

Soliton solutions and other analytical solutions of a new (3+1)-dimensional novel KP like equation

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Abstract

We study a new (3+1)-dimensional novel KP-like equation. We show that this equation admits topological soliton solutions. These will be attained via the aid of ansatz methods. Furthermore, mixed solutions consisting of singular and periodic solutions and others are derived. Moreover, other analytical solutions based on modern group analysis are derived. In addition, low-order conservation laws are constructed.

Keywords: Topological soliton solutions, Periodic solutions, Point symmetries, Conservation laws
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1 Introduction

Recently, many research activities [6, 2] on solitary waves theory, predominantly on integrable systems, have attracted a lot of researchers. This is due to the fact that solitary waves theory has found a lot applications in many areas of nonlinear science, such as engineering, plasma physics, biology and other fields of mathematical physics. In the past decade, researchers have confined their application of solitary waves theory to (1 +1) and (2 +1)-dimensional equations[14]. However, it was later found that solitary waves theory plays a significant role in the study of higher dimensional integrable equations. It is for this reason, that motivated authors in [8] to establish a new (3+1)-dimensional novel KP like equation. given by

$$\alpha u_{tz} - \beta u_{xxxy} - 3(u_{xx}u_y + u_xu_{xy}) + \gamma(u_{xx} + u_{xy} + u_{xz}) = 0, \quad (1.1)$$

where (α, β, γ) are arbitrary parameters. Authors in [8] used the simplified linear superposition principle to derive resonant multi-soliton solution of equation (1.1). It can easily be noticed that equation (1.1) is a natural extension of the famous B-type Kadomtsev–Petviashvili (BKP) equation[8] and references therein. To the best of our knowledge, topological soliton solutions, singular and periodic solutions and point symmetries of the aforesaid equation have not been reported in the literature.

The objectives of this work is two fold. Firstly, we will implement the ansatz methods so as to derive topological

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soliton solutions. Furthermore, we will employ the Tan-Cot method to attain singular and periodic solutions. In addition, symmetry method will be invoked to obtain some other analytical solutions and lastly construct some *low-order* conservation laws are constructed via the multiplier approach.

2 Soliton solutions

This section aims to compute topological 1-soliton solution of equation (1.1). This will be attained via a hypothesis method. In order to search for dark soliton solutions or shock waves or kinks, we begin our hypothesis [14, 12, 15, 13] in the form of

$$u(t, x, y, z) = \lambda \tanh^p \tau, \tag{2.1}$$

where the wave variable τ is defined by $\tau = \eta x + \delta y + \varphi z - \nu t$ while η, δ and φ are unknown free parameters representing the inverse width of the wave. ν is the velocity of the soliton and p is a positive exponent that will be determined. The hypothesis (2.1) yields

$$u_x = \lambda \eta p (\tanh^{p-1} \tau - \tanh^{p+1} \tau), \tag{2.2}$$

$$u_{xx} = \lambda \eta^2 p(p+1) \tanh^{p+2} \tau + \lambda \eta^2 p(p-1) \tanh^{p-2} \tau - 2\lambda \eta^2 p^2 \tanh^p \tau, \tag{2.3}$$

$$u_{xy} = \lambda \eta \delta p(p+1) \tanh^{p+2} \tau + \lambda \eta \delta p(p-1) \tanh^{p-2} \tau - 2\lambda \eta \delta p^2 \tanh^p \tau, \tag{2.4}$$

$$u_{xz} = \lambda \eta \varphi p(p+1) \tanh^{p+2} \tau + \lambda \eta \varphi p(p-1) \tanh^{p-2} \tau - 2\lambda \eta \varphi p^2 \tanh^p \tau, \tag{2.5}$$

$$u_{tz} = -\lambda \varphi \nu p(p+1) \tanh^{p+2} \tau - \lambda \varphi \nu p(p-1) \tanh^{p-2} \tau + 2\lambda \varphi \nu p^2 \tanh^p \tau, \tag{2.6}$$

$$u_y = \lambda \delta p \tanh^{p-1} \tau - \lambda \delta p \tanh^{p+1} \tau, \tag{2.7}$$

$$u_y u_{xx} = -\lambda^2 \eta^2 \delta p^2 (p+1) \tanh^{2p+3} \tau + \lambda^2 \eta^2 \delta p^2 (p-1) \tanh^{2p-3} \tau + \lambda^2 \eta^2 \delta p^2 (3p+1) \tanh^{2p+1} \tau - \lambda^2 \eta^2 \delta p^2 (3p-1) \tanh^{2p-1} \tau, \tag{2.8}$$

$$u_x u_{xy} = -\lambda^2 \eta^2 \delta p^2 (p+1) \tanh^{2p+3} \tau + \lambda^2 \eta^2 \delta p^2 (p-1) \tanh^{2p-3} \tau + \lambda^2 \eta^2 \delta p^2 (3p+1) \tanh^{2p+1} \tau - \lambda^2 \eta^2 \delta p^2 (3p-1) \tanh^{2p-1} \tau, \tag{2.9}$$

$$u_{xxxy} = -\lambda \eta^3 \delta p(p+3)(p+2)(p+1) \tanh^{p+4} \tau + \lambda \eta^3 \delta p(p-1)(p-2)(p-3) \tanh^{p-4} \tau - 4\lambda \eta^3 \delta p(p+1)(p^2+2p+2) \tanh^{p+2} \tau - 4\lambda \eta^3 \delta p(p-1)(p^2-2p+2) \tanh^{p-2} \tau + 2\lambda \eta^3 \delta p^2(3p^2+5) \tanh^p \tau. \tag{2.10}$$

The substitution of equations (2.2)–(2.10) into equation (1.1), gives

$$\begin{aligned} & -\lambda \eta^3 \delta \beta p(p+3)(p+2)(p+1) \tanh^{p+4} \tau - \lambda \eta^3 \delta \beta p(p-1)(p-2)(p-3) \tanh^{p-4} \tau \\ & + (4\eta^3 \delta \beta (p^2 + 2p + 2) - \alpha \varphi \nu + \gamma \eta(\eta + \delta + \varphi)) \lambda p(p+1) \tanh^{p+2} \tau \\ & + (4\eta^3 \delta \beta (p^2 - 2p + 2) - \alpha \varphi \nu + \gamma \eta(\eta + \delta + \varphi)) \lambda p(p-1) \tanh^{p-2} \tau \\ & + 6\lambda^2 \eta^2 \delta p^2 (p+1) \tanh^{2p+3} \tau - 6\lambda^2 \eta^2 \delta p^2 (p-1) \tanh^{2p-3} \tau \\ & - 6\lambda^2 \eta^2 \delta p^2 (3p+1) \tanh^{2p+1} \tau + 6\lambda^2 \eta^2 \delta p^2 (3p-1) \tanh^{2p-1} \tau \\ & + 2(\alpha \varphi \nu - \eta^3 \delta \beta (3p^2 + 5) - \gamma \eta(\eta + \delta + \varphi)) \tanh^p \tau = 0. \end{aligned} \tag{2.11}$$

To seek for the smallest positive integer p , we equate the highest linear term in \tanh with the least nonlinear term in $\tanh \tau$. This can be achieved by equating powers $\tanh^{p+4} \tau$ and $\tanh^{2p+3} \tau$, to get

$$p + 4 = 2p + 3,$$

therefore $p = 1$. Substituting $p = 1$ into the powers of $\tanh \tau$ only, equation (2.11) reduces to

$$\begin{aligned} & -\lambda \eta^3 \delta \beta p(p+3)(p+2)(p+1) \tanh^5 \tau - \lambda \eta^3 \delta \beta p(p-1)(p-2)(p-3) \tanh^{-3} \tau \\ & + (4\eta^3 \delta \beta (p^2 + 2p + 2) - \alpha \varphi \nu + \gamma \eta(\eta + \delta + \varphi)) \lambda p(p+1) \tanh^3 \tau \\ & + (4\eta^3 \delta \beta (p^2 - 2p + 2) - \alpha \varphi \nu + \gamma \eta(\eta + \delta + \varphi)) \lambda p(p-1) \tanh^{-1} \tau \\ & + 6\lambda^2 \eta^2 \delta p^2 (p+1) \tanh^5 \tau - 6\lambda^2 \eta^2 \delta p^2 (p-1) \tanh^{-1} \tau \\ & - 6\lambda^2 \eta^2 \delta p^2 (3p+1) \tanh^3 \tau + 6\lambda^2 \eta^2 \delta p^2 (3p-1) \tanh^{-1} \tau \\ & + 2(\alpha \varphi \nu - \eta^3 \delta \beta (3p^2 + 5) - \gamma \eta(\eta + \delta + \varphi)) \tanh \tau = 0. \end{aligned} \quad (2.12)$$

Splitting equation (2.12) with respect to the powers of $\tanh \tau$ and simplify, yields

$$\lambda = \frac{(p+2)(p+3)\beta\eta}{6p} \quad (2.13)$$

and

$$\nu = \frac{-3\beta\delta\eta^3((p+3)(p+2)(3p-1) - 2p(3p^2+5))}{6p\alpha\varphi} + \frac{\gamma\eta(\eta+\delta+\varphi)}{\alpha\varphi}. \quad (2.14)$$

Setting $p = 1$ into (2.13) and (2.14), we obtain

$$\lambda = 2\beta\eta \quad (2.15)$$

and

$$\nu = \frac{\gamma\eta(\eta+\delta+\varphi) - 4\beta\delta\eta^3}{\alpha\varphi}. \quad (2.16)$$

Consequently, the dark (optical) soliton solution for equation (1.1) is

$$u = 2\beta\eta \tanh \left(\eta x + \delta y + \varphi z - \frac{\gamma\eta(\eta+\delta+\varphi)t - 4\beta\delta\eta^3 t}{\alpha\varphi} \right). \quad (2.17)$$

Remark 1. We observe that equation (1.1) admits an optical or shock wave soliton solution if and only if $\alpha\varphi \neq 0$. To the best of our knowledge, this crucial observation is reported here for the first time. This observation cannot be found anywhere in the literature. We now present the profile solution of equation (2.17) subject to some choice of the arbitrary parameters.

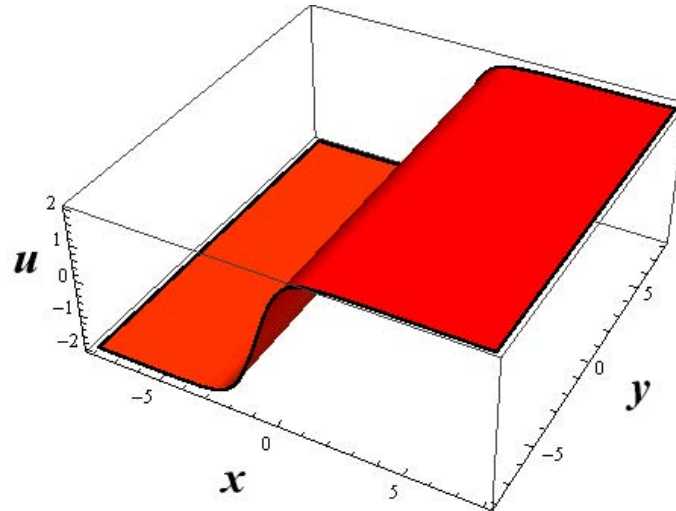


Figure 1: Kink shape soliton profile of (2.17) corresponding to $\eta = 1, \varphi = 1, \delta = 0.01, \gamma = 0.1, \alpha = 0.1, \beta = 1, t = 1, z = 1$.

In a like manner, we can derive the singular kink solution in the form

$$u = 2\beta\eta \coth\left(\eta x + \delta y + \varphi z - \frac{\gamma\eta(\eta + \delta + \varphi)t - 4\beta\delta\eta^3 t}{\alpha\varphi}\right) \quad (2.18)$$

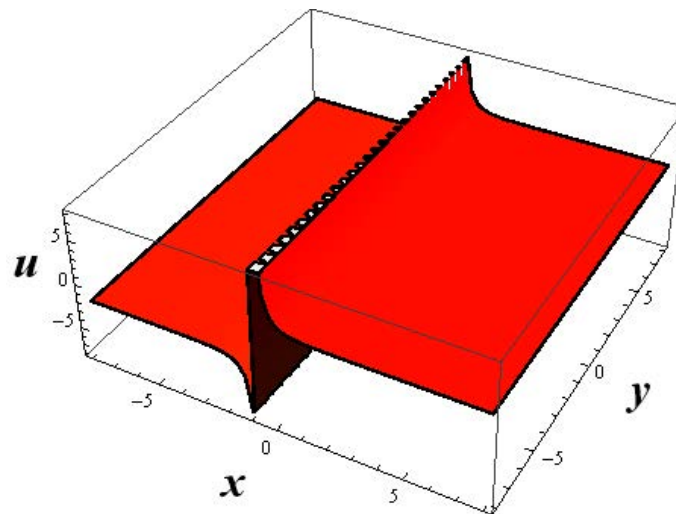


Figure 2: Singular kink shape soliton profile of (2.18) with $\eta = 1, \varphi = 1, \delta = 0.01, \gamma = 0.1, \alpha = 0.1, \beta = 1, t = 1, z = 1$.

Remark 2. It is worth noting that a singular kink soliton solution do exists for equation (1.1). However, this singular kink soliton solution can only exist provided that the product of α and φ is not zero. This is a very remarkable observation that is being mentioned here for the first time. Figure 2 below, gives a graphical presentation of solution (2.18) with respect to some choice of arbitrary constants.

3 Periodic solutions

In this section, we will implement the Tan-Cot ansatz method to derive periodic solutions of equation (1.1). The starting hypothesis is of the form

$$u(x, y, z, t) = \lambda \tan \tau, \quad (3.1)$$

where the wave variable τ is defined as $\tau = \eta x + \delta y + \varphi z - \nu t$. Proceeding as before and simplify, we obtain the amplitude and velocity of the wave as

$$\lambda = -2\beta\eta, \quad (3.2)$$

and

$$\nu = \frac{\gamma\eta(\eta + \delta + \varphi) + 4\beta\delta\eta^3}{\alpha\varphi}, \quad (3.3)$$

respectively.

This in turn gives the periodic solution of equation (1.1) as

$$u = -2\beta\eta \tan\left(\eta x + \delta y + \varphi z - \frac{\gamma\eta(\eta + \delta + \varphi)t + 4\beta\delta\eta^3 t}{\alpha\varphi}\right). \quad (3.4)$$

The graphical representation of solution (3.4) is presented in figure 3 below.

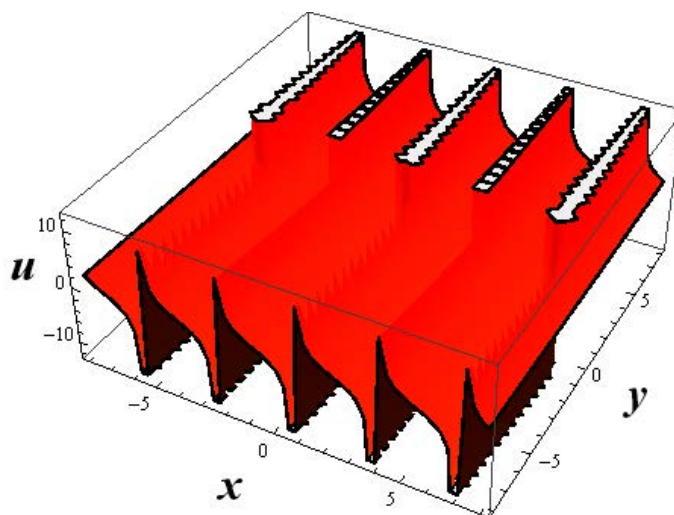


Figure 3: Periodic profile of solution (3.4) for $\eta = 1, \varphi = 1, \delta = 0.01, \gamma = 0.1, \alpha = 0.1, \beta = 1, t = 1, z = 1$.

Without loss of generality, the singular solution of equation (1.1) is

$$u = 2\beta\eta \cot\left(\eta x + \delta y + \varphi z - \frac{\gamma\eta(\eta + \delta + \varphi)t + 4\beta\delta\eta^3 t}{\alpha\varphi}\right). \quad (3.5)$$

The profile of solution (3.5) is presented in figure 4 subject to choices of arbitrary parameters.

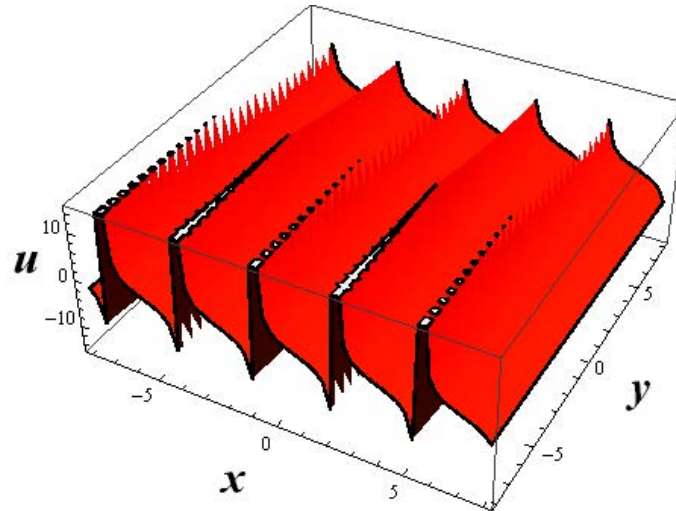


Figure 4: Periodic profile of solution (3.5) for $\eta = 1, \varphi = 1, \delta = 0.01, \gamma = 0.1, \alpha = 0.1, \beta = 1, t = 1, z = 1$.

Remark 3. A commendable observation indicate that periodic solutions due exist for equation (1.1). However, the existence of these solutions imply that the product of the parameters α and φ ($\alpha\varphi \neq 0$). To the authors knowledge, this outstanding observation is testified for the first time here.

4 Group invariant solutions

In order to derive group-invariant solutions of equation (1.1), one needs to obtained the admitted generators of equation (1.1). This is attained by considering the vector field of the form

$$X = \xi^1 \frac{\partial}{\partial t} + \xi^2 \frac{\partial}{\partial x} + \xi^3 \frac{\partial}{\partial y} + \xi^4 \frac{\partial}{\partial z} + \eta \frac{\partial}{\partial u}, \quad (4.1)$$

where $\xi^i (i = 1, 2, 3, 4)$ and η are functions of (t, x, y, z) . Applying the fourth extension of equation (4.1) to equation (1.1) and solve the resulting system of linear partial differential equations, we conclude that equation (1.1) admits infinitely many point symmetries spanned by

$$\begin{aligned} X_1 &= \frac{\partial}{\partial t}, X_2 = \frac{\partial}{\partial x}, X_3 = \frac{\partial}{\partial z}, X_4 = F_1(z) \frac{\partial}{\partial u}, X_5 = F_2(t) \frac{\partial}{\partial u}, X_6 = \frac{3}{3+\gamma} \frac{\partial}{\partial y} + \frac{\gamma}{3+\gamma} \frac{\partial}{\partial u}, \\ X_7 &= \alpha t \frac{\partial}{\partial t} + \gamma t \frac{\partial}{\partial x} - \alpha z \frac{\partial}{\partial z}, X_8 = 3\alpha t \frac{\partial}{\partial t} + 3\gamma t \frac{\partial}{\partial x} + 3\alpha y \frac{\partial}{\partial y} + \alpha \gamma y \frac{\partial}{\partial u}, \\ X_9 &= 9\alpha t \frac{\partial}{\partial t} + (3\alpha x + 6\gamma t) \frac{\partial}{\partial x} + (2\alpha \gamma x + \alpha \gamma y - 3\alpha u) \frac{\partial}{\partial u}. \end{aligned}$$

In the theory of Lie symmetries analysis [1, 10, 7], it is well-known that a combination of symmetries will always remain symmetries of the problem at hand. Thus, invoke the combination of X_1, X_2 and X_3 , and solving the associated Lagrange systems, we get four invariants, viz.,

$$f = y, \quad g = t - z, \quad h = x - z \quad \phi = u.$$

Using these invariants together with equation (1.1) and treating ϕ as function of (f, g, h) , we get

$$3\phi_h \phi_{fh} + 3\phi_f \phi_{hh} - \gamma \phi_{fh} + \beta \phi_{fhhh} + \alpha \phi_{gg} + \gamma \phi_{gh} + \alpha \phi_{gh} = 0.$$

This in turn gives the group-invariant solution of equation (1.1) as

$$u = 2\beta c_4 \tanh \left(y c_2 + \frac{(\gamma c_4 + \alpha c_4 - \Omega)(t - z)}{2\alpha} + (x - z) c_4 + c_1 \right) + c_5, \tag{4.2}$$

where

$$\Omega = \sqrt{\alpha^2 c_4^2 - 16\alpha\beta c_2 c_4^3 + 4\alpha\gamma c_2 c_4 + \alpha\gamma c_4^2 + \gamma^2 c_4^2}$$

and c_i are constants. The graphically representation of this solution is given in figure 5.

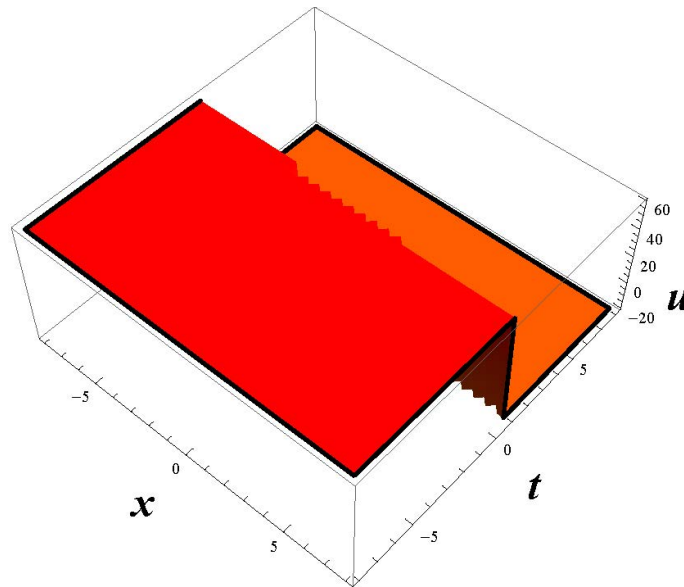


Figure 5: Anti-kink shape soliton profile of (4.2) for the values $y = 1, z = 1, \alpha = 1, \beta = -1, \mu = 1, \delta = 0.001, \gamma = 0.0001, c_1 = 10, c_2 = 5, c_4 = 20, c_5 = 20$

We now make use of the generator X_6 , which yields the following similarity variables

$$f = x, \quad g = z, \quad h = t \quad \phi = u - \frac{1}{3}\gamma y.$$

Consequently, equation (1.1) reduces to

$$\gamma \phi_{fg} + \alpha \phi_{gh} = 0.$$

Thus, we get

$$u = F_3(t, x) + F_4(z, -\alpha x + \gamma t) + \frac{1}{3}\gamma y, \tag{4.3}$$

as the group-invariant solution of equation (1.1) where F_i are arbitrary elements with respect to their arguments.

Thirdly, we employ linear combination of X_2 and X_4 , which leads to four invariants, namely

$$f = y, \quad g = z, \quad h = t \quad \phi = u - x, F_2(t),$$

which transform equation(1.1) into

$$\phi_{gh} = 0,$$

and the resulting group-invariant solution of equation (1.1) is

$$u = xF_2(t) + F_5(y, t) + F_6(y, z), \tag{4.4}$$

with F_i being arbitrary functions with respect to their arguments.

Lastly, we consider the symmetry combination of X_2 and X_6 and we get the following four invariants:

$$f = z, \quad g = t, \quad h = x - y - \frac{1}{3}\gamma y \quad \phi = u - \frac{1}{3}\gamma y,$$

which in turn reduce equation(1.1) into

$$6\gamma\phi_h\phi_{hh} - \gamma^2\phi_{hh} + \beta\gamma\phi_{hhhh} + 3\alpha\phi_{fg} + 3\gamma\phi_{fh} + 18\phi_h\phi_{hh} - 3\gamma\phi_{hh} + 3\beta\phi_{hhhh} = 0.$$

and the associated solution of equation (1.1) is

$$u = 2\beta c_4 \tanh\left(zc_2 + \frac{(\gamma^2 c_4 - 4\beta\gamma c_4^3 - 12\beta c_4^3 - 3\gamma c_2 + 3\gamma c_4)tc_4}{3\alpha c_2} + \left(x - y - \frac{1}{3}\gamma y\right)c_4 + c_1\right) + \frac{1}{3}\gamma y + c_5, \tag{4.5}$$

while the corresponding profile representation of solution (4.5) is give in figure 6.

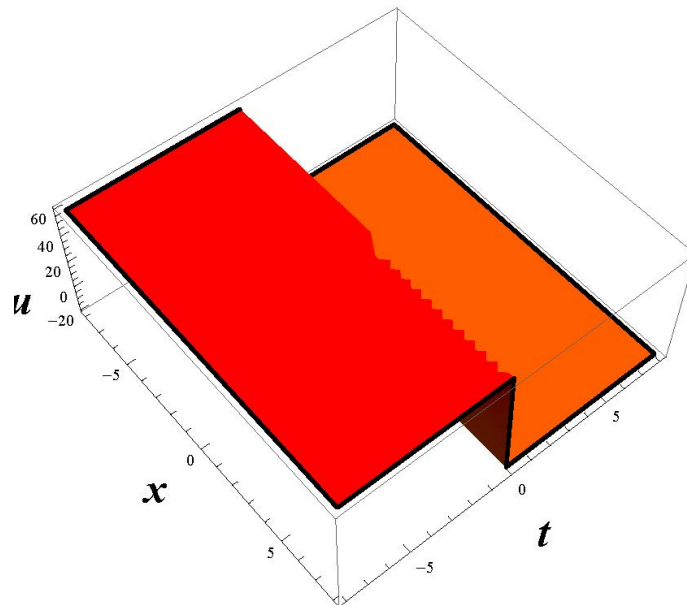


Figure 6: Anti-kink shape soliton profile of (4.5) for the values $y = 1, z = 1, \alpha = 1, \beta = -1, \gamma = 1, c_1 = 10, c_2 = 5, c_4 = 20, c_5 = 20$.

5 Local conservation laws

This section is devoted to the construction of *low-order* conservation laws of equation (1.1). This will be achieved via the multiplier method[9, 5, 4]. In order to derive the multiplier for equation (1.1), one needs to solve the resulting system of linear differential equation that arise from the expansion of

$$\frac{\delta(\Lambda E)}{\delta u} = 0, \tag{5.1}$$

where $\frac{\delta}{\delta u}$ is the Euler Lagrange operator, Λ denotes the multiplier function which in this context is assumed to be of order zero and E represent equation (1.1)[10, 11]. The analysis of equation (5.1) prompts the following lemma.

Lemma 1. Let Λ be a zeroth order conservation law multiplier, then a new (3+1)-dimensional novel KP like equation admits infinitely many zeroth-order multiplier of the form

$$\Lambda = C_1 u_x + F(y, z) + G(t, y),$$

with F and G being arbitrary functions with respect to their arguments.

Proof. A straightforward but lengthy computation from $\varepsilon_u(\Lambda E) = 0$. \square

Remark 4. It is worth pointing out that a new (3+1)-dimensional novel KP like equation admits *identical zeroth-order, first-order and second-order* multipliers. This is a commendable observation which is mentioned here for the first time. In general, if one increases the order of the multiplier, then one aims to get higher-order multipliers which in turn leads to higher order conservation laws, but this is not the case with the new (3+1)-dimensional novel KP like equation.

Without loss of generality, we now give the corresponding conserved vectors associated with the above multiplier, namely

$$\begin{aligned} T_1^t &= \frac{1}{4} \alpha u u_{xz} - \frac{1}{4} \alpha u_x u_z, \\ T_1^x &= \frac{1}{2} \gamma u_x^2 - \frac{3}{2} u_x^2 u_y - \frac{1}{8} \beta u_{xxx} u_y + \frac{3}{8} \beta u_{xx} u_{xy} - \frac{5}{8} \beta u_x u_{xxy} + \frac{1}{4} \gamma u u_{xy} + \frac{1}{4} \gamma u_x u_y \\ &\quad + \frac{1}{4} \gamma u_x u_z + \frac{1}{4} \gamma u u_{xz} + \frac{1}{2} \alpha u u_{tz} + \frac{1}{4} \delta u_t u_x - \frac{1}{8} \beta u u_{xxy}, \\ T_1^y &= -\frac{1}{4} \gamma u u_{xx} + \frac{1}{8} \beta u u_{xxx} - \frac{1}{2} u_x^3 - \frac{1}{4} \beta u_x u_{xxx} + \frac{1}{8} \beta u_{xx}^2 + \frac{1}{4} \gamma u_x^2 \\ T_1^z &= \frac{1}{4} \gamma u_x^2 - \frac{1}{4} \gamma u u_{xx} - \frac{1}{4} \alpha u u_{tx} + \frac{1}{4} \alpha u_t u_x; \\ T_F^t &= -\frac{1}{2} \alpha (u F_z(z, y) - F(z, y) u_z), \\ T_F^x &= \frac{3}{4} u u_x F_y(z, y) + \frac{3}{4} u u_{xy} F(z, y) - \frac{9}{4} u_x u_y F(z, y) + \frac{1}{4} \beta u_{xx} F_y(z, y) + \gamma u_x F(z, y) \\ &\quad - \frac{3}{4} \beta u_{xxy} F(z, y) - \frac{1}{2} \gamma u F_z(z, y) - \frac{1}{2} \gamma u F_y(z, y) + \frac{1}{2} \gamma u_y F(y, z) + \frac{1}{2} \gamma u_z F(y, z), \\ T_F^y &= -\frac{3}{4} u_x^2 F(z, y) - \frac{3}{4} u u_{xx} F(z, y) - \frac{1}{4} \beta u_{xxx} F(z, y) + \frac{1}{2} \gamma u_x F(z, y), \\ T_F^z &= \frac{1}{2} \alpha u_t F(z, y) + \frac{1}{2} \gamma u_x F(z, y); \end{aligned}$$

$$\begin{aligned} T_G^t &= \frac{1}{2} \alpha u_z G(t, y), \\ T_G^x &= \frac{3}{4} u u_{xy} G(t, y) - \frac{9}{4} u_x u_y G(t, y) + \gamma u_x G(t, y) - \frac{1}{2} \gamma u G_y(t, y) + \frac{1}{2} \gamma u_y(t, y) \\ &\quad + \frac{1}{2} \gamma u_z(t, y) - \frac{3}{4} \beta u_{xxy} G(t, y) + \frac{3}{4} u u_x G_y(t, y) - \frac{1}{4} \alpha \beta u_{xx} G_y(t, y), \\ T_G^y &= \frac{1}{2} \gamma u_x G(t, y) - \frac{3}{4} u_x^2 G(t, y) - \frac{3}{4} u u_{xx} G(t, y) - \frac{1}{4} \beta u_{xxx} G(t, y), \\ T_G^z &= -\frac{1}{2} \alpha u G_t(t, y) + \frac{1}{2} \gamma u_x G(t, y) + \frac{1}{2} \alpha u_t G(t, y). \end{aligned}$$

It is worth mentioning that due to the presence of the arbitrary elements in the conserved vectors, then the new (3+1)-dimensional novel KP like equation admits an infinite number of local conservation laws.

6 Conclusions

In this paper we obtained topological soliton solutions and periodic solutions of a new (3+1)-dimensional novel KP like equation. In addition, other analytical solutions based on Lie symmetries method have been attained. Furthermore,

conservation laws for the aforesaid equation were derived using the multiplier method. The correctness of the obtained solutions have been verified with Maple software package by back substitution. It is anticipated that the solution obtained can be used as benchmark with the numerical simulation.

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