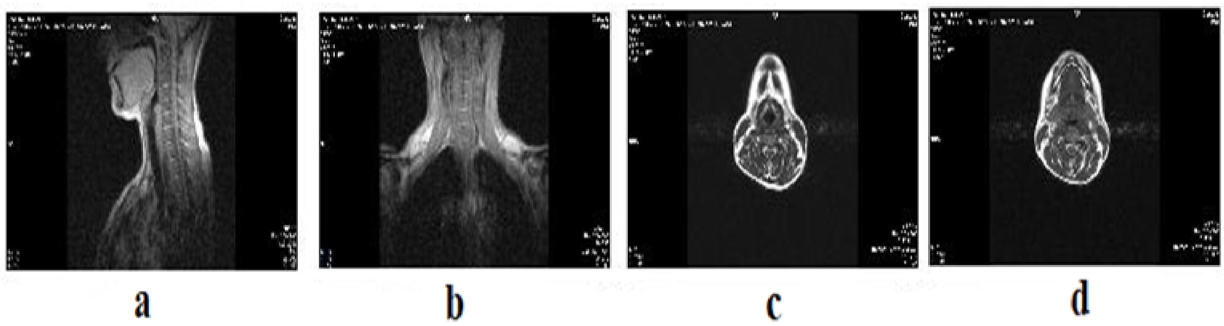


Figure 4: Medical Images Compressed by Haar Transformation

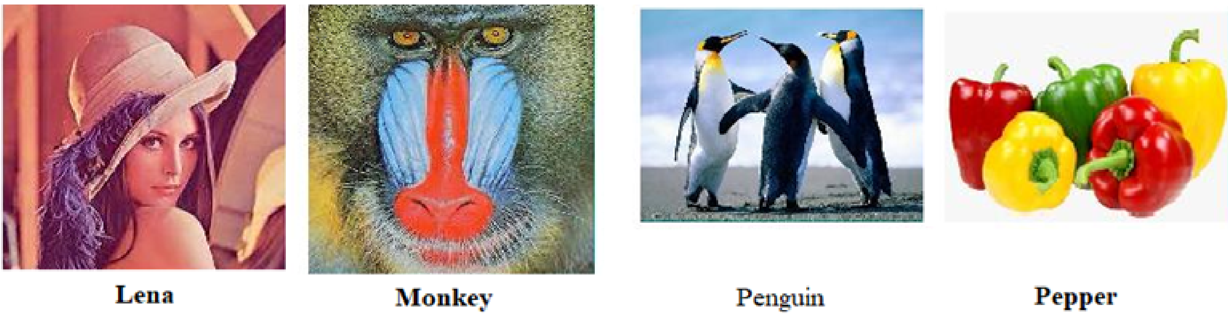


Figure 5: Color Images Compressed by New Wavelet Transformation

Figure 6: Color Images Compressed by Haar Transformation

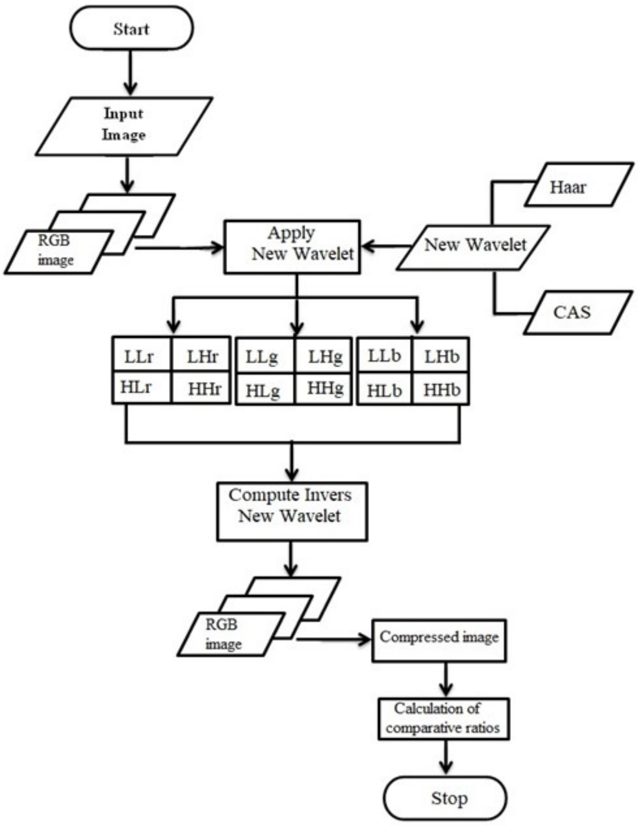


Figure 7: Flow Chart of the proposed work

8 Result and Discussion

This section evaluates and analyzes the execution of the proposal. The evaluation is based on the results using a number of the different medical images like MRI, CT, and color tests images tests as shown in Figures 1 and 2 (a, b, c, and d). All experiments are conducted using MATLAB 10 (2020) on a personal Laptop (Intel P4, 2.4 GHz, 4 GB).

After applying Haar, and the new wavelet to medical images (like CT and MRI) and after calculating the values, we found that the compressed file, compression ratios and RMSE of the proposed method are better than Haar when compared (see Tables 1 and 2) and Figure 8.

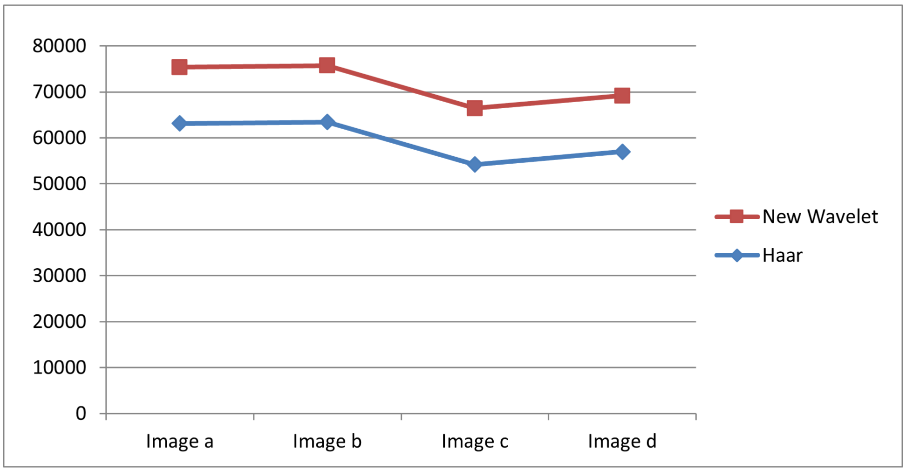


Figure 8: Medical Images Compressed by New Wavelet and Haar

Table 1: Results of different Medical Images using the Haar Transformation

Haar	File size	Compress	CR	RMSE	PSNR
Image a	99948	63134	1.583109	0.010829	201.335949
Image b	100255	63445	1.580188	0.011830	199.568352
Image c	93768	54193	1.730260	0.015373	194.327888
Image d	97842	56949	1.718064	0.015309	194.411508

Table 2: Results of different Medical Images using the New Wavelet Transformation

New Wavelet	File size	Compress	CR	RMSE	PSNR
Image a	99948	12234	8.169691	1.286985	105.779220
Image b	100255	12252	8.182746	1.294114	105.668738
Image c	93768	12224	7.670812	1.374850	104.458375
Image d	97842	12228	8.001472	1.408105	103.980377

As compressing color photos using the new wavelet, the compressed file shrinks dramatically, and the compression ratio is high and good, as well as RMSE and PSNR, when compared to utilizing Haar.

Table 3: Results of different Images Color using the Haar Transformation

Haar	File size	Compress	CR	RMSE	PSNR
Lena	278396	61012	4.562971	7.133207	71.530054
Monkey	404307	105204	3.843076	5.865495	75.443535
Penguin	244354	49973	4.889720	11.405318	62.143676
Pepper	120876	26782	4.513330	15.073774	56.566143

Table 4: Results of different Images Color using the New Wavelet Transformation

New wavelet	File size	Compress	CR	RMSE	PSNR
Lena	278396	8304	33.525530	1.679824	100.451486
Monkey	404307	8540	47.342740	2.591053	91.783984
Penguin	244354	6505	37.564028	2.561264	92.015255
Pepper	120876	5563	21.728564	1.724565	99.925773

9 Conclusion

In this paper, we proposed an efficient compression technique based on a new wavelet derived from torsion between Haar and CAS wavelets to compress color and medical images. Image compression reduces the size of an image so that it may be saved in less amount of space. Thus, it helps to reduce bits redundancy, which causes images to take up less space. Furthermore, waves fit nicely on time-limited data. Moreover, a wave-based compression technique improves image quality by reducing error and allowing good spatial and frequency domain localization. The proposed technique has been performed on real medical images.

According to the findings of the experiments compared to Haar wavelets, promising results were obtained regarding the quality of the reconstructed medical image and the preservation of important image details. Additionally, the proposed technique has achieved high compression rates and achieved suitable compression for color images when the proposed method is compared to Haar wavelets.

References

[1] A. Barzkar, P. Assari, and M.A. Mehrpouya, *Application of the cas wavelet in solving fredholm-hammerstein integral equations of the second kind with error analysis*, World Appl. Sci. J. **18** (2012), 1695–1704.

[2] G. Beylkin, R. Coifman, and V. Rokhlin, *Fast wavelet transforms and numerical algorithms i*, Commun. Pure Appl. Math. **44** (1991), no. 2, 141–183.

[3] C.K. Chui, *Wavelets: A mathematical tool for signal analysis*, SIAM, 1997.

[4] B.M.H. Gannam, *Mathematical theory of wavelets*, (2009).

[5] G. Hariharan, K. Kannan, and K.R. Sharma, *Haar wavelet method for solving fisher's equation*, Appl. Math. Comput. **211** (2009), no. 2, 284–292.

[6] P.K.M. Himani, *Medical image compression using block processing with dct*, Int. J. Adv. Res. Comput. Commun. Engin. **7** (2018).

[7] G.K. Jagatheswari, R. Murugesan, and G.V. Honnavar, *Image compression of 2-d continuous exponential functions, continuous periodic functions and product of sine and cosine functions using discrete wavelet transform*, J. Inf. Math. Sci. **9** (2017), no. 3, 781–790.

[8] A. Julius, *Medical image compression by using discrete Haar wavelet transform*, Int. J. Scien. Res. **6** (2013), no. 14, 1196–1200.

[9] R.N. Khatke, *Image compression using wavelet transform*, Imper. J. Interdiscip. Res. **2** (2016), no. 9, 82–84.

[10] A.M. Qasim, *New wavelet method for solving partial differential equations*, Turk. J. Comput. Math. Educ. **12** (2021), no. 13, 3951–3959.

[11] M.S. Ruchika, A.R. Singh, and A.R. Singh, *Compression of medical images using wavelet transforms*, Int. J. Soft Comput. Engin. **2** (2012), no. 2, 339–343.

[12] J.M. Sanches, J.C. Nascimento, and J.S. Marques, *Medical image noise reduction using the sylvester–lyapunov equation*, IEEE Trans. Image Process. **17** (2008), no. 9, 1522–1539.

[13] U. Sara, M. Akter, and M.Sh. Uddin, *Image quality assessment through fsim, ssim, mse and psnr—a comparative study*, J. Comput. Commun. **7** (2019), no. 3, 8–18.

[14] N. Sethi, R. Krishna, and R.P. Arora, *Image compression using haar wavelet transform*, Comput. Engin. Intell. Syst. **2** (2011), no. 3, 1–5.

[15] S.C. Shiralashetti and Lata Lamani, *Cas wavelets stochastic operational matrix of integration and its application for solving stochastic itô-Volterra integral equations*, Jordan J. Math. Statist. **14** (2021), no. 3, 555–580.

[16] S. Sridhar, P.R. Kumar, and K.V. Ramanaiah, *Wavelet transform techniques for image compression-an evaluation*, Int. J. Image Graphics Signal Process. **6** (2014), no. 2, 54.

[17] S. Tamboli and V. Udupi, *Image compression using haar wavelet transform*, Int. J. Adv. Res. Comput. Commun. Engin. **2** (2013), no. 8.

[18] K.S. Thyagarajan, *Still image and video compression with matlab*, John Wiley & Sons, 2011.