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Research Article

State Space Approach to Moore-Gibson-Thompson Generalized Piezo-Thermoelasticity with Memory-Dependent Derivative

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KEYWORDS

Memory-Dependent Derivative; Piezo-thermoelasticity; Moore-Gibson-Thomson equation; Two-temperature generalized thermoelasticity; State space approach

ABSTRACT

The aim of this research is to investigate the thermal and mechanical responses in an isotropic piezo-thermoelastic semi-infinite medium that is subjected to a moving heat source. The exploration has been carried out in the context of two-temperature Moore-Gibson-Thomson generalized thermoelasticity with memory-dependent derivative (MDD). The two-temperature approach is adopted to discern the separate evolution of temperature gradients, while a memory-dependent derivative is employed to capture the historical behavior of the material. The resulting system of partial differential equations is systematically solved in the transformed domain of Laplace using the state space approach, an advanced mathematical technique. The Fourier series expansion technique for numerical Laplace inversion is used to derive the solution for various thermophysical quantities in the real space-time domain. Parametric studies are conducted to explore the influence of the heat source speed and the parameter related to the memory-dependent derivative on the material's response. The outcomes of this work are presented graphically for a better understanding of the impacts of the parameters considered. Applications of this work extend to diverse areas, including material science, structural engineering, and thermal management systems.

Nomenclature

c_{ijkl} Isothermal elastic parameters
h_{ijk} piezoelectric moduli
γ_{ij} Thermal elastic coupling tensor
p_i pyroelectric moduli
ϵ_{ij} dielectric moduli
λ, μ Lame's constants
σ_{ij} Stress tensor
e_{ij} Strain tensor
u_i Displacement component vector
ρ Material density
α_t Thermal expansion coefficient
γ Thermal coupling parameter
θ_0 Reference temperature
θ Increase of thermodynamic temperature above θ_0
a_{ij} Temperature discrepancy tensor
ϕ Conductive temperature
φ Electric potential charge
E_i Electric field component vector
D_i Electric displacement
c_E Specific heat at constant strain
ω Delay time parameter

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K_{ij} Thermal conductivity tensor
K_{ij}^{*} Tensor for additional material constant
ϱ_e Free charge of the medium
Q Heat source

1. Introduction

Classical thermoelasticity theory suffers from two major imperfections. Firstly, the non-appearance of an elastic term in the heat conduction equation leads to failure in exploring heat generation due to elastic changes. Secondly, the parabolic nature of the heat conduction equation leads to the unrealistic observation of infinite speed of thermal signals. The first deficiency was addressed by Biot [1], who introduced an elastic term into the energy equation. This innovation resulted in the development of the classical coupled theory of thermoelasticity. However, despite this improvement, the second deficiency persisted in the form of the parabolic nature of the heat conduction equation, indicating non-feasible boundless speed of thermal wave propagation throughout the material. To address the second deficiency, various researchers have undertaken significant developments and modifications at different times, resulting in what is now known as the generalized theory of thermoelasticity. Pioneering contributions towards the formulation and advancement of these generalized thermoelastic theories were made by notable figures such as Lord and Shulman [2], Green and Lindsay [3], Green and Naghdi [4-6], Tzou [7], and S. K. Roy Choudhuri [8]. Comprehensive insights into these theories and their applications can be found in works by Hetnarski and Ignaczak [9] and Chandrasekharaiah [10]. Ismail et al. [11] have investigated the influence of variable thermal conductivity on generalized microelongation photo-thermoelasticity theory. Lotfy and El-Bary [12] have studied the effect of the magnetic field when the interaction between the microstretch (inner-structure) theory and the generalized magneto-photo-thermoelasticiy occurs. It is essential to note that the generalized theory of thermoelasticity is particularly valuable in addressing practical problems associated with high heat fluxes occurring within very short time intervals, which scenarios are commonly encountered in applications such as nuclear reactors, laser units, energy channels, and similar fields.

The theory of heat conduction in an elastic body, as elucidated by Chen and Gurtin [13], Chen et al. [14-15], and Gurtin and Williams [16-17], introduces the fundamental concept of two distinct temperatures: the conductive temperature and the thermodynamic temperature. The conductive temperature characterizes heat conduction originating from thermal processes within the body, capturing the transfer of heat energy through the material due to temperature gradients. In contrast, the thermodynamic temperature accounts for heat conduction arising from mechanical processes occurring between layers and particles within the elastic material. One of the main characteristics that separates classical thermoelasticity (CTE) from two-temperature thermoelasticity (2TT) is the temperature discrepancy parameter. Its presence accounts for non-equilibrium effects, and when $a = 0$ then the model seamlessly transitions to classical thermoelasticity (CTE). Youssef [18] has constructed the theory of two-temperature generalized thermoelasticity using the LS model and proved its uniqueness theorem. Building on this, El-Karamany and Ezzat [19] have introduced an analogous theory that utilises Green Naghdi model III, complemented by discussions on reciprocal and uniqueness theorems. Kumar et al. [20] have delved into the investigation of variational and reciprocal concepts within the framework of two-temperature generalized thermoelasticity. Lotfy et al. [21] have studied the photothermal excitation process during hyperbolic two-temperature theory for a magneto-thermoelastic semiconducting medium. Seth and Mallik [22] have investigated the thermoelastic interactions in a homogeneous, transversely isotropic infinite medium with a spherical cavity in the context of two temperature Lord-Shulman generalized theory of thermoelasticity, considering Eringen's nonlocal theory and memory-dependent derivative. Kumar et al. [23] have examined the behavior of plane wave propagation through the interface of an elastic half-space and a transversely isotropic piezoviscothermoelastic half-space composed of dual phase lag and hyperbolic two-temperature theory.

The exploration of coupling effects between different physical fields has become a significant area of research in materials science. This interdisciplinary approach has opened up new possibilities and applications, especially in the case of artificial materials like piezoelectric materials. The investigation of coupling effects for piezoelectric materials has led to numerous practical applications across different industries. The ability of this material to convert energy between different physical fields makes it valuable in various technological advancements and industrial applications. Kumari et al. [24] have studied the effect of gravity on piezo-thermoelasticity in the context of a phase lag model with two-temperature theory. Gupta et al. [25] have developed an innovative mathematical model to analyze the behavior of plane waves in piezo-thermoelastic materials and investigated the influence of moisture and temperature diffusivities as well as moisture content on the distribution of physical properties.

The introduction of memory-dependent derivative (MDD) by Wang and Li [26], as an alternative to fractional order derivatives, represents a significant development in mathematical modeling and particularly in generalized thermoelastic theories. Memory-dependent derivatives (MDD) are introduced as an integral form of a common derivative, incorporating a kernel function on a slipping interval. This integral nature of MDD, coupled with the inclusion of a kernel function, allows for a highly flexible representation of memory-dependent effects. In contrast to fractional derivatives, where the order is fixed as a fraction, MDD offers a remarkable degree of flexibility. The ability to choose both the kernel function and delayed time intervals based on the specific characteristics of the system enhances the adaptability of the model, providing a tailored approach to capturing memory-dependent behavior. According to Wang and Li [26], generalized thermoelastic theories utilizing MDD surpass those employing fractional derivatives in various aspects. The flexibility in choosing parameters and the integral form of MDD contribute to more accurate and versatile representations of physical systems. Notably, the order of the derivative in MDD is an integer, simplifying numerical calculations and enhancing computational efficiency compared to fractional order derivatives. This characteristic reflects the ease of implementation in numerical simulations. Furthermore, MDD is argued to better reflect the memory effect in physical systems. The ability to select the kernel function and delayed time intervals not only offers a more intuitive representation but also allows for a customizable portrayal of memory-dependent behavior. In essence, the definition of MDD is suggested to provide a more intuitive physical meaning, facilitating a deeper understanding of the underlying processes within the system.

The work of Yu et al. [27] introduces a novel generalized thermoelastic model that incorporates memory-dependent derivatives [26]. El-Karamany and Ezzat [28] have established variational principles, reciprocal theorems, and the uniqueness of solutions to account for the memory effect in a thermo-diffusive medium. Biswas [29] has studied the effect of the three-phase-lag model in the context of memory-dependent derivatives for an orthotropic infinite medium. Using an eigenvalue technique, Seth et al. [30] have investigated the thermoelastic interactions in a transversely isotropic unbounded medium with memory having a line heat source. These studies collectively contribute to advancing our understanding of generalized thermoelasticity by incorporating memory-dependent derivatives into the models, enabling a more comprehensive analysis of complex physical phenomena. Several other investigations relating to the generalized theory of thermoelasticity involving MDD have been presented by Abouelregal et al. [31], Barak et al. [32], Mondal and Sur [33], Othman et al. [34], and Seth and Mallik [35].

Moore-Gibson-Thomson thermoelasticity is a new mathematical framework of linear theory of thermoelasticity, introduced by R. Quintanilla [36]. He has further extended this theory for two-temperature thermoelasticity also [37]. Using the Moore-Gibson-Thomson theory of thermoelasticity, Bazarra et al. [38] have analyzed numerically a physical problem for a dielectric medium. Singh and Mukhopadhyay [39] have discussed the fundamental solution of the Moore-Gibson-Thomson theory of thermoelasticity. Lotfy et al. [40] have obtained an analytical solution for a semiconductor medium in the context of the Moore-Gibson-Thomson theory of thermoelasticity. Using the Moore-Gibson-Thomson theory of thermoelasticity, Sur [41] has solved a generalized thermoelastic thick plate problem. For more applications of Moore-Gibson-Thomson thermoelasticity on physical problems of various considerations, we may refer to Abouelregal et al. [42], El-Sapa et al. [43], Gupta et al. [44], Riadh et al. [45].

The potential function method was used to solve problems in thermoelasticity theory. However, there are a number of drawbacks to this, as listed by Bahar and Hetnarski [46]. Firstly, the boundary conditions for physical problems are directly related to the actual quantities being considered rather than the potential functions. Secondly, instead of focusing on the actual physical values, more rigorous assumptions must be established on the behaviour of potential functions. Finally, it was found that many integral representations of physical quantities are convergent in the classical sense, while their potential function representations only converge in the distributional sense. To overcome these difficulties, the state space approach is developed [46].

This research article delves into the exploration of a one-dimensional two-temperature generalized Moore-Gibson-Thomson problem for piezo-thermoelastic isotropic materials incorporating memory-dependent derivative and subjected to a moving heat source with a constant velocity. The study enriches the understanding of the thermal and mechanical response of a piezo-thermoelastic isotropic material, together with the aforementioned considerations. The two-temperature approach is adopted to discern the separate evolution of temperature gradients, while memory-dependent derivatives are employed to capture the historical behavior of the material. The resulting system of partial differential equations is systematically solved in the transformed domain of Laplace using the state space approach, an advanced mathematical technique. Parametric studies are conducted to explore the influence of the heat source speed and the parameter related to the memory-dependent derivative on the material's response. The outcomes of this work are presented graphically for a better understanding of the impacts of the parameters considered. The outcomes of this research contribute valuable insights to the field of thermoelasticity, particularly in understanding the role of MDD in modeling the behavior of piezo-thermoelastic materials. We can achieve the results for an analogous problem [47] which uses integer-order thermoelasticity theory as a special case of our own findings.

2. Basic Governing Equations

In a homogeneous anisotropic medium, the governing field equations for linear piezo-thermoelastic interactions are [36-37, 48]

(a) strain-displacement relations:

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad i,j = 1,2,3, \tag{1}$$

(b) the constitutive equations:

$$\sigma_{ij} = c_{ijkl}e_{kl} - h_{ijk}D_k - \gamma_{ij}\theta, \quad i,j,k,l = 1,2,3, \tag{2}$$

(c) stress-equations of motion in presence of body forces F_i:

$$\sigma_{ij,j} + F_i = \rho\ddot{u}_i, \quad i,j = 1,2,3, \tag{3}$$

(d) Gauss's equation and electric field relations:

$$D_{i,i} = \varrho_e, E_i = h_{ijkl}e_{jk} + \epsilon_{ik}D_k - p_i\theta, \quad i,j,k,l = 1,2,3, \tag{4}$$

(e) the heat conduction equation for Moore-Moore-Gibson-Thompson thermoelasticity with two temperatures and memory-dependent derivative:

$$\left(K_{ij}\frac{\partial}{\partial t} + K^*{}_{ij}\right)\phi_{,ij} = (1+\omega D_\omega)\left(\rho c_E\ddot{\theta} + \gamma\theta_0\ddot{e}_{ij} - \rho\dot{Q}\right), \quad i,j = 1,2,3, \tag{5}$$

(f) relation between thermodynamic temperature θ and conductive temperature ϕ:

$$\phi - \theta = a_{ij}\phi_{,ij}, \quad i,j = 1,2,3. \tag{6}$$

3. Formulation of the Problem

We consider a homogeneous, isotropic piezo-thermoelastic semi-infinite body whose boundary is assumed to be free from traction but is subjected to a thermal loading. We assume that the rectangular Cartesian coordinate system (x, y, z) as the coordinate axes with x-axis pointing towards the medium and perpendicular to the bounding surface. Then the body occupies the region $x \geq 0$. The direction of the piezoelectric is taken to be parallel to the x-axis. Considering a one-dimensional disturbance of the medium, the displacement vector $\vec{u}$, the thermodynamic temperature θ, and the conductive temperature ϕ can be taken in the following forms

$$\vec{u} = (u(x,t), 0,0), \theta = \theta(x,t), \quad \phi = \phi(x,t). \tag{7}$$

Then the strain components in this case become

$$e_{xx} = \frac{\partial u}{\partial x}. \tag{8}$$

The cubical dilatation e is given by

$$e = e_{xx} + e_{yy} + e_{zz} = e_{xx} = \frac{\partial u}{\partial x}. \tag{9}$$

In the context of linear theory of generalized thermoelasticity without body forces, the constitutive equation, the equation of motion, and the heat equation can be written as

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \gamma\theta - hD, \tag{10}$$

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial\theta}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{11}$$

$$K\frac{\partial^2\dot{\phi}}{\partial x^2} + K^*\frac{\partial^2\phi}{\partial x^2} = (1+\omega D_\omega)\left(\rho c_E\frac{\partial^2\theta}{\partial t^2} + \gamma\theta_0\frac{\partial^2 e}{\partial t^2} - \rho\frac{\partial Q}{\partial x}\right). \tag{12}$$

The relation between conductive temperature ϕ and thermodynamic temperature θ is

$$\phi - \theta = a\frac{\partial^2\phi}{\partial x^2}. \tag{13}$$

The following non-dimensional variables are now introduced:

$$x' = c_0\nu x, \ u' = c_0\nu u, \ t' = c_0^2\nu t, \ \theta' = \frac{\theta}{\theta_0},$$

$\phi' = \frac{\phi}{\theta_0}$, $\omega' = c_0^2 \nu\omega$, $\sigma'_{ij} = \frac{\sigma_{ij}}{(\lambda + 2\mu)}$,
$D' = \frac{h}{(\lambda+2\mu)} D$, $e' = e$, $Q' = \frac{\dot{Q}}{c_E \theta_0 c_0^4 \nu^2}$,
where $c_0^2 = \frac{\lambda+2\mu}{\rho}$ and $\nu = \frac{\rho c_E}{K}$.

Equations $(10) - (13)$ can be expressed in non-dimensional form using the dimensionless variables with the primes omitted as follows:

$$\sigma_{xx} = e - \alpha_1 \theta - D, \tag{14}$$

$$\frac{\partial^2 e}{\partial x^2} - \alpha_1 \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 e}{\partial t^2}, \tag{15}$$

$$\frac{\partial^2 \dot{\phi}}{\partial x^2} + C_T^2 \frac{\partial^2 \phi}{\partial x^2} = (1 + \omega D_\omega)\left(\frac{\partial^2 \theta}{\partial t^2} + \varepsilon \frac{\partial^2 e}{\partial t^2} - Q\right), \tag{16}$$

$$\phi - \theta = \eta \frac{\partial^2 \phi}{\partial x^2}, \tag{17}$$

$\alpha_1 = \frac{\gamma\theta_0}{\lambda+2\mu}$, $C_T = \sqrt{\frac{K^*}{\rho c_E c_0^2}}$, $\varepsilon = \frac{\gamma}{\rho c_E}$, $\eta = a c_0^2 \nu^2$.

The conditions for initialisation and regularity are given by

$e = \theta = \phi = 0$ at $t = 0$,

$\frac{\partial e}{\partial t} = \frac{\partial \theta}{\partial t} = \frac{\partial \phi}{\partial t} = 0$ at $t = 0$,

$e = \theta = \phi = 0$ as $x \to \infty, t > 0$.

The problem is to solve the equations $(14) - (17)$ subjected to the following boundary conditions:

(i) stress-free boundary:

$$\sigma_{xx}(0, t) = 0, \tag{18}$$

(ii) varying thermal load:

$$\phi(0, t) = \phi_0 = F(t), \tag{19}$$

where $F(t)$ is a known function of time t.

From now on, we consider the non-dimensional kernel function $K(t - \xi)$ as follows [26]:

$$k(t - \xi) = 1 - \frac{2b}{\omega}(t - \xi) + \frac{a^2}{\omega^2}(t - \xi)^2$$

$$= \begin{cases} 1, & if\ a = b = 0, \\ 1 - \left(\frac{t-\xi}{\omega}\right), & if\ a = 0,\ b = \frac{1}{2}, \\ \left(1 - \frac{t-\xi}{\omega}\right)^2, & if\ \ a = b = 1, \end{cases}$$

a and b being constants.

4. Analytical Solution in the Laplace-Transform Domain

To solve the problem, we utilise the Laplace transform, which is defined as

$$\bar{g}(x, s) = \int_0^\infty g(x, t) e^{-st} dt,\ \mathrm{Re}(s) > 0$$

on the equations $(14) - (17)$, then we get

$$\bar{\sigma}_{xx} = \bar{e} - \alpha_1 \bar{\theta} - \frac{D}{s}, \tag{20}$$

$$\frac{d^2 \bar{e}}{dx^2} - \alpha_1 \frac{d^2 \bar{\theta}}{dx^2} = s^2 \bar{e}, \tag{21}$$

$$(C_T^2 + s)\frac{d^2 \bar{\phi}}{dx^2} = \{1 + G(\omega, s)\}(s^2 \bar{\theta} + \varepsilon s^2 \bar{e} - \bar{Q}), \tag{22}$$

$$\bar{\phi} - \bar{\theta} = \eta \frac{d^2 \bar{\phi}}{dx^2}, \tag{23}$$

where

$$G_\omega(p) = 1 - \frac{2b}{\omega p} + \frac{2a^2}{\omega^2 p^2}$$

$$-e^{-p\omega}\left[(1 - 2b + a^2) + \frac{2(a^2 - b)}{\omega p} + \frac{2a^2}{\omega^2 p^2}\right].$$

The boundary conditions (18) and (19) in the transformed domain take the forms

$$\bar{\sigma}(0, s) = \bar{\sigma}_{xx} = 0, \tag{24}$$

$$\bar{\phi}(0, s) = \bar{\phi}_0 = \bar{F}(s). \tag{25}$$

We assume that the medium is subjected to a moving heat source of constant strength that constantly releases energy while moving along the positive direction of the x-axis with a constant velocity α. The following non-dimensional structure is assumed to be this moving heat source:

$$Q = Q_0 H(\alpha t - x), \tag{26}$$

where Q_0 is the heat source strength (constant) and $H(t)$ is the Heaviside unit step function. Applying Laplace transform on equation (26), we have

$$\bar{Q} = \frac{Q_0}{s} e^{-\frac{sx}{\alpha}}. \tag{27}$$

From equations (22) and (27), we get

$$(C_T^2 + s)\frac{d^2 \bar{\phi}}{dx^2} = \{1 + G(\omega, s)\}\left(s^2 \bar{\theta} + \varepsilon s^2 \bar{e} - \frac{Q_0}{s} e^{-\frac{sx}{\alpha}}\right). \tag{28}$$

Using equation (23) in (28), we get

$$\bar{\theta} = (1 - \eta m)\bar{\phi} - \eta m \varepsilon \bar{e} - \eta \alpha_2 e^{-\frac{sx}{\alpha}}, \tag{29}$$

again from equations (23) and (29)

$$\frac{d^2\bar{\phi}}{dx^2} = m\bar{\phi} + m\varepsilon\bar{e} + \alpha_2 e^{-\frac{sx}{\alpha}}, \tag{30}$$

where

$$m = \frac{\{1+G(\omega,s)\}s^2}{C_T^2 + s + \eta s^2\{1+G(\omega,s)\}},$$

$$\alpha_2 = \frac{Q_0\{1+G(\omega,s)\}}{s[C_T^2 + s + \eta s^2\{1+G(\omega,s)\}]}.$$

Putting the value of θ from equation (29) in equation (21) and using equation (30) we have

$$\frac{d^2\bar{e}}{dx^2} = M_1\bar{e} + M_2\bar{\phi} + M_3 e^{-\frac{sx}{\alpha}}, \tag{31}$$

where

$$M_1 = \frac{s^2 + \alpha_1\varepsilon(1-\eta m)m}{1+\alpha_1\eta m\varepsilon}, M_2 = \frac{\alpha_1(1-\eta m)m}{1+\alpha_1\eta m\varepsilon},$$

$$M_3 = \frac{\alpha_1\alpha_2(1-\eta m) + \eta\alpha_1\alpha_2\alpha^2}{1+\alpha_1\eta m\varepsilon}.$$

Equations (30) and (31) can be written in the form of a vector-matrix differential equation as follows

$$\frac{d^2\bar{X}(x,s)}{dx^2} = P(s)\bar{X}(x,s) + F(s)e^{-\frac{sx}{\alpha}}, \tag{32}$$

where

$$\bar{X}(x,s) = \begin{pmatrix}\bar{\phi}(x,s)\\ \bar{e}(x,s)\end{pmatrix}, \qquad P(s) = \begin{pmatrix} m & \varepsilon m\\ M_2 & M_1\end{pmatrix},$$

$$R(s) = \begin{pmatrix}\alpha_2\\ M3\end{pmatrix}.$$

State Space Approach

One way to express the formal solution to the system (32) is as follows:

$$\bar{X}(x,s) = C(s)e^{-\sqrt{P(s)}\,x} + D(s)e^{-hx}, \tag{33}$$

where

$$D(s) = \begin{pmatrix}D_1\\ D_2\end{pmatrix} = \{h^2 I - P(s)\}^{-1}R(s),$$

$$I = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, C(s) = \bar{X}(0,s) - D(s),$$

$$h = \frac{s}{\alpha}, \quad \bar{X}(0,s) = \begin{pmatrix}\bar{\phi}_0\\ \bar{e}_0\end{pmatrix},$$

$$\bar{\phi}_0 = \bar{F}(s), \qquad \bar{e}_0 = \bar{e}(0,s).$$

The characteristic equation of the matrix $P(s)$ is given by

$$\chi^2 - (m+M_1)\chi + (mM_1 - mM_2\varepsilon) = 0.$$

The roots of the characteristic equation χ_1 and χ_2 satisfy the relations:

$$\chi_1 + \chi_2 = m + M_1 \quad \text{and} \quad \chi_1\chi_2 = mM_1 - mM_2\varepsilon.$$

The spectral decomposition of $P(s)$ takes the form

$$P(s) = \chi_1 E_1 + \chi_2 E_2, \tag{34}$$

where E_1, E_2 are projectors of the matrix $P(s)$, given by

$$E_1 = \frac{1}{\chi_1 - \chi_2}\begin{pmatrix} m-\chi_2 & \varepsilon m\\ \frac{(\chi_1 - m)(\chi_2 - m)}{m\varepsilon} & \chi_1 - m\end{pmatrix},$$

$$E_2 = \frac{1}{\chi_1 - \chi_2}\begin{pmatrix} \chi_1 - m & -\varepsilon m\\ \frac{(\chi_1 - m)(\chi_2 - m)}{m\varepsilon} & m - \chi_2\end{pmatrix}.$$

Then, we have

$$Q(s) = \sqrt{P(s)} = \sqrt{\chi_1}E_1 + \sqrt{\chi_2}E_2$$

$$= \frac{1}{\sqrt{\chi_1}+\sqrt{\chi_2}}\begin{pmatrix} m + \sqrt{\chi_1\chi_2} & \varepsilon m\\ M_2 & M_1 + \sqrt{\chi_1\chi_2}\end{pmatrix}. \tag{35}$$

Now the solution (33) becomes

$$\bar{X}(x,s) = C(s)e^{-Q(s)x} + D(s)e^{-hx}. \tag{36}$$

The Cayley-Hamilton theorem allows us to express the matrix exponential $exp(-Q(s)x)$ in equation (36) as

$$exp(-Q(s)x) = a_0(x,s)I + a_1(x,s)Q(s), \tag{37}$$

where a_0, a_1 are the coefficients depending on s, x to be determined from the equations

$$exp(-J_1 x) = a_0 + a_1 J_1, \tag{38}$$

$$exp(-J_2 x) = a_0 + a_1 J_2, \tag{39}$$

where J_1, J_2 are the eigenvalues of the matrix Q(s).

Solving (38) and (39), we get a_0 and a_1 as follows:

$$a_0 = \frac{J_1 e^{-J_2 x} - J_2 e^{-J_1 x}}{J_1 - J_2}, a_1 = \frac{e^{-J_1 x} - e^{-J_2 x}}{J_1 - J_2}.$$

Hence, (37) can be written as

$$exp(-Q(s)x) = L(x,s) = \big(l_{ij}(x,s)\big), \quad i,j = 1,2, \tag{40}$$

where $l_{11} = \frac{(J_1^2 - m)e^{-J_2 x} - (J_2^2 - m)e^{-J_1 x}}{J_1^2 - J_2^2}$,

$$l_{12} = \frac{\varepsilon m(e^{-J_1x} - e^{-J_2x})}{J_1^2 - J_2^2},\ l_{21} = \frac{M_2(e^{-J_1x} - e^{-J_2x})}{J_1^2 - J_2^2},$$

$$l_{22} = \frac{(J_1^2 - M_1)e^{-J_2x} - (J_2^2 - M_1)e^{-J_1x}}{J_1^2 - J_2^2}.$$

Using equation (40), we can write the solution of equation (36) in the following form

$$\begin{pmatrix} \bar{\phi} \\ \bar{e} \end{pmatrix} = \begin{pmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} e^{-hx}, \tag{41}$$

$$c_1 = \bar{\phi}_0 - D_1, \qquad c_2 = \bar{e}_0 - D_2,$$

$$D_1 = \frac{\alpha_2(h^2 - M_2) + m\varepsilon M_3}{(h^2 - J_1^2)(h^2 - J_2^2)},$$

$$D_2 = \frac{M_3(h^2 - m) + \alpha_2 M_1}{(h^2 - J_1^2)(h^2 - J_2^2)}.$$

Hence,

$$\bar{\phi} = \phi_1 e^{-J_2x} - \phi_2 e^{-J_1x}, \tag{42}$$

$$\bar{e} = e_1 e^{-J_2x} - e_2 e^{-J_1x}, \tag{43}$$

where

$$\phi_1 = \frac{(J_1^2 - m)(\bar{\phi}_0 - D_1) - \varepsilon m(\bar{e}_0 - D_2)}{J_1^2 - J_2^2},$$

$$\phi_2 = \frac{(J_2^2 - m)(\bar{\phi}_0 - D_1) - \varepsilon m(\bar{e}_0 - D_2)}{J_1^2 - J_2^2},$$

$$e_1 = \frac{(J_1^2 - M_1)(\bar{e}_0 - D_2) - M_2(\bar{\phi}_0 - D_1)}{J_1^2 - J_2^2},$$

$$e_2 = \frac{(J_2^2 - M_1)(\bar{e}_0 - D_2) - M_2(\bar{\phi}_0 - D_1)}{J_1^2 - J_2^2}.$$

Using equation (8), we get from equation (43)

$$\bar{e}_{xx} = e_1 e^{-J_2x} - e_2 e^{-J_1x}. \tag{44}$$

Substituting equatiions (42) and (43) into equation (29), then we have

$$\bar{\theta} = \theta_1 e^{-J_2x} - \theta_2 e^{-J_1x} + \theta_3 e^{-hx}, \tag{45}$$

where

$$\theta_1 = (1 - \eta m)\phi_1 - \eta m\varepsilon e_1,$$

$$\theta_2 = (1 - \eta m)\phi_2 - \eta m\varepsilon e_2,$$

$$\theta_3 = (1 - \eta m)D_1 - \eta m\varepsilon D_2 - \eta\alpha_2.$$

Now the solution for stress σ_{xx} is obtained from equation (20) by using equations (43) and (45) as follows

$$\bar{\sigma}_{xx} = \sigma_1 e^{-J_2x} - \sigma_2 e^{-J_1x} + \sigma_3 e^{-hx} - \frac{D}{s}, \tag{46}$$

$$\sigma_1 = e_1 - \alpha_1\theta_1, \qquad \sigma_2 = e_2 - \alpha_1\theta_2,$$

$$\sigma_3 = D_2 - \alpha_1\theta_3.$$

Integrating equation (44) and using the regularity condition, we get

$$\bar{u} = \frac{e_2}{J_1} e^{-J_1x} - \frac{e_1}{J_2} e^{-J_2x} - \frac{D_2}{h} e^{-hx}, \tag{47}$$

which completes the solution in the Laplace transform domain.
We now solve this problem for thermal shock.

Thermal Shock Problem

Let, $F(t) = F_0 H(t)$ where F_0 is constant and $H(t)$ is the Heaviside unit step function. Taking the Laplace transform, we have

$$\bar{\phi}_0 = \bar{F}(s) = \frac{F_0}{s}. \tag{48}$$

Using boundary conditions (24), (25) on equations (20), (29) we get

$$\alpha_1\bar{\theta}_0 = \bar{e}_0 - \frac{D}{s},$$

$$\bar{\theta}_0 = (1 - \eta m)\bar{\phi}_0 - \eta m\varepsilon\bar{e}_0 - \omega\eta,$$

for the determination of $\bar{e}_0$ and $\bar{\theta}_0$.

Solving these,

$$\bar{e}_0 = \frac{1}{1 + m\eta\varepsilon\alpha_1}\left\{\frac{\alpha_1(1 - m\eta)F_0}{s} - \eta\alpha_1\alpha_2 + \frac{D}{s}\right\}, \tag{49}$$

and

$$\bar{\theta}_0 = \frac{1}{1 + m\eta\varepsilon\alpha_1}\left\{\frac{(1 - m\eta)F_0}{s} - \eta\alpha_2 - \frac{D\eta m\varepsilon}{s}\right\}. \tag{50}$$

Thus, we get a complete solution of the thermal shock problem in the Laplace transform domain using the equations (48) and (49) into the equations (42), (44), (45), (46) and (47).

From our results, one may derive all the results of [47] by taking $\omega \to 0, D = 0$ and $K_{ij} = 0$.

5. Numerical Results and Discussion

For a real space-time domain solution of the displacement component u, stress component σ_{xx}, strain component e_{xx}, conductive temperature ϕ and thermodynamic temperature θ, we have to apply the numerical inversion algorithm developed by Honig and Hirdes [49].

For computational purposes, the physical data for the chosen material is given below [50]

$K = 386$ N/Ks, $\alpha_t = 1.78 \times 10^{-5}$ K^{-1},

$c_E = 383$m^2/K, λ=7.76 $\times$ 10^{10}N/m^2, μ = 3.86 $\times$ 10^{10} N/m^2, $\rho =$ 8,954 kg/m^3,

$\theta_0 = 293$K, K^*=7.0, $F_0 = 1.0$, $\eta = 0.5$,

$\varepsilon = 0.003887$, $\alpha_1 = 0.036991$.

The computations were carried out for $t = 1$. For varying values of the delay time parameter ω and the heat source speed α, the displacement, conductive temperature, thermodynamic temperature, thermal stress, and strain distributions are graphically depicted.

Figures 1 − 5 are drawn to observe the effect of the heat source speed α on the different thermophysical quantities when the electric displacement $D = 0.5$, two temperature parameters $\eta = 0.5$, delay time parameter $\omega = 0.5$ and the kernel function $K(t - \xi) = 1 - \frac{(t-\xi)}{\omega}$.

Figure 1 demonstrates that the displacement u gets its maximum magnitude at the boundary $(x = 0.0)$ of the semi-infinite medium and the magnitude of u is larger for a smaller value of α. Effect of α on u appers in the region $0 \le x \le 0.5$ (approx).

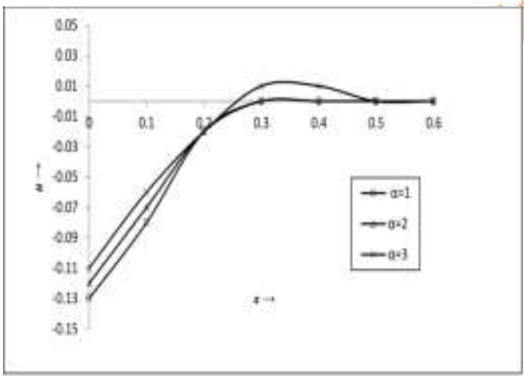

Fig. 1. Variation of u with respect to x.

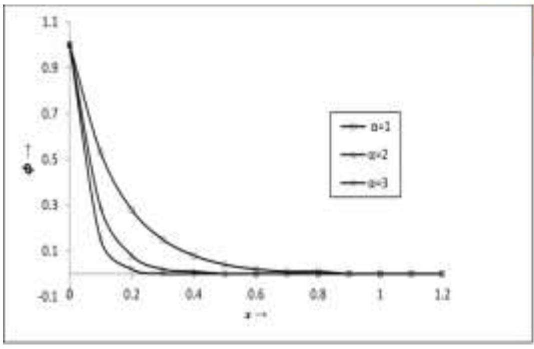

Fig. 2. Variation of ϕ with respect to x.

From figure 2 we see that the conductive temperature ϕ satisfies our assumed boundary condition. Each curve is concave upward, and the magnitude of ϕ is larger for a smaller value of the heat source speed α in the interval $0 < x < 0.5$. For a larger value of α, conductive temperature ϕ travels a smaller distance to vanish. For all curves, ϕ eventually approaches zero as x increases. This indicates that thermal disturbances do not propagate infinitely through the material; there is a limit beyond which temperature is essentially unaffected.

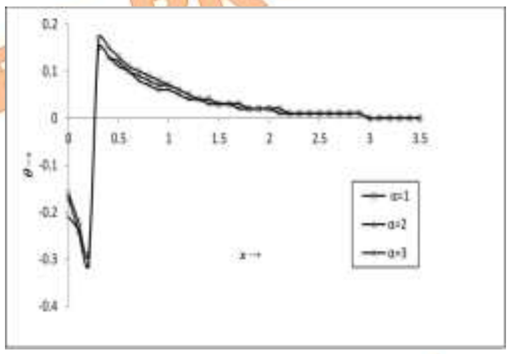

Fig. 3. Variation of θ with respect to x.

From figure 3 we see that thermodynamic temperature θ behaves oscillatory in $0 < x < 0.5$ (approx) and then decay to vanish.

From Figure 4, we see that the stress component σ_{xx} satisfies the assumed boundary condition. As α increases, the peak compressive stress becomes more negative in $-0.5 < x < -0.65$. A faster-moving heat source implies a more rapid localized heating and thermal expansion. This rapid expansion, when constrained, generates higher rates of thermal strain, which in turn leads to greater magnitudes of induced stress. The material has less time to deform or dissipate heat, leading to a more intense stress response. After $x = 1.8$, the magnitude of σ_{xx} increases to become stable.

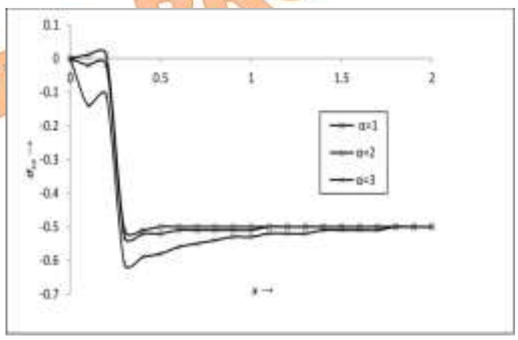

Fig. 4. Variation of σ_{xx} with respect to x.

From Figure 5 it is found that the strain component e_{xx} gets its greatest magnitude at the bounding plane $(x = 0.0)$. In the region $0.0 < x < 0.3$ (approx), the magnitude of e_{xx} is largervaluessmaller value of α, but in $0.3 < x < 1.8$ (approx) reverse phenomena are observed.

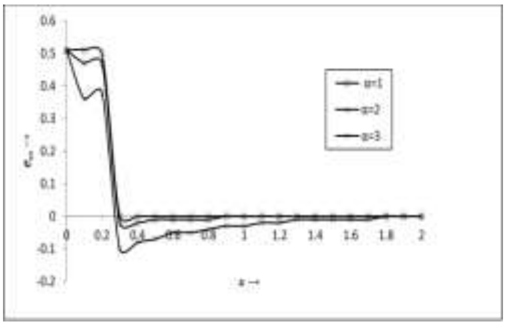

Fig. 5. Variation of e_{xx} with respect to x.

From Figure 6 it is clear that, as ω increases from 0.5 to 1.5, the magnitude of the initial negative displacement at $x = 0$ becomes significantly larger. Physically, this signifies that if the system is under a sustained load or has experienced a significant historical event, a longer memory means that the cumulative effect of that history is more strongly felt in the current displacement. Magnitude of u decreases in the interval $0.0 \leq x \leq 0.28$ (approx) and variation of u appears in the region $0.0 \leq x < 0.4$ (approx). When the value of the delay time parameter ω is larger then the magnitude of displacement component u is larger in $0.0 \leq x \leq 0.28$ (approx).

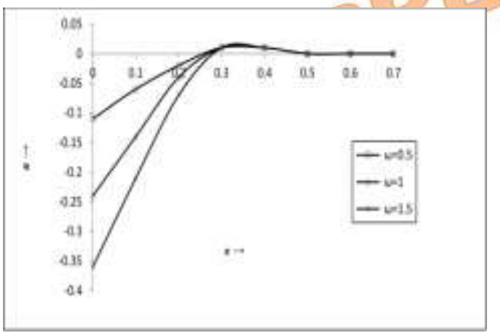

Fig. 6. Variation of u with respect to x.

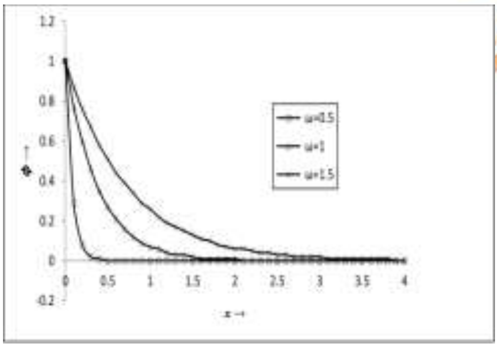

Fig. 7. Variation of ϕ with respect to x.

Figure 7 shows that the conductive temperature ϕ satisfies our assumed boundary condition in all considerations. For lesser value of ω the magnitude of ϕ is smaller, and each curve is concave upward. Over a larger span of x, the conductive temperature ϕ disappears for a smaller value of ω.

From figure 8 we see that, magnitude of the thermodynamic temperature θ is larger for a larger value of ω in $0 < x < 0.28$ (approx). The effect of ω on θ appears in the region $0.0 \leq x \leq 0.3$ (approx) and afterwards, with the increase of x, the magnitude of θ decreases to vanish for all choices of ω.

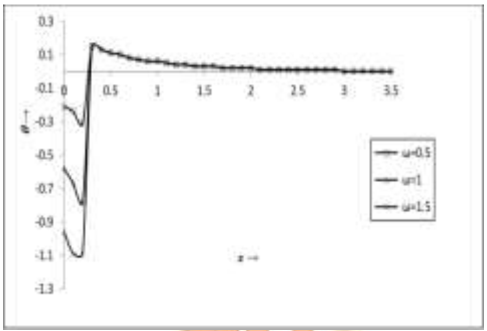

Fig. 8. Variation of θ with respect to x.

Figure 9 shows that the stress σ_{xx} satisfies our assumed boundary condition and they suddenly become compressive after $x = 0.2$ (approx). We see that the magnitude of σ_{xx} is greater for a greater value of ω in the region $x \geq 0.2$ (approx) and finally, they become stable.

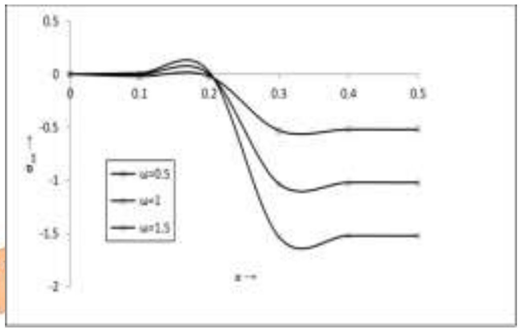

Fig. 9. Variation of σ_{xx} with respect to x.

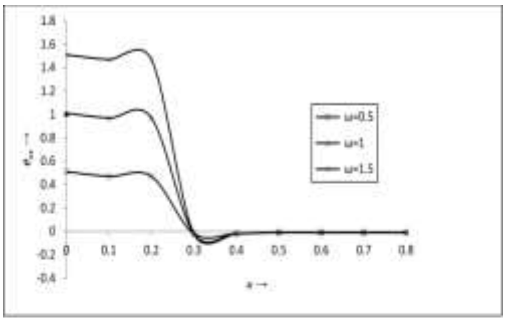

Fig. 10. Variation of e_{xx} with respect to x.

Figure 10 shows that strain e_{xx} gets its maximum value at the boundary $x = 0.0$. Strain gets its larger value for a larger value of ω in the interval $0.0 \leq x \leq 0.3$ (approx) and becomes negative near $x = 0.3$ before vanishing.

6. Conclusions

The problem of investigating the thermophysical quantities like stress, strain, displacement, conductive temperature, and thermodynamic temperature in a semi-infinite piezo-thermoelastic isotropic material is solved in the light of two temperature Moore-Gibson-Thompson thermoelasticity based on MDD. State space approach has been used to obtain the solution of the problem in the transformed domain of Laplace. Numerical inversion of the transformed solution has been performed using a method due to Honig and Hirdes [49]. The analysis of the results obtained allows us to make the following conclusions:

1. Thermophysical quantities like displacement, conductive temperature, thermodynamic temperature, and strain vanish travelling a certain distance in conformity with the generalized theory of thermoelasticity.
2. Significant effect of the delay time parameter ω and the heat source speed α are observed in the distribution of the thermophysical quantities.
3. The displacement component u gets its maximum magnitude at the boundary $x = 0.0$ for every choice of the heat source speed α and the delay time parameter ω.
4. Under all circumstances, the conductive temperature ϕ satisfies our assumed boundary condition.
5. The conductive temperature ϕ is concave upward for every choice of the heat source speed α and the delay time parameter ω.
6. The stress σ_{xx} satisfies our assumed condition in all considerations, and after travelling some distance (after $x = 0.2$) from origin, σ_{xx} becomes compressive for every choice of the delay time parameter ω and the heat source speed α.
7. For each combination of the delay time parameter ω and the heat source speed α, the strain e_{xx} reaches its maximum magnitude at the boundary $x = 0.0$.
8. From our results, one may derive all the results of [47] by taking $\omega \to 0, D = 0$ and $K_{ij} = 0$.

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Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this manuscript.

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