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Research Article

The Critical Conditions of Filtration Flow Blocking in a Porous Channel with Phase Transitions

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ABSTRACT

The paper studies the occurrence of kinetic phase transitions in porous media with thermodynamic phase transitions, which affect filtration flow through permeability or viscosity: in the simplest case, permeability is a step function of local temperature. The dependence of phase boundary under stationary heating and cooling processes has features that are concerned with the non-linearity of Stefan condition. These features result in limited stability of filtration flow. Calculations show that there exist critical temperatures in the vicinity of those stationary filtration flow becomes impossible due to the thermal interaction of phase transition and advection processes. The solutions form two branches; one of those is stable, and the other is unstable; the connection between the branches is a critical point. Several different setups are considered: flat and cylindrical channels, melting of the porous carcass, and crystallization of flowing liquid. Critical conditions for these cases are obtained in the form of approximate formulas that can be used to calculate heat transfer in thermal engineering units, including heat accumulators.

1. Introduction

Filtration flows with phase transitions are common objects of study in geophysics and thermal engineering [1, 2]. When the permeability of a porous medium depends on its phase state, then filtration flow may drastically change behaviour under a smooth change of control parameters. For example, agglomeration of polymer-containing waste combustion [3] and gas hydrates self-preservation [4] occur in this way. In some cases, phase transitions lead to the failure of heat-conducting elements [5]. To control or prevent these phenomena, detailed physico-chemical models are developed. At the same time, these phenomena have similarities that can be studied using relatively simple examples. In this paper, we investigate such simplified problems that allow exact solutions.

Heat transfer processes in granular beds are intensively studied in connection with thermal energy storage. Phase-changing materials heating by fluid heat carriers was experimentally and theoretically studied in works [6-8], where methods for the heating rate calculating under a steady flow were proposed. The influence of the domain and particle geometry was studied by authors [2, 9, 10] using numerical simulation. Models of processes in individual particles are proposed in papers [11, 12]. The issues of temperature stratification and the influence of free convection were considered in [13]. Thermochemical processes of energy storage were studied in works [14, 15].

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The object of the study is a porous medium domain with stationary heating/cooling conditions. In the papers listed above, porous structure was considered to be stable, which may not be the case. When the porous medium permeability is a step function of temperature, then introducing a non-linear boundary condition (namely, Stefan condition) results in critical phenomena: under fixed filtration flow velocity (i.e., fixed pressure drop), there exist limit temperatures dividing partial and full blocking of the porous medium section. These critical conditions have a simple geometric interpretation, and under realistic approximations, they can be written explicitly. The novelty of the paper consists of the new problem statement and new solutions.

2. Problem statement

Let us consider a domain of porous medium (for example, particle packing or a foam, see Fig. 1). Porous medium allows to assume the flat velocity profile (U(r) = const). Porous characteristics are reduced to the average fluid velocity through corresponding filtration relations [16]. Heat energy conservation is given by the equation:

$$c\rho U\frac{\partial T}{\partial z}=\lambda\left[\frac{\partial^2 T}{\partial z^2}+\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial T}{\partial r}\right)\right] \quad (1)$$

If the domain is relatively low or the filtration velocity is high, one can neglect the longitudinal temperature gradient compared to the radial gradient. This allows to average eq. (1) in z-direction:

$$\frac{c\rho U}{L}\left(T-T_0\right)=\lambda\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial T}{\partial r}\right) \quad (2)$$

The boundary conditions are as follows:

$$\frac{\partial T}{\partial r}(0)=0;-\lambda\frac{\partial T}{\partial r}(R)=\alpha\left(T-T_h\right) \quad (3)$$

here T_0 is the input flow temperature; T_h is the wall temperature.

Using characteristic temperature T^*, one can transform eqs. (2) and (3) to obtain their non-dimensional form:

$$\frac{1}{\xi^n}\frac{d}{d\xi}\left(\xi^n\frac{d\theta}{d\xi}\right)=Pe\left(\theta-\theta_{in}\right) \quad (2a)$$

$$\frac{d\theta}{d\xi}(0)=0,\ -\frac{d\theta}{d\xi}(1)=Bi\left[\theta(1)-\theta_w\right] \quad (3a)$$

here $\xi = r/R$; $\theta = (T - T^*)/(T_m - T^*)$; Pe is Peclet number ($Pe = c\rho UR^2/\lambda L$); and Bi is Biot number ($Bi = \alpha R/\lambda$). Characteristic temperature T^* depends on the problem statement (for example, T_{in} or T_h).

Similar problems were studied earlier in papers [17-19] for stationary volumetric heating. Experimental and theoretical studies of phase change materials melting in porous mediums without flow were conducted in papers [20-22]. Heat transfer model reduction was proposed in the works [23]. Natural convection problems are reviewed in [24]. In the present paper, we consider the approximate stationary linear advection term, which significantly changes the solution of (2a) compared to pure conduction heat transfer.

Phase transitions may change flow conditions in porous media. Two cases are considered here:

(1) Melting of porous material results in pore collapse and permeability drop;

(2) Crystallization of flowing liquid results in increasing viscosity and flow blocking. We are also interested in two basic geometries: flat porous channel (n = 0) and cylindrical porous channel (n = 1).

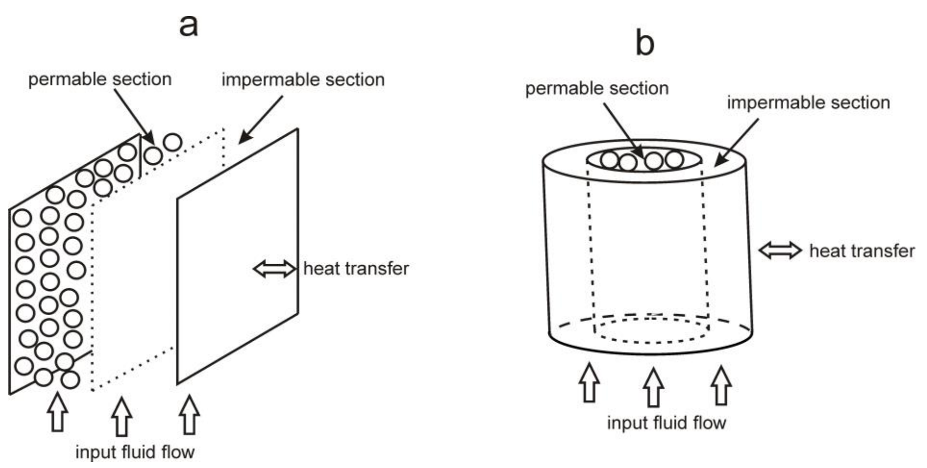

Fig. 1. Scheme of the porous channel: (a) flat case; (b) cylindrical case.

3. Single-Phase Solution and Melting Start

The characteristic temperature difference for the melting problem is the difference between the melting temperature and the reference temperature. The solution of the single-phase problem written as (2a) and (3a) is the following:

$$\theta(\xi)=\theta_{in}+\frac{(\theta_w-\theta_{in})\cosh\left(\xi\sqrt{Pe}\right)}{\cosh\left(\sqrt{Pe}\right)+\frac{\sqrt{Pe}}{Bi}\sinh\left(\sqrt{Pe}\right)}, n=0 \quad (4a)$$

$$\theta(\xi)=\theta_{in}+\frac{(\theta_w-\theta_{in})I_0\left(\xi\sqrt{Pe}\right)}{I_0\left(\sqrt{Pe}\right)+\frac{\sqrt{Pe}}{Bi}I_{-1}\left(\sqrt{Pe}\right)}, n=1 \quad (4b)$$

here I_k is the k-th order modified Bessel function of the first kind [25].

Let us consider θ_{in} = 0 (the domain is heated through the side wall). For non-zero Biot number, one can find the melting start condition using the following formulas:

$$\theta_w=1+\frac{\sqrt{Pe}}{Bi}\tanh\left(\sqrt{Pe}\right), n=0 \quad (5a)$$

$$\theta_w=1+\frac{\sqrt{Pe}}{Bi}\frac{I_{-1}\left(\sqrt{Pe}\right)}{I_0\left(\sqrt{Pe}\right)}, n=1 \quad (5b)$$

For a large Peclet number, the critical wall temperature can be approximately calculated:

$$\theta_w^*\simeq 1+\frac{\sqrt{Pe}}{Bi}. \quad (6)$$

When the Peclet number is close to zero, one can use Taylor expansion to obtain another approximation:

$$\theta_w^*\simeq 1+\frac{Pe}{Bi}. \quad (7)$$

In the limit of small Peclet and Biot numbers, melting starts when the temperature in the vicinity of the wall turns 1. Filtration flow and heat transfer give correction for the wall temperature which is proportional to flow velocity and heat transfer resistance.

Melting temperature is achieved in the center of the domain under the following conditions:

$$\theta_w^{**}=\cosh\left(\sqrt{Pe}\right)+\frac{\sqrt{Pe}}{Bi}\sinh\left(\sqrt{Pe}\right),\ n=0, \quad (8a)$$

$$\theta_w^{**}=I_0\left(\sqrt{Pe}\right)+\frac{\sqrt{Pe}}{Bi}I_{-1}\left(\sqrt{Pe}\right),\ n=1. \quad (8b)$$

4. Two-Phase Solution

When porous material melting results in pore blocking, then filtration flow velocity is a step function of spatial coordinate:

$$Pe(\xi)=\begin{cases}Pe_0, \xi<\xi_m\\ 0, \xi\ge\xi_m\end{cases}. \quad (9)$$

here Pe_0 is Peclet number in the permeable region.

In this case, temperature distribution in a flat channel can be found in the following form:

$$\theta(\xi<\xi_m)=\frac{\cosh\left(\xi\sqrt{Pe}\right)}{\cosh\left(\sqrt{Pe}\right)}, \quad (10a)$$

$$\theta(\xi\ge\xi_m)=1+\frac{\theta_w-1}{1-\xi_m+\frac{1}{Bi}}(1-\xi). \quad (10b)$$

To find the phase boundary, we are to solve the heat balance condition:

$$\sqrt{Pe}\tanh\left(\xi_m\sqrt{Pe}\right)=\kappa\frac{\theta_h-1}{1-\xi_m+\frac{1}{Bi}}. \quad (11)$$

here κ is a ratio of heat conductivities of phases. This transcendental equation can be solved numerically or graphically. To this end, let us transform eq. (11) to the form:

$$\sqrt{Pe}\left(1-\xi_m+\frac{1}{Bi}\right)\tanh\left(\xi_m\sqrt{Pe}\right)=\kappa(\theta_h-1). \quad (11a)$$

The left-hand side contains a concave function (Fig. 2). In the limit of large Biot numbers, this function has roots ξ_m = 0 and ξ_m = 1. The right-hand side is constant. Generally speaking, this equation may have up to two roots. When the right-hand side is small, then the phase boundary lies near the wall, i.e., the physical root is the bigger one.

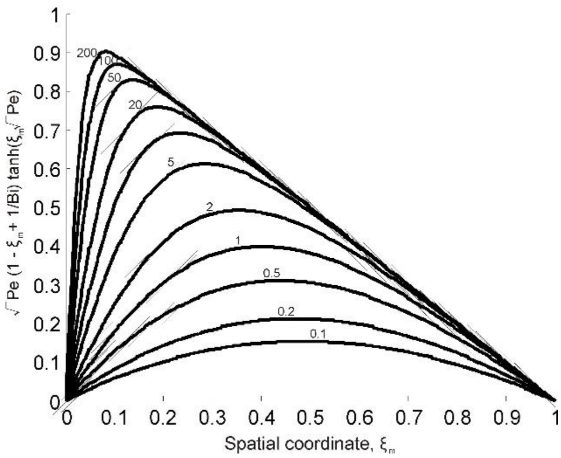

Fig. 2. Dependence of left hand side of eq. (11a) on spatial coordinate and Peclet number

When wall temperature gradually increases, then in tangency point of the left-hand side and the right-hand side of eq. (11a) the number of roots changes. That is, the phase boundary moves non-smoothly: at a given filtration flow velocity, there is a critical wall temperature, which corresponds to the conditions when the phase boundary cannot be held stationary (the whole domain melts, the filtration flow stops). Then, the full melting condition can be expressed as the tangency condition for the left-hand side concave function with the right-hand side constant level line.

Obviously, the tangency can occur only in the concave function maximum point, where its derivative becomes zero. At this point, the phase boundary becomes unstable and disappears at the point determined by the following equation (see Fig. 3):

$$1-\xi_m = \frac{1}{2\sqrt{Pe}}\sinh\left(2\xi_m\sqrt{Pe}\right). \tag{12}$$

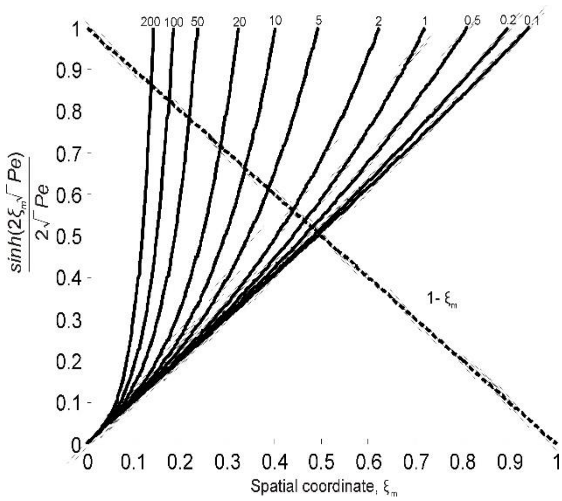

Fig. 3. Graphical solution of the eq. (12) at different Peclet numbers.

Under small Peclet numbers, the tangency point position is equal to 0.5. For large Peclet numbers, one can use the following approximate formula (Fig. 4):

$$\xi_m \simeq 1+\frac{1}{2\sqrt{Pe}}\left[1-W\left(\frac{e^{2\sqrt{Pe}+1}}{2}\right)\right] \tag{13}$$

here W is Lambert function. For finite Biot numbers, this formula has a correction of $1/Bi$.

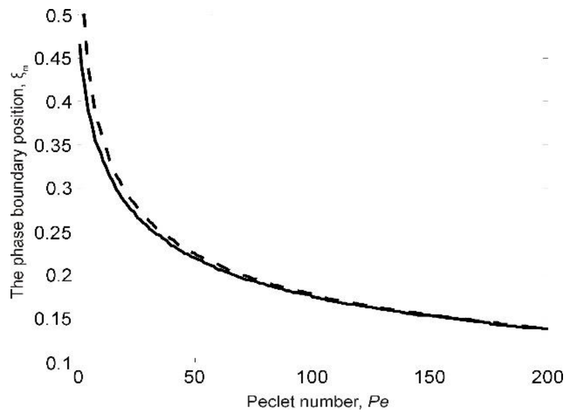

Fig. 4. Dependence of the critical phase boundary position on Peclet number (solid line – numerical calculation, dashed line – approximate formula (13)).

The wall temperature corresponding to the full melting can be found from the eq. (11) given phase boundary position from eq. (12). For large Peclet numbers, one can use a simple approximation:

$$\theta_w^{**} = 1+\frac{\sqrt{Pe}}{\kappa}\left(1-\xi_m\right) \tag{14}$$

Using approximate formula (13), one can obtain:

$$\theta_w^{**} = 1+\frac{1}{2\kappa}\left[W\left(\frac{e^{2\sqrt{Pe}+1}}{2}\right)-1\right] \tag{15}$$

The qualitative picture of the critical phenomenon does not change for cylindrical symmetry. The temperature distribution is as follows:

$$\theta\left(\xi<\xi_m\right) = \frac{I_0\left(\xi\sqrt{Pe}\right)}{I_0\left(\xi_m\sqrt{Pe}\right)}, \tag{16a}$$

$$\theta\left(\xi\ge\xi_m\right) = \theta_h + \frac{1-\theta_w}{\ln\xi_m-\dfrac{1}{Bi}}\ln\xi. \tag{16b}$$

Then Stefan condition gives the equation determining phase boundary position:

$$-\sqrt{Pe}\,\frac{I_{-1}\left(\xi_m\sqrt{Pe}\right)}{I_0\left(\xi_m\sqrt{Pe}\right)}\xi_m\left(\ln\xi_m-\frac{1}{Bi}\right) = \kappa\left(\theta_w-1\right). \tag{17}$$

As in the flat case, the left-hand side has roots $\xi_m = 0$ and $\xi_m = 1$ in the limit of large Biot numbers. In the unit segment, this function is not concave (see Fig. 5), although very close to concave (i.e., it has a "concavity margin" [26]).

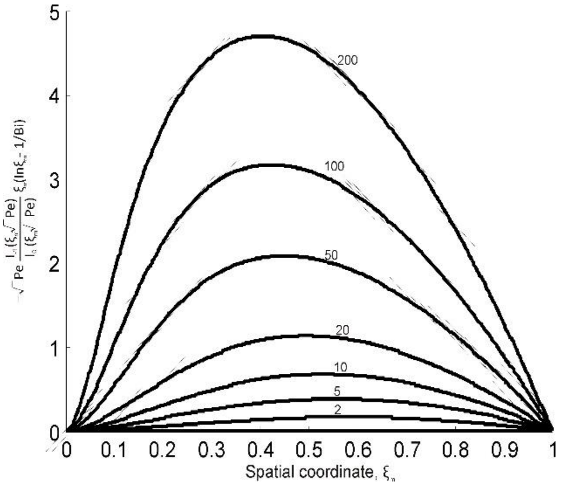

Fig. 5. Dependence of the left hand side of the eq. (17) on spatial coordinate and Peclet number.

For large Peclet numbers, the ratio of Bessel functions is close to 1, and the maximum position becomes close to e^{-1} (for finite Biot numbers – to $e^{-1+1/Bi}$). The critical wall temperature, corresponding to the full melting, can be approximately calculated using the following formula:

$$\theta_w^{**} \simeq 1 + \frac{1}{\kappa}\frac{\sqrt{Pe}}{e} \tag{18}$$

5. Liquid Crystallization in Pores

Now let us consider the reverse situation, when melt flows through the porous medium, and its crystallization blocks pores. That is, the input flow has a temperature higher than the melting point, and crystallization occurs due to cooling by the wall ($\theta_w = 0$).

The general solution for a flat channel is as follows:

$$\theta(\xi < \xi_m) = \theta_{in} - (\theta_{in} - 1)\frac{\cosh\left(\xi\sqrt{Pe}\right)}{\cosh\left(\xi_m\sqrt{Pe}\right)}, \tag{19a}$$

$$\theta(\xi \geq \xi_m) = 1 + \frac{\xi_m - \xi}{1 - \xi_m + \frac{1}{Bi}} \tag{19b}$$

Then Stefan condition again gives the equation determining phase boundary position:

$$\sqrt{Pe}\left(1 - \xi_m + \frac{1}{Bi}\right)\tanh\left(\xi_m\sqrt{Pe}\right) = \frac{\kappa}{\theta_{in} - 1}. \tag{20}$$

This equation is similar to eq. (11a) and can be treated the same way. Particularly, phase boundary position is given using the same approximate formula. However, the critical temperature formula slightly changes:

$$\theta_{in}^{*} \simeq 1 + \frac{\kappa}{(1 - \xi_m)\sqrt{Pe}}. \tag{21}$$

The Stefan condition in the cylindrical channel, in turn, looks similar to the eq. (17):

$$-\sqrt{Pe}\,\frac{I_{-1}\left(\xi_m\sqrt{Pe}\right)}{I_0\left(\xi_m\sqrt{Pe}\right)}\xi_m\left(\ln\xi_m - \frac{1}{Bi}\right) = \frac{\kappa}{(\theta_{in} - 1)}. \tag{22}$$

From there, the critical temperature can be found using a formula similar to (18):

$$\theta_{in}^{*} \simeq 1 + \frac{e\kappa}{\sqrt{Pe}}. \tag{23}$$

6. Discussion of Results

The main result of the study is the simple model of critical flow phenomena in porous media with phase transitions. Under heating and cooling, there are parameter ranges in which the phase boundary expands to the whole domain. Non-smooth dependences of the phase boundary position on the wall/input temperature and flow velocity are a consequence of the non-linearity of the Stefan condition. When phase transition worsens the heat transfer, it may lead to feedback: the more the section blocked for filtration, the further the phase boundary moves. Thermodynamic phase transition initiates kinetic phase transition [27].

In this regard, filtration flow with phase transitions becomes similar to non-linear chemical reactions. The effects of the interaction between chemical kinetics and heat and mass transfer are widely studied, such as thermal runaway and ignition stability [28]. Phase-changing materials may be used in the thermal control of chemically reacting media, so studying the interaction between exothermal reactions and melting materials has a practical interest [21, 29].

Qualitative effects concerned with the obtained solutions may explain processes of failure and agglomeration in heat exchangers and chemical reactors, for example, in heat-generating elements (in energy storage systems and electronics cooling [30-32]) and thermochemical conversion of unconventional fuels (plastic waste and fuels with low-melting ashes [33]). It should also be mentioned that we did not consider the kinetics of phase transitions, which may influence critical conditions of heat and mass transfer [34, 35].

It can be seen that when the flow is zero in the whole section, then the formulas reduce to the well-known results of heat transfer theory. In comparison to the results of [3], bed material in the present work is not supposed to degrade, so permeability restoration is not considered here. It is interesting to consider other heat sources, for example, internal heat generation [18] or lid heating [15, 36], which may cause more complex behaviour of bed packings.

7. Conclusions

In the present work, we studied the approximate heat transfer problem for a porous medium with melting and crystallization processes accompanied by flow permeability drop. Stefan problem is solved for the corresponding statements; approximate analytical formulas are derived. It is shown that the solutions to these problems have features that are concerned with non-linear phase boundary conditions. These features result in critical phenomena during the heating and cooling of porous channels when a fluid flow drops to zero in its entire section. The formulas are obtained that connect characteristic temperatures and filtration flow velocity at the stability edge for flat and cylindrical channels. These formulas allow to calculate heat transfer conditions corresponding to the meltdown or freezing of channels with phase transition.

Nomenclature

Bi	Biot number, $\alpha R/\lambda$
I_k	Modified Bessel function of the first kind
L	Channel length, m
Pe	Peclet number, $c\rho UR^2/\lambda L$
Pe_0	Peclet number in the permeable region
R	Channel size, m
T	Temperature, K
U	Fluid velocity, m/s
W	Lambert W-function
c	Heat capacity, J/kg/K
n	Geometric factor
m	Melting point
r	Radial coordinate, m
z	Axial coordinate, m
α	Heat transfer coefficient, W/m^2/K
κ	The ratio of thermal conductivities in solid and fluid phases
λ	Thermal conductivity, W/m/K
ρ	Densit, kg/m^3
θ	Dimensionless temperature
ξ	Dimensionless spatial coordinate

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Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

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