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Solving linear fractional transportation problems with interval cost, source and destination parameters

Abouzar Sheikhi[a], Seyed Mehdi Karbassi[b,*], Narges Bidabadi[b]

[a] *Department of Mathematics, Baghmalek Branch, Islamic Azad University, Baghmalek, Iran*

[b] *Department of Mathematics, Yazd University, Yazd, Iran*

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Abstract

In this paper, we focus on the fractional transportation problem where the cost coefficient of the objective functions, and the source and destination parameters have been expressed as interval values. The variable transformation solves the linear fractional transportation problem with interval coefficients in the objective function. In this method, instead of intervals in the function, using a convex combination of the left limit and right limit of the interval, linear fractional transportation problems with Interval Coefficients are reduced to a nonlinear programming problem. Finally, the nonlinear problem is transformed into a linear programming problem with two more constraints and one more variable compared to the initial problem. The constraints with interval source and destination parameters have been converted into deterministic ones. Numerical examples are presented to clarify the idea of the proposed approach for three possible cases of the original problem.

Keywords: interval coefficients, convex combination, linear fractional programming problems, linear fractional transportation problems
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1 Introduction

Fractional programming gains significant stature since many of the real-world problems are represented as fractional functions. These problems are often encountered in situations such as return on investment, current ratio, and actual capital to required capital. A linear fractional programming problem is one whose objective function is very useful in production planning, and financial and corporate planning. Fractional programming (FP) is a special case of nonlinear programming, which is generally used for modelling real-life problems with one or more objective(s) such as profit/cost, actual cost/standard, output/employee, etc, and it is applied to different disciplines such as engineering, business, finance, economics, etc. Linear fractional programming is a special class of fractional programming that can be transformed into a linear programming problem by the method of Charnes and Cooper [5]. Tantawy [15], proposed a new method for solving linear fractional programming problems. Wu [16], introduces four kinds of interval-valued optimization problems are formulated. In this paper, the linear fractional programming problem with interval coefficients in the objective function is considered. For solving the problem, a method based on variable transformation

*Corresponding author
Email addresses: abouzarsheikhi@gmail.com (Abouzar Sheikhi), second.author@email.address (Seyed Mehdi Karbassi), third.author@email.address (Narges Bidabadi)

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and a convex combination of intervals is used by Charnes and Cooper [5]. The reader is referred to Miettinen [8] about the theory and algorithms for MOPs. Fractional programming problems (FPPs) arise from many applied areas such as portfolio selection, stock cutting, game theory, and numerous decision problems. Stancu-Minasian [13], gives a survey on fractional programming which covers applications as well as major theoretical and algorithmic developments. Sheikhi et al. [12], have introduced a novel algorithm for solving bi-objective fractional transportation problems with fuzzy numbers. M. Borza et al. [3] Proposed a new approach for solving linear fractional programming problems with Interval coefficients in the objective function. Chanas and Kuchta [4] defined the transportation problem with a fuzzy cost coefficient and developed an algorithm for the solution. Steuer [14] and Shaocheng [9] have proposed linear programming models with interval objective functions. Ishibuchi and Tanaka [7] developed a concept for optimization of multiobjective programming problems with interval objective functions. Alefeld and Herzberger [1] introduction to Interval computations. The reader is referred to the recent book by Bajalinov [2] about linear-fractional programming, theory, methods, applications and software, which has examined the concepts of strategy and performance evaluation separately, see [10] and [11], for more details.

In this paper, we propose a new method for solving linear fractional transportation problems with Interval Coefficients in the Objective Function and the source and destination parameters. In the proposed method, by using a convex combination of the left limit and the right limit of intervals instead of intervals and also using variable transformation, the linear fractional transportation problem is transformed into a nonlinear programming problem which finally is changed into a linear programming problem which has two more constraints and one more variable compare to the initial problem. To illustrate the proposed solution method, numerical examples are provided.

2 Formulation of the problem

The general extended form of a linear fractional transportation problem with interval coefficients in the objective function is as follows:

$$\text{(LFTP1)}\quad \text{Max} \qquad Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}[p_{ij}^1, p_{ij}^2]\ x_{ij} + [p_0^1, p_0^2]}{\sum_{i=1}^{m}\sum_{j=1}^{n}[d_{ij}^1, d_{ij}^2]\ x_{ij} + [d_0^1, d_0^2]}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} = [a_{L_i}, a_{R_i}] \ \text{ for } i = 1, 2, \ldots, m \tag{2.1}$$

$$\sum_{i=1}^{m} x_{ij} = [b_{L_j}, b_{R_j}] \ \text{ for } j = 1, 2, \ldots, n \tag{2.2}$$

$$x_{ij} \geq 0,\ i = 1, 2, \cdots, m\ ;\ j = 1, 2, \ldots, n. \tag{2.3}$$

Here and in what follows we suppose that $D(x) > 0$, for all $x = (x_{ij}) \in S$, where S denotes a feasible set defined by constraints (2.1) to (2.3). Further, we assume that $a_i > 0$, $b_j > 0$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ and total demand equals to total supply, i.e.

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j}, \qquad \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j},$$

where $[p_{ij}^1, p_{ij}^2]$ and $[d_{ij}^1, d_{ij}^2]$, for $i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$ are interval representing the uncertain cost for the transportation problem; it can represent delivery time, quantity of goods delivered, under used capacity, etc. The source parameter lies between left limit a_{L_i} and right limit a_{R_i}. Similarly, destination parameter lies between left limit b_{L_j} and right limit b_{R_j}.

3 Formulation of crisp constraint

Consider the following fractional transportation problem:

$$\text{(FTP)}\quad \text{Max} \qquad Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}[p_{ij}^1, p_{ij}^2]\ x_{ij} + [p_0^1, p_0^2]}{\sum_{i=1}^{m}\sum_{j=1}^{n}[d_{ij}^1, d_{ij}^2]\ x_{ij} + [d_0^1, d_0^2]}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij}\ z \geq a_{L_i} \quad \text{for } i = 1, 2, \ldots, m \tag{3.1}$$

$$\sum_{j=1}^{n} x_{ij} \leq a_{R_i} \quad \text{for } i = 1, 2, \ldots, m \tag{3.2}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_{L_j} \quad \text{for } j = 1, 2, \ldots, n \tag{3.3}$$

$$\sum_{i=1}^{m} x_{ij} \leq b_{R_j} \quad \text{for } j = 1, 2, \ldots, n \tag{3.4}$$

$$x_{ij} \geq 0, \ i = 1, 2, \ldots, m\ ;\ j = 1, 2, \ldots, n \tag{3.5}$$

where a_{L_i}, a_{R_i}, b_{L_j}, and b_{R_j} are, as in Section 2.

Theorem 3.1. Problem IFTP1 with crisp objective functions and (FTP) are equivalent [6].

4 Proposed method for the solution of the problem

For solving problem IFTP1, we introduce variable

$$z = \frac{1}{D(x)} = \frac{1}{\sum_{i=1}^{m}\sum_{j=1}^{n}[d_{ij}^1, d_{ij}^2]\ x_{ij} + [d_0^1, d_0^2]} \tag{4.1}$$

and then we have

$$\text{(LFTP2)} \quad \text{Max} \qquad Q(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}[p_{ij}^1, p_{ij}^2]\ x_{ij}\ z + [p_0^1, p_0^2]\ z$$

$$\text{Subject to} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}[d_{ij}^1, d_{ij}^2]\ x_{ij}\ z + [d_0^1, d_0^2]\ z = 1 \tag{4.2}$$

$$\sum_{j=1}^{n} x_{ij}\ z - a_{L_i}\ z \geq 0 \quad \text{for } i = 1, 2, \ldots, m \tag{4.3}$$

$$\sum_{j=1}^{n} x_{ij}\ z - a_{R_i}\ z \leq 0 \quad \text{for } i = 1, 2, \ldots, m \tag{4.4}$$

$$\sum_{i=1}^{m} x_{ij}\ z - b_{L_j}\ z \geq 0 \quad \text{for } j = 1, 2, \ldots, n \tag{4.5}$$

$$\sum_{i=1}^{m} x_{ij}\ z - b_{R_j}\ z \leq 0 \quad \text{for } j = 1, 2, \ldots, n \tag{4.6}$$

$$x_{ij} \geq 0, \ i = 1, 2, \ldots, m\ ;\ j = 1, 2, \ldots, n, z \geq 0 \tag{4.7}$$

By introducing variables $y_{ij} = x_{ij}z$, for $i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n$ the problem IFTP2 is transformed into The following equivalent problem:

$$\text{(LFTP3)} \quad \text{Max} \qquad Q(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}[p_{ij}^1, p_{ij}^2]\ y_{ij} + [p_0^1, p_0^2]\ z$$

$$\text{Subject to} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}[d_{ij}^1, d_{ij}^2]\ y_{ij} + [d_0^1, d_0^2]\ z = 1 \tag{4.8}$$

$$\sum_{j=1}^{n} y_{ij} - a_{L_i}\ z \geq 0 \quad \text{for } i = 1, 2, \ldots, m \tag{4.9}$$

$$\sum_{j=1}^{n} y_{ij} - a_{R_i}\ z \leq 0 \quad \text{for } i = 1, 2, \ldots, m \tag{4.10}$$

$$\sum_{i=1}^{m} y_{ij} - b_{L_j}\ z \geq 0 \quad \text{for } j = 1, 2, \ldots, n \tag{4.11}$$

$$\sum_{i=1}^{m} y_{ij} - b_{R_j}\ z \leq 0 \quad \text{for } j = 1, 2, \ldots, n \tag{4.12}$$

$$y_{ij} \geq 0,\ \ i = 1, 2, \ldots, m\ ;\ j = 1, 2, \ldots, n \tag{4.13}$$

The linear combination of each interval yields to the following problem:

$$\text{(LFTP4) Max}\ \ Q(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}[(1-\lambda_{ij})\ p_{ij}^1 + \lambda_{ij}\ p_{ij}^2]\ y_{ij} + [(1-\lambda_0)\ p_0^1 + \lambda_0\ p_0^2]\ z$$

$$\text{Subject to}\ \ \sum_{i=1}^{m}\sum_{j=1}^{n}[(1-\beta_{ij})d_{ij}^1 + \beta_{ij}d_{ij}^2]\ y_{ij} + [(1-\beta_0)d_0^1 + \beta_0 d_0^2]\ z = 1 \tag{4.14}$$

$$\sum_{j=1}^{n} y_{ij} - a_{L_i}\ z \geq 0 \quad \text{for } i = 1, 2, \ldots, m \tag{4.15}$$

$$\sum_{j=1}^{n} y_{ij} - a_{R_i}\ z \leq 0 \quad \text{for } i = 1, 2, \ldots, m \tag{4.16}$$

$$\sum_{i=1}^{m} y_{ij} - b_{L_j}\ z \geq 0 \quad \text{for } j = 1, 2, \ldots, n \tag{4.17}$$

$$\sum_{i=1}^{m} y_{ij} - b_{R_j}\ z \leq 0 \quad \text{for } j = 1, 2, \ldots, n \tag{4.18}$$

$$y_{ij} \geq 0,\ \ i = 1, 2, \ldots, m\ ;\ j = 1, 2, \ldots, n \tag{4.19}$$

$$0 \leq \lambda_{ij} \leq 1\ ,\ \ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \tag{4.20}$$

$$0 \leq \beta_{ij} \leq 1\ ,\ \ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \tag{4.21}$$

$$z \geq 0,\ \ 0 \leq \lambda_0 \leq 1,\ \ 0 \leq \beta_0 \leq 1 \tag{4.22}$$

The equality constraint in problem IFTP4 can be further reduced to

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\beta_{ij}[d_{ij}^2 - d_{ij}^1]y_{ij} + \beta_0[d_0^2 - d_0^1]z + \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^1 y_{ij} + d_0^1 z = 1 \tag{4.23}$$

since

$$z \geq 0,\ 0 \leq \beta_0 \leq 1,\ d_0^2 - d_0^1 \geq 0,\ y_{ij} \geq 0,\ 0 \leq \beta_{ij} \leq 1,\ d_{ij}^2 - d_{ij}^1 \geq 0,\ i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n$$

therefore

$$\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^1\ y_{ij} + d_0^1\ z \leq 1 \tag{4.24}$$

and

$$\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^2\ y_{ij} + d_0^2\ z \geq 1. \tag{4.25}$$

Therefore on using (4.24) and (4.25), the problem(LFTP4) is transformed into the following equivalent problem:

$$\text{(LFTP5) Max}\ \ Q(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}[1 - \lambda_{ij}p_{ij}^1 + \lambda_{ij}p_{ij}^2]\ y_{ij} + 1 - \lambda_0 p_0^1 + \lambda_0 p_0^2]\ z$$

$$\text{Subject to} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^1 \ y_{ij} + d_0^1 \ z \leq 1 \tag{4.26}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^2 \ y_{ij} + d_0^2 \ z \geq 1 \tag{4.27}$$

$$\sum_{j=1}^{n} y_{ij} - a_{L_i} \ z \geq 0 \ \text{ for } i = 1, 2, \ldots, m \tag{4.28}$$

$$\sum_{j=1}^{n} y_{ij} - a_{R_i} \ z \leq 0 \ \text{ for } i = 1, 2, \ldots, m \tag{4.29}$$

$$\sum_{i=1}^{m} y_{ij} - b_{L_j} \ z \geq 0 \ \text{ for } j = 1, 2, \ldots, n \tag{4.30}$$

$$\sum_{i=1}^{m} y_{ij} - b_{R_j} \ z \leq 0 \ \text{ for } j = 1, 2, \ldots, n \tag{4.31}$$

$$y_{ij} \geq 0, \ i = 1, 2, \cdots, m \ ; \ j = 1, 2, \ldots, n \tag{4.32}$$

$$0 \leq \lambda_{ij} \leq 1, \ i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n \tag{4.33}$$

$$0 \leq \beta_{ij} \leq 1, \ i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n \tag{4.34}$$

$$z \geq 0, \ 0 \leq \lambda_0 \leq 1, \ 0 \leq \beta_0 \leq 1. \tag{4.35}$$

In addition, if we let$(\overline{y_{ij}}, \overline{z})$, for $i = 1, 2, \ldots, m \ ; \ j = 1, 2, \ldots, n$ be a point of feasible region of problem (LFTP5), with $0 \leq \beta_{ij} \leq 1, \ p_{ij}^2 - p_{ij}^1 \geq 0$, for $i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n, \ 0 \leq \beta_0 \leq 1, \ p_0^2 - p_0^1 \geq 0$, the objective function in problem (LFTP5) can be written as:

$$\begin{aligned} &\sum_{i=1}^{m}\sum_{j=1}^{n} \lambda_{ij} \ [p_{ij}^2 - p_{ij}^1] \ y_{ij} + \lambda_0 \ [p_0^2 - p_0^1] \ z + \sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^1 \ y_{ij} + p_0^1 \ z \\ \leq &\sum_{i=1}^{m}\sum_{j=1}^{n} [p_{ij}^2 - p_{ij}^1] \ y_{ij} + [p_0^2 - p_0^1] \ z + \sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^1 \ y_{ij} + p_0^1 \ z \\ = &\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^2 \ y_{ij} + p_0^2 \ z. \end{aligned}$$

The righthand side of the above equality can be considered as an upper bound for the objective function of the problem (LFTP5). Therefore, the problem (LFTP5) can be equivalently written as:

$$\text{(LFTP6)} \ \text{Max} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^2 \ y_{ij} + p_0^2 \ z$$

$$\text{Subject to} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^1 \ y_{ij} + d_0^1 \ z \leq 1 \tag{4.36}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^2 \ y_{ij} + d_0^2 \ z \geq 1 \tag{4.37}$$

$$\sum_{j=1}^{n} y_{ij} - a_{L_i} \ z \geq 0 \ \text{ for } i = 1, 2, \ldots, m \tag{4.38}$$

$$\sum_{j=1}^{n} y_{ij} - a_{R_i} \ z \leq 0 \ \text{ for } i = 1, 2, \ldots, m \tag{4.39}$$

$$\sum_{i=1}^{m} y_{ij} - b_{L_j} \ z \geq 0 \ \text{ for } j = 1, 2, \ldots, n \tag{4.40}$$

$$\sum_{i=1}^{m} y_{ij} - b_{R_j}\ z \leq 0 \ \text{ for } j = 1, 2, \ldots, n \tag{4.41}$$

$$y_{ij} \geq 0\ ,\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \tag{4.42}$$

$$z \geq 0. \tag{4.43}$$

The optimal solution(y^*_{ij}, z^*), for $i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$ of the linear programming problem (LFTP6) is same as the optimal solution of the problem (LFTP1) which can be easily obtained by $x_{ij} = \dfrac{y_{ij}}{z}$, for $i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$.

5 Cases of MITP

Three major cases that may arise in a interval fractional transportation problem can be described as:

- The coefficients p_{ij} and d_{ij} are in the form of interval, whereas source and destination parameters are deterministic.
- The source and destination parameters, i.e., a_i and b_j, are in the form of intervals but the objective functions coefficients p_{ij} and d_{ij} are deterministic.
- All parameters, i.e., objective functions' coefficients, the source (a_i) and destination (b_j) parameters are in the form of interval.

5.1 Case-I

When the objective functions coefficients p_{ij} and d_{ij} are in the form interval, i.e., $p_{ij} = [p^1_{ij}, p^2_{ij}]$ and $p_{ij} = [p^1_{ij}, p^2_{ij}]$ and the constraints are deterministic, i.e., the parameters a_i and b_j are deterministic, the multiobjective transportation problem is as follows:

$$\text{(LFTP1)}\ \ \text{Max} \qquad Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}[p^1_{ij}, p^2_{ij}]\ x_{ij} + [p^1_0, p^2_0]}{\sum_{i=1}^{m}\sum_{j=1}^{n}[d^1_{ij}, d^2_{ij}]\ x_{ij} + [d^1_0, d^2_0]}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} = a_i \ \text{ for } i = 1, 2, \ldots, m \tag{5.1}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \ \text{ for } j = 1, 2, \ldots, n \tag{5.2}$$

$$x_{ij} \geq 0,\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \tag{5.3}$$

the problem (LFTP1) can be equivalently written as:

$$\text{(LFTP} - \text{I)}\ \ \text{Max} \qquad \sum_{i=1}^{m}\sum_{j=1}^{n} p^2_{ij}\ y_{ij} + p^2_0\ z$$

$$\text{Subject to} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} d^1_{ij}\ y_{ij} + d^1_0\ z \leq 1 \tag{5.4}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} d^2_{ij}\ y_{ij} + d^2_0\ z \geq 1 \tag{5.5}$$

$$\sum_{j=1}^{n} y_{ij} - a_i\ z = 0 \ \text{ for } i = 1, 2, \ldots, m \tag{5.6}$$

$$\sum_{i=1}^{m} y_{ij} - b_j\ z = 0 \ \text{ for } j = 1, 2, \ldots, n \tag{5.7}$$

$$y_{ij} \geq 0,\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \tag{5.8}$$

$$z \geq 0. \tag{5.9}$$

The optimal solution (y^*_{ij}, z^*), for $i = 1, 2, \dots, m;\ j = 1, 2, \dots, n$ of the linear programming problem $(\mathrm{LFTP} - \mathrm{I})$ is same as the optimal solution of the problem (LFTP1).

Example 5.1. Let us consider following linear fractional transportation problem with interval coefficients in the objective function

$$\text{(LFTP1)} \qquad \text{Max} \qquad Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{3}\sum_{j=1}^{4}[p^1_{ij}, p^2_{ij}]\ x_{ij} + [p^1_0, p^2_0]}{\sum_{i=1}^{3}\sum_{j=1}^{4}[d^1_{ij}, d^2_{ij}]\ x_{ij} + [d^1_0, d^2_0]}$$

$$\text{Subject to} \quad \sum_{j=1}^{4} x_{ij} = a_i \quad \text{for } i = 1, 2, \dots, m \tag{5.10}$$

$$\sum_{i=1}^{3} x_{ij} = b_j \quad \text{for } j = 1, 2, \dots, n \tag{5.11}$$

$$x_{ij} \geq 0,\ i = 1, 2, 3;\ j = 1, 2, 3, 4. \tag{5.12}$$

where

$$P = \begin{bmatrix} [1,5] & [4,6] & [5,8] & [4,7] \\ [0,3] & [8,12] & [1,5] & [3,6] \\ [6,9] & [7,10] & [2,5] & [3,8] \end{bmatrix} \qquad D = \begin{bmatrix} [1,5] & [2,6] & [1,8] & [3,4] \\ [5,6] & [7,9] & [8,10] & [5,9] \\ [6,8] & [2,3] & [5,9] & [0,3] \end{bmatrix},$$

$$(a_1, a_2, a_3) = (9, 20, 17), \qquad (b_1, b_2, b_3, b_4) = (7, 9, 14, 16).$$

The above problem is transformed into the problem (LFTP6). Therefore we have:

$$\begin{aligned}
\text{Max} \quad & 5y_{11} + 6y_{12} + 8y_{13} + 7y_{14} + 3y_{21} + 12y_{22} + 5y_{23} + 6y_{24} + 9y_{31} + 10y_{32} + 5y_{33} + 8y_{34} \\
\text{Subject to} \quad & y_{11} + 2y_{12} + y_{13} + 3y_{14} + 5y_{21} + 7y_{22} + 8y_{23} + 5y_{24} + 6y_{31} + 2y_{32} + 5y_{33} \leq 1 \\
& 5y_{11} + 6y_{12} + 8y_{13} + 4y_{14} + 6y_{21} + 9x_{22} + 10y_{23} + 9y_{24} + 8y_{31} + 3y_{32} + 9y_{33} + 3y_{34} \geq 1 \\
& y_{11} + y_{12} + y_{13} + y_{14} - 9z = 0 \\
& y_{21} + y_{22} + y_{23} + y_{24} - 20z = 0 \\
& y_{31} + y_{32} + y_{33} + y_{34} - 17z = 0 \\
& y_{11} + y_{21} + y_{31} - 7z = 0 \\
& y_{12} + y_{22} + y_{32} - 9z = 0 \\
& y_{13} + y_{23} + y_{33} - 14z = 0 \\
& y_{14} + y_{24} + y_{34} - 16z = 0 \\
& y_{ij} \geq 0, \qquad i = 1, 2, 3;\ j = 1, 2, 3, 4.
\end{aligned}$$

The optimum solution of the above problem is

$$y_{13} = 0.06336, \quad y_{21} = 0.0493, \quad y_{22} = 0.05632, \quad y_{23} = 0.0352, \quad y_{32} = 0.00704, \quad y_{34} = 0.11264, \quad z = 0.00704$$

optimum solution of the linear fractional transportation problem with interval coefficients is

$$x_{13} = 9, \qquad x_{21} = 7, \qquad x_{22} = 8, \qquad x_{23} = 5, \qquad x_{32} = 1, \qquad x_{34} = 16.$$

5.2 Case-II

When the objective functions coefficients p_{ij} and d_{ij} are deterministic, and the constraints are in the form interval, i.e., $[a^i_L, a^i_R]$ and $[b^j_L, b^j_R]$, the fractional transportation problem is as follows:

$$\text{(LFTP1)} \ \ \text{Max} \qquad Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}\ x_{ij} + p_0}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}\ x_{ij} + d_0}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} = [a_{L_i}, a_{R_i}] \quad \text{for } i = 1, 2, \dots, m \tag{5.13}$$

$$\sum_{i=1}^{m} x_{ij} = [b_{L_j}, b_{R_j}] \quad \text{for } j = 1, 2, \dots, n \tag{5.14}$$

$$x_{ij} \geq 0,\ i = 1, 2, \dots, m;\ j = 1, 2, \dots, n. \tag{5.15}$$

Here and in what follows we suppose that $D(x) > 0$, for all $x = (x_{ij}) \in S$, where S denotes a feasible set defined by constraints (5.13) to (5.15). Further, we assume that $a_i > 0$, $b_j > 0$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ and total demand equals to total supply, i.e.

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j}, \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}$$

the problem (LFTP1) can be equivalently written as:

$$\text{(LFTP3)} \quad \text{Max} \qquad Q(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij}\ y_{ij} + p_0\ z$$

$$\text{Subject to} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}\ y_{ij} + d_0\ z = 1 \tag{5.16}$$

$$\sum_{j=1}^{n} y_{ij} - a_{L_i}\ z \geq 0 \quad \text{for } i = 1, 2, \ldots, m \tag{5.17}$$

$$\sum_{j=1}^{n} y_{ij} - a_{R_i}\ z \leq 0 \quad \text{for } i = 1, 2, \ldots, m \tag{5.18}$$

$$\sum_{i=1}^{m} y_{ij} - b_{L_j}\ z \geq 0 \quad \text{for } j = 1, 2, \ldots, n \tag{5.19}$$

$$\sum_{i=1}^{m} y_{ij} - b_{R_j}\ z \leq 0 \quad \text{for } j = 1, 2, \ldots, n \tag{5.20}$$

$$y_{ij} \geq 0,\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n \tag{5.21}$$

$$z \geq 0. \tag{5.22}$$

The optimal solution (y^*_{ij}, z^*), for $i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$ of the linear programming problem $(\mathrm{LFTP} - \mathrm{I})$ is same as the optimal solution of the problem (LFTP1).

Example 5.2. Let us consider following linear fractional transportation problem with interval coefficients in the objective function

$$\text{(LFTP1)} \qquad \text{Max} \qquad Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{3} \sum_{j=1}^{4} p_{ij}\ x_{ij} + p_0}{\sum_{i=1}^{3} \sum_{j=1}^{4} d_{ij}\ x_{ij} + d_0}$$

$$\textit{Subject to} \quad \sum_{j=1}^{4} x_{1j} = [18, 20]\ ,\ \sum_{j=1}^{4} x_{2j} = [21, 24]\ ,\ \sum_{j=1}^{4} x_{3j} = [27, 30] \tag{5.23}$$

$$\sum_{i=1}^{3} x_{i1} = [17, 18],\ \sum_{i=1}^{3} x_{i2} = [19, 21], \tag{5.24}$$

$$\sum_{i=1}^{3} x_{i3} = [14, 16],\ \sum_{i=1}^{3} x_{i4} = [16, 19] \tag{5.25}$$

$$x_{ij} \geq 0,\ i = 1, 2, 3;\ j = 1, 2, 3, 4 \tag{5.26}$$

where

$$P = \begin{bmatrix} 16 & 15 & 19 & 17 \\ 13 & 12 & 15 & 16 \\ 19 & 10 & 15 & 18 \end{bmatrix}$$

$$D = \begin{bmatrix} 15 & 16 & 18 & 14 \\ 16 & 19 & 10 & 19 \\ 18 & 13 & 19 & 13 \end{bmatrix}.$$

The above problem is transformed into the problem (LFTP6). Therefore we have:

$$\begin{array}{rl}
\text{Max} & 16\text{y}_{11} + 15y_{12} + 19y_{13} + 17y_{14} + 13y_{21} + 12y_{22} + 15y_{23} + 16y_{24} + 19y_{31} + 10y_{32} + 15y_{33} + 18y_{34} \\
\text{Subject to} & 15\text{y}_{11} + 16y_{12} + 18y_{13} + 14y_{14} + 16y_{21} + 19y_{22} + 10y_{23} + 19y_{24} + 18y_{31} + 13y_{32} + 19y_{33} + 13y_{34} = 1 \\
& y_{11} + y_{12} + y_{13} + y_{14} - 18z \geq 0 \\
& y_{11} + y_{12} + y_{13} + y_{14} - 20z \leq 0 \\
& y_{21} + y_{22} + y_{23} + y_{24} - 21z \geq 0 \\
& y_{21} + y_{22} + y_{23} + y_{24} - 24z \leq 0 \\
& y_{31} + y_{32} + y_{33} + y_{34} - 27z \geq 0 \\
& y_{31} + y_{32} + y_{33} + y_{34} - 30z \leq 0 \\
& y_{11} + y_{21} + y_{31} - 17z \geq 0 \\
& y_{11} + y_{21} + y_{31} - 18z \leq 0 \\
& y_{12} + y_{22} + y_{32} - 19z \geq 0 \\
& y_{12} + y_{22} + y_{32} - 21z \leq 0 \\
& y_{13} + y_{23} + y_{33} - 14z \geq 0 \\
& y_{13} + y_{23} + y_{33} - 16z \leq 0 \\
& y_{14} + y_{24} + y_{34} - 16z \geq 0 \\
& y_{14} + y_{24} + y_{34} - 19z \leq 0 \\
& y_{ij} \geq 0, \qquad i = 1, 2, 3; \quad j = 1, 2, 3, 4.
\end{array}$$

The optimum solution of the above problem is

$$y_{11} = 0.0010, \quad y_{12} = 0.0189, \quad y_{21} = 0.0050, \quad y_{24} = 0.0159, \quad y_{31} = 0.0110, \quad y_{34} = 0.0189, \quad z = 0.0010$$

optimum solution of the linear fractional transportation problem with interval coefficients is

$$x_{11} = 1, \quad x_{12} = 19, \quad x_{21} = 5, \quad x_{24} = 16, \quad x_{31} = 11, \quad x_{34} = 19.$$

5.3 Case-III

When the objective functions coefficients p_{ij}, d_{ij}, a_i and b_j are in the form interval, the multiobjective transportation problem is as follows:

$$\text{(LFTP1)}\ \text{Max} \qquad Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}[p_{ij}^1, p_{ij}^2]\ x_{ij} + [p_0^1, p_0^2]}{\sum_{i=1}^{m}\sum_{j=1}^{n}[d_{ij}^1, d_{ij}^2]\ x_{ij} + [d_0^1, d_0^2]}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} = [a_{L_i}, a_{R_i}] \ \text{ for } i = 1, 2, \ldots, m \tag{5.27}$$

$$\sum_{i=1}^{m} x_{ij} = [b_{L_j}, b_{R_j}] \ \text{ for } j = 1, 2, \ldots, n \tag{5.28}$$

$$x_{ij} \geq 0, \ i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n. \tag{5.29}$$

Here and in what follows we suppose that $D(x) > 0$, for all $x = (x_{ij}) \in S$, where S denotes a feasible set defined by constraints (5.27) to (5.29). Further, we assume that $a_i > 0$, $b_j > 0$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ and total demand equals to total supply, i.e.

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j}, \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}.$$

the problem (LFTP1) can be equivalently written as:

$$
\begin{aligned}
\text{(LFTP6) Max} \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^2 \ y_{ij} + p_0^2 \ z \\
\text{Subject to} \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^1 \ y_{ij} + d_0^1 \ z \leq 1 && (5.30)\\
& \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^2 \ y_{ij} + d_0^2 \ z \geq 1 && (5.31)\\
& \sum_{j=1}^{n} y_{ij} - a_{L_i} \ z \geq 0 \ \text{ for } i = 1,2,\ldots,m && (5.32)\\
& \sum_{j=1}^{n} y_{ij} - a_{R_i} \ z \leq 0 \ \text{ for } i = 1,2,\ldots,m && (5.33)\\
& \sum_{i=1}^{m} y_{ij} - b_{L_j} \ z \geq 0 \ \text{ for } j = 1,2,\ldots,n && (5.34)\\
& \sum_{i=1}^{m} y_{ij} - b_{R_j} \ z \leq 0 \ \text{ for } j = 1,2,\ldots,n && (5.35)\\
& y_{ij} \geq 0, \ i = 1,2,\ldots,m; \ j = 1,2,\ldots,n && (5.36)\\
& z \geq 0. && (5.37)
\end{aligned}
$$

The optimal solution (y_{ij}^*, z^*), for $i = 1,2,\ldots,m; \ j = 1,2,\ldots,n$ of the linear programming problem $(\mathrm{LFTP}-\mathrm{I})$ is same as the optimal solution of the problem (LFTP1).

Example 5.3. Let us consider following linear fractional transportation problem with interval coefficients in the objective function

$$
\begin{aligned}
\text{(LFTP1)} \quad \text{Max} \quad & Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{3}\sum_{j=1}^{4}[p_{ij}^1, p_{ij}^2] \ x_{ij} + [p_0^1, p_0^2]}{\sum_{i=1}^{3}\sum_{j=1}^{4}[d_{ij}^1, d_{ij}^2] \ x_{ij} + [d_0^1, d_0^2]} \\
\text{Subject to} \quad & \sum_{j=1}^{4} x_{1j} = [9,11], \ \sum_{j=1}^{4} x_{2j} = [20,23], \ \sum_{j=1}^{4} x_{3j} = [17,20] && (5.38)\\
& \sum_{i=1}^{3} x_{i1} = [7,8], \ \sum_{i=1}^{3} x_{i2} = [9,11], && (5.39)\\
& \sum_{i=1}^{3} x_{i3} = [14,16], \ \sum_{i=1}^{3} x_{i4} = [16,19] && (5.40)\\
& x_{ij} \geq 0, \ i = 1,2,3; \ j = 1,2,3,4, && (5.41)
\end{aligned}
$$

where

$$
P = \begin{bmatrix} [1,5] & [4,6] & [5,8] & [4,7] \\ [0,3] & [8,12] & [1,5] & [3,6] \\ [6,9] & [7,10] & [2,5] & [3,8] \end{bmatrix}
$$

$$
D = \begin{bmatrix} [1,5] & [2,6] & [1,8] & [3,4] \\ [5,6] & [7,9] & [8,10] & [5,9] \\ [6,8] & [2,3] & [5,9] & [0,3] \end{bmatrix}.
$$

The above problem is transformed into the problem (LFTP6). Therefore we have:

$$\begin{aligned}
\text{Max}\quad & 5y_{11}+6y_{12}+8y_{13}+7y_{14}+3y_{21}+12y_{22}+5y_{23}+6y_{24}+9y_{31}+10y_{32}+5y_{33}+8y_{34}\\
\text{Subject to}\quad & y_{11}+2y_{12}+y_{13}+3y_{14}+5y_{21}+7y_{22}+8y_{23}+5y_{24}+6y_{31}+2y_{32}+5y_{33}\leq 1\\
& 5y_{11}+6y_{12}+8y_{13}+4y_{14}+6y_{21}+9y_{22}+10y_{23}+9y_{24}+8y_{31}+3y_{32}+9y_{33}+3y_{34}\geq 1\\
& y_{11}+y_{12}+y_{13}+y_{14}-9z\geq 0\\
& y_{11}+y_{12}+y_{13}+y_{14}-11z\leq 0\\
& y_{21}+y_{22}+y_{23}+y_{24}-20z\geq 0\\
& y_{21}+y_{22}+y_{23}+y_{24}-23z\leq 0\\
& y_{31}+y_{32}+y_{33}+y_{34}-17z\geq 0\\
& y_{31}+y_{32}+y_{33}+y_{34}-20z\leq 0\\
& y_{11}+y_{21}+y_{31}-7z\geq 0\\
& y_{11}+y_{21}+y_{31}-8z\leq 0\\
& y_{12}+y_{22}+y_{32}-9z\geq 0\\
& y_{12}+y_{22}+y_{32}-11z\leq 0\\
& y_{13}+y_{23}+y_{33}-14z\geq 0\\
& y_{13}+y_{23}+y_{33}-16z\leq 0\\
& y_{14}+y_{24}+y_{34}-16z\geq 0\\
& y_{14}+y_{24}+y_{34}-19z\leq 0\\
& y_{ij}\geq 0,\qquad i=1,2,3;\quad j=1,2,3,4.
\end{aligned}$$

The optimum solution of the above problem is

$$y_{13}=0.0786,\quad y_{21}=0.0502,\quad y_{22}=0.0714,\quad y_{23}=0.0214,\quad y_{34}=0.1429,\quad z=2.8857$$

optimum solution of the linear fractional transportation problem with interval coefficients is

$$x_{13}=11,\quad x_{21}=7,\quad x_{22}=10,\quad x_{23}=3,\quad x_{32}=1,\quad x_{34}=19.$$

6 Conclusion

The present paper proposes a solution procedure to solve a linear fractional transportation problem, where the coefficient of the objective functions and the source and destination parameters have been considered as intervals. In the proposed method, by using a convex combination of the left limit and the right limit of intervals instead of intervals and also using variable transformation, the linear fractional transportation problem is transformed into a nonlinear programming problem, which finally is changed into a linear programming problem, which has two more constraints and one more variable compared to the initial problem. The method is such that each of the points at intervals is examined to obtain the optimal solution to the problem. The constraints with interval source and destination parameters have been converted into deterministic ones. Lastly, the solution procedure has been illustrated by one example for each case in three different situations in the interval fractional transportation problem.

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