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Research Article

Design and Performance Evaluation of a Multi-Stepped Absorber Plate Fin in a Flat Plate Solar Collector

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ABSTRACT

This research investigates the thermal performance and optimization of a new absorber plate fin design with a double step change in thickness (rectangular profile with double step changes in local thickness, RPDSLT). It compares the thermal performance of RPDSLT to fins with simpler geometries (rectangular, trapezoidal, and rectangular with a single step change) across a wide range of geometrical parameters. The main objective of researchers in this field is to determine the maximum fin efficiency with minimal fin material usage. Therefore, the focus is on understanding the impact of the additional step change in RPDSLT on heat transfer efficiency and examining whether the fin efficiency is enhanced or not. The study aims to identify the optimal values of the geometrical parameters (plate fin thickness ratios and dimensionless step lengths) that maximize efficiency. The absorber plate fin thickness plays a crucial role in solar collector efficiency. The research explores how the double step change in RPDSLT thickness affects collector efficiency compared to simpler fin designs. Detailed schematics of each fin profile and a solar collector demonstration are presented. The result show that the RPDSLT fin exhibits a maximum fin efficiency roughly 5% greater than the rectangular profile with step changes in local thickness (RPSLT) fin when their geometries is optimized. Compared to rectangular and trapezoidal fins, the RPDSLT offers a smaller efficiency improvement of 1-2%. Nevertheless, the superior thermal performance of the RPDSLT design could make it a preferred choice for flat plate solar collectors despite its more intricate fabrication. The study indicates that optimal efficiency is achieved with the minimum tested step-length ratios of 0.1 and the maximum tested thickness ratios of 0.9. Therefore, for efficient material utilization in the plate fin, it is recommended to use minimal step-length ratios and maximal thickness ratios.

1. Introduction

A flat plate solar collector is a type of heat exchanger designed to convert solar energy into usable heat energy for the transport medium carried out in the collector tube. Its primary application is the production of low-temperature energy. So, a collector turns absorbed solar radiation energy into useable heat energy, which is then transmitted to a fluid running through

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the tubes. Flat plate solar collectors are used for residential water heating, space heating, solar refrigeration systems, industrial process heating, and other purposes. An absorber plate fin converts solar energy into thermal energy. Thus, the thermal performance of an absorber plate strongly influences the fin efficiency of a flat plate solar collector of the plate fin. Several studies have also focused on improving the fabrication of the absorber plate fin.

Many studies have used a constant cross-sectional area for the plate fin in the construction of flat plate solar collectors [1, 3]. As solar radiation is received by an absorber plate fin, heat transfer increases along the direction of heat conduction. This absorbed energy is transmitted to the fluid. Hottel and Woertz [2] showed interest in investigating the usage of twin absorbers, where one is employed for maximal intake of morning solar and another for the afternoon. If the two absorbers are operated individually, one of them is not sufficiently exposed to release heat received by the other because diffuse sky radiation is sufficient to prevent reverse circulation. Such a dual system would be different from a bigger single absorber (except at noon), since the immediately exposed absorber would have greater temperatures.

Levinskii et al. [4] developed a theoretical formulation for the optimal thickness of the absorber plate fin, as well as the distance between two parallel collection tubes, which aids in determining optimal heat transfer via the plate fin. They discovered that the profile shape of an absorber plate fin in a properly designed collector should be divergent in the direction of heat conduction.

Researchers have previously conducted studies on a variety of non-rectangular plate fin designs, including parabolic and triangular forms. Kovarik [5] has expressed his opinions on parabolic profiles. They claimed that the parabolic profile of absorber plate fins is more effective for heat transfer and has a little greater heat transfer rate than the triangular and trapezoidal profiles with the same volume. However, Due to manufacturing challenges, these parabolic shapes with zero or negligible tip thickness and complex curvatures are rarely used. In contrast, Kovarik [5] stated that the plate fin of triangular shape takes about 44% less plate material, but because of the zero tip thickness, it is impossible to build. It is also crucial to note that due to the difficulty of manufacture and the need for unique manufacturing techniques, it is expensive. Therefore, for a given heat transfer rate, the trapezoidal shape is preferred since it requires the least amount of material.

Hollands and Stedman [6] introduced a novel profile, the rectangular profile with step changes in local thickness (RPSLT). They evaluated the thermal performance of the RPSLT profile. They assumed that heat flow is continuous at the intersection of the two portions of the absorber plate fin. This assumption ensures that the same amount of energy is transported by conduction in both portions of the plate fin. In other words, no solar energy is absorbed and lost at the junction of the absorber plate's extra thickness. However, the assumption of heat transfer continuity is not valid because of the energy interaction at the excess thickness junction.

To achieve fin efficiency, a changed energy balance at the junction is required. Aziz [7] conducted a literature assessment on the optimal dimensions of extended surfaces that lose heat to the surrounding environment by pure convection. His assessment covers longitudinal fins, annular or radial fins and spines with varying profile shapes. The optimal dimensions for each shape are defined in terms of material volume and heat dissipation. The major objectives of this study were to investigate the effects of tip heat loss, variable energy transfer coefficient, internal heat production, thermal conductivity of plate material, convection heat transfer through base, and optimal thickness at the primary surface on optimal dimensions.

It is observed from the literature review that fins should be constructed to reduce entropy generation rather than to achieve maximal heat transmission for a given amount of material.

Bliss Jr. [8] proposed a single "efficiency factor" that included all elements determining the efficiency of a flat-plate solar collector, commonly known as a heat exchanger. These efficiency factors, which are minimally affected by operating circumstances, are more or less design constants for the specific collector design. As a result, efficiency factors are particularly useful for conduction precise design and performance calculations. Mathematical derivations are provided, together with graphical data, for several of these efficiency factors for different types of solar collectors.

They are also immediately relevant to other forms of panel heat exchangers, such as floor or ceiling panels used to regulate interior temperatures. However, it may be inferred that these efficiency factors are not a novel notion in solar heat collector design, and they do not appear to have been employed as extensively as they could have been. Essentially, their notion may be utilized to reduce excessive empiricism in the design of heat exchangers used to harvest solar energy.

Badescu [9] suggested strategies for estimating and optimizing the construction of a solar collection system. His research uses four economic indices as objective functions, including net present value and internal return. He stressed the need to select the best devices from a given group of solar collectors using optimum non-uniformly distributed characteristics. According to his findings, unglazed, single-, and double-glazed collectors should be employed in the same collection area to achieve optimum results. Additionally, the bottom insulation thickness should be adjusted proportionately. The study found that unglazed, single, and double-glazed collectors with adjustable bottom insulation thickness provide optimum performance in the same collection area.

Lund [12] conducted study on the formulation of heat transmission in absorbers of flat-plate solar collector. His study includes a form component that considers flow and heat transfer in ducts. The work identifies analytical solutions in the form of perturbation series, which apply to actual collectors with lengths greater than the distance between flow ducts.

Solar collector performance is determined based on the effectiveness or number of transfer units. The researcher created heat transfer equations that account for various duct designs in a non-dimensional form, including a form component. These equations use perturbation series to account for axial variation in the duct's heat transfer coefficient for a certain flow and energy transfer development. To summarize, unlike previous procedures, this design method applies to all duct designs, heat transfer developments, and variables influencing plate thermal design when current methods are applied for particular cases.

Kundu [15] conducted a brief investigation of the numerous profile forms of absorber plates, including rectangular, trapezoidal, and RPSLT. He modified Hollands and Stedman [5] for the RPSLT profile. He thought that no energy was absorbed at the intersection of the extra thickness of the plate from the sun and thus it was lost. However, in actuality, energy exchange at the junction must occur between the environment and the absorber plate. He developed the governing equations with the necessary Hollands and Stedman [5] modifications for plate fin efficiency, plate volume, and optimal absorber plate design. The results clearly show that, among the three profiles, the trapezoidal profile is more effective at transferring energy than the rectangular profile with same thermo-physical parameters and constant plate volume. RPSLT absorber plate fin efficiency is somewhat lower than a rectangular shape at a same aspect ratio.

This review examines the design considerations for absorber plate fins in solar collectors. While non-rectangular shapes like parabolic and triangular fins might offer better heat transfer, their complex shapes and thin tips make them difficult to manufacture [16-19]. Trapezoidal fins provide a good compromise, requiring less material than rectangular fins for similar performance [20-22]. However, the assumption of continuous heat flow at junctions in some fin designs needs revision to account for energy exchange. Future research can explore the influence of various factors on optimal fin design, including heat loss at the tip, variations in heat transfer across the fin, and the material's thermal properties [23-28]. By considering these factors, researchers can develop more efficient and manufacturable fin designs for solar collectors [29-34].

In this research study, a new profile RPDSLT (Rectangular profile with double step changes in local thickness), also known as a multi-stepped absorber plate fin, is introduced. As a result, the primary focus of this study has been on the novel profile RPDSLT (multi-stepped absorber plate fin). The required theoretical formulation is briefly addressed, and the governing equation of efficiency for an absorber plate fin of RPDSLT has been developed. Thus, the performance of the RPDSLT absorber plate fin is assessed and shown by plate fin efficiency vs. plate parameter (Z0) graphs. The results also display the fin efficiency of RPSLT, both rectangular and trapezoidal, as well as a performance comparison. To optimize various geometrical factors such as thickness ratio and step length ratio of plate fin MATLAB was used. The optimal value of such ratios has been determined, at which maximum performance is achieved.

Main goal behind this research work to enhance thermal performance or fin efficiency and optimization of plate fin. The primary objective of optimization is to maximize the utilization of absorber plate fin material. It is obvious that volume of material may become less than a solid rectangular plate when step changes have been applied to it. However, it was quite challenging to determine the governing equation of the fin efficiency using boundary conditions due to complexities in design. But implementation of this research work has quite good effect on enhancement of thermal performance and saving plate material which economically sound good also. Previously some researchers conducted studies [5, 15] on the absorber plate fin with single step changes in length named as RPSLT which has shown good results in thermal performance than trapezoidal profile but not than rectangular profile. Now, in this research one more step change has been applied to that profile RPSLT which is named RPDSLT. Surprisingly, this profile of plate fin has shown better results than all existing profiles in thermal performances. Optimization of the geometrical parameters has also been successfully conducted as discussed earlier.

2. Theoretical Formulation

The theoretical formulation of performance analysis has been done for the multi-stepped absorber plate fin, or RPDSLT. Accordingly, mathematical formulations have been derived for fin efficiency and thoroughly investigated. A brief analysis of the design optimization of RPDSLT has also been conducted.

2.1. Performance Analysis

To accurately model the thermal performance of the novel multi-stepped absorber plate fin (RPDSLT), the following assumptions were made:

1. Plate fin has a steady heat transfer rate over all the fin surfaces.
2. The overall heat loss coefficient is constant across the absorber plate fin surface.
3. The temperature of the surrounding fluid is assumed to be uniform.
4. The thermal conductivity of the absorber plate fin material remains constant throughout the procedure.
5. The material possesses isotropic and homogenous properties, which are critical for temperature distribution.
6. The base temperature of the fin remains constant.
7. Newton's law of cooling can be utilized to calculate heat losses from the absorber plate fin.
8. To simplify calculations and minimize errors, the width of the absorber plate fin is considered to be unity.
9. The solar energy intensity or flux received from the sun remains constant.

The complete study for rectangular, trapezoidal, and RPSLT (rectangular profile with step change in local thickness) was conducted by Kovarik [5] and the equations for energy transfer and plate fin efficiency of rectangular, trapezoidal and RPSLT can be found in Appendix-A.

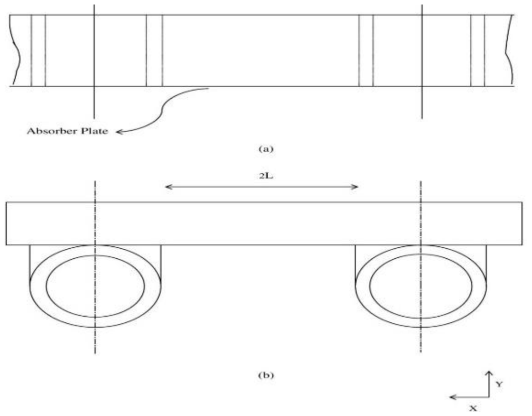

Fig. 1. Detailed geometry of an absorber plate fin of a rectangular profile

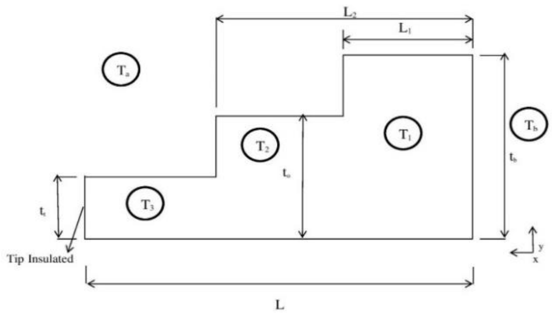

Fig. 2. Schematic geometry of RPDSLT

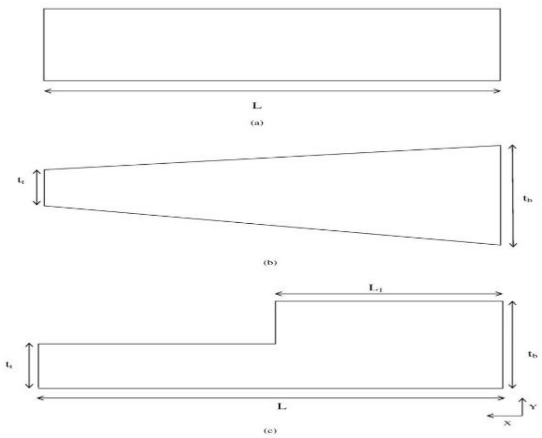

Fig. 3. Schematic geometry of a symmetric heat transfer element: (a) Rectangular profile; (b) Trapezoidal profile; (c) RPSLT. [5]

In case of RPDSLT, the energy equation for the absorber plate fin is derived. After that by applying some boundary conditions to the energy equation, required governing equation for energy transfer and plate fin efficiency can be obtained. The boundary conditions are established on the basis of some assumptions which are provided with the boundary conditions. Tip of the fin is assumed to be insulated and heat transfer occurs from base to tip through conduction process. As three-stepped absorber plate fin has been considered for a solar flat plate collector, there are two thickness ratios and two step length ratios considered (Figs. 1-3). Temperature distribution is a vital factor for analysis of efficiency of any fin. Temperature distribution for a conventional fin of rectangular profile has been shown below [5]. The behaviour of temperature distribution along the length of the plate fin has been shown through generated curve.

The energy equation for the plate fin may be written in dimensionless form as [5, 15]:

$$d^2\theta_1/dX^2 = Z_0^2\theta_1 \qquad for \quad 0 \le X \le \alpha_1 \quad (1)$$

$$d^2\theta_2/dX^2 = Z_0^2\theta_2/R_1 \qquad for \quad \alpha_1 \le X \le \alpha_2 \quad (2)$$

$$d^2\theta_3/dX^2 = Z_0^2\theta_3/R_2 \qquad for \quad \alpha_2 \le X \le 1 \quad (3)$$

$$\theta_1 = (T_1 - T_a - S/U_l)/(T_b - T_a - S/U_l) \quad (4)$$

$$\theta_2 = (T_2 - T_a - S/U_l)/(T_b - T_a - S/U_l) \quad (5)$$

$$\theta_3 = (T_3 - T_a - S/U_l)/(T_b - T_a - S/U_l) \quad (6)$$

$$Z_0 = \sqrt{Bi}/\delta \quad (7)$$

$$\mathrm{Bi} = \mathrm{U}_l t_b/\mathrm{k}_b \quad (8)$$

$$\delta = t_b/L \quad (9)$$

$$\alpha_1 = L_1/L \quad (10)$$

$$\alpha_2 = L_2/L \quad (11)$$

Solving the governing equations (Eq. 1 - Eq. 3) for the temperature distribution within the multi-stepped absorber plate fin requires the application of six boundary conditions. These conditions capture the thermal behavior at various crucial points of the fin:

1. The temperature at the top of the fin, directly above the fluid-carrying tubes, is assumed to be constant. This reflects the controlled heat input from the collector [35-36].
2. Energy balance is enforced at each point where two sections of the fin with different thicknesses meet. This ensures that heat flow across the junction is continuous, and the temperatures of the two sections at the junction are identical.
3. Due to the symmetrical design of the absorber plate, there is no heat transfer across the center plane of the fin. This simplifies the analysis by eliminating heat flow considerations along this plane.

These boundary conditions, expressed mathematically, provide the necessary constraints to solve the governing equations and determine the temperature distribution within the RPDSLT fin. This temperature profile is crucial for evaluating the fin's thermal performance and optimizing its design for solar collector applications [5, 15].

$$At\ X = 0, \quad \theta_1 = 1 \quad (12)$$

$$At\ X = \alpha_1, \quad \theta_1 = \theta_2 \quad (13)$$

$$At\ X = \alpha_1, \quad R_1\frac{d\theta_2}{dX} = \frac{d\theta_1}{dX} + (1-R_1)Z_0^2\delta\theta_1 \quad (14)$$

$$At\ X = \alpha_2, \quad \theta_2 = \theta_3 \quad (15)$$

$$\alpha_2 = \frac{L_2}{LAt}X = \alpha_2, \quad R_2\frac{d\theta_3}{dX} = \frac{d\theta_2}{dX} + (1-R_2)Z_0^2\delta\theta_2 \quad (16)$$

$$At\ X = 1, \quad \frac{d\theta_3}{dX} = 0 \quad (17)$$

Now, the boundary conditions are applied to solve Eq. (1), the following temperature distribution in the plate fin can be obtained.

$$\theta_1(X) = \left[\frac{\cosh y - \cosh x}{\sinh x - \sinh y}\right]\sinh(Z_0X) + \cosh(Z_0X) \quad (18)$$

$$\theta_2(X) = \left(\frac{P\sinh y - Q\cosh y}{(\sinh y)^2 - (\cosh y)^2}\right)\sinh y\left(\frac{Z_0X}{\sqrt{R_1}}\right) + \left(\frac{P\cosh y - Q\sinh y}{(\cosh y)^2 - (\sinh y)^2}\right)\cosh y\left(\frac{Z_0X}{\sqrt{R_1}}\right) \quad (19)$$

$$\theta_3(X) = \left[\frac{\left\{E\sinh y\left(\frac{Z_0\alpha_2}{\sqrt{R_1}}\right) + F\cosh y\left(\frac{Z_0\alpha_2}{\sqrt{R_1}}\right)\right\}\sinh y\left(\frac{Z_0}{\sqrt{R_2}}\right)}{\sinh y\left(\frac{Z_0\alpha_2}{\sqrt{R_2}}\right).\sinh y\left(\frac{Z_0}{\sqrt{R_2}}\right) - \cosh y\left(\frac{Z_0\alpha_2}{\sqrt{R_2}}\right).\cosh y\left(\frac{Z_0}{\sqrt{R_2}}\right)}\right].\sinh y\left(\frac{Z_0X}{\sqrt{R_2}}\right) + \left[\frac{\left\{E\sinh y\left(\frac{Z_0\alpha_2}{\sqrt{R_1}}\right) + F\cosh y\left(\frac{Z_0\alpha_2}{\sqrt{R_1}}\right)\right\}\cosh y\left(\frac{Z_0}{\sqrt{R_2}}\right)}{\cosh y\left(\frac{Z_0\alpha_2}{\sqrt{R_2}}\right).\cosh y\left(\frac{Z_0}{\sqrt{R_2}}\right) - \sinh y\left(\frac{Z_0\alpha_2}{\sqrt{R_2}}\right).\sinh y\left(\frac{Z_0}{\sqrt{R_2}}\right)}\right].\cosh y\left(\frac{Z_0X}{\sqrt{R_2}}\right) \quad (20)$$

$$E = \left(\frac{P\sinh y - Q\cosh y}{(\sinh y)^2 - (\cosh y)^2}\right) \tag{21}$$

$$F = \left(\frac{P\cosh y - Q\sinh y}{(\cosh y)^2 - (\sinh y)^2}\right) \tag{22}$$

$$P = U.\sinh x + \cosh x \tag{23}$$

$$Q = \left[\frac{U.\cosh x + \sinh x + (1 - R_1)Z_0\delta(U.\sinh x + \cosh x)}{\sqrt{R_1}}\right] \tag{24}$$

$$U = \left(\frac{\cosh y - \cosh x}{\sinh x - \sinh y}\right) \tag{25}$$

$$x = (Z_0\alpha_1) \tag{26}$$

$$y = \left(\frac{Z_0\alpha_1}{\sqrt{R_1}}\right) \tag{27}$$

After obtaining the temperature distribution in the absorber plate fin, the actual heat transfer rate can be calculated by the following equation.

$$Q = \frac{q}{k_p(T_b - T_a - S/U_l)} \tag{28}$$

The ideal energy transfer rate per unit width, Q_i may be calculated if the entire surface of the absorber plate fin were maintained at its base temperature i.e.; $T_1 = T_2 = T_3 = T_b$ (see Fig.2)

Absorber plate fin efficiency can be defined as the ratio of the energy transfer through the absorber plate fin to the energy transfer that would occur if the entire surfaces of fin at its base temperature.

Thus the plate fin efficiency for the RPDSLT may be expressed by the following equation

$$\eta = \frac{Q}{Q_i} \tag{29}$$

2.2. Optimization Analysis

Optimization can be defined as the procedure to obtain or achieve weight reduction through shape and size modifications which require optimal material usage to meet the design requirements. This technique ensures the optimal use of material used for engineering works. Mainly, optimization is achieved through weight reduction in materials where performance should also remain at a significant level. So, the objective of this step is mainly to execute the optimal use of absorber plate fin material where efficiency or performance of the collector is considered a criterion to determine the optimal value of the geometrical parameter like α_1, α_2, R_1, R_2 (thickness and length of plate fin are expressed as ratios). The main objective was to determine the optimal values of the thickness ratios R_1, R_2 and the dimensionless step lengths α_1, α_2, which were then graphically represented using appropriate MATLAB optimization solver code.

For optimization of all geometrical parameters, Eq. (29) was used as the objective function. Q (Energy transfer through the absorber plate fin) was determined by use of $\theta_1(X)$, $\theta_2(X)$, and $\theta_3(X)$ Eqs. (18), (19), and (20). Q_i (Energy transfer through the absorber plate fin if the entire surface of fin were at its base temperature) was taken as 3, since $Q_1 = Q_2 = Q_3 = 1$ when $\theta_1 = \theta_2 = \theta_3 = 1$.

Now, for optimization of each case, array indexing was applied in objective function. For optimization of thickness ratio R_1, R_1 and Z_0 were used as variable where other parameters (like R_2, α_1 and α_2) had fixed value. R_1 had a total nine values from minimum 0.1 to maximum 0.9 with step size of 0.1, while Z_0 had a total nine values from minimum 0.5 to maximum 2.9 with step size of 0.3.

Optimization process for R_2, α_1 and α_2 is similar to the optimization process for R_1. Here, one important fact has to be taken into consideration that minimum value of α_1 and α_2 (taken as 0.1) was chosen, as this dimensionless step length has a negative effect on fin efficiency. In the case of the optimization process for α_1 and α_2, value of R_1 and R_2 was taken as maximum (0.9), because R_1 and R_2 have a positive effect on fin efficiency.

3. Results and Discussion

The study is focused on a comparison of the performance between RPDSLT, rectangular, RPSLT and trapezoidal profile absorber plate fins of a flat-plate solar collector and optimization of its geometry. The above analysis may be applied to a multi stepped absorber plate fin or RPDSLT if L, $R_1, R_2, \alpha_1$and$\alpha_2$are taken between zero and one. However, values of R_1, R_2, α_1andα_2 cannot be taken exactly as zero or one to avoid the singularity of the mathematical formulation in above analysis. Here, a particular value of those geometrical parameters was taken just to show the differences in performance with respect to plate parameter (Z_0). In the case of other profile such as RPSLT, Rectangular and trapezoidal profiles, same procedure has been followed. For validation of the current analysis with consideration of the above fact, the result of the RPDSLT has been taken to compare with existing result of the RPSLT in the literature (Kundu, 2002 [15]). Figure 4 depicts the comparison on fin efficiency between RPDSLT (current analysis) and RPSLT (existing analysis).

Figure 4 demonstrates a high degree of agreement between the results obtained from the current analysis and those from existing studies. The maximum observed difference in fin efficiency is only 5%, indicating strong validation of the current model. In this validation procedure, values of thickness ratios R_1, R_2 and step length ratios α_1, α_2 were taken in such way that RPDSLT could be considered equivalent to RPSLT.

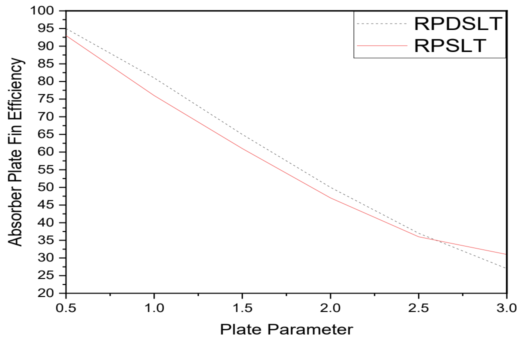

Fig. 4. Comparison between RPDSLT and RPSLT

In Figure 4, comparison between RPDSLT and RPSLT on thermal performance has been shown. This figure shows that fin efficiency is optimum at the initial or minimum value of plate parameter (Z_0) which is 0.5 for both profiles namely RPDSLT & RPSLT. With an increase in the value of Z_0 plate fin efficiency gradually decreases for both and becomes minimum at the highest value of Z_0 which is 3.0. Although there is a slight irregularity observed at the value of Z_0 = 2.5 where the fin efficiency of RPSLT is higher than of RPDSLT. But difference between optimum efficiency of RPDSLT and RPSLT is about 5% at Z_0 = 0.5. So, it is obvious that fin efficiency decreases with an increase in the value of the plate parameter. Plate parameter (Z_0) is mathematically expressed as $\frac{\sqrt{B_i}}{\delta}$ [15], where δ has a constant value of 0.05. It is worth mentioning that four possible cases exist to increase the plate parameter Z0 namely decrease of thermal conductivity of the absorber plate, the increase of plate length, the decrease of plate thickness at the root, and the increase of the overall plate loss coefficient. Obviously, increase of Z_0 reduces the plate efficiency [15]. It also may be said that fin efficiency will decrease when the Biot number increases, where the Biot number is defined as the ratio of conductive resistance within a body to convective resistance at its surface when the body is kept in the convective environment.

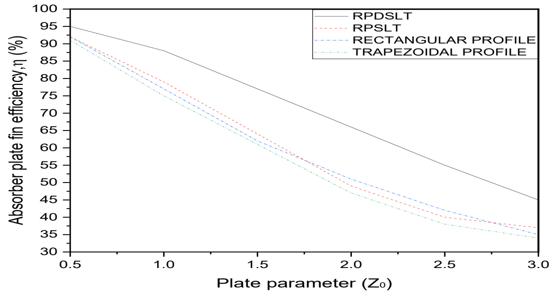

Fig. 5. Plate efficiency of different profiles of the absorber plate as a function of plate parameter Z_0 for $\delta = 0.05$. (%)

In Figure 5, comparison between RPDSLT, RPSLT, Rectangular and Trapezoidal profile plate fins in terms of thermal performance has been shown. Result has been determined in similar way to the previous case. Here some other profiles, such as Rectangular, Trapezoidal profiles are also taken into consideration from the previous study [15]. But, the variation of fin efficiency with respect to the plate parameter (Z_0) is the same as in the previous case for all profiles. It is clearly seen from Figure 5 that the differences in fin efficiency between the three profiles namely RPSLT, Trapezoidal and Rectangular are about 1% to 3%. But, the maximum difference in fin efficiency between RPDSLT and other profiles lies between the value of Z_0 = 2.0 to 2.5. The equations used from the existing analysis have been provided in Appendix A. Plate fin efficiency for these different profiles are listed in Tables 1-4.

Table 1. Plate fin efficiency of the rectangular profile at different thermo-physical parameters

δ	Z_0	η (%)
0.05	0.5	92
0.05	1	77
0.05	1.5	62
0.05	2	51
0.05	2.5	42
0.05	3	35

Table 2. Plate fin efficiency of the trapezoidal profile at different thermo-physical parameters

R	δ	Z_0	η (%)
0.6	0.05	0.5	91
0.6	0.05	1	75
0.6	0.05	1.5	61
0.6	0.05	2	47
0.6	0.05	2.5	38
0.6	0.05	3	34

Table 3. Plate fin efficiency of RPSLT at different thermo-physical parameters

R	α	δ	Z_0	η(%)
0.7	0.7	0.05	0.5	92
0.7	0.7	0.05	1	79
0.7	0.7	0.05	1.5	64
0.7	0.7	0.05	2	49
0.7	0.7	0.05	2.5	40
0.7	0.7	0.05	3	37

Table 4. Plate fin efficiency of RPDSLT at different thermo-physical parameters

R_1	R_2	α_1	α_2	δ	Z_0	η(%)
0.8	0.8	0.5	0.8	0.05	0.5	96
0.8	0.8	0.5	0.8	0.05	1	90
0.8	0.8	0.5	0.8	0.05	1.5	78
0.8	0.8	0.5	0.8	0.05	2	68
0.8	0.8	0.5	0.8	0.05	2.5	56
0.8	0.8	0.5	0.8	0.05	3	47

Figures 6 and 7 show the result of optimization of α_1 and α_2. The range of α_1 was taken from a minimum value of 0.1 to maximum value of 0.9. The, range of the plate parameter was also taken from a minimum value of 0.5 to maximum value of 2.9. The values of other variables like α_2, R_1, R_2, δ have been fixed. Aspect ratio (δ) has a fixed value of 0.05, the same in all cases. The same procedure has been followed for α_2. From Figures 6 and 7, it is clearly seen that maximum fin efficiency is obtained at the initial value of 0.1 of both α_1 and α_2. Hence, in the optimization process of R_1 and R_2, the same value of α_1 and α_2 was taken as 0.1 (Figures 8 and 9).

In the second part of this research, the emphasis is on the dimensional optimization of the plate fin where Figures 6 to 9 depict the results of dimensional optimization of the plate fin of the solar collector in terms of plate fin efficiency i.e., determination of geometrical parameters at which optimum fin efficiency may be obtained.

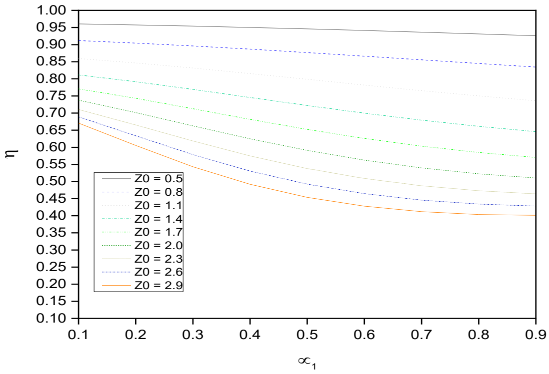

Fig. 6. Fin Efficiency vs. α_1

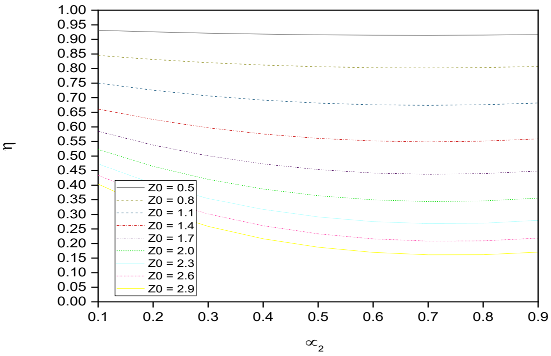

Fig. 7. Fin Efficiency vs. α_2

It shows the optimum value of thickness ratios (R_1 and R_2) and Step-length ratios (α_1 and α_2) where optimum fin efficiency may be obtained. This was done by MATLAB programming. These ratios were taken between values of 0.1 to 0.9. The value of Z_0 was taken between values of 0.5 to 2.9 at a rate of increase of 0.3 for each case.

In Figures 6 and 7, they show two results: first, at a certain value of Z_0 (for example, 0.5) and at a certain value of α_1 (Figure 6) or α_2 (Figure 7) (for example, 0.1) fin efficiency slightly decreases as about 0.2% to 0.5%. Secondly, fin efficiency decreases with an increase of α_1 and Z_0 (see Figure 6) or α_2 and Z_0 (See Figure 7). So, optimum fin efficiency may be obtained at the minimum value of α_1 and α_2.

In the case of Figures 8 and 9, they show two results: first, at a certain value of Z_0 (for example, 0.5) and at a certain value of R_1 (Figure 8) or R_2 (Figure 9) (for example, 0.1) fin efficiency slightly increases. Secondly, fin efficiency increases slightly with an increase of α_1 and Z_0 (Figure 6) or α_2 and Z_0 (Figure 7). So, optimum fin efficiency may be obtained at the minimum value of α_1 and α_2. So, it is clear that the thickness ratio and step-length ratio have adverse effect on fin efficiency with respect to each other.

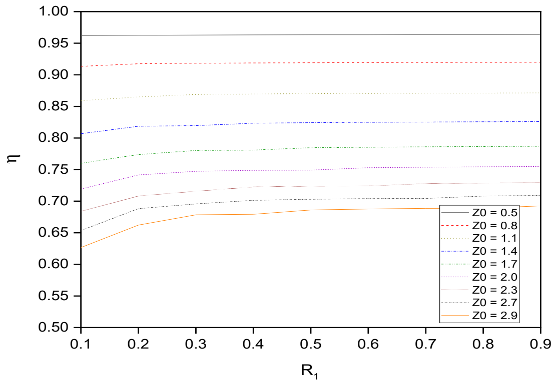

Fig. 8. Fin Efficiency vs. R_1

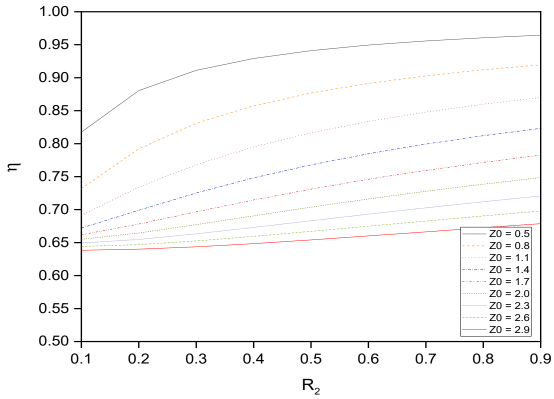

Fig. 9. Fin Efficiency vs. R_2

Figures 6-9 obtained as a result of optimization, conclude that the efficiency either increases or decreases in a regular manner with respect to a certain range of values of the thickness and step-length ratios. The results show that fin efficiency decreases slightly with an increase in value of α_1 at a particular value of the plate parameter (Z_0) (Figure 6). Fin efficiency also decreases slightly with an increase in the value of α_2 at a particular value of the plate parameter (Z_0) similar to α_1 (Figure 7). Fin efficiency increases slightly with an increase in the value of R_1 at a particular value of the plate parameter (Z_0) (Figure 8). Fin efficiency also increases slightly with an increase in value of R_2 at a particular value of the plate parameter (Z_0) similar to R_1. Hence, values of these geometrical parameters in ratio form may be easily selected for the optimal use of the absorber plate fin material (Figure 9).

4. Conclusions

Following a comprehensive analysis of the multi-stepped absorber plate fin's thermal performance, several key conclusions have been drawn, as detailed below:

- Comparison of plate fin efficiency between several profile shapes has been thoroughly conducted.
- Plate fin efficiency decreases with an increase in the plate parameter (Z_0).
- The efficiency of RPDSLT is higher than the other three profiles at each value of the plate parameter.
- As per result, the maximum fin efficiency of RPDSLT is higher than that of RPSLT about 5% at certain values of α_1,α_2, R_1,R_2 and Z_0, i.e., values were taken in such a way that maximum fin efficiency could be obtained for each profile. The difference in fin efficiency between RPSLT, Rectangular and trapezoidal profile is about 1% to 2%. So, it may be concluded that, instead of having manufacturing complexities compared to the rectangular profile, RPSLT and trapezoidal, RPDSLT may be recommended on the basis of thermal performance for a flat plate solar collector.
- Optimization has been conducted thoroughly using MATLAB.
- It has been observed that at the minimum value of both α_1, α_2 which is equal to 0.1 optimum efficiency has been observed. Optimum efficiency has been observed at maximum value of both R_1, R_2 which is equal to 0.9. Hence, for optimal use of plate fin materials, it may be said that the minimum value of the step length ratios (α_1, α_2) and the maximum value of the thickness ratios(R_1, R_2) should be taken in particular.
- In earlier studies optimization of geometrical parameters was discussed for RPSLT where plate thickness at the base and tip and step length were optimized for showing fin efficiency. Optimum fin efficiency was observed between 60% to 63% with respect to the optimized values of those geometrical parameters. In this new profile study of RPDSLT, thickness ratio and dimensionless step lengths are taken into consideration and optimum fin efficiency has been observed between 91% to 95% with respect to the optimized value of thickness ratios and step-length ratios.
- During the optimization process, it was found that at any change in the value of the variables maximum efficiency was obtained at the minimum value of the step-length ratios and the maximum value of the thickness ratios.

For future research, researchers can perform:

- Developing a simulation framework that incorporates dynamic environmental and operational parameters.

- Conducting a sensitivity analysis of the optimized design to changes in these operating conditions.
- Presenting case studies based on simulated scenarios that illustrate how the RPDSLT fin performs under specific sets of working conditions.

Nomenclature

B_i	Biot number based on the root thickness of the plate, $U_l t_b/k_p$ [-]
k_p	Thermal conductivity of the fin material, [W/m.K]
L	Semi-pitch length of the absorber plat fin, [m]
L_1	Distance at which first step change occurs, [m]
L_2	Distance at which second step change occurs, [m]
q	Actual heat transfer rate through the plate fin, [W]
Q	Dimensionless heat transfer rate in fraction, $\frac{q}{k_p(T_b - T_a - S/U_l)}$ [-]
R_1	Ratio of tip thickness to base thickness, $R_1 = t_o/t_b$ [-]
R_2	Ratio of tip thickness to base thickness, $R_2 = t_t/t_o$ [-]
S	Solar energy flux, absorbed by the absorber plate fin, [W/m^2]
T_b	Root thickness of the plate fin, [m]
T_b	Root thickness of the plate fin, [m]
T_t	Tip thickness of the plate fin, [m]
T_1	Local plate temperature for $0 \le X \le \alpha_1$, [K]
T_2	Local plate temperature for $\alpha_1 \le X \le \alpha_2$, [K]
T_3	Local plate temperature for $\alpha_2 \le X \le 1$, [K]
T_a	Ambient temperature, [K]
T_b	Plate temperature at the root, [K]
U_l	Overall loss coefficient, [W/m^{2}K]
x,y	Coordinates, [m]
X	Dimensionless coordinate, x/L [-]
Z_0	Plate parameter, $\frac{\sqrt{B_i}}{\delta}$ [-]

Greek letters

α_1	Dimensionless step length, $\alpha_1 = L_1/L$ [-]
α_2	Dimensionless step length, $\alpha_2 = L_2/L$ [-]
δ	Aspect ratio, $\delta = t_b/L$ [-]
η	Efficiency of the absorber plate
θ_1	Dimensionless temperature, $(T_1 - T_a - S/U_l)/(T_b - T_a - S/U_l)$ [-]
θ_2	Dimensionless temperature, $(T_2 - T_a - S/U_l)/(T_b - T_a - S/U_l)$ [-]
θ_3	Dimensionless temperature, $(T_3 - T_a - S/U_l)/(T_b - T_a - S/U_l)$ [-]

Subscripts

opt	Optimum

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Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this article.

Appendixes

The equations of dimensionless heat transfer rate and plate fin efficiency for the RPSLT, rectangular and trapezoidal profile shapes of absorber plate fin as follows:

RPSLT:

$$Q = \frac{q}{k_p(T_b - T_a - S/U_l)} = Z_0\delta\, D_3/D_2 \quad \text{(A1)}$$

where

$$D_2 = \begin{vmatrix} & \sqrt{R}\,(1-R)Z_0\delta & 1 \\ 1 & tanh\left[\frac{Z_0(1-\alpha)}{\sqrt{R}}\right] & 0 \\ 0 & cosh(Z_0\alpha) & sinh(Z_0\alpha) \end{vmatrix} \quad \text{(A2)}$$

$$D_3 = \begin{vmatrix} & \sqrt{R}\,(1-R)Z_0\delta & 1 \\ 1 & tanh\left[\frac{Z_0(1-\alpha)}{\sqrt{R}}\right] & 0 \\ 0 & sinh(Z_0\alpha) & cosh(Z_0\alpha) \end{vmatrix} \quad \text{(A3)}$$

$$B_i = U_l t_b/k_p \quad \text{(A4)}$$

$$Z_0 = \frac{\sqrt{B_i}}{\delta} \quad \text{(A5)}$$

$$\delta = t_b/L \quad \text{(A6)}$$

$$\eta = D_3/\{Z_0[1 + (1-R)\delta]\} \quad \text{(A7)}$$

Rectangular Profile:

$$Q = \frac{q}{k_p(T_b - T_a - S/U_l)} = \delta Z_0 tanh(Z_0) \quad \text{(A8)}$$

where

$$B_i = U_l t_b/k_p \quad Z_0 = \frac{\sqrt{B_i}}{\delta} \quad \delta = t_b/L \quad \text{(A9)}$$

$$\eta = tanh(Z_0)/Z_0 \quad \text{(A10)}$$

Trapezoidal Profile:

$$Q = \frac{q}{k_p(T_b - T_a - S/U_l)} = \delta(1-R)m\left[\begin{array}{c}\left(I_1(2m)K_1\left(2mR^{1/2}\right) - I_1\left(2mR^{1/2}\right)K_1(2m)\right) \\ /\left(I_1\left(2mR^{1/2}\right)K_0(2m) + I_0K_1\left(2mR^{1/2}\right)\right)\end{array}\right] \tag{A11}$$

where

$$m = Z_0(1-R)\left[1+\delta^2[(1-R)^2/4]\right]^{1/4} \tag{A12}$$

$$B_i = U_l t_b / k_p \tag{A13}$$

$$Z_0 = \frac{\sqrt{B_i}}{\delta} \tag{A14}$$

$$\delta = t_b/L \tag{A15}$$

$$R = t_t/t_b \tag{A16}$$

$$\eta = \frac{(1-R)m}{Z_0^2\,[1+\delta^2[(1-R)^2/4]]^{\frac{1}{2}}}\left[\begin{array}{c}\left(I_1(2m)K_1\left(2mR^{1/2}\right) - I_1\left(2mR^{1/2}\right)K_1(2m)\right) \\ /\left(I_1\left(2mR^{1/2}\right)K_0(2m) + I_0(2m)K_1\left(2mR^{1/2}\right)\right)\end{array}\right] \tag{A17}$$

Authors Contribution Statement

Gulshan Baharuddin Ahmed: Conceptualization; Methodology; Validation; Formal Analysis; Writing Original Draft.

Md Naim Hossain: Methodology; Validation; Investigation; Supervision; Formal Analysis.

Arijit Kundu: Resources; Data Curation; Visualization; Project Administration; Investigation.

Ashwani Kumar: Writing Original Draft; Writing Review and Editing, Visualization; Formal Analysis; Methodology; Data Curation.

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