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Buckling Analysis of Sandwich Timoshenko Nanobeams with AFG Core and Two Metal Face-Sheets

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KEYWORDS

Sandwich nanobeam;
Functionally graded core;
Nonlocal parameter;
Buckling;
First-order shear deformation theory.

ABSTRACT

This paper intends to introduce a new and simple technique to precisely assess the axial instability of a shear deformable sandwich nanobeam. The section of the considered beam element is composed of two metal face layers and an axially functionally graded (AFG) core. The power volume fraction law is utilized to describe the properties of spatially graded materials of the core. The coupled governing differential equations in terms of transverse displacement and angle of rotation due to bending are extracted within the context of first-order shear deformation theory and Eringen's nonlocal elasticity model. The resulting equilibrium equations are then combined and transformed into a unique fifth-order differential equation. Then, the numerical differential quadrature technique is used to estimate the endurable axial critical loads. The most beneficial feature of the proposed technique is to simplify and decrease the essential computational efforts to obtain the endurable axial buckling loads of sandwich shear-deformable nano-scale beams with AFG core. In the case of an axially loaded Timoshenko nanobeam subjected to simply supported end conditions, the obtained results are compared with those accessible in the literature to confirm the correctness and reliability of the proposed approach. Eventually, comprehensive parameterization research is performed to investigate the sensitivity of linear buckling resistance to slenderness ratio, nonlocal parameter, volume fraction exponent, and thickness ratio. The numerical outcomes indicate obviously that the stability strength of sandwich Timoshenko nanobeam is significantly affected by these parameters.

1. Introduction

These days, small-size structural components made of functionally graded materials (FGMs) have attracted a great practical application in different micro-and nanoscale susceptible engineering devices including probes, sensors, actuators, transistors, resonators, and nano/micro electro-mechanical systems (NEMS/MEMS). Due to the fast expansion of nanotechnology, different nonlocal continuum theories are established to accurately consider the small-scale effects along with the detailed study of the mechanical features of nanostructures [1-3]. In this paper, the well-known Eringen's nonlocal elasticity theory [3] is employed to extract the governing equations. According to this theory, the stress at a reference point is a function of the strains at all points in the body.

In recent years, several studies have been performed on the mechanical behavior of FG and/or homogenous nano-sized structural elements subjected to different loading cases. Regarding this and in the context of the first-order shear deformation theory (FSDT), Ghannadpour and Mohammadi [4,5] investigated the endurable buckling load and natural frequency of nanobeams employing the Chebyshev polynomials as the trial shape functions for implementing the Ritz method. In the framework of Eringen's nonlocal elasticity theory and the first order-shear deformation assumption, the bending, buckling, and free vibration analyses of nanobeam were performed

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by Roque et al. [6] via a meshless method. Based on the nonlocal Timoshenko beam theory (TBT), the effect of the Winkler-Pasternak foundation on the buckling strength of non-uniform FG nanobeam was studied by Robinson and Adali [7] with the material varying in the longitudinal direction. Kammoun et al. [8] investigated the effect of the nonlocal parameter, external electrical voltage, temperature change, and axial force on the vibrational response of graphene/piezoelectric/graphene sandwich Timoshenko nanobeams using the generalized differential quadrature method (GDQM). Chen et al. [9] used the differential quadrature method (DQM) to discuss the influence of flexoelectricity on the vibration behavior of functionally graded porous piezoelectric sandwich Euler-Bernoulli nanobeam reinforced by graphene platelets within the general modified strain gradient theory. Moreover, several numerical studies about the static and dynamic analyses of sandwich nanobeams with different shapes and geometries exposed to different loading conditions can be found in Refs. [10-13]. The linear buckling and free vibration characteristics of the functionally graded piezoelectric beams were investigated by Ren and Qing [14] using the nonlocal integral model and Euler–Bernoulli beam theory (EBT). Via the combination of the Jacobi–Ritz methodology, the transient response of functionally graded porous plates subjected to different end conditions is studied by Zhao et al. [15] in the context of the higher-order shear deformation theory (HSDT). Lezgy-Nazargah et al. [16-19] presented some useful works to analyze sandwich beam elements exposed to different external mechanical loads. For further numerical-based investigations on the bending, vibration, and buckling behaviors of nano-size structural elements made of different materials and subjected to various loadings, the reader is referred to [20-32]. Analysis of free vibration and linear stability of functionally graded porous material micro-beams under four different types of end conditions were comprehensively performed by Teng et al. [33]. The modified couple stress theory and the differential transformation method (DTM) are employed for this aim. Also, some important works as reported in Refs. [34-36] have been performed on the size-dependent analysis of nano-scale beams with axially varying materials. Finally, it should be noted that the following papers can be useful for more information on the application of the FSDT [37-48].

In this paper, the axial instability of a sandwich Timoshenko nanobeam is perused through a novel approach. It is supposed that the rectangular cross-section of the beam element is stacked as metal/ AFG materials/metal. Eringen's nonlocal elasticity in accordance with the classical first-order shear deformation theory is utilized for extracting the governing stability equations in terms of bending rotation and flexural deformation. Following the methodology proposed by Soltani et al. [40, 42], the pair of equilibrium equations is reduced to one fifth-order differential equation in terms of transverse displacement. In the next step, the resulting fifth-order differential equation with variable coefficients is solved numerically via the differential quadrature method as a powerful and accurate technique, and then the axial buckling load is calculated. To the author's best knowledge, the single governing equation formulated herein for linear buckling analysis of multi-layer Timoshenko nanobeam has never been derived before. Due to the uncoupling of the system of governing equations, the expanded formulation requires low computational cost which leads to a reduction in the central processing unit (CPU) time. In addition to the exactness of the presented approach, it can be applied to determine the sustainable buckling load of nano-size beams with different types of changes in material characteristics along the axis of the element. It is also believed that the developed technique is practicable for the optimal design of smart devices, such as oscillators, sensors, atomic force microscopes, and nano/micro-electro-mechanical systems. Numerical outcomes are eventually reported for a simply supported axially loaded three-layered shear deformable nanobeam. It is worth noting that the numerical example represents the core aspect of the work, with a preliminary validation of the proposed innovative approach compared to the existing results, and systematic parametric analysis to check the sensitivity of the linear buckling response at the considered nano-size structure for different input predominant parameters.

2. The Equilibrium Equations

For an axially loaded rectangular sandwich Timoshenko nanobeam with depth t, breadth b, and length L, as shown in Fig. 1, the Cartesian coordinate system (x, y, z) is selected. Let us consider x the longitudinal axis and y and z the first and second principal bending axes parallel to the breadth and depth. The origin of these axes (O) is positioned at the centroid of the cross-section. It is assumed that the cross-section of the nanobeam consists of two homogenous face sheets at the outer sides of an axially functionally graded core. As shown in Fig. 1, the total depth of the beam is, $t = t_c + 2t_f$, where t_c denotes the core thickness and t_f is the thickness of each face layer that is assumed to be perfectly bonded to the core material. Based on the FSDT and using the small displacements theory, the longitudinal (U^*) and the vertical (W^*) displacement components can be expressed as [38]:

$$U^*(x,y,z)=u_0(x)+z\,\theta(x)$$
$$W^*(x,y,z)=w_0(x) \tag{1}$$

In these equations, u_0 is the axial displacement at the midplane, which occurs only in the presence of external axial loading, w_0 represents the vertical displacement (in the z-direction), and θ is the angle of rotation of the cross-section due to bending.

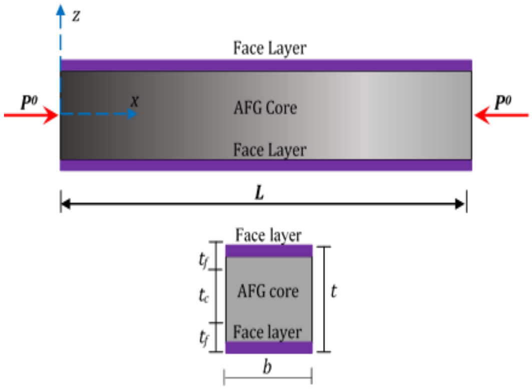

Fig. 1. An axially loaded sandwich Timoshenko beam with an AFG core and two outer isotropic layers.

To derive the equilibrium equations for an axially loaded sandwich Timoshenko nanobeam, in the first stage, the components of the strain tensor should be determined. Based on the displacement field given in Eq. (1), the non-zero components of the strain tensor consisting of the axial and shear terms are as follows [40]:

$$\varepsilon_{xx}^{l}=\frac{\partial U^*}{\partial x}=u_0'+z\theta'$$
$$\varepsilon_{xz}^{l}=\frac{1}{2}\left(\frac{\partial U^*}{\partial z}+\frac{\partial W^*}{\partial x}\right)=\frac{1}{2}(w_0'+\theta) \tag{2}$$
$$\varepsilon_{xx}^{*}=\frac{1}{2}\left(\frac{\partial W^*}{\partial x}\right)^2=\frac{1}{2}(w_0')^2$$

in which the parameters ε_{ij}^{l} and ε_{ij}^{*} indicate the linear and nonlinear strains. The resultant components of the Timoshenko beam involving the axial force N, the bending moment M, and the shear force Q are described as follows [40]:

$$N=\int_A\sigma_{xx}dA,M=\int_A\sigma_{xx}zdA,Q=\int_A\sigma_{xz}dA. \tag{3}$$

In the definition, the stress resultants σ_{ij} signify the components of the Piola–Kirchhoff stress tensor, consisting of σ_{xx} the normal stress and σ_{xz} the shear one.

For Timoshenko nanobeam, the nonlocal constitutive relations according to Eringen's elasticity theory can be rewritten as [49]:

$$\sigma_{xx}-\mu\frac{\partial^2\sigma_{xx}}{\partial x^2}=Q_{11}\varepsilon_{xx}^{l}$$
$$\sigma_{xz}-\mu\frac{\partial^2\sigma_{xz}}{\partial x^2}=2Q_{55}\varepsilon_{xz}^{l} \tag{4}$$

In these expressions, the term $\mu=(e_0a)^2$ is called the non-local parameter; in which e_0 is a material constant that is determined experimentally or approximated by matching the dispersion curves of the plane waves with those of the atomic lattice dynamics, a is an internal characteristic length of the material. Q_{ij} represents the stiffness coefficients. Using the superscripts $(\bullet)^c$ and $(\bullet)^f$ to present the core and face sheets, the afro-mentioned elastic constants can be expressed in terms of Young's modulus and Poisson's ratio as [50]:

$$Q_{11}^{f}=\frac{E_f}{1-\nu_f^2};\ Q_{55}^{f}=\frac{E_f}{2(1+\nu_f)}$$
$$Q_{11}^{c}(x)=\frac{E_c(x)}{1-\nu_c^2};\ Q_{55}^{c}(x)=\frac{E_c(x)}{2(1+\nu_c)} \tag{5}$$

in which E_f and ν_f are Young's modulus and Poisson's ratio of the face layers. Additionally, E_c and ν_c denote Young's moduli and Poisson's ratio of the FG core. Since in this work, the material properties of the core vary arbitrarily in the longitudinal direction, E_c is a function of the axial coordinate x, while ν_c is constant through the length [51]. It is assumed that the beam is made of ceramic and metal components, and the variation of Young's modulus along the longitudinal axis by taking the power-law gradient assumption is defined according to the following expression [40-42, 52]:

$$E_c(x)=E_{ceramic}+(E_{metal}-E_{ceramic})\left(\frac{x}{L}\right)^p \tag{6}$$

Here, $E_{ceramic}$ and E_{metal} denote ceramic and metal Young's modulus at the beginning and end of the member, respectively. Moreover, the parameter p is called the volume fraction index of the material, which determines how the volume fraction of ceramic and metal is combined in the longitudinal direction that $0\le p\le\infty$. It should be noted that the values of zero and infinity for this parameter represent pure metal and pure ceramic, respectively.

By introducing the linear components of strain tensor from Eq. (2) into Eq. (4) and using the definition of resultant components consisting of forces and moment (N,Q,M) given in Eq. (3), the stress resultants on the basis of non-local elasticity theory are thus obtained as [49]

$$N - \mu \frac{\partial^2 N}{\partial x^2} = \overline{EA}(x) u_0'$$
$$M - \mu \frac{\partial^2 M}{\partial x^2} = \overline{EI}(x) \theta' \tag{7}$$
$$Q - \mu \frac{\partial^2 Q}{\partial x^2} = \overline{GA}(x)(w_0' + \theta)$$

In the previous expressions, the terms $\overline{EA}(x)$, $\overline{EI}(x)$ and $\overline{GA}(x)$ signify respectively the axial rigidity, the flexural rigidity about the y-direction, and shear rigidity, including the contribution from both the AFG core and face layers, which are calculated as what follows:

$$\overline{EA}(x) = b\left(Q_{11}^f \int_{-0.5t_c - t_f}^{-0.5t_c} dz + Q_{11}^c(x) \int_{-0.5t_c}^{0.5t_c} dz + Q_{11}^f \int_{0.5t_c}^{0.5t_c + t_f} dz \right);$$
$$\overline{EI}(x) = b\left(Q_{11}^f \int_{-0.5t_c - t_f}^{-0.5t_c} z^2 dz + Q_{11}^c(x) \int_{-0.5t_c}^{0.5t_c} z^2 dz + Q_{11}^f \int_{0.5t_c}^{0.5t_c + t_f} z^2 dz \right); \tag{8}$$
$$\overline{GA}(x) = kb\left(Q_{55}^f \int_{-0.5t_c - t_f}^{-0.5t_c} dz + Q_{55}^c(x) \int_{-0.5t_c}^{0.5t_c} dz + Q_{55}^f \int_{0.5t_c}^{0.5t_c + t_f} dz \right)$$

where k is the shear correction factor and its value for rectangular cross-section is assumed as 5/6 [49].

Fig. 1 depicts an axially loaded member in which P^0 is the pre-buckling axial force. In this regard, the components of pre-buckling stresses are described as follows:

$$\sigma_{xx}^0 = \frac{P^0}{A}, \ \sigma_{xy}^0 = \sigma_{xz}^0 = 0 \tag{9}$$

where σ_{xx}^0 and ($\sigma_{xz}^0, \sigma_{xy}^0$) are the pre-buckling normal stress and shear stress, often called *the initial stresses*.

In this research, equilibrium equations and boundary conditions are derived from stationary conditions of the total potential energy. Based on this principle, the following relation is obtained [40]

$$\delta\Pi = \delta U_l + \delta U_0 = 0 \tag{10}$$

In this formulation, δ denotes a variational operator. In addition, U_l and U_0 represent the elastic strain energy and the strain energy due to the effects of the initial stresses, respectively. The previously-mentioned components could be computed using the following equations [40]:

$$\delta U_l = \int_V \sigma_{ij} \delta\varepsilon_{ij}^l dV = \int_0^L \int_A \sigma_{xx} \delta\varepsilon_{xx}^l dA dx + 2\int_0^L \int_A \sigma_{xy} \delta\varepsilon_{xy}^l dA dx + 2\int_0^L \int_A \sigma_{xz} \delta\varepsilon_{xz}^l dA dx$$
$$\delta U_0 = \int_V \sigma_{ij}^0 \delta\varepsilon_{ij}^* dV = \int_0^L \int_A \sigma_{xx}^0 \delta\varepsilon_{xx}^* dA dx + 2\int_0^L \int_A \sigma_{xy}^0 \delta\varepsilon_{xy}^* dA dx + 2\int_0^L \int_A \sigma_{xz}^0 \delta\varepsilon_{xz}^* dA dx \tag{11}$$

By inserting the variation form of the whole components of strain tensor involving linear and non-linear ones (Eq. (2)) along with Eq. (9), into Eqs. (11) and using Eq. (3), the first variational statement of total potential energy is obtained after essential integration over the cross-sectional area of the nanobeam as what follows [42]:

$$\delta\Pi = \int_L \left(N \delta u_0' + M \delta\theta' \right) dx + \int_L \left(Q(\delta w_0' + \delta\theta) \right) dx + \int_L \left(P^0 (w_0' \delta w_0') \right) dx = 0 \tag{12}$$

By gathering the coefficients of the virtual displacements ($\delta u_0, \delta w_0, \delta\theta$), and after equating them to zero, the following governing equations in the stationary state are obtained [42]

$$-N' = 0$$
$$(P^0 w_0')' - Q' = 0 \tag{13}$$
$$-M' + Q = 0$$

Subjected to the following boundary conditions at x=0 and x=L:

$$N = 0 \quad \text{Or} \quad \delta u_0 = 0$$
$$-P^0 w_0' + Q = 0 \quad \text{Or} \quad \delta w_0 = 0 \tag{14}$$
$$M = 0 \quad \text{Or} \quad \delta\theta = 0$$

By substituting nonlocal resultant components (Eq. (7)) into equation (13), the final nonlocal equilibrium equations in terms of the primary displacement field are acquired as follows:

$$\delta u_0 : \left(\overline{EA}(x) u_0' \right)' = 0 \tag{15}$$

$$\delta w_0 : \left(\overline{GA}(x)(w_0' + \theta) \right)' - P w_0'' + \mu P w_0^{iv} = 0 \tag{16}$$

$$\delta\theta : \left(\overline{EI}(x) \theta' \right)' - \overline{GA}(x)(w_0' + \theta) = 0 \tag{17}$$

The boundary conditions of the beam can be also expressed as:

$$N = 0 \quad \text{Or} \quad \delta u_0 = 0$$

$$\overline{GA}(w_0' + \theta) - P(w_0' - \mu w_0''') = 0 \quad \text{Or} \quad \delta w = 0 \tag{18}$$

$$\overline{EI}(x)\theta' + \mu P w_0'' = 0 \quad \text{Or} \quad \delta\theta = 0$$

Since the current study is concerned with stability analysis of an axially loaded sandwich Timoshenko nanobeam, the first equation (Eq. (15)) is not considered in the following.
In the line with the formulations proposed by Soltani et al. [40, 42], the governing equilibrium equation for the vertical displacement (16) can be rewritten as

$$\theta(x) = \frac{-\mu P w_0''' + (P - \overline{GA}(x))w_0'}{\overline{GA}(x)} \tag{19}$$

whose replacement in the third equilibrium Eq. (17) enables its redefinition in an uncoupled statement just dependent on the vertical deflection w_0, independently from the rotation θ, i.e.

$$\sum_{i=1}^{2} S_i(x)\frac{d^{i+1}w_0}{dx^{i+1}} + P\left(\sum_{i=1}^{3} R_i(x)\frac{d^i w_0}{dx^i} - \mu\sum_{i=1}^{3} R_i(x)\frac{d^{i+2}w_0}{dx^{i+2}}\right) = 0 \tag{20}$$

With

$$S_1(x) = -\left(\overline{GA}(x)\right)^3\left(\overline{EI}(x)\right)'$$
$$S_2(x) = -\overline{EI}(x)\left(\overline{GA}(x)\right)^3$$
$$R_1(x) = \begin{pmatrix} -\overline{EI}(x)\overline{GA}(x)\left(\overline{GA}(x)\right)'' + \\ 2\overline{EI}(x)\left(\overline{GA}(x)\right)'^2 - \\ \overline{GA}(x)\left(\overline{EI}(x)\right)'\left(\overline{GA}(x)\right)' - \\ P\left(\overline{GA}(x)\right)^3 \end{pmatrix} \tag{21}$$
$$R_2(x) = -\begin{pmatrix} 2\overline{EI}(x)\overline{GA}(x)\left(\overline{GA}(x)\right)' - \\ \left(\overline{GA}(x)\right)^2\left(\overline{EI}(x)\right)' \end{pmatrix}$$
$$R_3(x) = \overline{EI}(x)\left(\overline{GA}(x)\right)^2$$

The corresponding boundary conditions of a simply supported sandwich Timoshenko nanobeam can be expressed as

$$\text{at } x=0,\ L: \quad \begin{cases} -\overline{EI}(x)\left(\overline{GA}(x)\right)^2\dfrac{d^2w_0}{dx^2} + \\ P\begin{pmatrix} \overline{EI}(x)\left(\overline{GA}(x)\right)\dfrac{d^2w_0}{dx^2} - \\ \overline{EI}(x)\left(\overline{GA}(x)\right)'\dfrac{dw_0}{dx} \end{pmatrix} + \\ \mu P\begin{pmatrix} (\overline{EI}(x)\left(\overline{GA}(x)\right)' + \\ \left(\overline{GA}(x)\right)^2)\dfrac{d^3w_0}{dx^3} - \\ \overline{EI}(x)\left(\overline{GA}(x)\right)\dfrac{d^4w_0}{dx^4} \end{pmatrix} = 0, \\ w_0 = 0 \end{cases} \tag{22}$$

Referring to the author's knowledge, the resulting single non-local governing equation for linear buckling analysis of sandwich Timoshenko nanobeam having AFG core has never been acquired before. Due to the generality of the formula proposed herein, it also helps deal with estimating the buckling capacity of nano-scale sandwich Timoshenko beam with varying cross-sections. In addition, the acquired formula can simplify the computational effort necessary to calculate the critical axial load.

3. Solution Methodology

In this section, the numerical solution of a resulting fifth-order differential equation is developed. The GDQM is employed for this purpose and to calculate the axial critical loads. According to GDQM, the r^{th} order derivative of a function *f(x)* at an arbitrary point is described as [53]

$$\left.\frac{d^r f}{dx^r}\right|_{x=x_p} = \sum_{j=1}^{N} A_{ij}^{(r)} f(x_j) \quad for\ i = 1,2,\ldots,N \tag{23}$$

Here, N represents the number of grid points along the x direction. Regarding this x_j signifies the position of each sample point, which in this study is defined using the well-known Chebyshev–Gauss–Lobatto approach as

$$x_i = \frac{L}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right], \quad \text{if } 0 \le x \le L \quad i = 1,2,\ldots,N \tag{24}$$

The first-order derivative weighting coefficient ($A_{ij}^{(1)}$) is computed by the following algebraic formulations which are based on Lagrangian interpolation polynomials:

$$A_{ij}^{(1)} = \begin{cases} \dfrac{M(\xi_i)}{(\xi_i - \xi_j)M(\xi_j)} \; for \;\; i \neq j \\ -\sum_{k=1, k\neq i}^{N} A_{ik}^{(1)} \quad for \;\; i=j \end{cases} \quad i, j = 1, 2, ..., N \tag{25}$$

where

$$M(\xi_i) = \prod_{j=1, j\neq i}^{N} (\xi_i - \xi_j); \; for \;\; i = 1, 2, ..., N \tag{26}$$

Since the GDQM is based on the determination of the weighting coefficient, the r^{th}-order weighting coefficients $A_{ij}^{(r)}$ at the arbitrary sampling point x_i can be described as

$$A_{ij}^{(r)} = A_{ij}^{(1)} A_{ij}^{(r-1)} \qquad 2 \le r \le m-1 \tag{27}$$

Following the assumptions of the GDQM, the resulting nonlocal governing equation Eq. (20) can be rewritten in the following characteristics equation as

$$\left([K] + \lambda([K_G] - \mu[K_G^*])\right)\{w\} = 0 \tag{28}$$

The matrices $[K]$, $[K_G]$, and $[K_G^*]$ are of a size $N \times N$ and described by the following

$$\begin{aligned} [K] &= L^3[a][A]^{(2)} + L^2[b][A]^{(3)} \\ [K_G] &= L^4[c][A]^{(1)} + L^3[d][A]^{(2)} + L^2[e][A]^{(3)} \\ [K_G^*] &= L^2[c][A]^{(3)} + L[d][A]^{(4)} + [e][A]^{(5)} \end{aligned} \tag{29}$$

in which

$$\begin{aligned} & a_{jk} = (S_1|_{\xi=\xi_j})\delta_{jk}, \; b_{jk} = (S_2|_{\xi=\xi_j})\delta_{jk}, \\ & c_{jk} = (R_1|_{\xi=\xi_j})\delta_{jk}, \; d_{jk} = (R_2|_{\xi=\xi_j})\delta_{jk}, \\ & e_{jk} = (R_3|_{\xi=\xi_j})\delta_{jk}, \\ & \{w\}^T = \{w(\xi_1) \quad w(\xi_2) \quad ... \quad w(\xi_n)\}. \end{aligned} \tag{30}$$

here, the term δ_{jk} called Kronecker delta function. Also, ξ is the non-dimensional form of the longitudinal variable (x) and described as $\xi = x/L$. It is necessary to point out that the aforementioned parameter (ξ) is adopted to facilitate the mathematical procedure of solution of the equilibrium equation presented in Eq. (20) via applying the GDQM. After the accomplishment of associated boundary conditions of a simply supported given by Eq. (22), the buckling load for sandwich Timoshenko nanobeams with axially varying material core and two outer metal layers is derived using the eigenvalue solution of Eq. (28).

4. Numerical Results and Discussions

In this section, a comprehensive numerical analysis and discussion based on the proposed numerical model in the previous section are accomplished to discover the effect of different predominant parameters including, the ratio of core thickness to metal face-sheet, volume fraction exponent, nonlocality parameter, mode number and slenderness ratio on the buckling capacity of sandwich Timoshenko nanobeam subjected to simply supported end conditions.

The current section consists of two main parts. The first one aims to display the calculation correctness of the proposed method. The second part has been selected with the objective of investigating the impact of the previously mentioned parameters on the buckling characteristics of the selected nano-size sandwich member. Also, the following non-dimensional expression is adopted to represent the outcomes:

$$P_{nor} = \frac{P_{cr} L^2}{E_{ceramic} I} \tag{31}$$

Table 1. Comparison of P_{nor} of simply supported homogenous Timoshenko nanobeam for different slenderness ratios (*L/t*).

μ	L/t=10		L/t=20		L/t=100	
	Reddy [49]	Present	Reddy [49]	Present	Reddy [49]	Present
0.0	9.6228	9.6228	9.8067	9.8067	9.8671	9.8671
0.5	9.1701	9.1701	9.3455	9.3455	9.4031	9.4031
1	8.7583	8.7583	8.9258	8.9258	8.9807	8.9807
1.5	8.3818	8.3818	8.5421	8.5421	8.5947	8.5947
2.0	8.0364	8.0364	8.1900	8.19	8.2405	8.2405
2.5	7.7183	7.7183	7.8659	7.8659	7.9143	7.9143
3.0	7.4244	7.4244	7.5664	7.5664	7.613	7.613
3.5	7.1521	7.1521	7.2889	7.2889	7.3337	7.3337
4.0	6.899	6.899	7.0310	7.031	7.0743	7.0743
4.5	6.6633	6.6633	6.7907	6.7907	6.8325	6.8325
5.0	6.4431	6.4431	6.5663	6.5663	6.6068	6.6068

4.1. Validation of the Numerical Technique

In the first subsection, the validation of the established numerical methodology for stability analysis of simply supported prismatic isotropic beam in the context of nonlocal elasticity theory along with the first-order shear deformation theory is initially checked by comparing the calculated results with those reported by Reddy [49]. Referring to the author's knowledge of the DQ technique [54-58], twenty-one (*N=21*) are sufficient to estimate the buckling loads. Numerical results for axial critical load in the normalized form are presented in Table 1 for various values of nonlocal parameters and slenderness ratio. For comparison, the values for the required parameters of the nanobeam are considered as follows: $L=10$, $E=30\times10^6$, $\upsilon=0.3$.

Based on Table 1, excellent compatibility between the results obtained and the available results [49] is evident.

Subsequently, the validation of the current numerical formulations for axial instability analysis of AFG uniform Timoshenko beam subjected to simply-supported end conditions in the framework of classical elasticity theory is checked by comparing the calculated results with the ones presented in [41]. Regarding this, it is assumed that the functionally graded beam is composed of Zirconium dioxide (ZrO_2) and Aluminum (Al) with the following properties (ZrO_2: $E_{ceramic}$=200GPa; *Al*: E_{metal}=70GPa). Table 2 displays the variation of normalized endurable critical loads of AFG beams without face sheets in terms of the power-law index for various slenderness ratios (L/t) based on the local first-order shear deformation theory.

Table 2. Comparison of normalized buckling loads of AFG Timoshenko beams in terms of gradient parameters and various values of L/t.

Material	***L/t***	Normalized buckling load		
		Present	Ref. [41]	Δ (%)
Pure Ceramic	5	5.8715	5.8342	0.639
	10	6.2097	6.1894	0.328
	50	6.3177	6.3120	0.090
	100	6.3177	6.3159	0.028
p=1	5	3.5461	3.5125	0.956
	10	3.8132	3.7852	0.740
	50	3.8863	3.8804	0.152
	100	3.8880	3.8834	0.118

This comparison reveals that the established approach and corresponding numerical outcomes are in good agreement with Reference [41].

4.2. Parametric Study

In what follows, the parameterization investigation is performed to numerically understand the sensitivity of the stability strength to different factors such as in-homogeneous index, nonlocal parameter, thickness parameter, aspect ratio, mode number. Also, it should be noted that the AFG core is assumed to be made of Aluminum oxide (Al_2O_3) and Aluminum (Al) with the mechanical properties given in Table 3. Additionally, Aluminum is contemplated as the material of face sheets.

Table 3. Mechanical properties of ceramic and metallic components [59]

Properties of materials	Units	Alumina (Al_2O_3)	Aluminum (Al)
ρ	kg/m^3	3960	2702
E	GPa	380	70
υ	-	0.3	0.3

Through the expanded numerical approach, the normalized endurable buckling loads of simply-supported sandwich nano-size Timoshenko beam with a fixed slenderness ratio L/t=50 are estimated and reported in Table 4 to inspect the sensitivity of the stability resistance to power-law indices and nonlocal parameters. In this section, the ratio of core thickness to metal face sheet (t_c/t_f) is taken 8.

Table 4. Effect of FG power index on the first buckling load of a shear-deformable sandwich beam with different nonlocal parameters for the case: L/t=50, t_c/t_f=8.

p	μ=0	μ=0.5	μ=1	μ=1.5	μ=2
0	1.996	1.902	1.817	1.739	1.667
0.75	3.674	3.492	3.326	3.175	3.035
2.5	5.341	5.078	4.839	4.619	4.417
4	5.899	5.615	5.355	5.118	4.900

Graphical results are also shown in Fig. 2 where the variations of dimensionless buckling loads of nano-scale sandwich beam having ceramic-metal AFG core at constant thickness ratio t_c/t_f =10 with respect to volume fraction exponent (ranging from 0 to 5) for different non-locality parameters (μ= 0, 1, 2, 3 and 4 nm^2) and aspect ratios (L/t= 10, 20 and 100) is investigated.

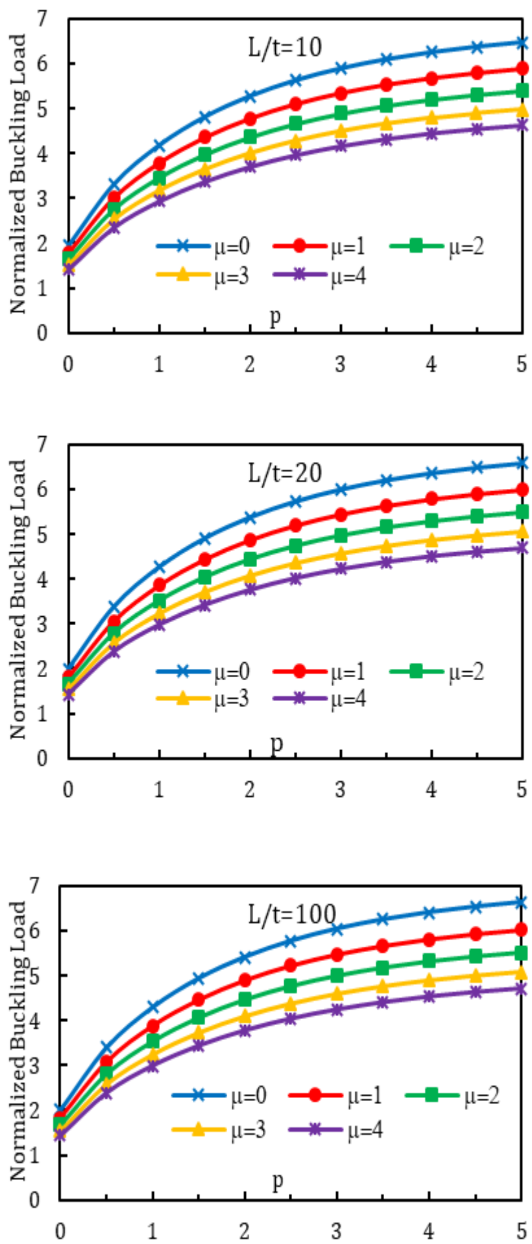

Fig. 2. Effect of Eringen's parameter on the dimensionless buckling load of simply supported sandwich nanobeam with respect to gradient index for different slenderness ratios (t_c/t_f=10).

Next, under the assumption t_c/t_f=10, Table 5 is devoted to examining the impact of aspect ratio (L/t) on the linear buckling response of the selected sandwich Timoshenko nanobeam for different power-law exponents, and various Eringen's nonlocality parameters (i.e. μ = 0, 0.5, 1, 1.5, 2 nm^2). Note that the compressive axial load is located at both the beam's ends without any eccentricities.

Table 5. Effect of slenderness ratio and Eringen's parameter on the normalized buckling load of a Timoshenko sandwich beam with FG power indices for the case: t_c/t_f=10.

L/t	μ	p=0	p=0.5	p=1	p=2	p=5	p=10
	0	1.799	3.062	3.842	4.831	5.972	6.377
5	2	1.528	2.563	3.195	4.012	5.015	5.385
	4	1.327	2.196	2.716	3.400	4.304	4.656
	0	1.945	3.308	4.175	5.264	6.460	6.839
10	2	1.631	2.745	3.449	4.351	5.389	5.727
	4	1.405	2.338	2.922	3.685	4.613	4.924
	0	1.984	3.372	4.262	5.376	6.588	6.962
20	2	1.659	2.791	3.513	4.436	5.484	5.815
	4	1.426	2.374	2.973	3.753	4.685	4.990
	0	1.992	3.384	4.278	5.397	6.611	6.985
30	2	1.664	2.800	3.525	4.450	5.500	5.831
	4	1.429	2.381	2.982	3.763	4.695	5.001
	0	1.996	3.390	4.286	5.407	6.623	6.996
50	2	1.667	2.804	3.530	4.458	5.508	5.838
	4	1.431	2.384	2.986	3.768	4.700	5.007
	0	1.997	3.392	4.288	5.409	6.626	6.999
70	2	1.668	2.805	3.532	4.459	5.510	5.841
	4	1.432	2.385	2.987	3.769	4.701	5.008
	0	1.998	3.393	4.289	5.411	6.628	7.001
100	2	1.668	2.806	3.532	4.460	5.511	5.842
	4	1.432	2.385	2.988	3.770	4.702	5.009

Next, to examine the effect of thickness parameter t_c/t_f on the variations of normalized buckling loads of shear deformable sandwich nanobeam having a ceramic-metal functionally graded core with respect to Eringen's parameter (ranging from 0 to 4) with two different values of volume fraction exponents (p=1, and 3) are respectively plotted in Figs. 3 and 4 for L/t = 10, and L/t = 100. Each of the depictions of these figures illustrated six different plots relating to t_c/t_f =5, 10, 15, 20, 25, and 50.

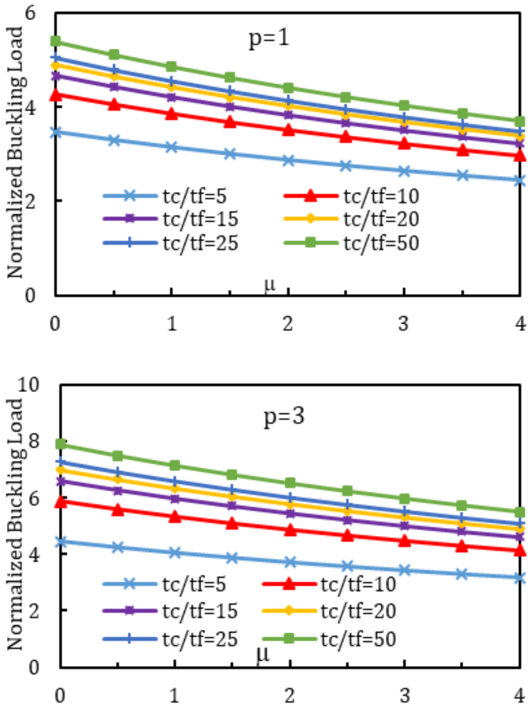

Fig. 3. Variation of the Normalized buckling load of simply supported sandwich Timoshenko nanobeam with nonlocal parameter and thickness ratio for different material indexes (L/t=10)

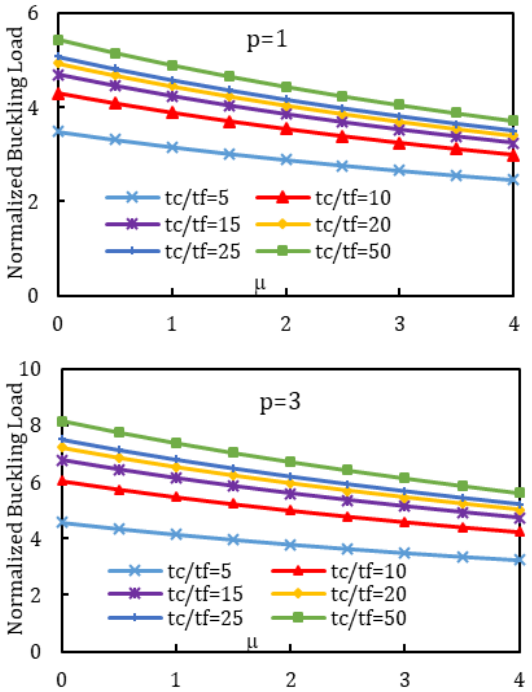

Fig. 4. Variation of the Normalized buckling load of simply supported sandwich beam with nonlocal parameter and thickness ratio for different material indexes (L/t=100)

The aspect ratio significantly affects the axial buckling capacity of three-layered shear deformable nano-size beams, as seen in the above figures and tables. An enhancement in the axial buckling strength is observed when the value of the slenderness ratio (*L*/*t*) increases. It is noteworthy that the buckling strength of the selected sandwich Timoshenko nanobeam is greater with *L*/*t*=100 due to the reduction in the shear deformation. In other words, all of the cases studied show that a beam having the value of $L/t = 5$ provides the least amount of resistance to stability. Furthermore, the effect of the slenderness ratio on the buckling loads is negligible for long and slender sandwich nanobeam (i.e., $L/t \geq 30$). For more information please see [40, 41, 55].

Inspection of the preceding figures and tabulations reveals that an increase in the value of the non-dimensional ratio of AFG core thickness to the metal-layer t_c/t_f leads to an increase in the normalized buckling loads. This is due to the fact that a larger amount of thickness ratio corresponds to a sandwich beam that is closer to a ceramic-metal FG beam without Aluminum face layers. Under this condition, the stiffness of the member is increased and consequently higher buckling load is obtained. This could yield a different beneficial effect on the overall structural response of many nano-engineering components such as scanning tunneling microscopes, oscillators, or sensors.

Additionally, it is demonstrated that the axial buckling load improves noticeably with the increase in volume fraction exponent. Based on Eq. (6), one can conclude that with an increase in the value of *p*, the portion of ceramic through the beam element increases, and due to this fact, the amount of elasticity modulus and consequently the quantity of bending and shear stiffnesses are enhanced. Therefore, the maximum and minimum magnitudes of buckling load are respectively obtained for shear-deformable beams with full ceramic core ($p \to \infty$) and pure metal core ($p = 0$). Besides, it is obvious that the normalized critical loads increase sharply for the FG power-law index in the range of $0 \leq p \leq 2$. For larger values of volume fraction exponent *p*>2, the endurable buckling load increases monotonically and slightly.

Based on graphical and tabular results, it is observed that the buckling capacity decreases significantly with the increase in the nonlocality parameter related to Eringen's nonlocal elasticity theory as expected because of the reduction in the value of the stiffness and rigidity of the member. In general, the inclusion of the influence of the nonlocal parameter (μ) increases the deflection, which in turn leads to a noticeable decrease in the buckling capacity of the member, and consequently, a more unstable member is obtained. This confirms the findings from the literature, for which classical formulations overestimate the results compared to nonlocal formulations.

In the subsequent part, the variation of the first four dimensionless axial critical loads of simply-supported sandwich Timoshenko nanobeam versus Eringen's parameter (μ) for ceramic core and AFG one with (p=2) is presented in Fig. 5. Other required parameters for the problem are considered as L/t=20, and t_c/t_f=5.

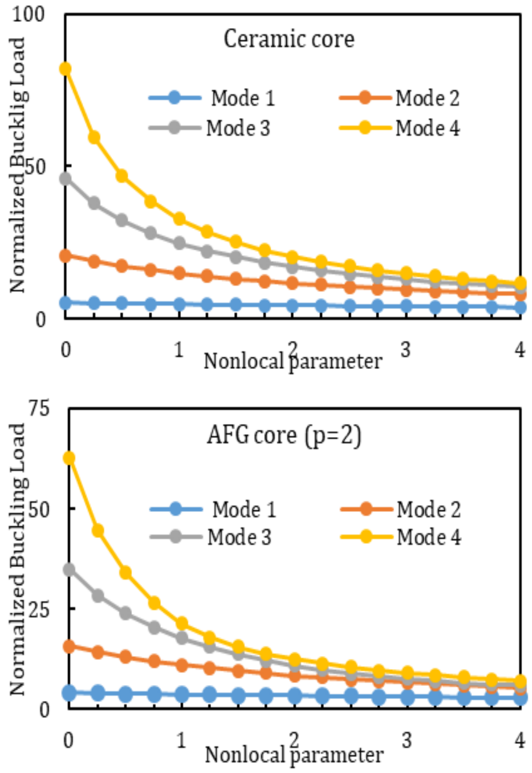

Fig. 5. Effect of the nonlocal parameter on the first four buckling loads, for various mode numbers.

Regarding these illustrations, it is found that the nonlocal parameter has more influence on higher buckling modes compared with the lower ones. It can be stated that it is necessary and crucial to contemplate the nonlocal theory for the exact estimation of sustainable axial buckling loads related to higher modes of nano-size sandwich shear deformable beams. In addition, it is easily observed that the impact of μ is more when the nonlocal parameter changes from zero to one.

5. Conclusions

This article presents an efficient and innovative approach for analyzing the linear stability behavior of shear deformable sandwich nanobeams, as useful for smart devices, such as oscillators, sensors, atomic force microscopes, and nano/micro-electro-mechanical systems. The cross-section of the considered nano-size beam consists of two metal sheets at the outer sides of an FG core, where the material properties vary continuity in the length direction. The equilibrium differential equations in terms of the vertical displacement and the rotation angle are extracted for axially loaded multi-layer beams within the framework of small displacements and rotations through Eringen's nonlocal elasticity and the first-order shear deformation beam theory. The coupled governing equations are then mixed and converted into a single and new fifth-order differential equation with variable coefficients. Due to uncoupling the system of nonlocal governing equations and its transformation into only one differential equation, it is believed that the expanded formula requires a lower computational cost as well as less CPU time. Another advantage of the suggested methodology is the ability to obtain the sustainable buckling load of sandwich Timoshenko nanobeam with desired axial changes in the properties of the core material.

In the following, the resulting equation is solved via the differential quadrature method as a powerful numerical technique, and finally, the axial buckling load is calculated. The key point of the adopted numerical technique, indeed, lies in the accurate approximation of a general-order derivative of a smooth function through a linear combination of its values assumed at the selected collocation points discretizing the domain, even for a reduced number of 21 grid points. The accuracy of the proposed approach is confirmed by comparing our results with the exciting numerical ones. Finally, a detailed parameterization study is performed to peruse the effect of slenderness ratio, power-law index, Eringen's parameter, and core thickness to face-layer ratio on the buckling characteristics of a simply supported sandwich Timoshenko nanobeam.

In addition to the mentioned results in the text, it can be stated that the numerical methodology established herein can accurately estimate the endurable axial critical load of sandwich shear deformable nanobeam with different types of longitudinal variations in the properties of the core material. Therefore, the proposed approach can be applied for an optimized design of smart devices.

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