Evaluating educational services using TOPSIS method with eigne value methodology based on GIS

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Abstract

Choosing the right site for creating educational services is a complex problem that involves evaluating many criteria, therefore, there is a need for a multi criteria decision-making tool that can make a reliable, coherent decision. The aim of this paper is to develop a methodological plan that can help decision-makers in determining the locations of the most important schools to build new schools by adopting seven basic criteria: The population's actual need for schools, the distance between the school and the main street, the proximity to health centers, the number of students in each school, the number of teachers in each school, the proportion of teachers to students, the number of students in each class(.that were selected by a group of specialists and experts. The use of the (Eigen value) method to calculate the relative importance of these criteria, combined with the A multi-criteria decision-making technique (TOPSIS), The final results showed that the relative priority of the criteria is respectively) 46.6%, 15.9%, 15.0%, 5.5%, 4.10%, 8.8%, 4.2%, (And the percentage of schools that were in the four categories, respectively (6%,10%,13%,26%), One of the most important recommendations is to bridge the deficit experienced by the educational services provided within the study area by adopting the results that were reached above, by choosing sites to build new schools that take into account the basic standards.

Keywords: Eigenvalue, proximity, educational services
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1 Introduction

TOPSIS technique is one of the multi-criteria decision-making methods that was first introduced by [7, 11]. and later developed by several authors including [6, 8, 14, 19], the main idea of TOPSIS is the chosen alternative should have the shortest Euclidean distance from the positive ideal solution (PIS) and the longest Euclidean distance from the negative ideal solution (NIS) or nadir, it can be compared to the principle of eastern and western religions; i.e. leading human beings toward heaven and away from hell, where this method assumes that the ideal positive solution (PIS) is to characterized as the maximum possible benefits and the lowest possible cost for all alternatives, while the ideal negative solution (NIS) is the solution that contains Minimum possible benefits and maximum costs [9].

2 TOPSIS Features

- A sound logic that represents the rational of human choice [15].

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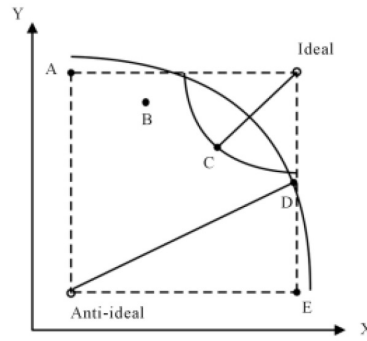
Figure 1: [9]

- The number of criteria does not affect the number of steps and thus obtaining results in a record time.

- In recent years, TOPSIS has been successfully applied as decision-making tools in various fields including water management, transportation planning, human resources, mechanical engineering, manufacturing engineering and policy development [1].

- A scalar value that accounts for both the best and worst alternatives ability to measure the relative performance for each alternative in a simple mathematical form [14].

- The relative performance measurement of all substituents can be represented as a polyhedron, for at least any two dimensions which makes it easy to identify positive and negative ideal solutions.

2.1 Mathematical model of TOPSIS [12]

Initially, after normalizing the values of parameters x_{ij} using one of the normalization methods that will be covered within the steps of implementing TOPSIS, we assume that a_{ij} are the values of the standard parameters, each option A_i is expressed as the point $A_i(a_{i1}, a_{i2}, ..., a_{in}) \in R^N$, and when choosing the optimal value $a_j^* \in (a_{1j}, a_{2j}, ..., a_{mj})$ for each parameter x_j, we determine the positive ideal solution $A_i^* = (a_1^*, a_2^*, ..., a_n^*)$ and the opposite is Negative ideal solution $A_i^\Delta = (a_1^\Delta, a_2^\Delta, ..., a_n^\Delta)$ The positive and negative ideal solution can also be referred to, respectively, A^+ and A^-, then the decision is made about the order of the options taking into account the order of the options numbers.

$$D_i^* = \frac{d(A_i, A^\Delta)}{d(A_i, A^\Delta) + d(A_i, A^*)} = \frac{1}{d(A_i, A^*)/d(A_i, A^\Delta)} \tag{2.1}$$

where A_{i1} is the best solution if $\max\{D_1^*, D_2^*, ..., D_m^*\} = D_{i1}^*$ and A_{i2} is the worst solution if $\min\{D_1^*, D_2^*, ..., D_m^*\} = D_{i2}^*$ the other options are between these two extremes, and the maximum distance $D^* = \max D_i^*$, $i = \{1, 2, ..., m\}$ is usually called the TOPSIS scale.

3 Eigen Value Method for Assessing Weight [17]

Eigenvectors are a special set of vectors related to a linear system of equations (in the form of a matrix) which are sometimes also known as characteristic vectors, appropriate vectors, or latent vectors, the dominant eigenvalue describes the main component of the model's behavior.

Multiplying the matrix A and the vector v is defined as the product of the scalar quantity λ and the given vector, such that [4]:

$$Av = \lambda v \tag{3.1}$$

$$|A - \lambda I| = 0 \tag{3.2}$$

This "characteristic equation" $\det.(A - \lambda I) = 0$ involves only, not v. When A is n by n, the equation has degree n. Then A has n eigenvalues and each λ leads to v [16]

The Eigenvector shows the relative weights between each criterion by calculating the arithmetic average of all criteria. So, we can observe that the sum of all values from the vector is always equal to one [2].

$$\sum_{i=1}^{n} V_i = 1, \quad 0 \le V_i \le 1 \tag{3.3}$$

4 Coupling MCDM method with GIS

Here is a considerable amount of publications on the usage of MCDA and GIS combinations [13], in (2011) many research on MCDA and GIS applications developed and listed the main factors that affect the choice and implementation of MCDA methods. The goal of using spatial solutions based on MDCA procedures is to facilitate the complex problem on multiple criteria location problem (MCLP), and This type of problems can be tackled using GIS-based Multi criteria Decision Analysis [10], (GIS-MCDA) procedures allows us to view, understand, question, interpret, and visualize data in many ways that reveal relationships, patterns, and trends in the form of maps, reports, and charts. A GIS helps us answer questions and solve problems by looking at your data in a way that is quickly understood and easily shared.

5 TOPSIS- EIGEN value Methodology

To achieve the main principle of TOPSIS technique that the chosen alternative must have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution and assessing weight by the Eigen value methodology, We follow the following steps:

1. Building a decision matrix and making pairwise comparisons between criteria
   We assume that there is a multi-criteria decision-making problem with m alternatives and n criteria, so we have to create a set of pairwise comparison matrices, this pairwise comparisons are carried out for all factors to be considered, usually not more than 7, and the matrix is completed [17], to compare each element in the corresponding level and calibrate them on the numerical scale requires $n(n-1)/2$ comparisons, where n is the number of elements using a verbal scale for judgment preference which is consistent with Saaty's 1 to 9 numerical recommendations to measure quantitative as well as qualitative criteria [18]. The scale ranges from "equal" (number 1) to "absolutely more important than" (number 9) as shown in table 1, the decision matrix is as follows:

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \tag{5.1}$$

Table 1:

Intensity of importance	Definition
1	Equal importance
3	Moderate importance of one over another
5	Essential or strong importance
7	Very strong importance
9	Extreme importance
2,4,6,8	Intermediate values

The preferred criterion cell of the matrix has the value and the other has the inverted value (1 / value) as shown in Formula 5.2.

$$A = \begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ 1/a_{21} & 1 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{n1} & 1/a_{n2} & \dots & 1 \end{bmatrix} \tag{5.2}$$

2. Calculate the standard decision matrix according to the Formula (3.1):

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \ i = \{1, 2, ..., m\}, \ j = \{1, 2, ..., n\} \tag{5.3}$$

Thus, the matrix will be as shown below

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \tag{5.4}$$

3. Calculating the maximum Eigenvalue ($\lambda_{\max}$), Consistency index (CI), Consistency Ratio (CR).
   From the matrix above, we can calculate $\lambda_{\max}$ and its associated eigenvector. And this eigenvector shows the relative "weights" of each criterion, the components of the eigenvector are understood as an expression of the preference between the elements under investigation and the sum of these components equals 1, i.e. $\sum_{j=1}^{n} w_j = 1$. The value of ($\lambda_{\max}$), the maximum eigenvalue of the comparison matrix, is an important validating parameter. It is used as a reference index to screen information by calculating the consistency ratio (CR) of the estimated vector (λ) is obtained by averaging the value of the consistency vector, formulated using equation (3.3):

$$\lambda_{\max} = \sum_{i=1}^{n} Cv_{ij} \tag{5.5}$$

where $\lambda_{\max}$ The main eigenvalue of v, which is a random variable whose distribution depends on the distribution of v_{ij}.
Then we calculate a Consistency index (CI) as deviation or degree of consistency using the following formula:

$$CI = (\lambda_{\max} - n)/(n - 1) \tag{5.6}$$

To determine Consistency Ratio (CR) which is a comparison between Consistency Index and Random Consistency Index, or in formula (5.6) to measure how consistent the judgments, If the CR is much in excess of 0.1 the judgments are untrustworthy because they are too close for comfort to randomness and the exercise is valueless or must be repeated [3].

$$CR = CI/RI \tag{5.7}$$

The average random consistency index of sample size $N \leq 10$ is shown in the table below:

Table 2:

N	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49

4. Calculate the standard weighted decision matrix according to the equation:

$$v_{ij} = w_j r_{ij}, \ i = \{1, 2, ..., m\}, \ j = \{1, 2, ..., n\} \tag{5.8}$$

where w_j represents the weight of the jth standard, and the sum of the weights of the standards is one, $\sum_{j=1}^{n} w_j = 1$

5. Determine the optimal positive solution and the optimal negative solution according to the two equations below. the deviation from each alternative can be measured [9] by using a distance scale for measuring physical distance in a two- or three-dimensional space, or it may extend to a multi-dimensional space called Euclidean distance [5] which ensures that the highest-ranking alternative is closest to the ideal solution and farthest from its equivalent solution, it represents the shortest distance between two points and this distance depends on the weight of each parameter in measuring the distance, because all the alternatives are compared with the corresponding ideal solutions, and not directly between them, as shown in [5, 10].

$$WED_i^+ = \sqrt{\sum_{j=1}^{n} \{w_i(z_{ij} - a_j^*)\}^2}, \ i = \{1, 2, ..., m\} \tag{5.9}$$

$$WED_i^- = \sqrt{\sum_{j=1}^{n}\{w_i(z_{ij} - b_j^*)\}^2}, \ i = \{1, 2, ..., m\} \tag{5.10}$$

6. Calculate the relative proximity to the optimal solution according to the equation below:

$$D_i^* = \frac{WED_i^-}{WED_i^- + WED_i^+}, \ 0 \le D_i^* \le 1, \ i = 1, 2, ..., m \tag{5.11}$$

7. Convert and classify the measured data via GIS:
   convert the scaled values into a final suitability map with the new crisp values that could be easier to visually interpret and classified into four main categories according to the value of relative affinity coefficient (D_i^*), indicating the highest and least suitability, respectively.

6 Apply the Mythology of (TOPSIS-EIGEN VALUE) and represent the results within (GIS)

To apply TOPSIS method based on proposed methodology, and Preparing a matrix of reciprocal pairwise comparisons for seven criteria using the data obtained from the preferences of sample of experts and specialists, as shown in Formula (5.2), we get:

Table 3:

Criteria	c1	c2	c3	c4	c5	c6	c7
c1	1	7	5	3	7	5	9
c2	0.14	1	1	3	3	5	3
c3	0.2	1	1	3	5	3	3
c4	0.33	0.33	0.33	1	1	0.33	1
c5	0.14	0.33	0.2	1	1	0.33	1
c6	0.2	0.2	0.33	3	3	1	3
c7	0.11	0.33	0.33	1	1	0.33	1

Table 4. represents the standard decision matrix according to the formula (5.3):

Table 4:

Criteria	c1	c2	c3	c4	c5	c6
c1	0.472	0.687	0.611	0.2	0.333	0.334
c2	0.066	0.098	0.122	0.2	0.143	0.334
c3	0.094	0.098	0.122	0.2	0.238	0.2
c4	0.156	0.032	0.04	0.067	0.048	0.022
c5	0.066	0.032	0.024	0.067	0.048	0.022
c6	0.094	0.02	0.04	0.2	0.143	0.067
c7	0.052	0.032	0.04	0.067	0.048	0.022
Sum	1	1	1	1	1	1

by calculating the arithmetic average of all criteria Using the eigenvalue method to find the probability weights for each criterion, we get:

Table 5: shown the probability weights for each criterion

Criteria	Priority	Rank
c1	46.60%	1
c2	15.90%	2
c3	15.00%	3
c4	5.50%	4
c5	4.10%	5
c6	8.80%	6
c7	4.20%	7
Sum	100%	

Ranks were given for each criterion, and as shown in Fig. 2, the criterion of the actual population need for schools is the most important criterion for determining the most appropriate location for establishing a high schools with

a probability weight (46.60%), followed by the second criterion, which is the school's proximity to the main street with a probability weight of (15.90%), and the value of $\lambda_{\max} = 7.742, CI = 0.124, CR = 0.092$ by using formulas (5.4),(5.5),(5.6) respectively and As we note that the value of that $CR = 0.092 < 0.1$ which proves reciprocal matrix consistency.

Finally, Converts all previously defined criteria by using the Tools 3D Analyst command within the Spatial Analysis Tools in the ArcGIS 10.4 environment, and the Raster Interpolation - IDW application to categories the results into four levels in descending order from the highest to the lowest and according to the value of the proximity coefficient index and display them in the form of a layer on the map as shown in Fig. 2.
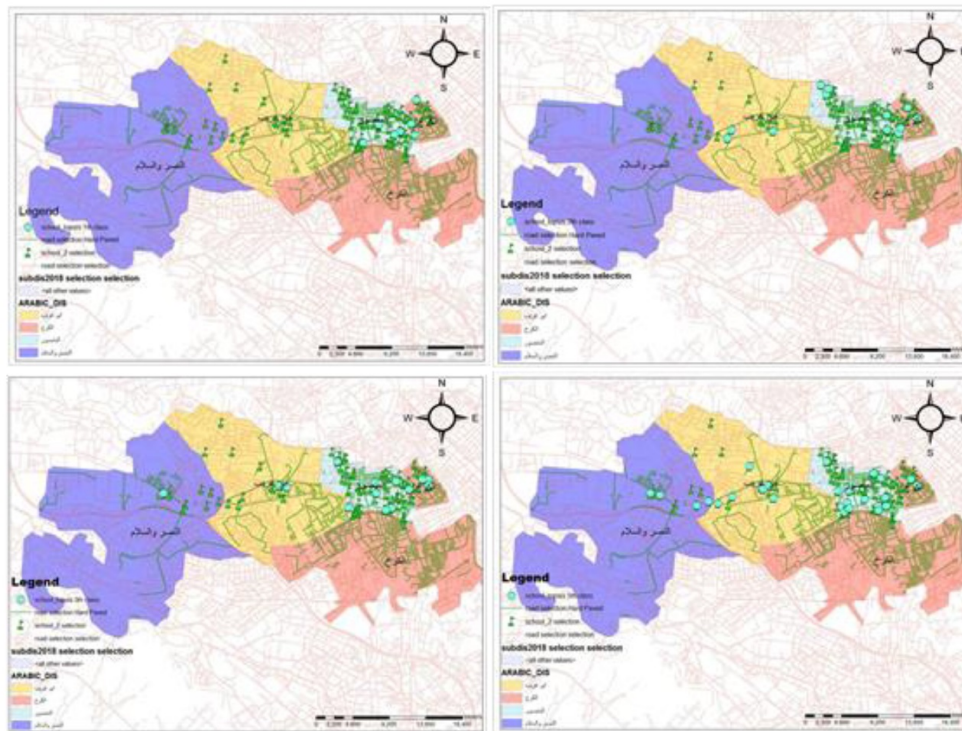

Figure 2:

Table 6 summarizes the four categories, whereas, the numbers of higher schools are (7, 12, 16, 31) categorized into (Extremely Importance, More Importance, Importance, Less Importance), respectively. And It should be noted that the degree of importance here means the most need to establish new schools near the selected sites.

Table 6:

No.	Category	High school	Percentage
1	Extremely importance	2	2%
2	More importance	7	6%
3	Importance	19	16%
4	Less importance	31	26%
5		59	49%

7 Conclusions

1. Relying on the modified pairwise decision matrix, it was found that the main criterion that gained the highest importance in adopting the resulting probabilistic weights is the population's actual need for schools by obtaining a percentage (46.6%), followed by the criterion near the school to the health center with a percentage of (15.9%), followed by The standard ratio of the number of students in each school (5.5%), the standard ratio of the number of teachers in each school (4.10%), the standard ratio of the number of teachers to the number of students (8.8%), the standard ratio of the number of students in each class (4.2%), And finally, the standard ratio for the number of students in each school is (15%), as shown in table 5.

2. The number of schools classified as extremely importance important, meaning that they are the most in need of building new schools, compared to other categories (7) higher schools, with a percentage of (6%) of the total number, as shown in table 7.

Table 7:

id	Na_school	Region	WED^+	WED^-	Di
1	aljamiea	Mansour	2.8548	7.6991	0.7295
2	Alfirdaws	Mansour	3.13	7.644	0.7095
3	Dajalah	Mansour	2.5522	7.2846	0.7405
4	H.almansur	Karkh District Center	2.7897	7.908	0.7392
5	M.almansur	Karkh District Center	2.8263	7.4206	0.7242
6	H.aleamiria	Mansour	2.7112	8.7512	0.7635
7	H.alkhadra	Mansour	2.7737	7.529	0.7308

3. Higher schools classified as more important, that is, they are less in need of building new schools, compared to the category before it, which is (12) schools, at a rate of (10%) of the total number, as shown in table 8.

Table 8:

id	Na_school	Region	WED^+	WED^-	Di
1	M. Asamaa bin zayd	Mansour	3.6396	6.1034	0.6264
2	H. Albayan	Mansour	3.4953	6.8334	0.6616
3	H. Almansur	Karkh District Center	3.3689	6.6018	0.6621
4	H. Almamwn	Mansour	3.8686	5.8976	0.6039
5	H. Abi gharib	Abu Gharib District Center	3.0522	6.8164	0.6907
6	H. Abi gharib	Karkh District Center	3.2922	6.6044	0.6673
7	H. Alkandiu	Mansour	3.6576	6.0842	0.6245
8	Muhamad mahdi albusayr	Abu Gharib District Center	3.9005	6.12	0.6107
9	Dar Alsalam	Mansour	3.3418	6.6715	0.6663
10	Omar almukhtar	Karkh District Center	3.5519	6.4212	0.6439
11	Amaar bn yasir	Mansour	3.9581	6.6326	0.6263
12	Alhadara	Abu Gharib District Center	4.1331	6.6469	0.6166

4. higher schools classified as important, meaning that they are less in need of building new schools, compared to the category before them, which are (16) schools, at a rate of (13%) of the total number, as shown in table 9.

Table 9:

id	Na_school	X	Y	Region	WED^+	WED^-	Di
1	Aleirfan	44.345028	33.326139	Mansour	4.1868	5.9499	0.587
2	Almutamayizin	44.339528	33.296056	Mansour	4.8587	5.0643	0.5104
3	H. Alqadisia	44.380667	33.32875	Karkh District Center	4.7861	4.8543	0.5035
4	Almamwn	44.316472	33.324333	Mansour	4.7785	4.846	0.5035
5	M. Alqadisia	44.318139	33.312583	Mansour	4.7708	4.8678	0.505
6	Alwathba	44.331278	33.293528	Karkh District Center	4.7588	4.9275	0.5087
7	Albutula	44.187444	33.32025	Abu Gharib District Center	4.534	5.1843	0.5335
8	Alhikmat	44.362472	33.316972	Karkh District Center	4.5137	5.2249	0.5365
9	M. aljamiea	44.28075	33.296083	Mansour	4.4456	5.3273	0.5451
10	Alshabab	44.197944	33.318528	Abu Gharib District Center	4.7212	5.1092	0.5197
11	Alyaman	44.265722	33.34875	Mansour	4.1127	5.5943	0.5763
12	Alhawra	44.265583	33.347889	Mansour	4.7603	5.0099	0.5128
13	M. alrahmania	44.388167	33.328611	Karkh District Center	4.0208	5.724	0.5874
14	Alqahtania	44.029778	33.309889	Alnasr walsalam	4.6252	5.3115	0.5345
15	Alkarar	44.389583	33.321833	Karkh District Center	4.6751	5.1858	0.5259
16	Alrasafii	44.031444	33.310583	Alnasr walsalam	5.0558	5.1407	0.5042

5. Higher schools classified as less important, meaning that they are less in need of building new schools, compared to the category before it, which is (31) schools, at a rate of (26%) of the total number.

6. Adoption quantitative methods in government institutions (DM), when solving complex problems, helps specialists in the decision support process with a sound scientific approach, which leads to raising the level of performance of these institutions.

7. Using other methods and making a comparison with the results that have been reached to distinguish the best (MCDM) methods in implementing educational services or any other type of services.

8. Filling the deficit suffered by the educational services provided within the study area by adopting the results that were reached above

9. Creating an accurate and comprehensive database for all schools that can be relied upon and continuously updated to help specialists and decision makers to avoid problems in the future.
10. Classification of criteria that affect the determination of educational services sites according to their importance by a group of experts and specialists that can be considered as a resource for researchers when conducting future studies.
11. Balance the number of teachers according to the number of students and the needs of the schools.
12. Taking into account the basic criteria when choosing the appropriate location for the establishment of new schools, in order to meet the actual need for educational services.

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