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Inference for the Pareto Type-I distribution using upper record ranked set sampling scheme

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Abstract

In some real-life situations, we will face restrictions of time and sample size which cause a researcher to not have access to all of the data. Therefore, it is valuable to study the estimation of parameters based on information of available data. In such situations, using appropriate sampling schemes, to more efficient estimators are important. The aim of the present paper is to study the Bayes estimators of parameters of the Pareto type-I model under different loss functions and compare among them as well as with the classical estimator named maximum likelihood estimator based on upper record ranked set sampling scheme. Here the informative Gamma prior is used as the conjugate prior distribution for finding the Bayes estimator. We also used symmetric loss functions such as squared error loss function and asymmetric loss functions such as linear-exponential loss function. We present the analysis of a Monte Carlo simulation to compare the performance of the estimators with respect to their risks (average loss over sample space) based on upper record ranked set sampling. Finally, one real data set is analyzed to illustrate the performance of the proposed estimators.

Keywords: Pareto type-I model, Bayesian estimator, Upper record ranked set sampling, Loss function, Maximum likelihood estimator
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1 Introduction

The problem of estimating the parameters of a model is an important estimation problem having wide practical applications in various quality control, agricultural, industrial, and medical experiments. Estimation problems discussed in the literature can be divided into two types, referred to as classical and Bayesian estimation. In the classical approach, the parameters are considered to be fixed. However, in many real-life situations, the parameters cannot be treated as constant. In view of this, we propose a Bayesian estimation procedure. The Bayesian approach as an alternative to the classical approach is in statistical inference. In the Bayesian approach, the unknown parameter is regarded as a realization of a random variable which belongs to some prior distribution. When prior information is available, sensitivity analysis focuses on the structure of the prior distribution; when noninformative or low informative priors are used, it focuses on how different choices of prior parameters may influence the posterior inference. The parameters of a prior distribution are called hyperparameters. In the Bayesian inference procedures, the performance of the estimator depends on the prior distribution and also on the loss function used. We specify a loss function $L_(\theta, \hat{\theta})$

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that describes the loss incurred by making an estimate $\hat{\theta}$ when the true value of the parameter is θ. In the Bayesian problem, the most commonly used loss function is the squared error (SE) loss function. This loss is symmetric and its use is very popular, perhaps, because of its mathematical simplicity. The symmetric nature of this function gives equal weight to overestimation as well as underestimation, while in the estimation of parameters of the lifetime model, overestimation may be more severe than underestimation or vice-versa. For more regard see, chapter 10 of [3] and chapter 4 of [9]. An asymmetric loss function is also useful. For example, in the estimation of reliability and failure rate functions, an overestimation is usually much more serious than an underestimate.

In this paper author obtained the maximum likelihood (ML) estimator of parameter and computed Bayes estimators by assuming gamma prior distribution based on upper record ranked set sampling (RRSS) as an informative sample with respect to both symmetric loss function such as SE loss function and asymmetric loss function such as linear-exponential (Linex) loss function for different values of hyperparameters. In order to compare estimators, we have considered the risk estimators.

Many authors have studied estimation problems by considering different sampling schemes and selecting various distributions. For example, Muttlak et al [11] studied estimation for $Pr(Y < X)$ by considering a bivariate exponential model, Salehi and Ahmadi [20] considered the estimation of stress and strength using upper RRSS from the exponential distribution. They also, with the collaboration of Dey [21], made a comparison between the RRSS scheme and the ordinary record statistics in estimating the unknown parameter of the proportional hazard rate model. Safaryian et al. [19] proposed some improved estimators including the preliminary test estimator as well as stein-type shrinkage estimator for stress-strength reliability using record ranked set sampling scheme. Sadeghpour et al. [18] presented the estimation of the parameter using a lower record ranked set sampling scheme under the generalized exponential distribution. Recently, Golzade Gervi et al. [7] considered the comparison of empirical Bayesian estimations and predictions based on record-ranked set sampling scheme with an inverse sampling scheme.

Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed (i.i.d) random variables with absolutely continuous cumulative distribution function (cdf) $F(x; \theta)$ and corresponding probability density function (pdf) $f(x; \theta)$ where θ is possibly a vector real-valued parameter. We say X_j is an upper record value of this sequence if its value is greater than that of all previous observations. Thus X_j is an upper record of this sequence if $X_i < X_j$ for all $i < j$. By definition, X_1 is the first upper record, which is referred to as the reference value or the trivial record. There is a similar definition for lower record values by considering the observations that fall below all previous observations. For a more comprehensive review and details on the records, see [1, 12, 2]. The most important use of record values arises in experiments in which specified characteristics measurements of a unit are made sequentially and only values that exceed or fall below the current extreme value are recorded. So the only available observations are record values. Record values arise in a wide variety of practical situations. Examples include oil and mining surveys, quality control, hydrology, sports achievements, seismology, the strength of materials, economics, industry, and climatology. In all the situations mentioned above any statistical inference must be done using record values. The number of ordinary records is often small but the RRSS scheme overcomes this problem. The sampling method in this article is the RRSS scheme (Salehi and Ahmadi [22]). In some practical experiments, considering suitable sampling schemes, in order to reduce the cost and increase efficiency are crucial. To this end, first, we give a brief description of RRSS scheme: consider n independent random sequences where the ith sequence sampling scheme is terminated whenever the ith upper record is observed. The only observations available for the analysis are the last upper record value in each sequences. In this sampling scheme, let us denote the last upper record for the ith sequence by $R_{i,i}$, then the available observations are $\mathbf{R} = (R_{1,1}, R_{2,2}, \cdots, R_{n,n})'$. Then, $\mathbf{R}$ is a record ranked set sample of size n. The following diagram can be used to describe this observational process:

$$\begin{array}{lllll} 1: \ \underline{R_{(1)1}} & & & & \to R_{1,1} = R_{(1)1} \\ 2: \ R_{(1)2} & \underline{R_{(2)2}} & & & \to R_{2,2} = R_{(2)2} \\ \quad \vdots & \vdots & \vdots & \vdots & \vdots \\ n: \ R_{(1)n} & R_{(2)n} & \cdots & \underline{R_{(n)n}} & \to R_{n,n} = R_{(n)n} \end{array} \tag{1.1}$$

where $R_{(i)j}$ is the ith ordinary upper record in the jth sequence. From [22], it can be seen that unlike the ordinary records, here $R_{i,i}$'s are independent random variables but not necessarily ordered with probability one. Then, by using the marginal density of ordinary records (see, [2]), the joint density of $\mathbf{R}$ is given by

$$f_{\mathbf{R}}(\mathbf{r}; \theta) = \prod_{i=1}^{n} \frac{\{-\log(1 - F(r_{i,i}; \theta))\}^{i-1}}{(i-1)!} f(r_{i,i}; \theta), \quad \theta \in \Theta, \tag{1.2}$$

where $\mathbf{r} = (r_{1,1}, r_{2,2}, \cdots, r_{n,n})'$ is the observed vector of $\mathbf{R} = (R_{1,1}, R_{2,2}, \cdots, R_{n,n})'$ and Θ is the parameter space. In fact, Eq (1.2) is the joint pdf of an upper RRSS of size n.

The RRSS scheme was also investigated by the other researchers. Among them, Eskandarzadeh et al. [5] described information measures for the RRSS scheme. Paul and Thomas [13] proposed concomitant the RRSS for situations that measuring of the variable of interest is costly or even impossible. Hassan et al. [8] discussed Bayesian comparison of the RRSS and the ordinary records based on generalized inverted exponential distribution. Recently, Golzade Gervi et al. [6] presented an overview of Bayesian prediction of future record statistics using upper record ranked set sampling scheme.

In this article, let us consider the Pareto type-I distribution. The Pareto distribution was originated by Vilfredo Pareto [14] as a model in such widely diverse areas as economics, reliability, hydrology, survival analysis, engineering, business and insurance, see [16, 17, 23]. The Pareto distribution is a power-law probability distribution that is used in description of social, quality control, scientific, geophysical, actuarial, and many other types of observable phenomena. The principle originally applied to describing the distribution of wealth in a society, fitting the trend that a large portion of wealth is held by a small fraction of the population. The Pareto type-I distribution (denoted by $X \sim Pa\ (\alpha, \beta)$) has the pdf

$$f(x; \alpha, \beta) = \frac{\alpha}{\beta}(\frac{x}{\beta})^{-(\alpha+1)}\ , \quad x \geqslant \beta, \tag{1.3}$$

and the cdf

$$F(x; \alpha, \beta) = 1 - (\frac{\beta}{x})^{\alpha}\ , \quad x \geqslant \beta, \tag{1.4}$$

where $\alpha(> 0)$ is a shape parameter and $\beta(> 0)$ is a scale parameter. We can easily observe that the s-th raw moment (i.e., moment about zero) and variance of the distribution are

$$E(X^s) = \begin{cases} \infty & \text{for } \alpha \leq s, \\ \frac{\alpha\beta^s}{\alpha - s} & \text{for } \alpha > s, \end{cases}$$

and

$$V(X) = \begin{cases} \infty & \text{for } \alpha \in (1, 2], \\ (\frac{\beta}{\alpha-1})^2 \frac{\alpha}{\alpha-2} & \text{for } \alpha > 2, \end{cases}$$

respectively. Henceforth, in the following sections, without lose of generality we consider $\beta = 1$. The layout of this paper is organized as follows. Bayesian estimation of the unknown shape parameter is considered in Section 2. The Bayes estimates are obtained under SE and LINEX loss functions based on upper RRSS. ML estimation based on upper RRSS is discussed in Section 3. A Monte Carlo simulation study is carried out in Section 4 to illustrate the results. A real data set is also analysed in Section 5. Section 6 concludes the paper with a brief discussion.

2 Bayesian estimation based on upper RRSS

In this section, we focus the Bayesian estimation of α of the $Pa\ (\alpha, 1)$ based on upper RRSS. Let $\mathbf{r} = (r_{1,1}, r_{2,2}, \cdots, r_{n,n})'$ be the observation of the random vector $\mathbf{R} = (R_{1,1}, R_{2,2}, \cdots, R_{n,n})'$, an upper RRSS of size n from $Pa\ (\alpha, 1)$. Substituting (1.3) and (1.4) into (1.2), the likelihood function of α given $\mathbf{r}$ will be obtained as

$$L(\alpha|\mathbf{r}) \propto \alpha^N \prod_{i=1}^{n} (\frac{1}{r_{i,i}})^{\alpha}, \tag{2.1}$$

where $N = \frac{n(n+1)}{2}$ and $r_{i,i}$ (upper record values from RRSS scheme) plays a key role in all inference procedures that can be achieved. In the Bayesian process we need a prior distribution for the parameter of α. In case of an informative prior, the use of prior information is equivalent to adding a number of observations to a given sample size, and therefore leads to a reduction of the variance/posterior risk of the Bayes estimates. Because α is nonnegative, we can use the following informative gamma prior with pdf

$$\pi_1(\alpha) = \frac{b^a}{\Gamma(a)} \alpha^{a-1} e^{-b\alpha}. \tag{2.2}$$

The hyperparameters $a, b(> 0)$ are assumed to be positive constants and $\Gamma(.)$ the complete gamma function. When, no prior information is available, it is better to consider the noninformative prior for the Bayesian analysis. Here, the Jeffrey's noninformative prior

$$\pi_2(\alpha) \propto \frac{1}{\alpha},$$

is specified by letting $a \to 0$ and $b \to 0$. Although it implies an improper prior on α, but the corresponding posterior is proper. Now, the posterior distribution of α given $\mathbf{r}$ denoted by $\Pi(\alpha|\mathbf{r})$, can be obtained using Bayes theorem for combining (2.1) and (2.2) as

$$\Pi(\alpha|\mathbf{r}) \propto \alpha^{N+a-1} e^{-\alpha(b+\sum\limits_{i=1}^{n} \log r_{i,i})}. \tag{2.3}$$

In other words, $\alpha|\mathbf{R} \sim \Gamma(N + a, b + \sum\limits_{i=1}^{n} \log r_{i,i})$. The posterior distribution summarizes available probabilistic information on the parameters in the form of prior distribution and the sample information contained in the likelihood function. The likelihood principle suggests that the information on the parameter should depend only on its posterior distribution. For parameter Bayes estimation, it is most common to use SE loss function, defined as

$$L_1(\alpha, \hat{\alpha}) = (\hat{\alpha} - \alpha)^2, \tag{2.4}$$

where, $\hat{\alpha}$ is the estimator for the parameter α. It is well known that the Bayes estimator of α, under this loss function, say $\hat{\alpha}_S$, is the mean of the posterior distribution. So, from (2.3), we obtain

$$\hat{\alpha}_S = E(\alpha|\mathbf{r}) = \int_0^\infty \alpha\Pi(\alpha|\mathbf{r})d\alpha = \frac{N + a}{b+\sum\limits_{i=1}^{n} \log r_{i,i}}. \tag{2.5}$$

Another useful loss function is the linear-exponential (LINEX) loss function. The Linex loss function is a convex but asymmetric loss function, and defined as

$$L_2(\alpha, \hat{\alpha}) = d(e^{c\Delta} - c\Delta - 1), \tag{2.6}$$

where $d > 0$ is the scale parameter and $c \neq 0$ is the shape parameter. Here, Δ represents the estimation error i.e., $\Delta = \hat{\alpha} - \alpha$. At initial, without loss of generality, it can be assumed that $d = 1$. A positive value of c is used when the overestimation is more serious than an underestimation while a negative values of c is vice-versa. For c close to zero, this loss function is approximately SE and therefore almost symmetric. Under the Linex loss function, the Bayes estimator for α is

$$\hat{\alpha}_L = -\frac{1}{c} \log E(e^{-c\alpha}|\mathbf{r}) = -\frac{1}{c} \log \big[\int_0^\infty e^{-c\alpha}\Pi(\alpha|\mathbf{r})d\alpha \big], \tag{2.7}$$

provided that $E(e^{-c\alpha}|\mathbf{r})$ exist and is finite (see, [24, 25]). $\hat{\alpha}_L$ can not be obtained in explicit form and hence must be solved numerically. In the next section, we will present ML estimation of α based on upper RRSS .

3 ML estimation based on upper RRSS

The maximum likelihood (ML) is one of the most important and widely used methods in statistics. The idea behind maximum likelihood parameter estimation is to determine the parameters that maximize the probability (likelihood) of the sample data. Furthermore, ML estimators are consistent and asymptotically normally distributed under some certain regularity conditions. In order to estimating of α based on upper RRSS via ML method, differentiating the log-likelihood function (2.1) with respect to α then setting to zero, we get

$$\frac{\partial \log L(\alpha|\mathbf{r})}{\partial \alpha} = \frac{N}{\alpha} - \sum_{i=1}^{n} \log r_{i,i} = 0, \tag{3.1}$$

from (3.1), it can be obtained a closed-form expression for the ML estimation of the parameter α, say $\hat{\alpha}_{ML}$, as a function of $r_{i,i}$ as

$$\hat{\alpha}_{ML} = \frac{N}{\sum_{i=1}^{n} \log r_{i,i}} \quad . \tag{3.2}$$

4 Monte Carlo simulation study

Simulation is a flexible methodology to analyze the behavior of proposed estimators. In present section, we conduct a Monte Carlo simulation to compare the performance of the classical and Bayesian methods of estimation based on upper RRSS. The performances of the estimators are compared by using estimated risks. To achieve this purpose, we use the following Algorithm:

•**Algorithm:**

Step 1. For given values of the hyperparameters a and b we generate α from prior distribution (2.2).

Step 2. For generated α an upper RRSS of size n, $\mathbf{r} = (r_{1,1}, r_{2,2}, \cdots, r_{n,n})'$, is generated from (1.3).

Step 3. For given $\mathbf{r}$ in **Step 2**, compute $\hat{\alpha}_{ML}$ and the Bayes estimators with respect to both symmetric and asymmetric loss functions in (2.5), (2.7) and (3.2), respectively.

Step 4. Compute for B replications, the ERs for all the Bayes estimators obtained in **Step 3**.

The acronym ER stands for the estimated risk (average loss over sample space). Thus

$$ER(SE) = \frac{1}{B}\sum_{i=1}^{B} L_1(\alpha_i, \hat{\alpha_i}) \quad \& \quad ER(Linex) = \frac{1}{B}\sum_{i=1}^{B} L_2(\alpha_i, \hat{\alpha_i}),$$

where $\hat{\alpha_i}$ denotes the estimator of α_i. We assumed that here the simulated data has been generated from the Pareto type-I distribution with parameters 1/5 and 1. Also, we took $n = 2(1)5$. For gamma prior, in first case, we use $(a = b = 2)$ and in the second case, we take $(a = b = 3)$, which are denoted by cases I-II, respectively. We call this prior as prior1. Since we do not have any prior information, we consider Jeffrey's prior with $a = b = 0.001$. We call this prior as prior0. The values of parameter in Linex loss function is considered to be $c = 1, 2$. In this simulation design, all of the computations are performed using the R-statistical programming language based on B=1000 replications. The simulated ER of the estimators obtained in the Table 1.

Table 1: Estimated risk of the estimators for $n = 2(1)5$ based on Monte Carlo simulation.

n	a	b	$Squared\ error$	$Linex(c=1)$	$Linex(c=2)$
2	0.001	0.001	77.20	24.29	24.34
	2	2	0.89	0.63	0.64
	3	3	0.76	0.17	0.55
3	0.001	0.001	51.86	18.13	20.36
	2	2	0.87	0.11	0.22
	3	3	0.64	0.05	0.19
4	0.001	0.001	43.70	17.07	19.04
	2	2	1.97	0.88	1.65
	3	3	0.55	0.26	0.61
5	0.001	0.001	42.32	16.27	19.01
	2	2	4.93	4.14	7.43
	3	3	0.39	1.43	2.35

On the light of this simulation results we can conclude:

1. In both loss functions, the ER's decreases with the joint increase of a and b. Notice that, from (2.2) the priors have the same means. However, the variance of case II is smaller than the case I. For this reason, case II works better than case I. The results under SE loss function are plotted in Figure 1. The corresponding result under Linex loss function is similar to that of SE loss function.

2. The estimators obtained based on gamma prior namely prior1 have smaller ER as compared to Jeffrey's prior namely prior0, when sample size become large. For instance, we have plotted the results under SE and Linex loss functions in Figure 2, for $n = 2(1)5$ and $c = 1$.

3. Almost everywhere, the estimated risks decrease as n increases. Moreover, when n increases, estimators under the Linex loss function performs better than the SE loss function. The estimated risks in Linex loss function are increasing with respect to c. For instance, see the results in Figure 3. The corresponding plot based on $c = 2$ is similar to that of $c = 1$.

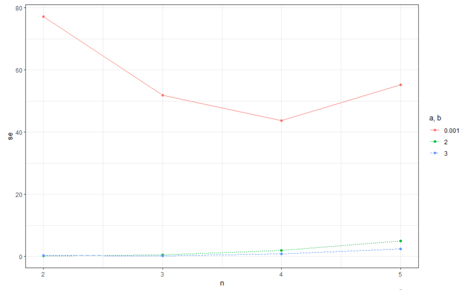

Figure 1: The plot of ER versus n, with the joint increase of a and b under SE loss function.

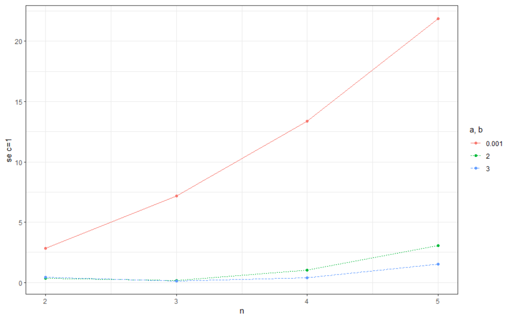

Figure 2: The plot of ER versus n under SE and Linex loss functions for $n = 2(1)5$ and $c = 1$.

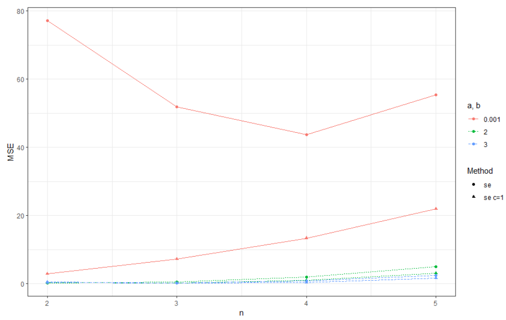

Figure 3: The comparison of ER under SE and Linex loss functions for $n = 2(1)5$ and $c = 1$.

5 A real example

Here, in order to demonstrate the results in the previous section, we intend to present the analysis of one real data set. In this regard, we consider the time of successive failures of the air conditioning system of each member of a fleet of Boing 720 jet airplanes data (say, X) reported by Proschan [15]. We have used three tests, namely the Kolmogorov-Smirnov (K-S), the Anderson-Darling and the Cramer Von-Mises in order to see whether Pareto type-I model is adequate to be fittted on this data set. First of all, the Kolmogorov-Smirnov (K-S) test has been employed. The resulting p-value of the K-S test for the data, X, is 0.626. We observe that the fit Pareto type-I model to the above data set is reasonable. Also, the Pareto type-I model was one of the models fitted on X based on suggestions of Easy Fit software. In more detail, based on the maximum likelihood approach, we have $X \sim Pa(0.8533, 2.8163)$. For more certainly, based on Anderson-Darling test, the test statistic equals 4.211 and the corresponding p-value equals 0.072. Since p-value is more than 0.05, we can conclude that we have sufficient evidence to say data X, does follow a Pareto type-I distribution. Also, under Cramer Von-Mises test, test statistic equals 2.18 and the corresponding p-value equals 0.0014. Since The p-value of Cramer Von-Mises test turns out to be extremely small, we can conclude that we have sufficient evidence to say data X, does not follow a normal distribution. This matches the result we expected since we know (Based on K-S test and Anderson-Darling test) that our data actually follows a Pareto distribution. The extracted records from a part of this dataset, is $\mathbf{r} = (r_{1,1} = 194, r_{2,2} = 447, r_{3,3} = 310)$, so we observed a RRSS of size $n = 3$. The MLE of the parameter α for planes 7907, 7908 and 7909 based on $\mathbf{r}$ and X, is $\hat{\alpha}_{ML} = \frac{N}{\sum_{i=1}^{n} \log r_{i,i}} = 0.8075$. In the following, we can see

$$\hat{\alpha}_S = \frac{N + a}{b + \sum_{i=1}^{n} \log r_{i,i}} = \begin{cases} 0.8076 & ; \alpha = \beta = 0.001, \\ 0.8484 & ; \alpha = \beta = 2, \\ 0.8629 & ; \alpha = \beta = 3, \end{cases}$$

It is observed as it was expected, the Bayes estimator of α based on SE loss function and the case of Jeffrey's prior $(a = b = 0.001)$, is very close to the $\hat{\alpha}_{ML}$. As mentioned earlier in section 2, the Bayes estimator of θ under Linex loss function must be solved by an appropriate numerical method. Clearly, the mean squared error of the estimators does not have a closed form. As the mean squared errors of the point estimators are a functions of α, we estimated the values of the mean squared errors by substituting their respective the Bayes estimator of α. In order to check the performances of the estimators, the bootstrap method is employed once again to re-sample the data and so provide more observations for the estimators. We have

$$\hat{\alpha}_L = \begin{cases} (0.8535), \ 0.8342 & ; \alpha = \beta = 0.001, \\ (0.8493), \ 0.7938 & ; \alpha = \beta = 2, \\ (0.8472), \ 0.7921 & ; \alpha = \beta = 3. \end{cases}$$

It is clear that, the above estimated values became close to the exact value, $\alpha = 0.8533$, in most situations. The corresponding Boxplots of the observations are plotted in Figure 4 for $a = b = 0.001$.

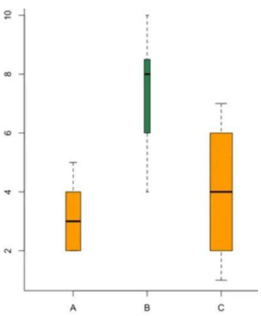

Figure 4: Comparison of ER (A: Linex(c = 1), B: Se, C: Linex(c = 2)).

It can be seen that the interquartile range in Linex(c = 1) is less than that of Linex(c = 2) and Se. The Boxplot for $a = b = 2$ and 3 is similar to that of $a = b = 0.001$. The estimated risk of the estimators are reported in Table 2. The values in parentheses reefers to the results for $c = 1$.

Table 2: The estimated risk based on real data.

			Squared error	Linex
n	a	b	$ER(\hat{\alpha_S})$	$ER(\hat{\alpha_L})$
3	0.001	0.001	0.4876	(0.3667) 0.4468
	2	2	0.4187	(0.3582) 0.4081
	3	3	0.3508	(0.3317) 0.3410

Therefore, as we have already observed in the simulation study, in Figure 4 and Table 2 too, for fixed n, the performance of the estimators under Linex loss function is better than Se loss function.

6 Discussion

In this paper, we obtained Bayes and maximum likelihood estimations for the parameter on the basis of upper record ranked set sampling from the pareto type-I distribution. We have derived Bayes estimators via informative and noninformative priors under both symmetric and asymmetric loss functions. These point estimators have been compared according to the criteria estimated risk by using Monte Carlo simulation. The simulation results indicate that almost everywhere, the estimated risks are decreasing with respect to n. By comparing symmetric and asymmetric loss functions, we concluded that, the estimator under the linear-exponential loss function is better than the squared error loss function as n increases. Also, evaluating the performance of informative and noninformative priors, one can observe that informative priors namely prior1 have smaller estimated risk as compared to noninformative prior namely prior0. The estimated risks are decreasing as the prior parameters increase, and consequently as expected the Bayes estimators obtained based on case II is more efficient than the Bayes estimators based on case I. Because Bayesian inference is sensitive to the choice of hyperparameters for informative priors, care must be taken in the selection of values. In the Bayesian estimation, if the hyperparameters are unknown, they can be estimated by using the empirical Bayes method (Maritz and Lewin [10]), or the hierarchical method (Bernardo and Smith [4]). Finally, the efficiency of some of the obtained results is illustrated throughout using real data. The results in the real data section confirm the results of the simulation section.

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