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Optical soliton solutions to the new Hamiltonian amplitude equation

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Abstract

In this study, the efficiency of the exponential rational function method for the new Hamiltonian amplitude equation has been investigated and the results have been graphically tested and analyzed using Mathematica software techniques. What is important is the high efficiency and accuracy of this method and the variety of answers that provide us with a wide range of answers.

Keywords: Hamiltonian amplitude equation, exponential rational function, Soliton solution
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1 Introduction

As nonlinear partial differential equation, the new Hamiltonian amplitude equation has and importance in much of area. This system can be used to describe various types of wave phenomena in mathematical physics. In the aim to describe new nonlinear phenomena which appear in many systems, several analytical methods of resolution have been developed and proposed in the literature. We can list the Biswas–Arshed equation [8, 13], Gerdjikov–Ivanov equation [4, 9], Radhakrishnan–Kundu–Lakshmanan equation [1, 11], Fokas–Lenells equation [2, 5], Kundu–Mukherjee–Naskar equation [6, 12], Chen–Lee–Liu equation [7, 14] and some other authors [3, 10]. All of these methods are used to solve a particular class of equations. This is to say that it does not exist a method to solve in a general way all the nonlinear equations that appear today. In this manuscript, we study the new Hamiltonian amplitude equation. It takes the following form:

$$iu_x + u_{tt} + 2\sigma|u|^2u - \varepsilon u_{xt} = 0,\ \sigma = \pm 1,\ \varepsilon \ll 1, \tag{1.1}$$

In the following, we will first introduce the exponential rational function, then we will state the application of the above method for the new Hamiltonian amplitude equation and the graphic analysis corresponding to each answer, and finally, we will state the result of the discussion.

1. ***Algorithm*** **of GERFM method**

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The main steps of GERFM are outlined below. At first we consider the following nonlinear PDE

$$\mathcal{L}(\psi, \psi_x, \psi_t, \psi_{xx}, \ldots) = 0. \tag{1.2}$$

Using the transformations $\psi = \psi(\xi)$ and$\xi = \sigma x - lt$, we reduce the nonlinear partial differential equation to the following ordinary differential equation:

$$\mathcal{L}(\psi, \psi', \psi'', \ldots) = 0, \tag{1.3}$$

where the values of σ and $l$will be found later. Now we consider that Eq. (1.3) has solution of the form

$$\psi(\xi) = A_0 + \sum_{k=1}^{M} A_k \Psi(\xi)^k + \sum_{k=1}^{M} B_k \Psi(\xi)^{-k}, \tag{1.4}$$

where

$$\Psi(\xi) = \frac{p_1 e^{q_1 \xi} + p_2 e^{q_2 \xi}}{p_3 e^{q_3 \xi} + p_4 e^{q_4 \xi}}. \tag{1.5}$$

The values of constants$_{p_i, q_i (1 \le i \le 4)}$, $_{A_0, A_k}$ and $_{B_k (1 \le k \le M)}$ are determined, in such a way that solution (1.4) always persuade Eq. (1.3). By considering the homogenous balance principle the value of M is determined [15].

1. ***Method* application:**

In order to obtain travelling wave solutions of Equation (1.1), we take the transformation by using the wave variables:

$$u(x,t) = e^{i\theta} u(\xi), \ \theta = \alpha x - \beta t, \ \xi = k(x - \lambda t). \tag{1.6}$$

By applying (1.6) into (1.1) we have imaginary and real parts as follows

$$i[(1 + 2\beta\lambda + \lambda\alpha\varepsilon + \beta\varepsilon) u'(\xi)] = 0, \tag{1.7}$$

and

$$k^2(\lambda^2 + \lambda\varepsilon) u''(\xi) - (\alpha + \beta^2 + \varepsilon\alpha\beta) u(\xi) + 2\sigma u^3(\xi) = 0. \tag{1.8}$$

Equating the coefficients of Eq. (1.7) to zero provides

$$1 + 2\beta\lambda + \lambda\alpha\varepsilon + \beta\varepsilon = 0.$$

In this case we apply the method for real part. By applying the homogenous balance method, balancing $u''$and u^3 in Eq. (1.8), we get$m = 1$. So, we can introduce the solution as (1.5):

$$u(\xi) = A_0 + A_1 \varphi(\xi) + B_1 \varphi(\xi)^{-1}. \tag{1.9}$$

Set 1: in this set we consider $r =$ [1; 1; 1; 1]and$s =$ [1, 1, 1, 1], then we have equation (1.5) as follows

$$\varphi(\xi) = -\frac{\cosh(\xi)}{\sinh(\xi)} \tag{1.10}$$

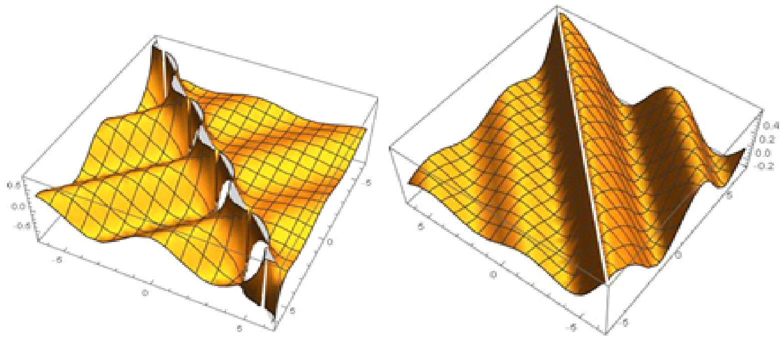
Inserting (1.10) into (1.9) then substituting in (1.8) and method outlined we have

$$A_0 = \frac{\sqrt{10}}{10\sigma} \sqrt{\sigma(\varepsilon\alpha\beta + \beta^2 + \alpha)}, A_1 = \frac{\sqrt{30}}{30\sigma} \frac{(\varepsilon\alpha\beta + \beta^2 + \alpha)}{\sqrt{\sigma(\varepsilon\alpha\beta + \beta^2 + \alpha)}}, B_1 = \frac{\sqrt{30}}{30\sigma} \sqrt{\sigma(\varepsilon\alpha\beta + \beta^2 + \alpha)}.,$$

So exact soliton solution of Eq. (1.1) determined as follows

$$u_1(x,t) = \tfrac{\sqrt{10}}{10\sigma} \sqrt{\sigma(\varepsilon\alpha\beta + \beta^2 + \alpha)} e^{i(\alpha x - \beta t)} - \tfrac{\sqrt{30}}{30\sigma} \tfrac{(\varepsilon\alpha\beta + \beta^2 + \alpha)}{\sqrt{\sigma(\varepsilon\alpha\beta + \beta^2 + \alpha)}} \left(\tfrac{\cosh k(x - \lambda t)}{\sinh k(x - \lambda t)}\right) e^{i(\alpha x - \beta t)} - \tfrac{\sqrt{30}}{30\sigma} \sqrt{\sigma(\varepsilon\alpha\beta + \beta^2 + \alpha)} \left(\tfrac{\sinh k(x - \lambda t)}{\cosh k(x - \lambda t)}\right) e^{i(\alpha x - \beta t)}$$

Given that we have$e^{i(\alpha x-\beta t)} = \cos(\alpha x-\beta t) + i\sin(\alpha x-\beta t)$. Therefore, each answer will have two real and imaginary parts, which we will have separate graphical analysis according to each part as follows.



A B

Image 1: graphical behavior related to $u_1(x,t)$- **A**: Real and **B**: imaginary

Set 2: In order we consider $r=[-1,0,1,1]$ and$s=[1,0,1,0]$, then we have equation (1.6) as follows

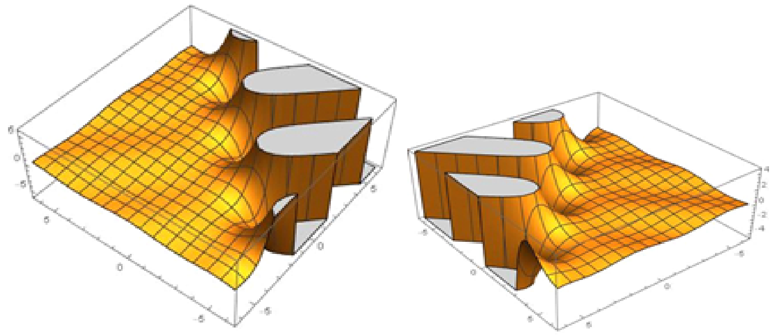
$$\varphi(\xi) = -\frac{1}{1+e^{\xi}}. \tag{1.11}$$

Inserting (1.11) into (1.9) then substituting in (1.8) and method outlined we have

$$A_0 = \frac{\sqrt{-6\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}}{6\sigma}, A_1 = \frac{3}{2}\frac{(\varepsilon\alpha\beta+\beta^2+\alpha)}{\sqrt{-6\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}}, B_1 = \frac{\sqrt{-6\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}}{12\sigma}.$$

So exact soliton solution of Eq. (1.1) determined as follows

$$u_2(x,t) = \frac{\sqrt{-6\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}}{6\sigma}e^{i(\alpha x-\beta t)} - \frac{3}{2}\frac{(\varepsilon\alpha\beta+\beta^2+\alpha)}{\sqrt{-6\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}}\left(\frac{1}{1+e^{k(x-\lambda t)}}\right)e^{i(\alpha x-\beta t)} - \frac{\sqrt{-6\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}}{12\sigma}\left(1+e^{k(x-\lambda t)}\right)e^{i(\alpha x-\beta t)}$$



A B

Image 2: graphical behavior related to $u_2(x,t)$- **A**: real and **B**: imaginary

Set 3: in this set we consider $r=[3;2;1;1]$and$s=$ $[1;0;1;0]$, then we have equation (1.6) as follows

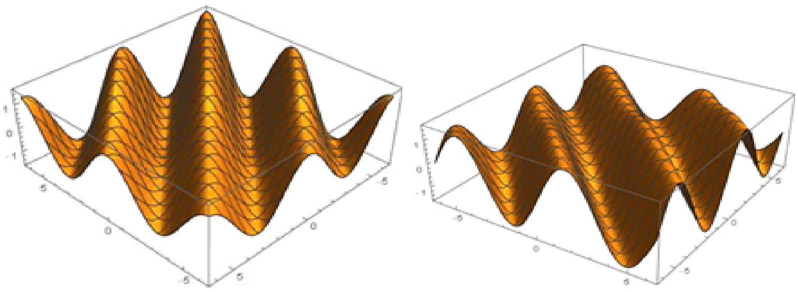
$$\varphi(\xi) = \frac{3e^{\xi}+2}{1+e^{\xi}}. \tag{1.12}$$

Inserting (1.12) into (1.9) then substituting in (1.8) and method outlined we have

$$A_0 = -\frac{6\sqrt{2}}{\sigma}\sqrt{\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}, A_1 = \frac{3\sqrt{2}}{2\sigma}\sqrt{\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}, B_1 = \frac{6\sqrt{2}}{\sigma}\sqrt{\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}.$$

So exact soliton solution of Eq. (1.1) determined as follows

$$u_3(x,t) = -\frac{6\sqrt{2}}{\sigma}\sqrt{\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}e^{i(\alpha x-\beta t)} + \frac{3\sqrt{2}}{2\sigma}\sqrt{\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}\left(\frac{3e^{k(x-\lambda t)}+2}{1+e^{k(x-\lambda t)}}\right)e^{i(\alpha x-\beta t)} + \frac{6\sqrt{2}}{\sigma}\sqrt{\sigma(\varepsilon\alpha\beta+\beta^2+\alpha)}\left(\frac{1+e^{k(x-\lambda t)}}{3e^{k(x-\lambda t)}+2}\right)e^{i(\alpha x-\beta t)},$$



A B

Image 3: graphical behavior related to $u_3\left(x,t\right)$- **A**: real and **B**: imaginary

Set 4: in this set we consider $r=[3,1,1,1]$and$s=\ [1,1,1,1]$, then we have equation (1.6) as follows

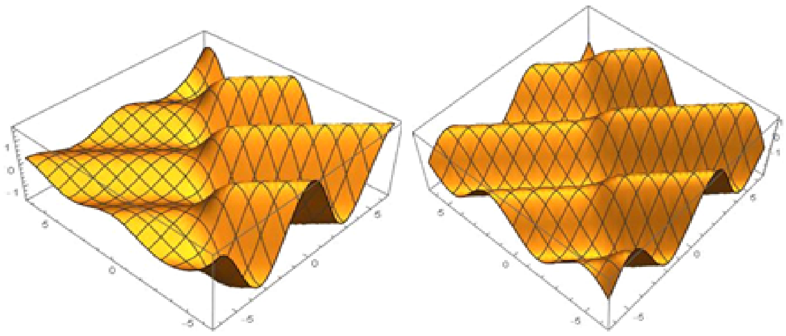
$$\varphi\left(\xi\right)=-\frac{2\cosh\left(\xi\right)+\sinh\left(\xi\right)}{\cosh\left(\xi\right)}, \tag{1.13}$$

Inserting (1.13) into (1.9) then substituting in (1.8) and method outlined we have

$$A_0=-\frac{2\sqrt{15}}{15\sigma}\sqrt{\sigma\left(\varepsilon\alpha\beta+\beta^2+\alpha\right)}, A_1=\frac{\sqrt{15}}{30\sigma}\sqrt{\sigma\left(\varepsilon\alpha\beta+\beta^2+\alpha\right)}, B_1=\frac{13\sqrt{15}}{30\sigma}\sqrt{\sigma\left(\varepsilon\alpha\beta+\beta^2+\alpha\right)},$$

So exact soliton solution of Eq. (1.1) determined as follows

$$u_4\left(x,t\right)=-\tfrac{2\sqrt{15}}{15\sigma}\sqrt{\sigma\left(\varepsilon\alpha\beta+\beta^2+\alpha\right)}e^{i(\alpha x-\beta t)}-\tfrac{\sqrt{15}}{30\sigma}\sqrt{\sigma\left(\varepsilon\alpha\beta+\beta^2+\alpha\right)}\left(\tfrac{2\cosh(k(x-\lambda t))+\sinh(k(x-\lambda t))}{\cosh(k(x-\lambda t))}\right)e^{i(\alpha x-\beta t)}-\tfrac{13\sqrt{15}}{30\sigma}\sqrt{\sigma\left(\varepsilon\alpha\beta+\beta^2+\alpha\right)}\left(\tfrac{\cosh(k(x-\lambda t))}{2\cosh(k(x-\lambda t))+\sinh(k(x-\lambda t))}\right)e^{i(\alpha x-\beta t)},$$



A B

Image 4: graphical behavior related to $u_4\left(x,t\right)$- **A**: real and **B**: imaginary

1. **Conclusion:**

The robust performance of the exponential rational function method gives us the ability to use this method for many equations, including the Hamiltonian amplitude equation used in this paper. The solutions obtained in this study are new and less complicated compared to other methods.

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