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Large deviation principle for a mixed fractional, jump and local process

Ibrahima Sané*, Raphaêl Diatta, Clément Manga, Alassane Diédhiou

Université Assane Seck, Département de Mathématiques, Laboratoire Mathématiques et Applications, B.P. 523, Ziguinchor, Sénégal

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Abstract

We study the asymptotic behaviour of a solution of a mixed differential equation driven by an independent fractional Brownian motion with Hurst index $H \in (0;1)$ and compensated Poisson process and a local time. This study consists in determining the uniform Freidlin-Wentzell estimates in a temporal distribution space $\mathcal{S}'(\mathbb{R})$. The approach is purely probabilistic.

Keywords: Large deviations principle, Fractional Brownian motion, Principle contraction, Poisson process, Skorohod problem
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1 Introduction

let's consider the stochastic differential equation (SDE) in short, defined in the probability space $(\mathcal{S}'(\mathbb{R}), \mathcal{B}(\mathcal{S}'(\mathbb{R})), \mathbb{P})$

$$\begin{cases} X_t^\varepsilon = x_0 + \int_0^t b(X_r^\varepsilon)dr + \varepsilon \int_0^t \sigma(X_r^\varepsilon)dB_r^H + \varepsilon \int_0^t \int_{\mathbb{R}^*} K(x, X_r^\varepsilon)\bar{N}(dx, dr) + L_t^\varepsilon, t \in [0;T] \\ X_0^\varepsilon = x_0 \end{cases} \tag{1.1}$$

where the following assertions hold:

- ⋆ $x_0 \in \mathbb{R}^*$ is a measurable random variable to value in the tempered distribution space $\mathcal{S}'(\mathbb{R})$, dual space of Schwartz space on which $\mathcal{B}(\mathcal{S}'(\mathbb{R}))$ is a Borel algebra ;
- ⋆ b, σ : $[0;T] \times \mathcal{S}'(\mathbb{R}) \to \mathcal{S}'(\mathbb{R})$ and $K : [0;T] \times \mathbb{R}^* \times \mathcal{S}'(\mathbb{R}) \to \mathbb{R}^* \times \mathcal{S}'(\mathbb{R})$ are measurable functions such that the integrals are defined as white noise integral (see Siaska [18]). b, σ and K satisfy the following assumptions:

Assumption 1.1. For almost all $t \in [0;T]$ and for $\Phi, \Psi \in \mathcal{S}'(\mathbb{R})$ there exist constants M and L such that

(i) $|b(\Phi)| \leq M, \qquad |\sigma(\Phi)| \leq M, \qquad |K(x, \Phi)| \leq M;$

*Corresponding author
Email addresses: i.sane2318@zig.univ.sn (Ibrahima Sané), r.diatta1232@zig.univ.sn (Raphaêl Diatta), cmanga@univ-zig.sn (Clément Manga), adiedhiou@univ-zig.sn (Alassane Diédhiou)

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(ii) $|b(\Phi)-b(\Psi)| \leq L|\Phi-\Psi|, \quad |\sigma(\Phi)-\sigma(\Psi)| \leq L|\Phi-\Psi|, \quad |K(x,\Phi)-K(x,\Psi)| \leq L|\Phi-\Psi|.$

⋆ L_t^ε is non-decreasing continuous process such that

$$L_t^\varepsilon = \begin{cases} 0 \ if \ t=0 \\ \\ \int_0^t 1_{\{X_r^\varepsilon=0\}}(X_r^\varepsilon)dL_r^\varepsilon \ \ if \ \ t\in[0;T] \end{cases} \tag{1.2}$$

The local time L_t^ε is increase when and only when the process is zero.

On the one hand, if the local time is zero in (1.1), Bai and Mai [2] have proved the existence and uniqueness of solution. On the other hand, when the Poisson process is zero in (1.1), Diédhiou and al. [10] have established a large deviation principle. Many authors have established the large deviations principle for a SDE driven by a Brownian motion and a Poisson process (see Yumeng Li [16]) To the best of our knowledge, no study of the large deviations principle on SDE driven by a fractional Brownian motion, a Poisson process and a local time simultaneously has been done. This is the motivation behind our study.

The paper is organized as follows: Section 2 contains some definitions and theorems of the fractional Brownian motion, Poisson process and large deviation principle which we need for our results, Section 3 contain our main results.

2 Preliminaries

Consider a white noise space $(\mathcal{S}'(\mathbb{R}), \mathcal{B}(\mathcal{S}'(\mathbb{R})), \mathbb{P})$ and denote $\langle .,. \rangle$ the scalar product. We define the spaces of continuous functions of integrable squares by:

$$L^2_\phi(\mathbb{R}) = \left\{ f\in\mathcal{S}'(\mathbb{R}) \text{ such that } s,t\in[0,T], |f|^2_{\phi,t} = \int_0^t\int_0^s f(r)f(u)\phi(r,u)dudr < +\infty \right\}$$

$$L^2(\mathbb{R}^*\times\mathbb{R}) := \left\{ \varphi\in\mathcal{S}'(\mathbb{R}^*\times\mathbb{R}), \Delta(\varphi) = \langle 1\otimes\dot{\nu}, \lambda(\varphi)1_{[0,t]}\rangle = \int_0^t\int_{\mathbb{R}^*}\lambda(\varphi(x,r))\nu(dx)dr < +\infty \right\}$$

where $\lambda(\varphi) = \varphi e^\varphi - e^\varphi + 1$.

Definition 2.1. (see Biagini and al [4]; Hu and al [14]) Let B_t^H be a fractional Brownian motion (fBm in short), for $\omega\in\mathcal{S}'(\mathbb{R})$, the process

$$\langle \omega, f1_{[0,t]}\rangle = \int_0^t f(r)dB_r^H \quad \text{is Gaussian process with covariance}$$

$$|f|^2_{\phi,t} = \langle f1_{[0,t]}, f1_{[0,s]}\rangle_\phi = \int_0^t\int_0^s f(r)f(u)\phi(r,u)dudr \ , \quad \forall \ f\in L^2_\phi(\mathbb{R}).$$

where

$$\phi(t,s) = \frac{\partial^2\mathbb{E}(B_t^HB_s^H)}{\partial t\partial s} = H(2H-1)|t-s|^{2H-2}.$$

Definition 2.2. (see Lokka [17]) For $\eta\in\mathcal{S}'(\mathbb{R}^*\times\mathbb{R})$, the stochastic integral of $\varphi\in L^2(\mathbb{R}^*\times\mathbb{R})$ with respect to $\bar{N}$ is defined by

$$\langle \eta - 1\otimes\dot{\nu}, \varphi 1_{[0,t]}\rangle := \int_0^t\int_{\mathbb{R}^*}\varphi(x,r)\bar{N}(dx,dr). \tag{2.1}$$

Definition 2.3. (Dembo and Zeitouni [7], Deuschel and Stroock [8]) The family $(X_t^\varepsilon)_{\varepsilon>0}$ of probability $\mathbb{P}^\varepsilon$ is said to satisfy large deviation principle if there exists a rate function I defined on $\mathbb{L}^2$ and a speed ε tending to 0 such that:

(i) $0\leq I(h)\leq+\infty$, for all $h\in\mathbb{L}^2$;

(ii) I is lower semi-continuous that is, for all $a<+\infty$, $\{h: I(h)\leq a\}$ is a closed of $\mathbb{L}^2$;

(iii) for all $a<+\infty$, $\{h: I(h)\leq a\}$ is a compact of $\mathbb{L}^2$, in which case I is a good rate function;

(iv) for any closed set $C \subset \mathbb{L}^2$,

$$\lim_{\varepsilon\to 0}\sup \varepsilon \log \mathbb{P}^\varepsilon(X_t^\varepsilon \in C) \leq -\inf_{h\in C} I(h) \tag{2.2}$$

(v) for any open set $O \subset \mathbb{L}^2$,

$$\lim_{\varepsilon\to 0}\inf \varepsilon \log \mathbb{P}^\varepsilon(X_t^\varepsilon \in O) \geq -\inf_{h\in O} I(h) \tag{2.3}$$

Theorem 2.4. (Dembo and Zeitouni[7]) Let E_1 and $E_2 \subset \mathbb{L}^2$ and $g : E_1 \to E_2$ is a continuous function. If the family $(X_t^\varepsilon)_{\varepsilon>0}$ satisfies a large deviations principle of a rate function I on E_1, then the family $g((X_t^\varepsilon)_{\varepsilon>0})$ satisfies the LDP on E_2 with a rate function J defined by:

$$J(z) = \inf\{I(h) : h \in E_1,\ z = g(h)\}, \quad \text{for each} \quad z \in E_2.$$

3 Main Results

The aim of this paper is to establish the large deviations principle for solution $(X_t^\varepsilon, L_t^\varepsilon)$ (1.1) by assuming that the fBm B_t^H and the Poisson process $\bar{N}_t$ are independent. We use the Azencott [1] method according to the Freidlin-Wentzell [13] estimates. The main result of this article is the Theorem (3.7), the proof of which is based on the following propositions, lemmas and theorems.

Consider the family $(\varepsilon B_t^H + \varepsilon \bar{N}_t)_{(\varepsilon>0)}$, with $t \in [0;T]$ obtained from the stochastic differential equation (1.1) without the term L_t^ε, when we suppose that the drift is zero and the diffusion coefficients are equal to the identity and we denote by $\mathbb{P}^\varepsilon$ its probability.

Assume that $\mathbb{P}^\varepsilon = \mathbb{P}_\phi^{H,\varepsilon} \times \nu^\varepsilon$ where $\mathbb{P}_\phi^{H,\varepsilon}$ is the probability measure of the family (εB_t^H) and ν^ε is the probability measure of $(\varepsilon \bar{N}_t)$. Let $\mathbb{L}^2 = L_\phi^2(\mathbb{R}) \times L^2(\mathbb{R}^* \times \mathbb{R})$ denote the space of integrable square functions $h : [0,T] \to \mathbb{R}$, with the norm $\|.\|_{\mathbb{L}^2}$ is defined by $\|h\|_{\mathbb{L}^2} = \sup_{0\leq r\leq t} |h(r)|$, for all $h \in \mathbb{L}^2$ and $t \in [0,T]$. It is well know (see [10]) that:

Theorem 3.1. The family $(\varepsilon B_t^H + \varepsilon \bar{N}_t)_{(\varepsilon>0)}$ satisfies the large deviations principle with the good rate function $I : \mathbb{L}^2 \to [0,+\infty]$ given by

$$I(f,\psi) = \begin{cases} \frac{1}{2}|f|_\phi^2 + \Delta(\varphi), & \text{if } (f,\varphi) \in \mathbb{L}^2 \\ \\ +\infty & \text{otherwise.} \end{cases} \tag{3.1}$$

In other word:

* I is a good rate function;
* for all closed set $C \subset \mathbb{L}^2$,

$$\lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \mathbb{P}^\varepsilon(\varepsilon B_t^H + \varepsilon \bar{N}_t \in C) \leq -[\frac{1}{2}|f|_\phi^2 + \Delta(\varphi)];$$

* for any open set $O \subset \mathbb{L}^2$,

$$\lim_{\varepsilon\to 0}\inf \varepsilon^2 \log \mathbb{P}^\varepsilon(\varepsilon B_t^H + \varepsilon \bar{N}_t \in O) \geq -[\frac{1}{2}|f|_\phi^2 + \Delta(\varphi)].$$

Proof . see [11] □

For $\Lambda \in \mathbb{L}^2$, define an operator $\Gamma : \mathbb{L}^2 \to \mathbb{L}^2$ by

$$\Gamma\Lambda_t = \Lambda_t - \inf_{0\leq s\leq t}(\Lambda(s) \wedge 0) \tag{3.2}$$

for $t \in [0;T]$ satisfying the following inequality:

$$\sup_{0\leq r\leq t} |\Gamma\psi_1(r) - \Gamma\psi_2(r)| \leq 2 \sup_{0\leq r\leq t} |\psi_1(r) - \psi_2(r)|,$$

see [10], for more details. By the reflection principle, the solution of (1.1) is given by

$$\begin{cases} X_t^\varepsilon = \Gamma Z_t^\varepsilon \\ \\ L_t^\varepsilon = \Gamma Z_t^\varepsilon - Z_t^\varepsilon \end{cases} \tag{3.3}$$

where Z^ε is a solution of the following stochastic differential equation:

$$Z_t^\varepsilon = x_0 + \int_0^t b(\Gamma Z_r^\varepsilon)dr + \int_0^t \sigma(\Gamma Z_r^\varepsilon)dB_r^H + \int_0^t \int_{\mathbb{R}^*} K(x, \Gamma Z_r^\varepsilon)\bar{N}(dx, dr),\ t \in [0;T] \tag{3.4}$$

and we denote the probability law of X_t^ε by $\mu^\varepsilon = \mathbb{P}^\varepsilon \circ F^{-1}$ where

⋆ $\mathbb{P}^\varepsilon$ is the probability law of $\varepsilon(B_t^H + \bar{N}_t)$;

⋆ F is a deterministic function associated to f and φ by solutions of the following ordinary differential equations:

$$\begin{cases} F(f,\varphi)(t) = h_t = x_0 + \int_0^t b(h_r)dr + \int_0^t \sigma(h_r)f_r\phi(r,s)dr + \int_0^t \int_{\mathbb{R}^*} K(x,h_r)(e^{\varphi(x,r)} - 1)\nu(dx)dr + \eta(h_t) \\ F(f,0)(t) = F_\phi(f_t) = z_t = x_0 + \int_0^t b(z_r)dr + \int_0^t \sigma(z_r)f_r\phi(r,s)dr + \eta(h_t) \\ F(0,\varphi)(t) = F_\nu(\varphi_t) = g_t = x_0 + \int_0^t b(g_r)dr + \int_0^t \int_{\mathbb{R}^*} K(x,g_r)(e^{\varphi(x,r)} - 1)\nu(dx)dr + \eta(g_t) \\ F(0,0)(t) = m_t = x_0 + \int_0^t b(m_r)dr + \eta(m_t) \end{cases} \tag{3.5}$$

where $\eta(h_t) = \int_0^t \chi_{\{h_r=0\}}d\eta_r$ is an increasing continuous function. Similar as (3.3), F can also be written as for $f \in L^2_\phi(\mathbb{R})$ and $\varphi \in L^2(\mathbb{R})$

$$\begin{cases} F(f,\varphi)(t) = \Gamma\Lambda_t(f,\varphi) \\ \eta(f,\varphi)(t) = \Gamma\Lambda_t(f,\varphi) - \Lambda_t(f,\varphi) \end{cases} \tag{3.6}$$

where Λ is a solution of the following stochastic equations:

$$\begin{cases} \Lambda_t(f,\varphi) = x_0 + \int_0^t b(\Gamma\Lambda_r(f,\varphi))dr + \int_0^t \sigma(\Gamma\Lambda_r(f,\varphi))f_r\phi(r,s)dr + \int_0^t \int_{\mathbb{R}^*} K(x,\Gamma\Lambda_r(f,\varphi))(e^{\varphi(x,r)} - 1)\nu(dx)dr \\ \Lambda_t(f,0) = x_0 + \int_0^t b(\Gamma\Lambda_r(f,0))dr + \int_0^t \sigma(\Gamma\Lambda_r(f,0))f_r\phi(r,s)dr \\ \Lambda_t(0,\varphi)(t) = x_0 + \int_0^t b(\Lambda_r(0,\varphi))dr + \int_0^t \int_{\mathbb{R}^*} K(x,\Lambda_r(0,\varphi))(e^{\varphi(x,r)} - 1)\nu(dx)dr \\ \Lambda_t(0,0) = x_0 + \int_0^t b(\Gamma\Lambda_r(0,0))dr \end{cases} \tag{3.7}$$

for which $(f,\varphi) \in \mathbb{L}^2$ are induced respectively by LDP of the fBm and Bm.

Proposition 3.2. Assume $\Lambda_t(0;0)$ defined in (3.7) and under (1.1), then for $R > 0$ and $\delta > 0$ there exists $\alpha > 0$ such that

$$\lim_{\varepsilon\to 0} \sup \varepsilon^2 \log \mu^\varepsilon\{\|Z_t^\varepsilon - \Lambda_t(0;0)\|_{\mathbb{L}^2} > \delta, \|B_t^H + \bar{N}_t\|_{\mathbb{L}^2} < \alpha\} < -R. \tag{3.8}$$

Proof . For $R > 0$ and $\delta > 0$, we have

$$\begin{aligned} |Z_t^\varepsilon - \Lambda_t(0;0)| &\le \int_0^t |b(\Gamma Z_r^\varepsilon) - b(\Gamma\Lambda_r(0;0))|dr + |\varepsilon \int_0^t \sigma(\Gamma Z_r^\varepsilon)dB_r^H + \varepsilon \int_0^t \int_{\mathbb{R}^*} K(x,\Gamma Z_r^\varepsilon)\bar{N}(dx,dr)| \\ &\le L \int_0^t |\Gamma Z_r^\varepsilon - \Gamma\Lambda_r(0;0)|dr + \varepsilon M|B_t^H + \bar{N}_t|. \end{aligned}$$

Then

$$\begin{aligned} \sup_{0\le r\le t} |Z_t^\varepsilon - \Lambda_t(0;0)| &\le L \int_0^t \sup_{0\le r\le t} |\Gamma Z_r^\varepsilon - \Gamma\Lambda_r(0;0)|dr + \varepsilon M \sup_{0\le r\le t} |B_t^H + \bar{N}_t| \\ &\le 2L \int_0^t \sup_{0\le r\le t} |Z_r^\varepsilon - \Lambda_r(0;0)|dr + \varepsilon M \sup_{0\le r\le t} |B_t^H + \bar{N}_t|. \end{aligned}$$

By Gronwall's Lemma, we have

$$\|Z_t^\varepsilon - \Lambda_t(0;0)t\|_{\mathbb{L}^2} \le \varepsilon M \sup_{0\le t\le T} |B_t^H + \bar{N}_t| e^{2LT}.$$

In light of [10], we have

$$\mu^\varepsilon\{\|Z_t^\varepsilon - \Lambda_t(0;0)\|_{\mathbb{L}^2} > \delta\} \le \mu^\varepsilon\left\{\|B_t^H + \bar{N}_t\|_{\mathbb{L}^2} > \frac{\delta e^{-2LT}}{\varepsilon M}\right\}$$
$$\le 4\exp\left\{-\frac{\delta^2 e^{-4LT}}{2\varepsilon^2 M^2(t^{2H}+t)T^2}\right\}.$$

Hence,

$$\mu^\varepsilon\left\{\|Z_t^\varepsilon - \Lambda_t(0;0)\|_{\mathbb{L}^2} > \delta, \|B_t^H + \bar{N}_t\|_{\mathbb{L}^2} < \alpha\right\} \le \mu^\varepsilon\left\{\|B_t^H + \bar{N}_t\|_{\mathbb{L}^2} > \frac{\delta e^{-2LT}}{\varepsilon M}, \|B_t^H + \bar{N}_t\|_{\mathbb{L}^2} < \alpha\right\}$$
$$\le 4\exp\left\{-\frac{\delta^2 e^{-4LT}}{2\varepsilon^2 M^2(t^{2H}+t)T^2}\right\}.$$

Put $R = \dfrac{\delta^2 e^{-4LT}}{2M^2(t^{2H}+t)T^2}$, thus

$$\mu^\varepsilon\left\{\|Z_t^\varepsilon - \Lambda_t(0;0)\|_{\mathbb{L}^2} > \delta, \|B_t^H + \bar{N}_t\|_{\mathbb{L}^2} < \alpha\right\} \le 4\exp\left\{-\frac{R}{\varepsilon^2}\right\}.$$

This implies that

$$\lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \mu^\varepsilon\left\{\|Z_t^\varepsilon - \Lambda_t(0;0)\|_{\mathbb{L}^2} > \delta, \|B_t^H + \bar{N}_t\|_{\mathbb{L}^2} < \alpha\right\} < -R.$$

□

Lemma 3.3. Let σ be a bounded Lipschitz function and f be bounded and continuous function. Then there exist $c > 0$ and $N > 0$ such that

$$|f(t)\phi(t,s)| \le c \qquad \text{and} \qquad |\sigma(h(t))\phi(t,s)| \le N, \quad \forall s,t \in [0,T]. \tag{3.9}$$

Proof . Since f is a bounded function, there exists δ such that $|f| \le \delta$. We have for $s,t \in [0;T]$

$$|f(t)\phi(s,t)| = |f(t)||\phi(s,t)| = |f||H(2H-1)|t-s|^{2H-2}|$$
$$\le \delta H|(2H-1)|T^{2H} = c.$$

Moreover, from boundedness of σ, there exists M such that $|\sigma(\Gamma\Lambda_t)| \le M$, for all $h \in \mathbb{L}^2$, we have for $s,t \in [0;T]$

$$|\sigma(\Gamma\Lambda_t)\phi(s,t)| = |\sigma(\Gamma\Lambda_t)||\phi(s,t)| = |\sigma(\Gamma\Lambda_t)||H(2H-1)|t-s|^{2H-2}|$$
$$\le MH|(2H-1)|T^{2H} = N.$$

□

Proposition 3.4. Assume that $\Lambda_t(0,\varphi)$ defined in (3.7), under (1.1) and

$$\Psi_t = \int_0^t \int_{\mathbb{R}^*} (e^{\varphi(x,r)} - 1)\nu(dx)dr \quad \text{for} \quad \varphi \in L^2(\mathbb{R}^* \times \mathbb{R}).$$

Then for $R' > 0$ and $\delta' > 0$ there exists $\alpha > 0$ such that

$$\lim_{\varepsilon\to 0}\sup \varepsilon^2 \log \mu^\varepsilon\{\|Z_t^\varepsilon - \Lambda_t(0,\varphi)\|_{\mathbb{L}^2} > \delta', \|B_t^H + \bar{N}_t - \frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2} < \alpha\} < -R'. \tag{3.10}$$

Proof . For $(0,\varphi) \in \mathbb{L}^2$,

$$|Z_t^\varepsilon - \Lambda_t(0,\varphi)| \le \int_0^t |b(\Gamma Z_r^\varepsilon) - b(\Gamma\Lambda_r(0,\varphi))|dr + |\varepsilon\int_0^t \sigma(X_r^\varepsilon)dB_r^H + \varepsilon\int_0^t\int_{\mathbb{R}^*} K(x,\Gamma Z_r^\varepsilon)\bar{N}(dx,dr)$$
$$- \int_0^t\int_{\mathbb{R}^*} K(x,\Gamma\Lambda_r(0,\varphi))(e^{\varphi(x,r)}-1)\nu(dx)dr|$$
$$\le L\int_0^t |\Gamma Z_r^\varepsilon - \Gamma\Lambda_r(0,\varphi))|dr + M|\varepsilon\int_0^t dB_r^H + \varepsilon\int_0^t\int_{\mathbb{R}^*}\bar{N}(dx,dr) - \int_0^t\int_{\mathbb{R}^*}(e^{\varphi(x,r)}-1)\nu(dx)dr|$$
$$\le L\int_0^t |\Gamma Z_r^\varepsilon - \Gamma\Lambda_r(0,\varphi)|dr + \varepsilon M|B_t^H + \bar{N}_t - \frac{1}{\varepsilon}\Psi_t|.$$

Then,

$$\sup_{0\le t\le T}|Z^\varepsilon_t-\Lambda_t(0,\varphi)| \le L\int_0^t \sup_{0\le r\le t}|\Gamma Z^\varepsilon_r-\Gamma\Lambda_r(0,\varphi)|dr+\varepsilon M\sup_{0\le t\le T}|B^H_t+\bar N_t-\frac{1}{\varepsilon}\Psi_t|$$
$$\le 2L\int_0^t \sup_{0\le r\le t}|Z^\varepsilon_r-\Lambda_r(0,\varphi)|dr+\varepsilon M\sup_{0\le t\le T}|B^H_t+\bar N_t-\frac{1}{\varepsilon}\Psi_t|.$$

This implies that

$$\|Z^\varepsilon_t-\Lambda_t(0,\varphi)\|_{\mathbb{L}^2}\le \varepsilon M\sup_{0\le t\le T}|B^H_t+\bar N_t-\frac{1}{\varepsilon}\Psi_t|e^{2LT}.$$

So, we have

$$\mu^\varepsilon\left\{\|Z^\varepsilon_t-\Lambda_t(0,\varphi)\|_{\mathbb{L}^2}>\delta'\right\} \le \mu^\varepsilon\left\{\|B^H_t+\bar N_t-\frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2}>\frac{\delta' e^{-2LT}}{\varepsilon M}\right\}$$
$$\le \exp\left\{-\frac{I(f,\varphi)}{\varepsilon^2}\right\}\times\tilde\mu^\varepsilon\{\|B^H_t+\bar N_t-\frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2}>\frac{\delta' e^{-LT}}{\varepsilon M}\right\}.$$

Hence,

$$\begin{aligned}
&\mu^\varepsilon\left\{\|Z^\varepsilon_t-\Lambda_t(0,\varphi)\|_{\mathbb{L}^2}>\delta',\|B^H_t+\bar N_t-\frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2}<\alpha\right\}\\
\le&\exp\left\{-\frac{I(f,\varphi)}{\varepsilon^2}\right\}\times\tilde\mu^\varepsilon\left\{\|B^H_t+\bar N_t-\frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2}>\frac{\delta' e^{-LT}}{\varepsilon M},\|B^H_t+\bar N_t-\frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2}<\alpha\right\}\\
=&\exp\left\{-\frac{I(f,\varphi)}{\varepsilon^2}\right\}\tilde\mu^\varepsilon\left\{\|B^H_t+\bar N_t-\frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2}>\frac{\delta' e^{-LT}}{\varepsilon M},\|B^H_t+\bar N_t-\frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2}<\alpha\right\}\\
=&\exp\left\{-\frac{I(f,\varphi)}{\varepsilon^2}\right\}\tilde\mu^\varepsilon\left\{\|\tilde B^H_t+\tilde{\bar N}_t\|_{\mathbb{L}^2}>\frac{\delta' e^{-2LT}}{\varepsilon M},\|\tilde B^H_t+\tilde{\bar N}_t\|_{\mathbb{L}^2}<\alpha\right\}\\
\le&4\exp\left\{-\frac{I(f,\varphi)}{\varepsilon^2}\right\}\times\exp\left\{-\frac{R}{\varepsilon^2}\right\}\\
=&4\exp\left\{-\frac{I(f,\varphi)+R}{\varepsilon^2}\right\}=4\exp\left\{-\frac{R'}{\varepsilon^2}\right\}.
\end{aligned}$$

Thus,

$$\lim_{\varepsilon\to 0}\sup\varepsilon^2\log\mu^\varepsilon\{\|Z^\varepsilon_t-\Lambda_t(0,\varphi)\|_{\mathbb{L}^2}>\delta',\|B^H_t+\bar N_t-\frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2}<\alpha\}<-R'.$$

□

Proposition 3.5. The functions F and η defined by (3.5) are continuous on a compact subset of $L^2_\phi(\mathbb{R})$.

Proof .

- Let's first show F is continuous. Let $F(f_1,\varphi_1)=\Gamma\Lambda(f_1,\varphi_1)$ and $F(f_2,\varphi_2)=\Gamma\Lambda(f_2,\varphi_2)$. Then

$$\sup_{0\le t\le T}|F(f_1,\varphi_1)(t)-F(f_2,\varphi_2)(t)|=\sup_{0\le t\le T}|\Gamma\Lambda_t(f_1,\varphi_1)-\Gamma\Lambda_t(f_2,\varphi_2)|$$
$$\le 2\sup_{0\le t\le T}|\Lambda_t(f_1,\varphi_1)-\Lambda_t(f_2,\varphi_2)|$$

with

$$\Lambda_t(f,\varphi)=x_0+\int_0^t b(\Gamma\Lambda_r(f,\varphi))dr+\int_0^t\sigma(\Gamma\Lambda_r(f,\varphi))f_r\phi(r,s)dr+\int_0^t\int_{\mathbb{R}^*}K(x,\Gamma\Lambda_r(f,\varphi))(e^\varphi-1)d\nu(dx)dr.$$

Then,

$$\begin{aligned}\Lambda_t(f_1,\varphi_1)-\Lambda_t(f_2,\varphi_2)=&\int_0^t[b(\Gamma\Lambda_r(f_1,\varphi_1))-b(\Gamma\Lambda_r(f_2,\varphi_2))]dr+\int_0^t\sigma(\Gamma\Lambda_r(f_1,\varphi_1))f_1(r)\phi(r,s)dr\\&-\int_0^t\int_{\mathbb{R}^*}K(x,\Gamma\Lambda_r(f,\varphi_2))(e^{\varphi_2}-1)\nu(dx)dr-\int_0^t\int_{\mathbb{R}^*}K(x,\Gamma\Lambda_r(f,\varphi_2))(e^{\varphi_2}-1)\nu(dx)dr\\=&\int_0^t[b(\Gamma\Lambda_r(f_1,\varphi_1))-b(\Gamma\Lambda_r(f_2,\varphi_2))]dr+\int_0^t[\sigma(\Gamma\Lambda_r(f_1,\varphi_1))-\sigma(\Gamma\Lambda_r(f_2,\varphi_2))]f_1(r)\phi(r,s)dr\\&+\int_0^t\sigma(\Gamma\Lambda_r(f_2,\varphi_2))\phi(r,s)[f_1(r)-f_2(r)]dr+\int_0^t\int_{\mathbb{R}^*}K(x,\Gamma\Lambda_r(f_2,\varphi_2))[e^{\varphi_1}-e^{\varphi_2}]\nu(dx)dr\\&+\int_0^t\int_{\mathbb{R}^*}[K(x,\Gamma\Lambda_r(f_1,\varphi_1))-K(x,\Gamma\Lambda_r(f_2,\varphi_2))](e^{\varphi_1}-1)\nu(dx)dr.\end{aligned}$$

Hence,

$$\begin{aligned}|\Lambda_t(f_1,\varphi_1)-\Lambda_t(f_2,\varphi_2)|\leq&\int_0^t|b(\Gamma\Lambda_r(f_1,\varphi_1))-b(\Gamma\Lambda_r(f_2,\varphi_2))|dr+\int_0^t|\sigma(\Gamma\Lambda_r(f_2,\varphi_2))\phi(r,s)||f_1(r)-f_2(r)|dr\\&+\int_0^t|\sigma(\Gamma\Lambda_r(f_1,\varphi_1))-\sigma(\Gamma\Lambda_r(f_2,\varphi_2))||f_1(r)\phi(r,s)|dr\\&+\int_0^t\int_{\mathbb{R}^*}[K(x,\Gamma\Lambda_r(f_1,\varphi_1))-K(x,\Gamma\Lambda_r(f_2,\varphi_2))](e^{\varphi_1}-1)\nu(dx)dr\\&+\int_0^t\int_{\mathbb{R}^*}K(x,\Gamma\Lambda_r(f_2,\varphi_2))[e^{\varphi_1}-e^{\varphi_2}]\nu(dx)dr\\\leq&L\int_0^t|\Gamma\Lambda_r(f_1,\varphi_1)-\Gamma\Lambda_r(f_2,\varphi_2)|dr+Lc\int_0^t|\Gamma\Lambda_r(f_1,\varphi_1)-\Gamma\Lambda_r(f_1,\varphi_1)|dr\\&+N\int_0^t|f_1(r)-f_2(r)|dr+Lc\int_0^t|\Gamma\Lambda_r(f_1,\varphi_1)-\Gamma\Lambda_r(f_2,\varphi_2)|dr\\&+N\int_0^t\int_{\mathbb{R}^*}|e^{\varphi_1}-e^{\varphi_2}|\nu(dx)dr\\\leq&L(1+2c)\int_0^t|\Gamma\Lambda_r(f_1,\varphi_1)-\Gamma\Lambda_r(f_2,\varphi_2)|dr+2N\delta T.\end{aligned}$$

This implies that

$$\begin{aligned}\sup_{0\leq t\leq T}|\Lambda_t(f_1,\varphi_1)-\Lambda_t(f_2,\varphi_2)|&\leq L(1+2K)\int_0^t\sup_{0\leq r\leq t}|\Gamma\Lambda_r(f_1,\varphi_1)-\Gamma\Lambda_r(f_2,\varphi_2)|dr+2N\delta T\\&\leq 2L(1+2c)\int_0^t\sup_{0\leq r\leq t}|\Lambda_r(f_1,\varphi_1)-\Lambda_r(f_2,\varphi_2)|dr+2N\delta T.\end{aligned}$$

Thus,

$$\|\Lambda_t(f_1,\varphi_1)-\Lambda_t(f_2,\varphi_2)\|_{\mathbb{L}^2}\leq 2N\delta Te^{2L(1+2K)T}\qquad\text{and}\qquad\|F(f_1,\varphi_1)-F(f_2,\varphi_2)\|_{\mathbb{L}^2}\leq 2N\delta Te^{2L(1+2c)T}.$$

Hence F is continuous.

- According to [10], $\eta_t(f,\varphi)=\Gamma\Lambda_t(f,\varphi)-\Lambda_t(f,\varphi)$, so

$$\eta_t(f_1,\varphi_1)-\eta_t(f_2,\varphi_2)=\Gamma\Lambda_t(f_1,\varphi_1)-\Lambda_t(f_1,\varphi_2)-\Gamma\Lambda_t(f_2,\varphi_2)+\Lambda_t(f_2,\varphi_2).$$

Then

$$|\eta_t(f_1,\varphi_1)-\eta_t(f_2,\varphi_2)|\leq|\Gamma\Lambda_t(f_1,\varphi_1)-\Gamma\Lambda_t(f_2,\varphi_2)|+|\Lambda_t(f_1,\varphi_1)-\Lambda_t(f_2,\varphi_2)|$$

and we have

$$\begin{aligned}\sup_{0\le t\le T}|\eta_t(f_1,\varphi_1)-\eta_t(f_2,\varphi_2)| &\le \sup_{0\le t\le T}|\Gamma\Lambda_t(f_1,\varphi_1)-\Gamma\Lambda_t(f_2,\varphi_2)|+\sup_{0\le t\le T}|\Lambda_t(f_1,\varphi_1)-\Lambda_t(f_2,\varphi_2)|\\ &\le 2\sup_{0\le t\le T}|\Lambda_t(f_1,\varphi_1)-\Lambda_t(f_2,\varphi_2)|+\sup_{0\le t\le T}|\Lambda_t(f_1,\varphi_1)-\Lambda_t(f_2,\varphi_2)|\\ &\le 3\sup_{0\le t\le T}|\Lambda_t(f_1,\varphi_1)-\Lambda_t(f_2,\varphi_2)|.\end{aligned}$$

This implies that

$$\|\eta_t(f_1,\varphi_1)-\eta_t(f_2,\varphi_2)\|_{\mathbb{L}^2}\le 2N\delta T e^{6L(1+2c)T},$$

because $\Lambda\in\mathbb{L}^2$. Hence η is continuous. □

Theorem 3.6. Assume g defined in (3.5). Then for $R'>0$ and $\delta'>0$ there exists $\alpha>0$ such that

$$\lim_{\varepsilon\to 0}\sup\varepsilon^2\log\mu^\varepsilon\{\|X_t^\varepsilon-g_t\|_{\mathbb{L}^2}+\|L_t^\varepsilon-\eta_t\|_{\mathbb{L}^2}>\delta',\|B_t^H+\bar{N}_t-\frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2}<\alpha\}<-R'. \tag{3.11}$$

Proof . For $R'>0$, $\delta'>0$, then by (3.3) and (3.5), we have

$$\begin{aligned}|X_t^\varepsilon-g_t|+|L_t^\varepsilon-\eta_t| &= |\Gamma Z_t^\varepsilon-\Gamma\Lambda_t(0,\varphi)|+|\Gamma Z_t^\varepsilon-Z_t^\varepsilon-\Gamma\Lambda_t(0,\varphi)+\Lambda_t(0,\varphi)|\\ &\le |\Gamma Z_t^\varepsilon-\Gamma\Lambda_t(0,\varphi)|+|\Gamma Z_t^\varepsilon-\Gamma\Lambda_t(f,\varphi)|+|Z_t^\varepsilon-\Lambda_t(0,\varphi)|.\end{aligned}$$

Then

$$\begin{aligned}\sup_{0\le t\le T}|X_t^\varepsilon-g_t|+\sup_{0\le t\le T}|L_t^\varepsilon-\eta_t| &\le \sup_{0\le t\le T}|\Gamma Z_t^\varepsilon-\Gamma\Lambda_t(0,\varphi)|+\sup_{0\le t\le T}|\Gamma Z_t^\varepsilon-\Gamma\Lambda_t(0,\varphi)|+\sup_{0\le t\le T}|Z_t^\varepsilon-\Lambda_t(0,\varphi)|\\ &\le 2\sup_{0\le t\le T}|Z_t^\varepsilon-\Lambda_t(0,\varphi)|+2\sup_{0\le t\le T}|Z_t^\varepsilon-\Lambda_t(0,\varphi)|+\sup_{0\le t\le T}|Z_t^\varepsilon-\Lambda_t(0,\varphi)|\\ &\le 5\sup_{0\le t\le T}|Z_t^\varepsilon-\Lambda_t(0,\varphi)|.\end{aligned}$$

Thus,

$$\|X_t^\varepsilon-g_t\|_{\mathbb{L}^2}+\|L_t^\varepsilon-\eta_t\|_{\mathbb{L}^2}\le 5\varepsilon M\|B_t^H+\bar{N}_t-\frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2}e^{2LT}.$$

This implies that

$$\lim_{\varepsilon\to 0}\sup\varepsilon^2\log\mu^\varepsilon\{\|X_t^\varepsilon-g_t\|_{\mathbb{L}^2}+\|L_t^\varepsilon-\eta_t\|_{\mathbb{L}^2}>\delta',\|B_t^H+\bar{N}_t-\frac{1}{\varepsilon}\Psi_t\|_{\mathbb{L}^2}<\alpha\}<-R'.$$

□

We can now formulate the main theorem of this article.

Theorem 3.7. The family $(X_t^{H,\varepsilon},L_t^\varepsilon)_{\varepsilon>0}$ of the stochastic differential equation (1.1) satisfies a large deviation principle with a good rate function given by

$$J(z,\eta,g)=\begin{cases}\frac{1}{2}[|\sigma^{-1}(z)[\dot{z}-b(z)-\chi_{\{z=0\}}(z)\dot{\eta}]|^2_{\phi^{-1}}+\inf_g\{\Delta(\varphi),F_\nu(\varphi)=g\} & \text{if } (z,g)\in\mathbb{L}^2\\ \\ +\infty & \text{otherwise.}\end{cases} \tag{3.12}$$

In other words:

- J is lower semi-continuous and $\{(z,g)\in\mathbb{L}^2, and\ a\in\mathbb{R}_+, J(z,\eta,\varphi)\le a\}$ is a compact subset of $\mathbb{L}^2$;
- For all closed set $C\subset\mathbb{L}^2$,
  $\lim_{\varepsilon\to 0}\sup\varepsilon^2\log\mu^\varepsilon[(X_t^\varepsilon,L_t^\varepsilon)\in C]=\lim_{\varepsilon\to 0}\sup\varepsilon^2\log\mathbb{P}^\varepsilon oF^{-1}[(X_t^\varepsilon,L_t^\varepsilon)\in O]\le -J(z,\eta,g)$;
- For any open set $O\subset\mathbb{L}^2$,
  $\lim_{\varepsilon\to 0}\inf\varepsilon^2\log\mu^\varepsilon[(X_t^\varepsilon,L_t^\varepsilon)\in O]=\lim_{\varepsilon\to 0}\inf\varepsilon^2\log\mathbb{P}^\varepsilon oF^{-1}[(X_t^\varepsilon,L_t^\varepsilon)\in O]\ge -J(z,\eta,g)$.

Proof . J is a good rate function for the LDP of the process X_t^ε because J is the sum of two good rate functions: $\frac{1}{2}|\sigma^{-1}(z)[\dot{z}-b(z)-\chi_{\{z=0\}}(z)\dot{\eta}]|^2_{\phi^{-1}}$ (see Diatta and al [11]) and $\inf_g\{\Delta(\varphi), F_\nu(\varphi)=g\}$ (see Dadashi [6]) for all $(z,g)\in\mathbb{L}^2$. The family $\varepsilon(B_t^H+\bar{N}_t)$ of probability measure $\mathbb{P}^\varepsilon$ satisfies an LDP with the good rate function $I(f,\varphi)=\dfrac{1}{2}[|f|^2_\phi+|\Delta(\varphi)]$ for $(f,\varphi)\in\mathbb{L}^2$ and F is a continuous function. So by the contraction principle (2.4), for a close set $C\subset\mathbb{L}^2$, we have:

$$\begin{aligned}
\limsup_{\varepsilon\to 0}\varepsilon^2\log\mu^\varepsilon[(X_t^\varepsilon,L_t^\varepsilon)\in C] &= \limsup_{\varepsilon\to 0}\varepsilon^2\log\mathbb{P}^\varepsilon oF^{-1}[(X_t^\varepsilon,L_t^\varepsilon)\in C]\\
&= \limsup_{\varepsilon\to 0}\varepsilon^2\log\mathbb{P}^\varepsilon[F^{-1}[(X_t^\varepsilon,L_t^\varepsilon)\in C]]\\
&= \limsup_{\varepsilon\to 0}\varepsilon^2\log\mathbb{P}^\varepsilon[F^{-1}[(X_t^\varepsilon,L_t^\varepsilon)]\in F^{-1}(C)]\\
&= \limsup_{\varepsilon\to 0}\varepsilon^2\log\mathbb{P}^\varepsilon[\varepsilon(B_t^H+\bar{N}_t)\in F^{-1}(C)]\\
&\le -\inf_{f\in F^{-1}(C)} I(f,\varphi)\\
&= -\inf_{(z,g)\in C}\{\inf I(f,\varphi),(f,\varphi)\in\mathbb{L}^2, F_\phi=z, F_\nu(\varphi)=g\}\\
&= \frac{1}{2}|\sigma^{-1}(z)[\dot{z}-b(z)-\chi_{\{z=0\}}(z)\dot{\eta}]|^2_{\phi^{-1}}+\inf_g\{\Delta(\varphi),F_\nu(\varphi)=g\}\\
&= -J(z,\eta,g).
\end{aligned}$$

Hence $\limsup_{\varepsilon\to 0}\varepsilon^2\log\mu^\varepsilon[(X_t^\varepsilon,L_t^\varepsilon)\in C]\le -J(z,\eta,g)$. For an open set $O\subset\mathbb{L}^2$,

$$\begin{aligned}
\liminf_{\varepsilon\to 0}\varepsilon^2\log\mu^\varepsilon[(X_t^\varepsilon,L_t^\varepsilon)\in O] &= \limsup_{\varepsilon\to 0}\varepsilon^2\log\mathbb{P}^\varepsilon oF^{-1}[(X_t^\varepsilon,L_t^\varepsilon)\in O]\\
&= \liminf_{\varepsilon\to 0}\varepsilon^2\log\mathbb{P}^\varepsilon[F^{-1}[(X_t^\varepsilon,L_t^\varepsilon)\in O]]\\
&= \liminf_{\varepsilon\to 0}\varepsilon^2\log\mathbb{P}^\varepsilon[F^{-1}[(X_t^\varepsilon,L_t^\varepsilon)]\in F^{-1}(O)]\\
&= \liminf_{\varepsilon\to 0}\varepsilon^2\log\mathbb{P}^\varepsilon[\varepsilon(B_t^H+\bar{N}_t)\in F^{-1}(O)]\\
&\ge -\inf_{f\in F^{-1}(O)} I(f,\varphi)\\
&= -\inf_{(z,g)\in O}\{\inf I(f,\varphi),(f,\varphi)\in\mathbb{L}^2, F_\phi=z, F_\nu(\varphi)=g\}\\
&= \frac{1}{2}|\sigma^{-1}(z)[\dot{z}-b(z)-\chi_{\{z=0\}}(z)\dot{\eta}]|^2_{\phi^{-1}}+\inf_g\{\Delta(\varphi),F_\nu(\varphi)=g\}\\
&= -J(z,\eta,g)
\end{aligned}$$

Hence $\liminf_{\varepsilon\to 0}\varepsilon^2\log\mu^\varepsilon[(X_t^\varepsilon,L_t^\varepsilon)\in O]\ge -J(z,\eta,g)$. Now, let us show that

$$J(z,\eta,\varphi)=\frac{1}{2}|\sigma^{-1}(z)[\dot{z}-b(z)-\chi_{\{z=0\}}(z)\dot{\eta}]|^2_{\phi^{-1}}+\inf_g\{\Delta(\varphi),F_\nu(\varphi)=g\}.$$

According to (3.5), we have:

$$z_t=x+\int_0^t b(z_r)dr+\int_0^t\sigma(z_r)f_r\phi(r,s)dr+\eta(z_t).$$

Then,

$$\dot{z}_t=b(z_t)+\sigma(z_t)f_t\phi(t,s)+\chi_{\{z_t=0\}}(z_t)\dot{\eta}.$$

So, we have

$$f_t=\frac{\dot{z}_t-b(z_t)-\dot{\eta}(z_t)}{\sigma(z_t)\phi(t,s)}=\sigma^{-1}(z_t)[\dot{z}_t-b(z_t)-\chi_{\{z_t=0\}}(z_t)\dot{\eta}]\phi^{-1}(t,s)$$

.

Then

$$\begin{aligned}
J(z,\eta,\varphi) =&\frac{1}{2}|\sigma^{-1}(z_t)[\dot{z}_t - b(z_t) - \chi_{\{z_t=0\}}(z_t)\dot{\eta}]\phi^{-1}(t,s)|^2_{\phi} + \inf_g\{\Delta(\varphi), F_\nu(\varphi)=g\}\\
=&\frac{1}{2}\int_0^t\int_0^s \sigma^{-1}(z_r)[\dot{z}_r - b(z_r) - \chi_{\{z_r=0\}}(z_r)\dot{\eta}]\phi^{-1}(r,u)\sigma^{-1}(z_u)[\dot{z} - b(z_u) - \chi_{\{z_u=0\}}(z_u)\dot{\eta}]\\
&\quad\times\phi^{-1}(r,u)\phi(u,r)dudr + \inf_g\{\Delta(\varphi), F_\nu(\varphi)=g\}\\
=&\frac{1}{2}\int_0^t\int_0^s \sigma^{-1}(z_r)[\dot{z} - b(z_r) - \chi_{\{z_r=0\}}(z_r)\dot{\eta}]\sigma^{-1}(z_u)[\dot{z} - b(z_u) - \chi_{\{z_u=0\}}(z_u)\dot{\eta}]\phi^{-1}(r,u)dudr\\
&+\inf_g\{\Delta(\varphi), F_\nu(\varphi)=g\}\\
=&\frac{1}{2}|\sigma^{-1}(z)[\dot{z} - b(z) - \chi_{\{z=0\}}(z)\dot{\eta}]|^2_{\phi^{-1}} + \inf_g\{\Delta(\varphi), F_\nu(\varphi)=g\}.
\end{aligned}$$

□

4 Conclusion

In the present paper, we have established the asymptotic behaviour of a solution of a mixed differential equation driven by an independent fractional Brownian motion, a compensated Poisson process and a local time under the assumption of low regularity. This construction is carried out in the tempered distribution space $\mathcal{S}'(\mathbb{R})$ using the method of Freidlin-Wentzell [13] and Azencott [1]. So it would be very interesting to do this in a space larger than that considered here. For example it is interesting to study the process in a higher dimensional space.

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