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Structural Damage Identification through an Optimal Sensor Placement Approach and the Modal Strain Energy Based Index

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ABSTRACT

In this study, an efficient method for determining the accurate location of damage to structures is introduced using an optimal sensor placement (OSP) and modal strain energy-based index (MSEBI). The research is implemented in two main stages. In the first stage, a correlation function between the reconstructed mode shapes using the iterated improved reduced system (IRS) method and the complete mode shapes of a structure is defined and then the function is minimized via the binary differential evolution (BDE) algorithm to find the optimal sensor placement of the structure. In the second stage, the location of damage is determined using MSEBI based on the optimum place of sensors previously obtained. In order to assess the efficiency of the proposed method, two standard examples, including a two-dimensional (2-D) frame structure with 45 elements and a 2-D truss with 47 elements, are examined. Numerical results, considering different conditions, demonstrate that the integration of OSP method and MSEBI can provide an efficient tool for accurate and rapid identification of the damage location. The parametric study shows that the proposed method has a low sensitivity to the number of modes and noise level, and it can properly identify damage by considering a few modes and the high level of noise.

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1. Introduction

Structural damage identification has attracted a significance attention given that the timely detection and repair of such structural damage can increase the service life and avert the overall failure of the structure. The cornerstone of most damage identification procedures in structures is based on the inspection of changes in structural responses, since the occurrence of damage comparatively reduces the stiffness, and thus alters the static and dynamic properties of the structure [1–3]. Many damage identification methods are founded on utilizing the notion of modal strain energy (MSE). The MSE appears to be more sensitive to damage than the natural frequencies and mode shapes. Thereby, MSE is basically defined as a damage index to figure the location and level of the damage [4–7]. Nevertheless, one of the downsides of such methods is that they generally require structural responses at all degrees of freedom (DOFs) of the structure to identify damage. Although some researchers have used the responses of limited DOFs of the structure so as to expand them to other DOFs [8–10], choosing the right master DOFs or the appropriate placement of sensors can still significantly affect the efficiency of the approach. For this purpose, optimization methods can be employed to determine the optimal sensor placement (OSP).

Over the last years, several investigations have been carried out concerning the identification of damage in structural systems based on various form of strain energy. A two-stage method to identify the location and severity of structural damage was proposed by Seyedpoor (2012). The modal strain energy-based index (MSEBI) located potential damage at first. Then, the actual site and severity of the damage were determined using the particle swarm optimization (PSO) algorithm established on the results of the first stage. Numerical examples revealed that the method

can offer a robust tool to identify multiple damage cases in structures [11]. Nobahari and Seyedpoor (2013) devised a sophisticated procedure to locate multiple damages in structures on the basis of strain energy and flexibility matrix principles [12]. Seyedpoor and Yazdanpanah (2014) proposed an index for damage localization using the damage-induced static strain energy (SSE) change. The proposed index was then compared with MSEBI to evaluate the efficiency of the method [13]. Chen and Yu (2015) implemented a two-stage method to identify the location and severity of damage. In their work, MSEBI localized damage in the first phase, and then in the second phase, the PSO algorithm was coupled with the improved Nelder-mead algorithm to identify the severity of damage in single and multiple damage scenarios [14]. Wei et al. (2016) used a two-stage method to detect damage in moderately thin plates. In the first step, the damage location was identified through the ratio of MSE change. Then, the extent of damage was obtained through the sensitivity of MSE change, being updated on the basis of the IRS method where the reduced model was utilized to eliminate the rotational DOFs [8]. Wang et al. (2018) proposed a two-step method to identify the location and extent of damage in anisotropic structures such as composite laminates by introducing an anisotropic reduction factor. First, the MSE based method was utilized to locate the damage, by which the candidate damaged elements can be selected. Taking advantage of the IRS method, the damage localization method could be completed with incomplete modal information. Second, the optimization method based on a direct search algorithm is adopted to identify the damage extent of the candidate elements. The results signified the high capacity of the methodology in one experimental and two numerical examples by considering the noise [9].

Moreover, some progress has been made related to limiting the number of sensors or achieving optimal placement of sensors. In 2015, Sun and Büyüköztürk optimized the position of the sensors on several models using the artificial bee colony optimization algorithm and the IRS approach. The results showed the efficacy of the proposed method, indicating that the method can potentially be used in optimizing the placement of sensors in structural health monitoring (SHM) [15]. Zare Hosseinzadeh et al. (2016) introduced an efficient objective function formulated on the modal assurance criterion (MAC) and the modal flexibility matrix to locate structural damage. They utilized the democratic PSO algorithm to minimize the objective function, which was a modified version of the original PSO and led to the evaluation of damage in different types of structures. On this wise, the generalized Neumann series expansion was used to limit the number of sensors [16]. Dinh-Cong et al. (2018) conducted an optimal sensor placement (OSP) using the iterated IRS method. The OSP strategy was then implemented through the Jaya algorithm by formulating and solving an optimization problem to find the best sensor position [17]. In another study in 2018, Dinh-Cong et al. used a two-stage method to identify damage. In the first step, the damage was localized using the normalized MSEBI. In the second step, the severity of damage was determined by the teaching-learning-based optimization (TLBO) algorithm. To further enhance the procedure, a model reduction method based on the Neumann series expansion (NSEMR-II) has also been employed to limit the number of sensors on the structures [18]. A sensor placement algorithm for structural health monitoring with redundancy elimination model (REM) based on sub-clustering strategy was proposed by Yang et al. (2019). In order to overcome the previous limitations in sensor distribution, the REM considered the global and local sensor distribution effect. The

method was verified by a simple example and also two engineering numerical examples including space solar power satellite and reusable launch vehicle were applied to demonstrate the validity of the proposed sensor placement algorithm [19]. A modified cross model cross mode (CMCM) algorithm was developed to identify and localize the damage of a steel platform frame by Zargarzadeh et al. (2020). In the study, a SAR method was used to reduce the model. The numerical model was updated employing the improved CMCM method and the results were compared to those of the traditional CMCM method. Based on the results, the improved method showed higher accuracy in detecting damage than the traditional method [20]. A Strategy for sensor number determination and placement optimization with incomplete information based on interval possibility model and clustering avoidance distribution index was proposed by Yang et al. (2020). The performance of sensor configuration with optimum sensor number and less redundant information was improved by considering two types of incomplete information simultaneously, namely uncertainty and redundancy. The effectiveness of the proposed method was verified through three engineering numerical examples [21]. The contribution of stiffness, stability, strength and fatigue in structural service performance using a weighting coefficient was measured by Shi et al. (2020). Based on a weighting coefficient termed the weighted standard deviation norm (WSDN), calculated by the residual performance after degradation of each component, a novel OSP method was proposed to decrease the error of damage identification under uncertainty. Through the optimization of sensor placement using the proposed method, the identified probability damage index of the damaged unit can be larger led to the damaged element could be easily distinguished [22]. An adaptive sensor placement algorithm for SHM based on multi-objective iterative optimization

using weight factor updating was proposed by Yang (2021). Considering different optimal sensor placement methods from their own perspectives, a novel combined fitness function using weight factors and normalization was constructed and solved by a genetic algorithm. Three numerical examples were considered to demonstrate the effectiveness of the proposed algorithm under five sensor placement criteria, including the sensor distribution index and the ratio of the same positions [23].

In order to optimally install the sensors on structures for damage identification, it is principally important to find the DOFs to be used to measure the most proper responses of the structure among all active DOFs. Accordingly, in the first step of the current study, the optimal position of the sensors is determined by minimizing a correlation function, between the actual mode shapes of the structure and those estimated by the iterated IRS method, using the BDE algorithm. Then, in the second step, the damage location is identified through the MSEBI constructed on optimal position of sensors.

2. Modal strain energy based index

In this study, the modal strain energy based index (MSEBI) is used to localize damage in structures [11]. As specified by the method, the modal strain energy (MSE) of the e th element of the structure in the i th mode is expressed by Eq. (1) as:

$$mse_i^e = \frac{1}{2} \varphi_i^{eT} K^e \varphi_i^e, \quad (1)$$

$$i = 1, \dots, ndf$$

$$e = 1, \dots, nte$$

where K^e is the stiffness matrix of e th element and φ_i^e shows the corresponding mode shape vector of the e th element in the i th mode. Also, nte is the total number of elements and ndf is the total number of active degrees of freedom. Furthermore, the MSE of the i th mode of the

whole structure is determined from the total strain energy of all elements via Eq. (2):

$$mse_i = \sum_{e=1}^{nte} mse_i^e \quad (2)$$

$$i = 1, \dots, ndf$$

In order to achieve more efficiency, the strain energy of each element is normalized relative to the MSE of the whole structure by Eq. (3):

$$nmse_i^e = \frac{mse_i^e}{mse_i} \quad (3)$$

where $nmse_i^e$ is the normalized strain energy of the e th element in the i th mode.

Now the mean of Eq.(3), considering the first nm modes of the structure, is used as an efficient parameter to detect damage in an element, as given by Eq. (4):

$$mnmse^e = \frac{\sum_{i=1}^{nm} nmse_i^e}{nm}, \quad (4)$$

$$e = 1, \dots, nte$$

The occurrence of damage typically leads to an increase in MSE, thus increasing the efficient parameter $mnmse^e$ in the element. Denoting the efficient parameter for an intact element and damaged element of the structure by $(mnmse^e)^h$ and $(mnmse^e)^d$, respectively, the MSEBI is expressed by Eq. (5) as [11]:

$$MSBEI^e = \max \left[0, \frac{(mnmse^e)^d - (mnmse^e)^h}{(mnmse^e)^h} \right] \quad (5)$$

$$e = 1, \dots, nte$$

In this study, to candidate the damaged elements, the trivial measures of $MSBEI^e$ are removed by applying a universal threshold, given by Eq. (6):

$$T = \sigma \sqrt{2 \ln(nte)} \quad (6)$$

where σ represents the standard deviation of $MSBEI^e$ ($e = 1, \dots, nte$). The $MSBEI^e$ outputs are considered as damaged elements if they meet the limiting requirement T ; if not, they are put as intact elements.

3. The standard improved reduced system

The standard improved reduced system (IRS) method is based on the static reduction method introduced by Guyan [24]. The IRS method was later extended to an iterative version by Friswell et al. [25] named as the iterated IRS method. According to Guyan method, the displacement (D) and force (f) vectors, as well as the stiffness (K) and mass (M) matrices of the structure, are subdivided into vectors and matrices associated with the preserved principal (master) DOFs and those eliminated dependent ones (slave). Assuming that no force is applied to the slave DOFs, and the amount of damping is trivial, the equation of motion of the structure is characterized by Eq. (7).

$$\begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{D}_m \\ \ddot{D}_s \end{Bmatrix} + \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \begin{Bmatrix} D_m \\ D_s \end{Bmatrix} = \begin{Bmatrix} f_m \\ 0 \end{Bmatrix} \quad (7)$$

where subscripts m and s , respectively correspond to the master and slave DOFs. Disregarding the expressions related to inertia in the second set of Eq. (7), one can obtain Eq. (8) as:

$$K_{sm}D_m + K_{ss}D_s = 0 \quad (8)$$

where it can be applied to eliminate the dependent DOFs such that

$$\begin{Bmatrix} D_m \\ D_s \end{Bmatrix} = \begin{bmatrix} I \\ -K_{ss}^{-1}K_{sm} \end{bmatrix} \{D_m\} = [T_s]\{D_m\} \quad (9)$$

where T_s is the static transfer matrix between the vector of the DOFs of the whole structure and the vector of the principal ones.

The reduced mass and stiffness matrices are immediately defined by Eq. (10) as:

$$M_R = T_s^T M T_s \quad , \quad K_R = T_s^T K T_s \quad (10)$$

Eq. (7) for a sinusoidal excitation with frequency ω reads as [24]:

$$[K_{ss} - \omega^2 M_{ss}]D_s = -[K_{sm} - \omega^2 M_{sm}]D_m \quad (11)$$

By rewriting the equation, the vector of the slave DOFs can be expressed in terms of the master DOFs as follows

$$D_s = -K_{ss}^{-1} [K_{sm} + \omega^2 (M_{ss} K_{ss}^{-1} k_{sm} - M_{sm}) + o(\omega^4)] D_m \quad (12)$$

where $o(\omega^4)$ signifies the error of order ω^4 .

By ignoring the expressions related to ω^4 and higher orders in Eq. (12), it can be expressed as

$$D_s = [-K_{ss}^{-1}K_{sm} + K_{ss}^{-1} (M_{sm} - M_{ss}K_{ss}^{-1}K_{sm})M_R^{-1} K_R] D_m \quad (13)$$

Eq. (13) represents a transformation to express the dependent DOFs in terms of the reference ones. It can hence be generalized to Eq. (14) as follows

$$\begin{Bmatrix} D_m \\ D_s \end{Bmatrix} = \begin{bmatrix} I \\ -K_{ss}^{-1}k_{sm} + K_{ss}^{-1} (M_{sm} - M_{ss}K_{ss}^{-1}k_{sm})M_R^{-1} K_R \end{bmatrix} \{D_m\} = T_{IRS} D_m \quad (14)$$

where the transfer matrix T_{IRS} is expressed by

$$T_{IRS} = T_s + S M T_s M_R^{-1} K_R \quad (15)$$

$$\text{in which } S = \begin{bmatrix} 0 & 0 \\ 0 & K_{ss}^{-1} \end{bmatrix}.$$

Subsequently, the reduced mass and stiffness

$$K_{IRS} = T_{IRS}^T K T_{IRS} \quad , \quad M_{IRS} = T_{IRS}^T M T_{IRS} \quad (16)$$

matrices obtained through the IRS approach take the form of Eq. (16).

3.1. Iterated IRS technique

The transfer matrix given by Eq.(15) relies on reduced matrices of mass and stiffness

resulting from the static reduction. After calculating the transformation, there is an improved estimation of these reduced matrices from the existing Eq. (16). These improved estimates can be used to define the IRS transformation of Eq. (15) for a more accurate transformation. The transformation is given by Eq. (15) for the first iteration and for the next iterations is [25]:

$$T_{IRS,i+1} = T_S + SMT_{IRS,i} M_{IRS,i}^{-1} K_{IRS,i} \quad (17)$$

where the subscript i shows the i th iteration. In Eq. (17) the transformation $T_{IRS,i}$ is the current IRS transformation and $M_{IRS,i}$ and $K_{IRS,i}$ are the associated reduced mass and stiffness matrices given by Eq. (16). A new transformation, $T_{IRS,i+1}$ is obtained which then becomes the current IRS transformation for the next iteration.

4. The proposed method

In this study, an efficient two-step method for identifying damage to structures is proposed. The optimal position of the sensors is initially determined by minimizing a correlation function, between the complete mode shapes of the structure and those reconstructed by the iterated IRS method, using BDE algorithm. The damage is then localized via MSEBI based on installing the sensors at their optimal positions. The noticeable feature of the proposed method is to quickly determine the exact location of the damaged elements of the structure.

4.1. Optimal sensor placement method

In this study, an optimization method is used to properly select the master DOFs of the structure. In order to determine the appropriate master DOFs named here as the optimal sensor placement (OSP), an optimization problem is defined as:

$$\begin{aligned} \text{Find: } X^T &= \{x_1, x_2, \dots, x_n\} \\ \text{Minimize: } &w(X) \\ x_i &\in \{0,1\} \end{aligned} \quad (18)$$

where X is the vector of DOFs disposed to the placement of sensors, x_i indicates the position of the sensor in the i th potential DOF of the structure, which can take the value 0 or 1. In this respect, 0 denotes the absence of a sensor, and 1 represents the presence of the sensor in that DOF. Further, w denotes an objective function needs to be minimized.

The objective function plays a significant role in optimization problems so that selecting an appropriate objective function, along with increasing the convergence, prevents the algorithm from getting stuck in local optima. In the present study, the objective function is defined as

$$w(X) = - \frac{(R_A^T \cdot R_R(X))^2}{(R_A^T \cdot R_A)(R_R(X)^T \cdot R_R(X))} \quad (19)$$

where R_A is the complete mode shape vector of the healthy structure obtained through an analytical model based on the finite element method, and R_R is the reconstructed mode shape vector of the healthy structure attained by the iterated IRS method when the position of sensors is defined according to the X vector. It should be noted that if more than one mode is considered in the process, all mode shapes are defined as a column vector.

4.2. Optimization algorithm

In this study, the BDE algorithm is used to solve the optimization problem given by Eq. (18). The differential evolution (DE) algorithm [26] is originally a continuous optimization algorithm and it needs to be modified as a binary version for solving the optimization problems with discrete variables. The BDE algorithm involves the following steps:

Step 1: Initial population

The initial population is randomly generated by a uniform distribution in the search space according to Eq. (20):

$$x_{i,j} = \text{round}(\text{rand}), \quad (20)$$

$$i = 1, \dots, n, \quad j = 1, \dots, np$$

where rand is a function to produce a uniformly distributed random number in the interval $[0,1]$, round is a function to round the number, np is the total number of initial population and n is the total number of solution dimensions. Each candidate solution of the population is a vector of n components taken the value 0 or 1.

Step 2: Mutation and recombination

Generation of new offspring involves mutation and recombination operators. For each target vector, $X_{j,G}$ ($j = 1, 2, \dots, np$) a mutant vector is generated as given by Eq. (21):

$$V_{j,G+1} = X_{r_1,G} + \text{mf} \cdot (X_{r_2,G} - X_{r_3,G}) \quad (21)$$

where r_1, r_2, r_3 are three different integers belong to $\{1, 2, \dots, np\}$, randomly chosen, mf refers to the mutation factor, which is a value between 0 and 2, and G represents the number of generations.

Then, a new combination of phenotypes is produced by recombination operator as:

$$u_{i,j,G+1} = \begin{cases} v_{i,j,G+1} & \text{rand}_{i,j} \leq cr \quad \text{or} \quad j = I_{\text{rand}} \\ x_{i,j,G} & \text{rand}_{i,j} > cr \quad \text{and} \quad j \neq I_{\text{rand}} \end{cases} \quad (22)$$

$$i = 1, 2, \dots, n \quad j = 1, 2, \dots, np$$

where $cr \in [0,1]$ is the crossover ratio, $\text{rand}_{i,j}$ is a random real number $\in [0,1]$ and I_{rand} is a random integer number among numbers $[1, 2, \dots, n]$.

Step 3: Discretization

The discretization scheduling is performed on vector U_j through Eq. (23) [27]:

$$u_{i,j,G+1} = \begin{cases} 1 & \text{if } \text{rand} \leq \frac{1}{1+e^{-(u_{i,j,G+1})}} \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

$$i=1,\dots,n \quad j=1,\dots,np$$

Step 4: Selection

The selection operator, as expressed by Eq. (24), compares two vectors $X_{j,G}$ and $U_{j,G+1}$ in terms of the value of the objective function so as to determine the vector to be propagated to the next generation. The vector with smaller objective function is utilized in the next generation population.

$$X_{j,G+1} = \begin{cases} U_{j,G+1} & w(U_{j,G+1}) \leq w(X_{j,G}) \\ X_{j,G} & \text{otherwise} \end{cases} \quad (24)$$

$$j = 1, 2, \dots, np$$

Step 5: Convergence

In this step, the optimization process will be stopped if the convergence appears. Otherwise, the process would repeat from Step 2.

4.3. Damage identification using modal strain energy based index

Once the optimal placement of the sensors is determined, the damage can then readily be localized via the modal strain energy based index (MSEBI) defined by Eq. (5). Thus, any element with $MSEBI < T$ is considered as intact element, and that with $MSEBI \geq T$ is introduced as damaged one. Fig. 1 illustrates the general flow of the proposed method.

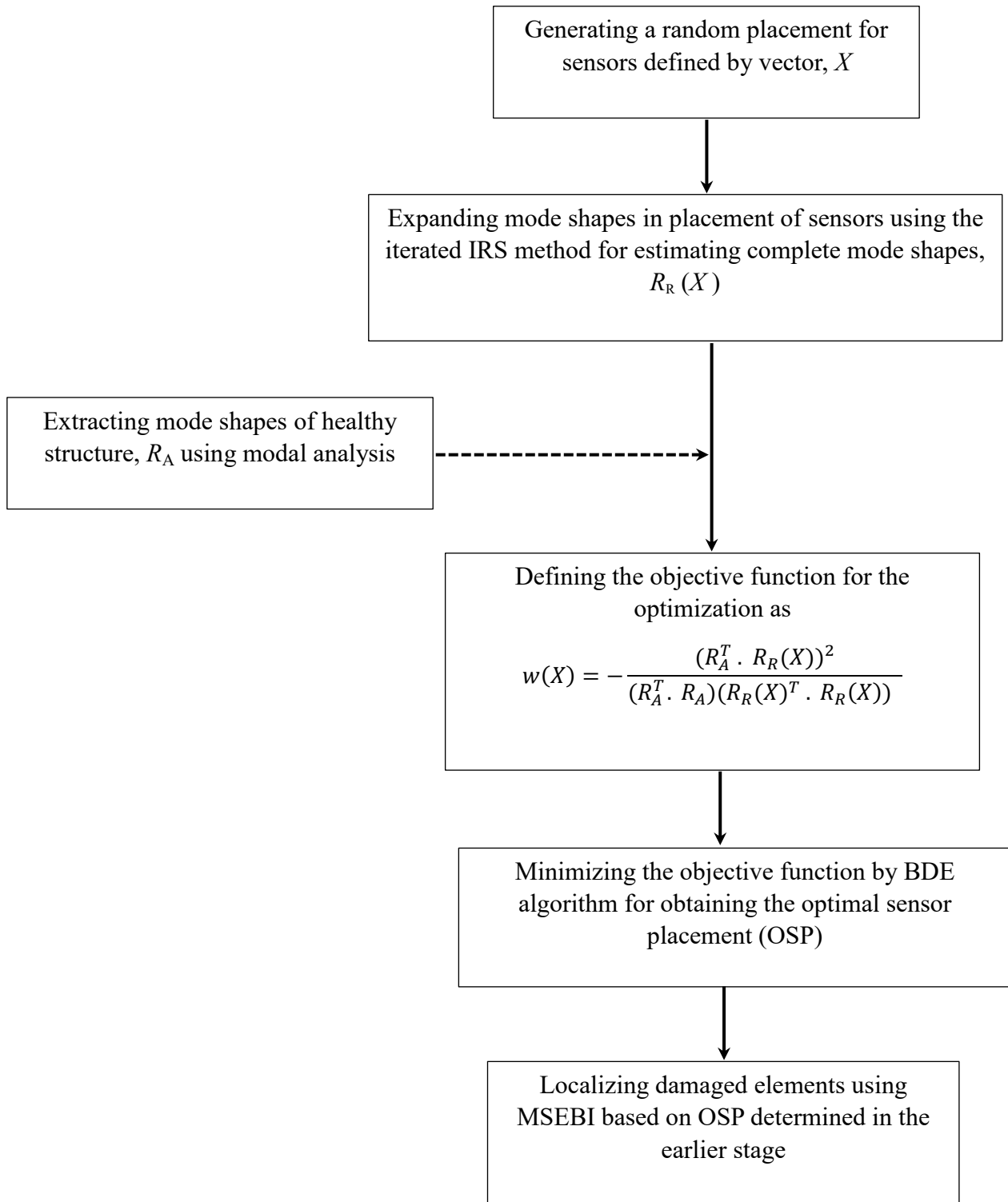


Fig. 1. General steps of the proposed method.

5. Numerical examples

In order to assess the efficiency of the proposed method for the structural damage identification, two numerical examples,

including a 2-D frame with 45 elements and a 2-D truss with 47 elements are considered. The feasibility of the method is assessed for single, double, and triple damage scenarios in the presence of noise. Besides, the performance of the approach, which uses responses at limited

DOFs of structures, is compared with that of a method considering responses at all DOFs [11].

5.1. Frame with 45 elements

A four-span five-story frame, as shown in Fig. 2, is considered as the first example [28]. The

steel sections used for beams and columns are ($W12 \times 87$) and ($W14 \times 145$), respectively. The elastic modulus and mass density for the material are 210 GPa and 7780 kg/m^3 , respectively.

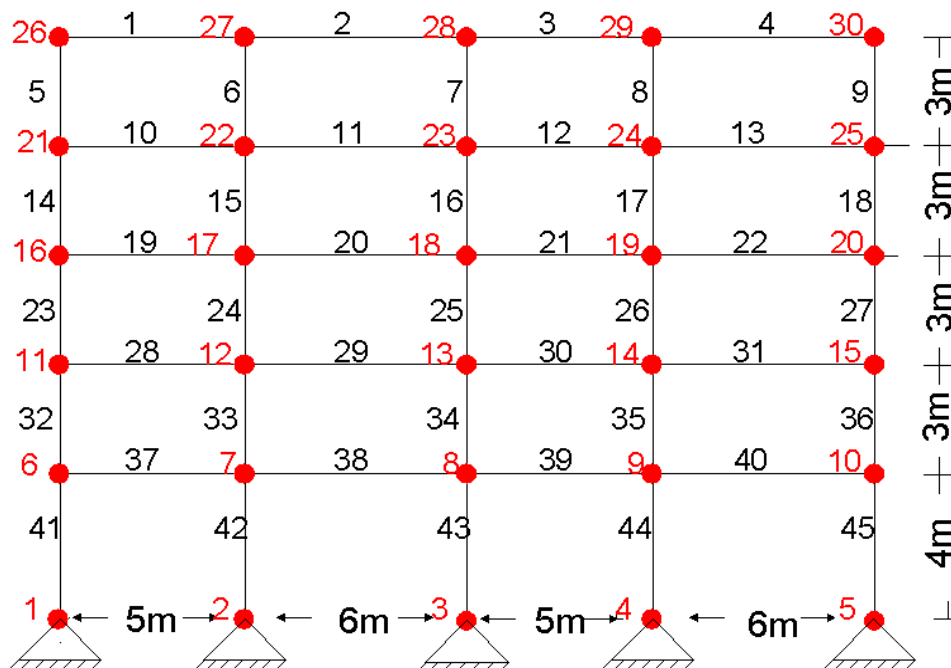


Fig. 2. Configuration of the frame with 45 elements.

Five damage cases listed in Table 1 are considered. For damage detection, 3 modes contaminating 3% noise are considered. In order to evaluate the robustness of the proposed method, the damage identification results of the present approach are compared with those of the method presented by Seyedpoor [11].

The optimal placement of sensors obtained by solving the optimization problem, Eq. (18) is based on the healthy state of the structure and is completely independent of the damage applied to the structure. The number of sensors for the frame is assumed to be 3. Based on the randomness nature of the optimization algorithm, there are different solutions minimizing the objective function to -1. The

optimal placement of sensors for five different optimization runs and the final sensor placement are given in Table 2. These positions indicate the optimal state in terms of the correlation between the mode shapes estimated through the iterated IRS method and the actual ones. The final placement of sensors is selected here from these placements based on the engineering judgment. In the table, the number 1 for DOFs corresponds to the horizontal displacement of the node.

Damage identification results of 45-element frame for various damage scenarios are depicted in Fig. 3 using the OSP of the first stage of the current study as well as MSEBI when considering 3 modes with 3% noise. The figure compares the results of the proposed

approach with the method provided by Ref. [11]. It is revealed from Fig. 3 that the approach proposed here has a higher accuracy in detecting damage while considering a limit number of DOFs and optimizing the sensor

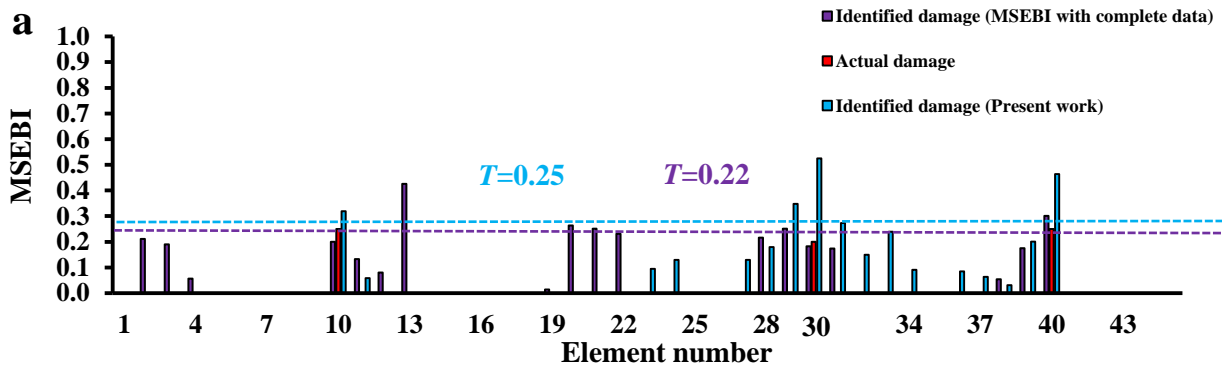
position. In contrast, the procedure presented in Ref. [11] localizes elements other than those containing damage whereas using all DOFs of the structure.

Table 1. Damage cases applied to the frame with 45 element.

Damage Case 1		Damage Case 2		Damage Case 3	
Element No.	Damage Extent	Element No.	Damage Extent	Element No.	Damage Extent
10	0.25	9	0.25	14	0.30
30	0.20	15	0.30	-	-
40	0.25	-	-	-	-
Damage Case 4		Damage Case 5			
Element No.	Damage Extent	Element No.	Damage Extent		
14	0.35	9	0.30		
28	0.30	18	0.20		
38	0.35	36	0.25		

Table 2. Optimal placement of sensors in 45-element frame for five optimization runs.

Run 1		Run 2		Run 3	
Node No.	DOF	Node No.	DOF	Node No.	DOF
6	1	6	1	6	1
18	1	18	1	19	1
30	1	29	1	29	1
Run 4		Run 5		Final sensor placement	
Node No.	DOF	Node No.	DOF	Node No.	DOF
6	1	6	1	6	1
16	1	17	1	18	1
28	1	28	1	29	1



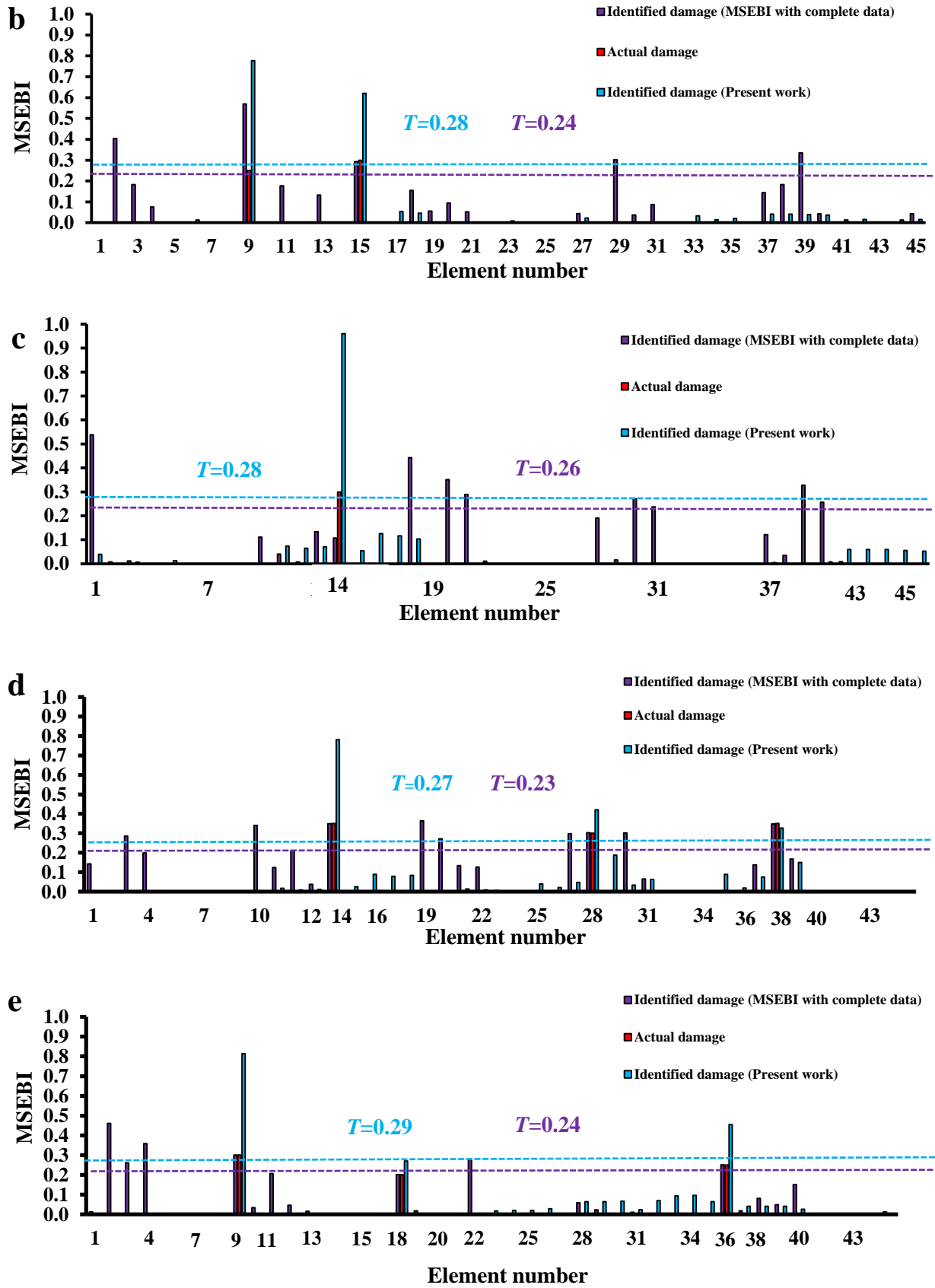


Fig. 3. Comparison of damage identification results for 45-element frame via the proposed approach and the method presented in Ref. [11] (a) Damage Case 1; (b) Damage Case 2; (c) Damage Case 3; (d) Damage Case 4 and (e) Damage Case 5.

5.2. Truss with 47 elements

A planar truss with 47 elements, as shown in Fig. 4, is studied as the second example [29].

The structure consists of 47 members and 22 nodes and it is manufactured from steel with specific mass and modulus of elasticity 0.3 lb/in³ and 30,000 ksi, respectively.

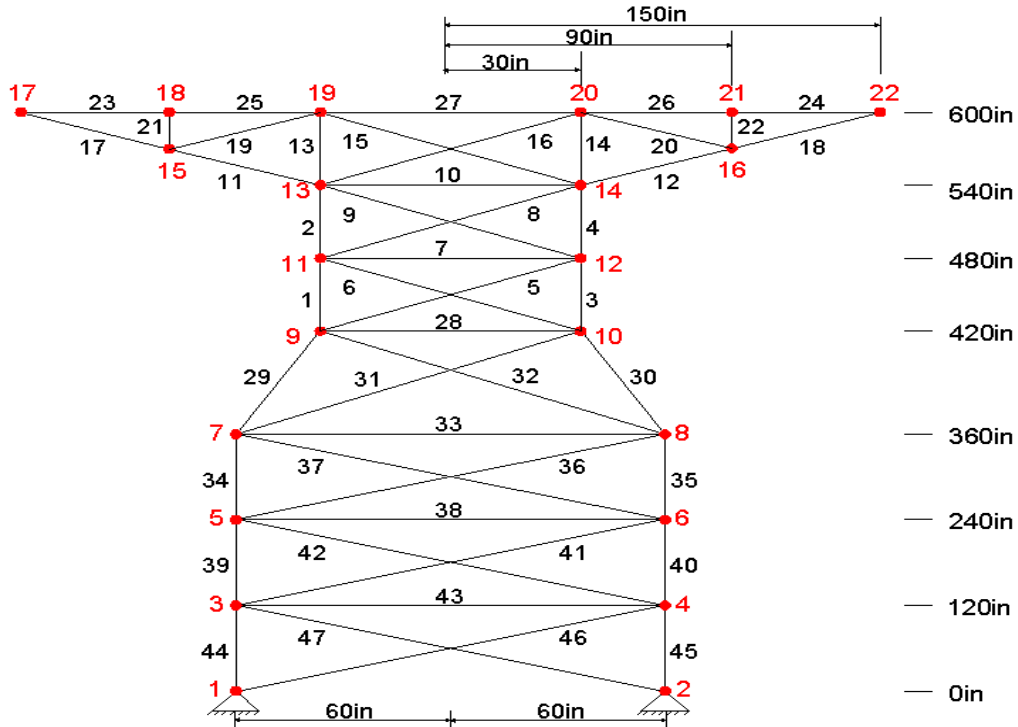


Fig. 4. Configuration of the truss with 47 elements.

Three damage scenarios provided in Table 3 are investigated by considering 3 modes polluting with 3% noise. For the efficiency assessment of the method, the damage

identification results of the present approach are compared with those of the method presented in Ref. [11].

Table 3. Damage cases applied to truss with 47 elements.

Damage Case 1		Damage Case 2		Damage Case 3	
Element No.	Damage Extent	Element No.	Damage Extent	Element No.	Damage Extent
11	0.3	10	0.3	10	0.3
20	0.2	30	0.3	-	-
38	0.3	-	-	-	-

Table 4. Optimal placement of sensors in 47-element truss for five optimization runs.

Case 1		Case 2		Case 3	
Node No.	DOF	Node No.	DOF	Node No.	DOF
8	1	10	1	9	1
12	2	12	2	13	2
20	1	15	1	17	1
Case 4		Case 5		Final sensor placement	
Node No.	DOF	Node No.	DOF	Node No.	DOF
9	2	7	1	7	1
12	1	12	2	12	2
18	1	16	1	16	1

The number of sensors for the truss is also assumed to be 3. Table 4 summarizes the optimal placement of sensors obtained by the proposed method for the five optimization runs and those selected here as final sensor placement. In the table, the number 1 and 2 for a DOF corresponds to the horizontal and vertical displacement of the node, respectively. Damage identification results of 47-element truss for various damage scenarios are shown in Fig. 5 using OSP achieved as well as

MSEBI when considering 3 modes with 3% noise. As can be observed in Fig. 5, the efficiency of the proposed method for accurately localizing damaged elements is higher than that of the method provided in Ref. [11] while the present method needs a limit number of sensors for damage localization. Moreover, the MSEBI with complete data of the structures localizes some intact elements as damaged ones.

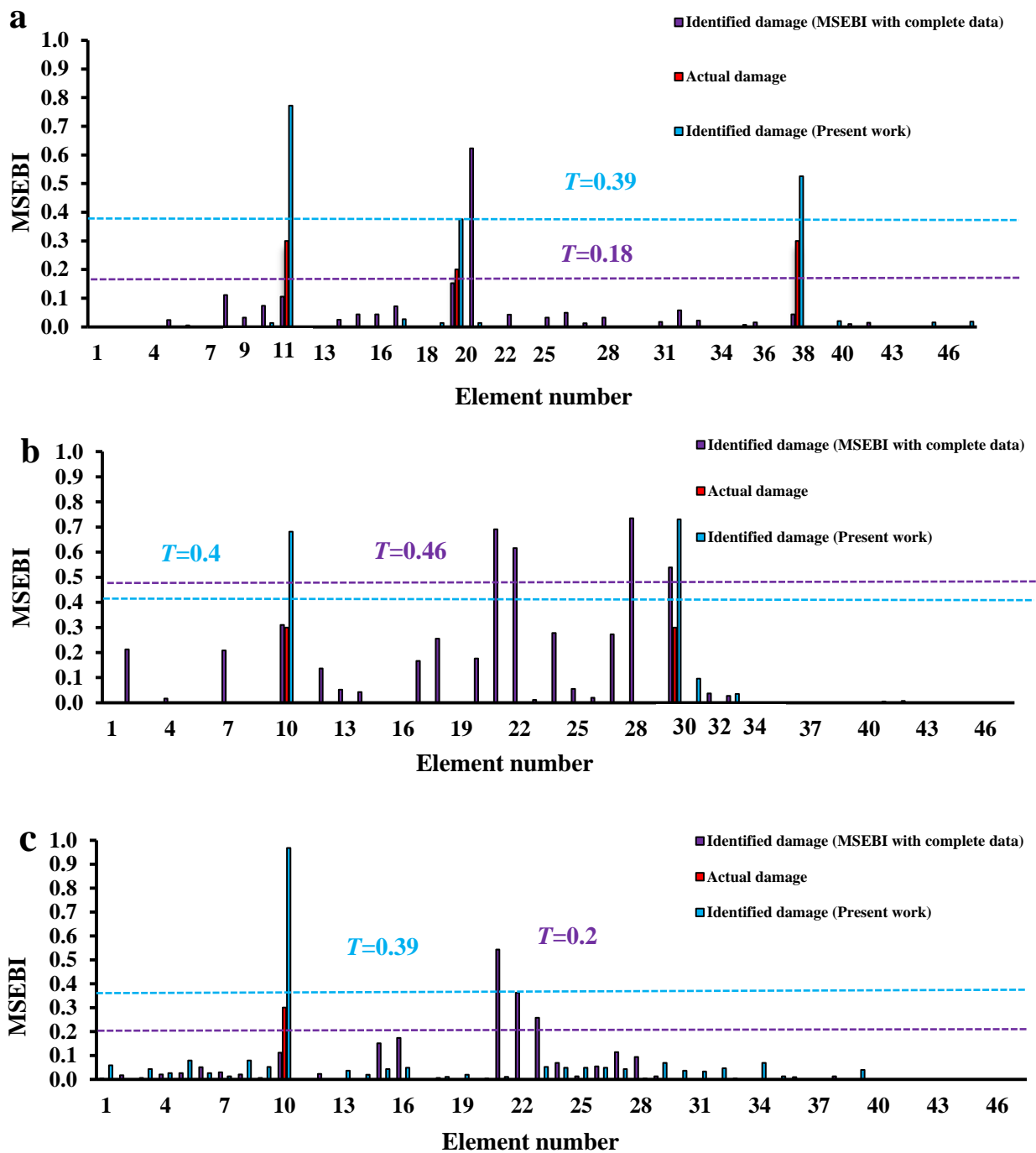


Fig. 5. Comparison of damage identification results of the truss achieved by the current study and the method presented in Ref. [11]: (a) Damage Case 1; (b) Damage Case 2 and (c) Damage Case 3.

5.3. The parametric study

In order to assess the effects of mode number and noise level on the effectiveness of the proposed method, the damage identification results of 47-element truss for considering the

different number of mode shapes and various noise levels are investigated. Fig. 6 shows the damage identification results when 1,3,5,7 and 9 modes are considered for the truss while the noise level is assumed to be 3%.

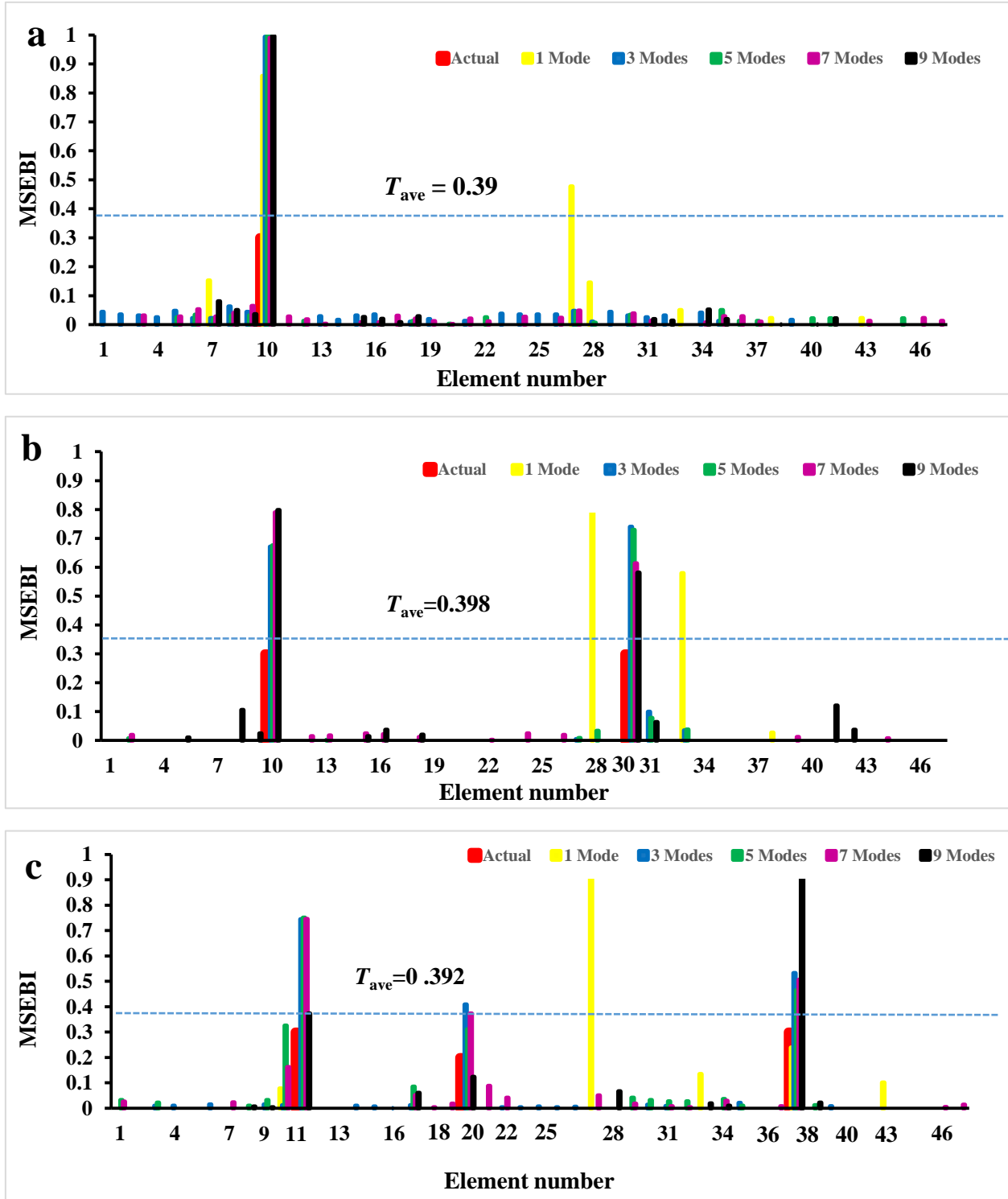


Fig. 6. Investigating the effect of mode number on the efficiency of the proposed method in identifying the damage of the truss: (a) single damage; (b) double damage and (c) triple damage.

Moreover, the damage identification results by considering the noise level 1, 4, 8, 12 and 16% while the number of modes is 3 are shown in Fig. 7. It should be noted that, since increasing the number of modes and noise level does not have a significant effect on the value of threshold, accordingly, the average value of thresholds signified in the figures by T_{ave} is used. As shown in Fig. 6, the proposed approach is not

equipped to localize damage using only a single mode. Nonetheless, the methodology gains more competence with increasing the number of modes so that the damage location is accurately identified by considering 3 modes. As a matter of fact, the correct arrangement of sensors by dint of the proposed method leads to reducing the destructive effects of other modes and increasing detection veracity.

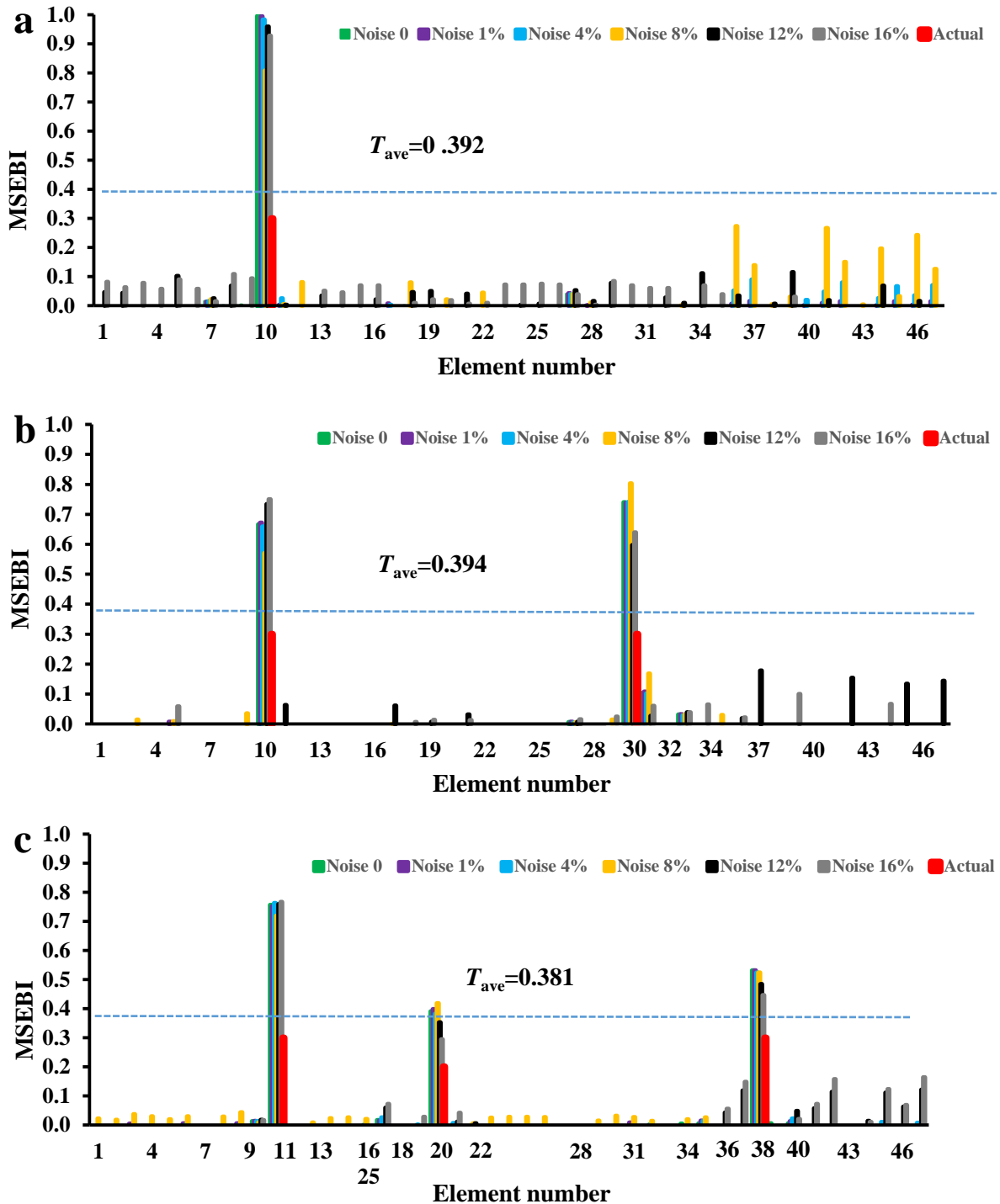


Fig. 7. Investigating the effect of noise level on the efficiency of the proposed method in identifying the damage of the truss: (a) single damage; (b) double damage and (c) triple damage.

Additionally, based on Fig. 7, once the truss is examined for the three damage scenarios by applying different noise levels up to 16 %, it is perceived that increasing noise level has no considerable effect on the reduction of detection accuracy, so that the proposed method clearly localizes damage even when 16% noise is considered. As a result, the increase of modes and noises in the structure has a slight effect on the index.

6. Conclusions

In this paper, an efficient two-stage method was proposed for damage assessment in truss and frame structures using noise-contaminated modal data gathered from a limited number of sensors. In the first step, the optimal position of sensors was determined by minimizing a correlation function between the reconstructed mode shapes through the iterated IRS method and the complete mode shapes of the structure via the BDE algorithm. In the second stage, the damage was localized using optimal position of the sensors already obtained and applying MSEBI. Two illustrative examples were selected from the technical literature, so as to appraise the viability of the proposed method in identifying damage. The following results were thus drawn:

- It was shown that the combination of MSE, iterated IRS, and BDE can be considered as an influential tool for accurately identifying damage.
- The performance of the proposed method for damage identification, compared to MSEBI provided in the literature was higher where MSEBI localized some spots other than the exact location of the damage.
- As the best modal information was achieved by placing the sensors in their optimal points, therefore, the method could accurately localize damage considering only 3 mode shapes.
- By adding different levels of the noise to mode shapes even up to 16%, the proposed method could precisely localize the damage with the least possible error for all scenarios.

The proposed two-stage method is an applied practice for damage assessment in truss and frame structures so as to it needs only a limited number of sensors for extracting modal data. However, a practical investigation to verify the suggested methodology is highly advantageous in the future research. Another challenging issue is how to minimize the number of DOF masters and related locations altogether, which can significantly affect damage identification results. This is likely to shape up the progress of future investigations.

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors contribution statement

Coding was done using MATLAB by Ali Asghar Mofidi. The output from the code, analyses and the drawing of graphs were done by Reza Yahyapour, all items was checked and then the article was written by Seyed Mohammad Seyedpoor and finally edited by Saeed Fallahian.

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