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Result on value distribution of meromorphic functions

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Abstract

In this article, we deal with value distribution of transcendental meromorphic functions with finite order and obtain some results which improve previous theorems given by Y.Liu, J.P. Wang and F.H. Liu [14].

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1 Introduction

In this paper, meromorphic functions are always defined as meromorphic functions in the complex plane. We adopt the standard notations of Nevanlinna's theory of meromorphic functions as explained in [9], [12] and [17]. For a non-constant meromorphic function f, we denote by $T(r, f)$ the Nevanlinna characteristic function of f and by $S(r, f)$ any quantity satisfying $S(r, f) = o\{T(r, f)\}$ for all r outside a possible exceptional set of the finite logarithmic measure, $S(f)$ denotes the family of all meromorphic functions α such that $T(r, \alpha) = S(r, f)$, where $r \to \infty$ outside a possible exceptional set of finite logarithmic measure.

Let $f(z)$ and $g(z)$ be two nonconstant meromorphic functions. If for some $a \in \mathbb{C} \cup \{\infty\}$, the zeros of $f(z) - a$ and $g(z) - a$ (if $a = \infty$, zeros of $f(z) - a$ and $g(z) - a$ are the poles of $f(z)$ and $g(z)$, respectively) coincide in locations and multiplicities, we say that $f(z)$ and $g(z)$ share the value a CM (counting multiplicities) and if they coincide in locations only, we say that $f(z)$ and $g(z)$ share a IM (ignoring multiplicities). In 2010, Qi et al. [15] studied the uniqueness of the difference monomials and obtained the following result:

Theorem A. Let $f(z)$ and $g(z)$ be transcendental entire functions with finite order, c a non–zero complex constant, and $n \geq 6$ an integer. If $E(1, f^n(z)f(z + c)) = E(1, g^n(z)g(z + c))$, then $f(z) \equiv t_1 g(z)$ or $f(z)g(z) = t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$.

In 2015, Y. Liu, J.P.Wang and F.H. Liu [14] obtained the following results.

Theorem B. Let $c \in \mathbb{C} \setminus \{0\}$. Let $f(z)$ and $g(z)$ be two transcendental meromorphic functions with finite order, and $n(\geq 14), k(\geq 3)$ be two positive integers. If $E_k(1, f^n(z)f(z + c)) = E_k(1, g^n(z)g(z + c))$, then $f(z) \equiv t_1 g(z)$ or $f(z)g(z) = t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$.

Theorem C. Let $c \in \mathbb{C}$ and $n \geq 16$ be an integer. Let $f(z)$ and $g(z)$ be two transcendental meromorphic functions with finite order. If $E_2(1, f^n(z)f(z+c)) = E_2(1, g^n(z)g(z+c))$, then $f(z) \equiv t_1 g(z)$ or $f(z)g(z) = t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$.

Theorem D. Let $c \in \mathbb{C}$ and $n \geq 22$ be an integer. Let $f(z)$ and $g(z)$ be two transcendental meromorphic functions

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with finite order. If $E_1(1, f^n(z)f(z+c)) = E_1(1, g^n(z)g(z+c))$, then $f(z) \equiv t_1 g(z)$ or $f(z)g(z) = t_2$, for some constants t_1 and t_2 that satisfy $t_1^{n+1} = 1$ and $t_2^{n+1} = 1$.

Let $P(z) = a_n z^n + a_{n-1}z^{n-1} + ... + a_0$ be a non-zero polynomial where $a_n(\neq 0), a_{n-1}, ..., a_0$ are complex constants. We denote Γ_0 by $\Gamma_0 = m_1 + m_2$ respectively, where m_1 is the number of simple zeros of $P(z)$ and m_2 is the number of multiple zeros of $P(z)$. Also m is the total number of zeros of $P(z)$, so $m = m_1 + m_2$.

This paper will investigate the value distribution of meromorphic functions and obtain the following result.

Theorem 1.1. Let f and g be transcendental meromorphic functions with finite order and c be a nonzero complex constant. Let $P(z) = a_n z^n + a_{n-1}z^{n-1} + ... + a_0$ be a nonzero polynomial, where $a_0, a_1, ...a_n(\neq 0)$ are complex constants and let $n > 2\Gamma_0 + 9$, where $\Gamma_0 = m_1 + m_2$, m_1 is the number of the simple zero of $P(z)$ and m_2 is the number of multiple zeros of $P(z)$. If $E_l(1, P(f)f(z+c)) = E_l(1, P(g)g(z+c))$ and l, n, m are integers satisfying one of the following conditions:

(I) $l = 2, n > 2\Gamma_0 + \frac{m}{2} + 11 - \lambda$;

(II) $l = 1, n > 2\Gamma_0 + 2m + 13 - 2\lambda$;

(III) $l = 0, n > 2\Gamma_0 + 3m + 17 - 6\lambda$;

(IV) $l \geq 3, n > 2\Gamma_0 + 11$.

Then one of the following results holds:

(i) $f \equiv tg$ for a constant t such that $t^d = 1$, where $d = GCD\{n+1, n, n-1, ..., 1\}$.

(ii) f and g satisfy the algebraic equation $R(f, g) \equiv 0$, where $R(\omega_1, \omega_2) = P(\omega_1)\omega_1(z+c) - P(\omega_2)\omega_2(z+c)$.

(iii) $fg \equiv \mu$, where μ is a complex constant satisfying $a_n^2 \mu^{n+1} \equiv 1$.

The following example exhibits that Theorem 1.1 improves Theorems B-D respectively by relaxing the nature of sharing and by reducing the lower bound of n.

Example 1.2. Let $P(z) = (z-1)^6(z+1)^6 z^{11}$, $f(z) = sinz$, $g(z) = cosz$, $k = 0$ and $c = 2\pi$. It immediately follows that $n > 2\Gamma_2 + 1$ and $P(f)f(z+c) = P(g)g(z+c)$. Therefore $P(f)f(z+c)$ and $P(g)g(z+c)$ share 1 CM and hence they share $(1, 2)$.

Here f and g satisfy the algebraic equation $R(f, g) = 0$,where $R(w_1, w_2) = P(w_1)w_1(z+c) - P(w_2)w_2(z+c)$.

2 Preliminaries

To prove our result, we need following lemmas:

Lemma 2.1. [4] Let f and g be two meromorphic functions and let l be a positive integer. If $E_l(1; f) = E_l(1; g)$, then one of the following cases must occur:

(i)

$$\begin{aligned} T(r,f) + T(r,g) \leq\ & \overline{N}_2(r,f) + \overline{N}_2(r,g) + \overline{N}_2(r,\frac{1}{f}) + \overline{N}_2(r,\frac{1}{g}) + \overline{N}(r,\frac{1}{f-1}) + \overline{N}(r,\frac{1}{g-1}) \\ & - N_{11}(r,\frac{1}{f-1}) + \overline{N}_{(l+1}(r,\frac{1}{f-1}) + \overline{N}_{(l+1}(r,\frac{1}{g-1}) + S(r,f) + S(r,g); \end{aligned} \tag{2.1}$$

(ii)

$$f \equiv \frac{(b+1)g + (a-b-1)}{bg + (a-b)},$$

where $a(\neq 0), b$ are two constants.

Lemma 2.2. [4] Let f and g be two meromorphic functions. If f and g share 1 IM, then one of the following cases must occur:

(i)

$$T(r,f)+T(r,g)\leq 2[N_2(r,f)+N_2(r,\frac{1}{f})+N_2(r,g)+N_2(r,\frac{1}{g})]+3\overline{N}_L(r,\frac{1}{f-1})+3\overline{N}_L(r,\frac{1}{g-1})+S(r,f)+S(r,g); \quad (2.2)$$

(ii)

$$f\equiv\frac{(b+1)g+(a-b-1)}{bg+(a-b)},$$

where $a(\neq 0), b$ are two constants.

Lemma 2.3. [18] Let $f(z)$ be a transcendental meromorphic function of zero order and q a nonzero complex constant. Then

$$T(r,f(qz))=(1+o(1))T(r,f(z))$$

and

$$N(r,f(qz))=(1+o(1))N(r,f(z))$$

on a set of logarithmic density 1.

Lemma 2.4. [14] Let $f(z)$ be a transcendental meromorphic function of zero order and q,η two nonzero complex constants. Then

$$T(r,f(qz+\eta))=T(r,f(z))+S(r,f),$$
$$N(r,\frac{1}{f(qz+\eta)})\leq N(r,\frac{1}{f})+S_1(r,f),$$
$$N(r,f(qz+\eta))\leq N(r,f)+S_1(r,f),$$
$$\overline{N}(r,\frac{1}{f(qz+\eta)})\leq \overline{N}(r,\frac{1}{f})+S_1(r,f),$$
$$\overline{N}(r,f(qz+\eta))\leq \overline{N}(r,f)+S_1(r,f).$$

Lemma 2.5. [14] Let f be a trancendental meromorphic function of zero order, $q(\neq 0),\eta$ complex constants, and let $P(z)=a_nz^n+a_{n-1}z^{n-1}+...+a_1z+a_0$ be a polynomial, where $a_0,a_1,a_2...a_n(\neq 0)$ are complex constants. Then we have

$$(n-1)T(r,f)+S_1(r,f)\leq T(r,P(f)f(qz+\eta))\leq (n+1)T(r,f)+S_1(r,f).$$

If f is a transcendental entire function of zero order, we have

$$T(r,P(f)f(qz+\eta))=T(r,P(f)f)+S_1(r,f)=(n+1)T(r,f)+S_1(r,f).$$

Lemma 2.6. [14] Let $f(z)$ and $g(z)$ be transcendental entire functions of zero order, $P(z)=a_nz^n+a_{n-1}z^{n-1}+\cdots+a_1z+a_0$ be a polynomial, where $a_0,a_1,a_2,...,a_n(\neq 0)$ are complex constants. If $n\geq 2$, and $P(f)f(qz+\eta)P(g)g(qz+\eta)=t$ where $q(\neq 0)$, $\eta,t(\neq 0)$ are constants, then we have $fg=\mu$, where $a_n^2\mu^{n+1}=t$.

Lemma 2.7. [14] Let $f(z)$ be a nonconstant zero order meromorphic function and $q\in\mathbb{C}\setminus\{0\}$. Then $m(r,\frac{f(qz+\eta)}{f(z)})=S(r,f)$, on a set of logarithmic density 1.

Lemma 2.8. [16] Let f be a nonconstant meromorphic function and $P(f)=a_0+a_1f+a_2f^2+...+a_nf^n$, where $a_0,a_1,a_2,...,a_n$ are constants and $a_n\neq 0$. Then

$$T(r,P(f))=nT(r,f)+S(r,f).$$

3 Proof of the Theorem 1.1

Let $F(z) = P(f)f(z+c)$ and $G(z) = P(g)g(z+c)$. From the assumptions of Theorem 1.1, we have $E_l(1; F(z)) = E_l(1; G(z))$.

(I) $l = 2$. Since

$$\begin{aligned}&\overline{N}(r, \frac{1}{F-1}) + \overline{N}(r, \frac{1}{G-1}) - N_{11}(r, \frac{1}{F-1}) + \frac{1}{2}\overline{N}_{(l+1}(r, \frac{1}{F-1}) + \frac{1}{2})\overline{N}_{(l+1}(r, \frac{1}{G-1}) \\ &\leq \frac{1}{2}N(r, \frac{1}{F-1}) + \frac{1}{2}N(r, \frac{1}{G-1}) \leq \frac{1}{2}T(r, F) + \frac{1}{2}T(r, G) + S(r, F) + S(r, G).\end{aligned} \tag{3.1}$$

$$\begin{aligned}\overline{N}_{(l+1}(r, \frac{1}{F-1}) &\leq \frac{1}{2}N(r, \frac{F}{F'}) = \frac{1}{2}N(r, \frac{F'}{F}) + S(r, F) \\ &\leq \frac{1}{2}\overline{N}(r, \frac{1}{F}) + S(r, F) \\ &\leq \frac{m}{2}T(r, f) + \frac{1}{2}\overline{N}(r, \frac{1}{f}) + S_1(r, f).\end{aligned}$$

So,

$$\frac{1}{2}\overline{N}_{(l+1}(r, \frac{1}{F-1}) \leq \frac{m}{4}T(r, f) + \frac{1}{4}\overline{N}(r, \frac{1}{f}) + S_1(r, f),$$

$$\frac{1}{2}\overline{N}_{(l+1}(r, \frac{1}{G-1}) \leq \frac{m}{4}T(r, g) + \frac{1}{4}\overline{N}(r, \frac{1}{g}) + S_1(r, g).$$

Case 1. If $F(z), G(z)$ satisfy Lemma 2.1(i), from transcendental meromorphic function $f(z), g(z)$ and (3.1), we have

$$\begin{aligned}T(r, F(z)) + T(r, G(z)) \leq\ & 2N_2(r, F) + 2N_2(r, G) + 2N_2(r, \frac{1}{F}) + 2N_2(r, \frac{1}{G}) + \frac{m}{2}T(r, f) + \frac{m}{2}T(r, g) \\ &+ \frac{1}{2}\overline{N}(r, \frac{1}{f}) + \frac{1}{2}\overline{N}(r, \frac{1}{g}) + S_1(r, f) + S_1(r, g).\end{aligned}$$

From Lemma 2.5 and $\lambda = \min\{\Theta(0, f), \Theta(0, g)\}$, we have

$$\begin{aligned}(n-1)(T(r, f) + T(r, g)) \leq\ & 8(T(r, f) + T(r, g)) + 2(T(r, f) + T(r, g)) + 2\Gamma_0(T(r, f) + T(r, g)) \\ &+ \frac{m}{2}(T(r, f) + T(r, g)) + \frac{1}{2}\left\{\overline{N}(r, \frac{1}{f}) + \overline{N}(r, \frac{1}{g})\right\} + S_1(r, f) + S_1(r, g)\end{aligned}$$

$$(n - 2\Gamma_0 - \frac{m}{2} - 11 + \lambda)[T(r, f) + T(r, g)] \leq S_1(r, f) + S_1(r, g). \tag{3.2}$$

Since $n > 2\Gamma_0 + \frac{m}{2} + 11 - \lambda$ and f, g are transcendental functions, we can get a contradiction.

Case 2. If $F(z), G(z)$ satisfy Lemma 2.1(ii), that is,

$$F \equiv \frac{(b+1)G + (a-b-1)}{bG + (a-b)}, \tag{3.3}$$

where $a(\neq 0), b$ are two constants. We consider three cases as follows.

Subcase 2.1. $b \neq 0, -1$. If $a - b - 1 \neq 0$, then by (3.3), we know

$$\overline{N}(r, \frac{1}{G + \frac{a-b-1}{b+1}}) = \overline{N}(r, \frac{1}{F}).$$

Since f, g are meromorphic functions of zero order, by the Second Fundamental Theorem and Lemmas 2.3 and 2.4, we have

$$
\begin{aligned}
T(r,G) &\le \overline{N}(r,G) + \overline{N}(r,\frac{1}{G}) + \overline{N}(r,\frac{1}{G+\frac{a-b-1}{b+1}}) + S(r,g) \\
&\le \overline{N}(r,P(g)g(z+c)) + \overline{N}(r,\frac{1}{P(g)g(z+c)}) + \overline{N}(r,\frac{1}{F}) + S(r,g) \\
&\le (2+m+1)T(r,g) + mT(r,f) + \overline{N}(r,\frac{1}{f}) + S_1(r,f) + S_1(r,g).
\end{aligned}
$$

Then, from Lemma 2.5, we have

$$(n-m-4)T(r,g) \le mT(r,f) + \overline{N}(r,\frac{1}{f}) + S_1(r,f) + S_1(r,g).$$

Similarly, we have

$$(n-m-4)T(r,f) \le mT(r,g) + \overline{N}(r,\frac{1}{g}) + S_1(r,f) + S_1(r,g).$$

From the above two inequalities, we have

$$(n-2m-6+2\lambda)[T(r,f)+T(r,g)] \le S_1(r,f) + S_1(r,g). \tag{3.4}$$

From the definitions of m and Γ_0, we have $m = m_1 + m_2$. Since $2\Gamma_0 + \frac{m}{2} + 11 - \lambda - (2m+6-2\lambda) \ge 0$ that is, $n > 2\Gamma_0 + \frac{m}{2} + 11 - \lambda \ge (2m+6-2\lambda)$. From (3.4) and since f, g are transcendental, we can get a contradiction.

If $a-b-1=0$, then by (3.3) we know $F = ((b+1)G)/(bG+1)$. Since f, g are meromorphic functions, we get that $\frac{-1}{b}$ is a Picard's exceptional value of $G(z)$. By the Second fundamental theorem, we have

$$T(r,G) \le \overline{N}(r,\frac{1}{G}) + S(r,G) \le (m+1)T(r,g) + S_1(r,g).$$

Then, from Lemma 2.5 and $n > 2\Gamma_0 + \frac{m}{2} + 11 - \lambda$, we know $T(r,g) \le S_1(r,g)$, a contradiction.

Subcase 2.2. $b=-1$. Then (3.3) becomes $F = a/(a+1-G)$. If $a+1 \ne 0$, then $a+1$ is a Picard's exceptional value of G. Similarly to the discussion in Subcase 2.1, we can deduce a contradiction again.

If $a+1=0$, then $FG=1$, that is,

$$P(f)f(z+c)P(g)g(z+c) \equiv 1.$$

Since $n > 2\Gamma_0 + \frac{m}{2} + 11 - \lambda \ge 2$, by Lemma 2.6, we can get that $fg = \mu$ for a constant μ such that $a_n^2\mu^{n+1} \equiv 1$.

Subcase 2.3. $b=0$. Then (3.3) becomes $F = (G+a-1)/a$.

If $a-1 \ne 0$, then $\overline{N}(r,\frac{1}{G+a-1}) = \overline{N}(r,\frac{1}{F})$. Similarly to the discussion in Subcase 2.1, we can deduce a contradiction again.

If $a-1=0$, then $F \equiv G$, that is,

$$P(f)f(z+c) \equiv P(g)g(z+c). \tag{3.5}$$

Set $h = \frac{f}{g}$. If h is not a constant, from (3.5), we can get that f and g satisfy the algebraic equation $R(f,g) \equiv 0$, where $R(\omega_1,\omega_2) = P(\omega_1)\omega_1(z+c) - P(\omega_2)\omega_2(z+c)$. If h is a constant. Substituting $f = gh$ into (3.5), we can get

$$g(z+c)[a_ng^n(h^{n+1}-1) + a_{n-1}g^{n-1}(h^n-1) + ... + a_0(h-1)] \equiv 0, \tag{3.6}$$

where $a_n(\ne 0), a_{n-1}, ..., a_0$ are constants. Since g is transcendental meromorphic function, we have $g(z+c) \not\equiv 0$. Then, from (3.6), we have

$$a_ng^n(h^{n+1}-1) + a_ng^{n-1}(h^n-1) + ... + a_0(h-1) \equiv 0. \tag{3.7}$$

If $a_n \ne 0$ and $a_{n-1} = a_{n-2} = ... = a_0 = 0$, then from (3.7) and g being a transcendental function, we can get $h^{n+1} = 1$. Since $a_n \ne 0$ and there exists $a_i \ne 0 (i \in \{0,1,2,...n-1\})$. Suppose that $h^{n+1} \ne 1$, by Lemma 2.8 and (3.7), we have $T(r,g) = S(r,g)$ which is a contradiction with a transcendental function g. Then $h^{n+1} = 1$. Similarly to this discussion, we can get that $h^{j+1}=1$. when $a_j\neq 0$ for some $j=0,1,...,n$. Thus, from the definition of d, we can get that $f\equiv tg$, where t is a constant such that $t^d=1$, $d=GCD\{n+1,n,n-1,...,1\}$.

(II) $l=1$. Since

$$\begin{aligned}\overline{N}(r,\frac{1}{F-1})+\overline{N}(r,\frac{1}{G-1})-N_{11}(r,\frac{1}{F-1})&\leq\frac{1}{2}N(r,\frac{1}{F-1})+\frac{1}{2}N(r,\frac{1}{G-1})\\&\leq\frac{1}{2}T(r,F)+\frac{1}{2}T(r,G)+S(r,F)+S(r,G).\end{aligned}\tag{3.8}$$

From Lemma 2.4, we have

$$\begin{aligned}\overline{N}_{(2}(r,\frac{1}{F})&\leq N(r,\frac{F}{F'})=N(r,\frac{F'}{F})+S(r,f)\leq\overline{N}(r,\frac{1}{F})+S(r,f)\\&\leq mT(r,f)+\overline{N}(r,\frac{1}{f})+S_1(r,f),\end{aligned}\tag{3.9}$$

and

$$\overline{N}_{(2}(r,\frac{1}{G})\leq mT(r,g)+\overline{N}(r,\frac{1}{g})+S_1(r,g).\tag{3.10}$$

Case 1. If $F(z),G(z)$ satisfy Lemma 2.1(i), from f,g as meromorphic functions and (3.8)-(3.10), we have

$$\begin{aligned}T(r,F)+T(r,G)\leq&2N_2(r,F)+2N_2(r,G)+2N_2(r,\frac{1}{F})+2N_2(r,\frac{1}{G})+2m(T(r,f)+T(r,g))+2\overline{N}(r,\frac{1}{f})\\&+2\overline{N}(r,\frac{1}{g})+S_1(r,f)+S_1(r,g)\\\leq&8(T(r,f)+T(r,g))+2(\Gamma_0+m+1)(T(r,f)+T(r,g))+(2-2\lambda)(T(r,f)+T(r,g))\\&+S_1(r,f)+S_1(r,g).\end{aligned}$$

From Lemma 2.5 and $\lambda=\min\{\Theta(0,f),\Theta(0,g)\}$, we have

$$[n-2\Gamma_0-2m-13+2\lambda][T(r,f)+T(r,g)]\leq S_1(r,f)+S_1(r,g).\tag{3.11}$$

Since $n>2\Gamma_0+2m+13-2\lambda$, from (3.11) and since f,g are transcendental, we can get a contradiction.

Case 2. If $F(z),G(z)$ satisfy Lemma 2.1(ii). Similarly to the proof of Case 2 in (I) we can get the conclusion of Theorem 1.1.

(III) $l=0$, that is, $F(z),G(z)$, share 1 IM. From the definition of $F(z),G(z)$, we have

$$\begin{aligned}\overline{N}_L(r,\frac{1}{F-1})&\leq N(r,\frac{F}{F'})=N(r,\frac{F'}{F})+S(r,F)\leq\overline{N}(r,\frac{1}{F})+S(r,F)\\&\leq mT(r,f)+\overline{N}(r,\frac{1}{f})+S_1(r,f)\end{aligned}\tag{3.12}$$

Similarly, we have

$$\overline{N}_L(r,\frac{1}{G-1})\leq mT(r,g)+\overline{N}(r,\frac{1}{g})+S_1(r,g).\tag{3.13}$$

Case 1. Suppose that $F(z),G(z)$ satisfy Lemma 2.2(i). From (3.12) and (3.13), we have

$$\begin{aligned}T(r,F(z))+T(r,G(z))\leq&2N_2(r,F)+2N_2(r,G)+2N_2(r,\frac{1}{F})+2N_2(r,\frac{1}{G})+3mT(r,f)+3mT(r,g)+3\overline{N}(r,\frac{1}{f})\\&+3\overline{N}(r,\frac{1}{g})+S_1(r,f)+S_1(r,g)\\\leq&8(T(r,f)+T(r,g))+(2\Gamma_0+3m+2)(T(r,f)+T(r,g))+3(2-2\lambda)(T(r,f)+T(r,g))\\&+S_1(r,f)+S_1(r,g).\end{aligned}$$

From Lemma 2.5, we have

$$(n-2\Gamma_0-3m-17+6\lambda)(T(r,f)+T(r,g))\leq S_1(r,f)+S_1(r,g)$$

Since $n>2\Gamma_0+3m+17-6\lambda$, we get a contradiction.

Case 2. Suppose that $F(z),G(z)$ satisfy Lemma 2.2(ii). Similarly to the proof of the Case 2 in (I), we can easily get the conclusion of Theorem 1.1.

(IV) $l\geq 3$. Since

$$\begin{aligned}\overline{N}(r,\frac{1}{F(z)-1})+\overline{N}(r,\frac{1}{G(z)-1})+\overline{N}_{(l+1}(r,\frac{1}{F(z)-1})+\overline{N}_{(l+1}(r,\frac{1}{G(z)-1})-N_{11}(r,\frac{1}{F(z)-1})\\ \leq\frac{1}{2}N(r,\frac{1}{F(z)-1})+\frac{1}{2}N(r,\frac{1}{G(z)-1})+S(r,F)+S(r,G)\\ \leq\frac{1}{2}T(r,F)+\frac{1}{2}T(r,G)+S(r,F)+S(r,G)\end{aligned}\tag{3.14}$$

Case 1. Suppose that $F(z),G(z)$ satisfy Lemma 2.1(i). From Lemmas 2.4 and 2.7, we have

$$(n-1)(T(r,f)+T(r,g))\leq(8+2\Gamma_0+2)[T(r,f)+T(r,g)]+S_1(r,f)+S_1(r,g)$$

that is,

$$(n-11-2\Gamma_0)[T(r,f)+T(r,g)]\leq S_1(r,f)+S_1(r,g).\tag{3.15}$$

Since $n>2\Gamma_0+11$ and f,g are transcendental functions, we can get a contradiction.

Case 2. Suppose that $F(z),G(z)$ satisfy Lemma 2.1(ii). Similarly to the proof of Case 2 of (I), we can easily get the conclusions of Theorem 1.1.

Thus, the proof of Theorem 1.1 is complete.

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