Mechanics of Advanced Composite Structures 6 (2019) 95 – 104

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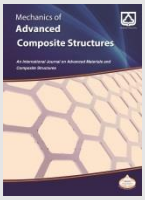
Mechanics of Advanced Composite Structures

journal homepage: http://MACS.journals.semnan.ac.ir



Elasto-Thermodiffusive Response in a Two-Dimensional Transversely Isotropic Medium

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PAPER INFO

Paper history:
Received 2017-12-26
Received in revised form 2018-08-18
Accepted 2018-09-23

Keywords:
Generalized thermoelastic diffusion
Finite wave speed
Normal mode analysis
Transversely isotropic material
Green–Lindsay model

ABSTRACT

The present article investigates the elasto-thermodiffusive interactions in a transversely isotropic elastic medium in the context of thermoelasticity with one relaxation time parameter and two relation time parameters. The resulting non-dimensional coupled equations are applied to a specific problem of a half-space in which the surface is free of tractions and is subjected to time-dependent thermal and chemical loadings. The analytical expressions for the displacement components, stresses, temperature, strain, mass diffusion, and chemical potential are obtained in the physical domain by employing the normal mode analysis as a tool. These expressions are calculated for a copper-like material and the results are depicted graphically. A comparative study of a diffusive medium and a thermoelastic medium show that diffusion has a significant effect on the thermophysical quantities. Furthermore, in the absence of the effect of thermodiffusion, the results agree with the existing literature.

1. Introduction

Thermoelastic diffusion, which is also known as elasto-thermodiffusion in elastic solids, deals with the coupling effects of the fields of temperature, mass diffusion, and strain, in addition to heat and mass exchange with the environment. It has extensive applications in geophysics and other industrial fields, including the extraction of oil from oil deposits. In recent years, the subject of thermoelastic diffusion has received serious attention. The theory of thermoelastic diffusion was first developed by Nowacki [1–4]. Gawinecki et al. [5] proved a theorem about the existence, uniqueness, and regularity of the solutions for a nonlinear parabolic thermoelastic diffusion problem. Gawinecki and Szymaniec [6] established a theorem about the global existence of the solution for the same problem. In the theory developed by Nowacki [1–4], the classical coupled thermoelastic model was used [7]. It should be mentioned that the theory of coupled dynamical thermoelasticity predicts an infinite speed for thermal signals, which is physically unrealistic. There has been an increased interest in the field of heat propagation to remove this unrealistic prediction, which has led to the development of well-established theories of generalized thermoelasticity. Generalized thermoelasticity theories involve hyperbolic-type governing equations and predict the finite speed of thermal signals. For example, Lord and Shulman [8] proposed the generalized thermoelasticity theory, which is known as the LS model, involving one relaxation time. Green and Lindsay [9] developed the temperature-rate-dependent thermoelasticity model (GL model) involving two relaxation times.

Recently, Sherief et al. [10] developed a generalized thermoelastic diffusion theory with

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DOI: 10.22075/MACS.2018.13517.1134

one relaxation time, which allowed waves to propagate at finite speeds. Sherief and Saleh [11] investigated the problem of a thermoelastic half-space in the context of the generalized thermoelastic diffusion theory with one relaxation time. Singh [12] discussed the reflection wave phenomena from the free surface of an elastic solid with generalized thermodiffusion with one relaxation time and two relaxation times in a later study [13]. Aouadi [14] studied diffusion in an infinitely long solid cylinder and in an infinite elastic body with a spherical cavity [15]. The uniqueness and reciprocity theorems for the equations of a generalized thermoelastic diffusion problem in isotropic media were proven by Aouadi [16] on the basis of the Laplace transform method. Kumar and Gupta [17] studied the wave propagation at the boundary surface of an inviscid fluid under thermoelastic diffusion. Recently, Othman et al. [18] analyzed the effects of diffusion on a two-dimensional problem of generalized thermoelasticity in the context of the Green–Naghdi theory. Deswal and Choudhary [19–21] also analyzed a two-dimensional thermoelastic diffusion problem using the same theory. Kumar and Kansal [22] discussed the propagation of waves on the free surface of a transversely isotropic body under generalized thermoelastic diffusion. Kothari and Mukhopadhyay [23, 24] investigated thermoelastic diffusion inside a spherical shell under three different theories. Wang et al. [25] studied the thermoelastic dynamic solution of a multilayered spherically isotropic hollow sphere for spherically symmetric problems. Such a body is said to possess transverse isotropy about any radius vector drawn from the center to a given point of material. Recently, several researchers, including El-Sayed [26], Karmakar and Kanoria [27], and Bhattacharya and Kanoria [28, 29], have used the elasto-thermodiffusive response to solve several problems. In addition, a few remarkable works on generalized thermoelastic diffusion have been published [30–33].

In the present analysis, we study the generalized thermoelastic diffusion in a transversely isotropic two-dimensional thermoelastic medium subjected to a prescribed temperature and chemical loading in which the boundary is free of traction. The analysis compares the thermoelastic diffusion model with two relaxation times, also known as the Green–Lindsay model with diffusion (GLD), and the thermoelastic diffusion model with one relaxation time, also known as the Lord–Shulman model with diffusion (LSD) for a thermodiffusive medium. Introducing a normal mode analysis, the governing equations have been expressed and solved in terms of normal modes. The numerical estimates for the thermal stresses, temperature, mass concentration, and chemical potential have been computed for a copper-like material and depicted graphically; the most significant points arising from our analysis have also been highlighted. In the absence of thermodiffusion, the LSD and GLD have been compared with the Lord–Shulman (LS) heat transfer model and the Green–Lindsay (GL) heat transfer model.

2. Formulation of the Problem

We consider a transversely isotropic elastic medium in a two-dimensional xy plane subjected to thermal and chemical loadings on the plane $y = 0$. The displacement components μ and v in the x and y directions are given as:

$$u = u(x, y, t),\ v = v(x, y, t) \tag{1}$$

The stress–strain temperature relations for the present problem are given as:

$$\sigma_{xx} = C_{11}e_{xx} + C_{12}e_{yy} - \beta_1\left(T + \tau_1\dot{T}\right) - \beta_2\left(C + \tau^1\dot{C}\right) \tag{2}$$

$$\sigma_{yy} = C_{12}e_{xx} + C_{11}e_{yy} - \beta_1(T + \tau_1\dot{T}) - \beta_2(C + \tau^1\dot{C}), \tag{3}$$

$$\sigma_{xy} = \left(C_{11} - C_{12}\right)e_{xy}. \tag{4}$$

where $C_{ij}(i, j = 1,2)$ is the elastic coefficient, $\sigma_{ij}(i, j = 1,2)$ is the stress tensor, $e_{ij}(i, j = 1,2)$ is the strain tensor, β_1 and β_2 are the tensors of thermal and diffusion moduli, respectively, and τ_1 and τ^1 are the thermal and diffusion relaxation times, respectively.

The equations of motion in the x and $y$directions are given by:

$$C_{11}\frac{\partial^2 u}{\partial x^2} + \frac{C_{11} - C_{12}}{2}\frac{\partial^2 u}{\partial y^2} + \frac{C_{11} + C_{12}}{2}\frac{\partial^2 v}{\partial x \partial y} = \rho\frac{\partial^2 u}{\partial t^2} - \beta_1\frac{\partial}{\partial x}(T + \tau_1\dot{T}) - \beta_2\frac{\partial}{\partial x}(C + \tau^1\dot{C}), \tag{5}$$

$$\frac{C_{11} - C_{12}}{2}\frac{\partial^2 v}{\partial x^2} + C_{11}\frac{\partial^2 v}{\partial y^2} + \frac{C_{11} + C_{12}}{2}\frac{\partial^2 u}{\partial x \partial y} = \rho\frac{\partial^2 v}{\partial t^2} - \beta_1\frac{\partial}{\partial y}(T + \tau_1\dot{T}) - \beta_2\frac{\partial}{\partial y}(C + \tau^1\dot{C}), \tag{6}$$

where ρ is the density.

The heat conduction equation corresponding to the problem, introduced by some unified parameters, is defined as:

$$K\nabla^2 T = \rho c_v\left(\dot{T} + \alpha_0 \ddot{T}\right) + T_0\beta_1\left(1 + \chi\frac{\partial}{\partial t}\right)\left(\frac{\partial \dot{u}}{\partial x} + \frac{\partial \dot{v}}{\partial y}\right) + cT_0(\dot{C} + \alpha_1\ddot{C}) \quad (7)$$

where α_0 and α_1 are the thermal and diffusion relaxation times satisfying the relations $\tau_1 \geq \alpha_0 \geq 0$ and $\tau^1 \geq \alpha_1 \geq 0$. For $\chi = 0$ from Equation (7), we have the GLD model. However, if $\alpha_0 = \alpha_1 = \tau_0$, we have the LSD model in the presence of thermodiffusion; τ_0 is the relaxation time for the LS model.

The chemical potential P is given by:

$$P = -\beta_2 e_{kk} + dC - cT \quad (8)$$

where c and d are the measures of the thermo-diffusion effect and the diffusive effect, respectively. The mass flux η_i is given by:

$$\eta_i = -DP_{,j} \quad (9)$$

where D is the diffusive constant. The diffusion equation is given by:

$$D\beta_2\nabla^2 e + Dc\nabla^2 T + \dot{C} = Dd\nabla^2 C \quad (10)$$

The following non-dimensional variables are as follows:

$$x' = c_1\eta x, y' = c_1\eta y, u' = c_1\eta u, v' = c_1\eta v, t' = c_1^2\eta t, \tau^{1'} = c_1^2\eta\tau^1, \tau'_1 = c_1^2\eta\tau_1, \alpha'_1 = c_1^2\eta\alpha_1, \alpha'_0 = c_1^2\eta\alpha_0, \theta' = \frac{T\beta_1}{C_{11}}, \eta = \frac{\rho c_v}{K}, c_1^2 = \frac{C_{11}}{\rho}, C' = \frac{\beta_2 C}{C_{11}}, P' = \frac{P}{\beta_2}$$

After removing the primes, the above equations can be written in a non-dimensional form as:

$$\sigma_{xx} = \frac{\partial u}{\partial x} + (a_2 - a_1)\frac{\partial v}{\partial y} - (\theta + \tau_1\dot{\theta}) - (C + \tau^1\dot{C}) \quad (11)$$

$$\sigma_{yy} = (a_2 - a_1)\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - (\theta + \tau_1\dot{\theta}) - (C + \tau^1\dot{C}) \quad (12)$$

$$\sigma_{xy} = \left(\frac{1 + a_1 - a_2}{2}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad (13)$$

where

$$a_1 = \frac{C_{11} - C_{12}}{2C_{11}}$$

$$a_2 = \frac{C_{11} + C_{12}}{2C_{11}}$$

Therefore, the equations of motion in the x and y directions are given by:

$$\frac{\partial^2 u}{\partial x^2} + a_1\frac{\partial^2 u}{\partial y^2} + a_2\frac{\partial^2 v}{\partial x\partial y} = \frac{\partial^2 u}{\partial t^2} + \frac{\partial}{\partial x}(\theta + \tau_1\dot{\theta}) + \frac{\partial}{\partial x}(C + \tau^1\dot{C}) \quad (14)$$

$$a_1\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + a_2\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial^2 v}{\partial t^2} + \frac{\partial}{\partial y}(\theta + \tau_1\dot{\theta}) + \frac{\partial}{\partial y}(C + \tau^1\dot{C}) \quad (15)$$

The heat conduction equation is given by:

$$\nabla^2\theta = \left(\dot{\theta} + \alpha_0\ddot{\theta}\right) + \varepsilon\left(1 + \chi\frac{\partial}{\partial t}\right)\dot{e} + \delta\left(\dot{C} + \alpha_1\ddot{C}\right) \quad (16)$$

where

$$\varepsilon = \frac{T_0\beta_1^2}{KC_{11}}$$

$$\delta = \frac{c\beta_1 T_0}{\beta_2}$$

The chemical potential is given by:

$$P = -e_{kk} + \alpha_3 C - \alpha_1\theta \quad (17)$$

The diffusion equation takes the form:

$$\nabla^2 e + \alpha_1\nabla^2\theta + \alpha_2\dot{C} = \alpha_3\nabla^2 C \quad (18)$$

where

$$\alpha_1 = \frac{cC_{11}}{\beta_1\beta_2}$$

$$\alpha_2 = \frac{C_{11}}{\eta\beta_2^2 D}$$

$$\alpha_3 = \frac{dC_{11}}{\beta_2^2}$$

3. Normal Mode Analysis

First, we choose the following equation:

$$\begin{aligned}&\big(u, v, \theta, \sigma_{ij}, P, C\big)(x, y, t) = \\ &(u^*, v^*, \theta^*, \sigma^*{}_{ij}, P^*, C^*)(y)\, exp(\omega t + iax)\end{aligned} \tag{19}$$

where ω is the complex time constant and a is the wave number in the x direction. Therefore, employing the normal mode analysis, the above equations can be written as:

$$\begin{aligned}&[a_1 D^2 - (a^2 + \omega^2)]u^*(y) + \\ &a_2 iaDv^*(y) = ia(1 + \tau_1\omega)\theta^*(y) + \\ &ia(1 + \tau^1\omega)C^*(y)\end{aligned} \tag{20}$$

$$\begin{aligned}&[D^2 - (a_1 a^2 + \omega^2)]v^*(y) + \\ &a_2 iaDu^*(y) = (1 + \tau_1\omega)D\theta^*(y) + \\ &(1 + \tau^1\omega)DC^*(y)\end{aligned} \tag{21}$$

$$\begin{aligned}&[D^2 - (a^2 + \omega + \alpha_0\omega^2)]\theta^*(y) = \\ &\varepsilon\omega(1 + \chi\omega)iau^*(y) + \varepsilon\omega(1 + \\ &\chi\omega)Dv^*(y) + \delta[\omega + \alpha_1\omega^2]C^*(y)\end{aligned} \tag{22}$$

$$\begin{aligned}&P^*(y) = -iau^*(y) - Dv^*(y) + \\ &\alpha_3 C^*(y) - \alpha_1\theta^*(y)\end{aligned} \tag{23}$$

$$\begin{aligned}&[iaD^2 - ia^3]u^*(y) + [D^3 - \\ &a^2 D]v^*(y) + \alpha_1[D^2 - a^2]\theta^*(y) = \\ &[\alpha_3 D^2 - (\alpha_3 a^2 + \alpha_2\omega)]C^*(y)\end{aligned} \tag{24}$$

When we eliminate$C^*(y)$ from Equations (20)–(22) and (24), they can be simplified to the following:

$$\begin{aligned}&[a_1 D^3 - c_{41}]u^*(y) = [c_{42}D^2 - \\ &c_{43}]v^*(y)\end{aligned} \tag{25}$$

$$\begin{aligned}&[c_{51}D^2 - c_{52}]u^*(y) + c_{53}Dv^*(y) = \\ &[c_{54}D^2 - c_{55}]\theta^*(y)\end{aligned} \tag{26}$$

$$\begin{aligned}&[c_{61}D^4 - c_{62}D^2 + c_{63}]u^*(y) = \\ &[c_{64}D^2 - c_{65}]\theta^*(y) + [c_{66}D^3 - \\ &c_{67}D]v^*(y)\end{aligned} \tag{27}$$

where

$$c_{41} = a^2 - a_2 a^2 + \omega^2$$

$$c_{42} = ia(1 - a_2)$$

$$c_{43} = ia(a_1 a^2 + \omega^2)$$

$$c_{51} = \delta a_1(\omega + \alpha_1\omega^2)$$

$$\begin{aligned}&c_{52} = \delta(\omega + \alpha_1\omega^2)(a^2 + \omega^2) - \\ &\varepsilon\omega a^2(1 + \tau^1\omega)(1 + \chi\omega)\end{aligned}$$

$$\begin{aligned}&c_{53} = a_2 ia\delta(\omega + \alpha_1\omega^2) + \varepsilon\omega ia(1 + \\ &\tau^1\omega)(1 + \chi\omega)\end{aligned}$$

$$c_{54} = ia(1 + \tau^1\omega)$$

$$\begin{aligned}&c_{55} = ia(1 + \tau^1\omega)(a^2 + \omega + \alpha_0\omega^2) - \\ &ia(\omega + \alpha_1\omega^2)(1 + \tau_1\omega)\end{aligned}$$

$$\begin{aligned}&c_{61} = a_1\alpha_3, c_{62} = a_1(\alpha_3 a^2 + \alpha_2\omega) + \\ &\alpha_3(a^2 + \omega^2) - a^2(1 + \tau^1\omega)\end{aligned}$$

$$c_{63} = (a^2 + \omega^2)(\alpha_3 a^2 + \alpha_2\omega) - a^4$$

$$c_{64} = \alpha_1 ia(1 + \tau^1\omega) + ia\alpha_3(1 + \tau_1\omega)$$

$$\begin{aligned}&c_{65} = \alpha_1 ia^3(1 + \tau^1\omega) + ia(1 + \\ &\tau_1\omega)(\alpha_3 a^2 + \alpha_2\omega)\end{aligned}$$

$$c_{66} = ia(1 + \tau^1\omega) - iaa_2\alpha_3$$

$$\begin{aligned}&c_{67} = ia^3(1 + \tau^1\omega) - a_2 ia(\alpha_3 a^2 + \\ &\alpha_2\omega)\end{aligned}$$

Furthermore, if we eliminate $u^*(x)$ and $\theta^*(x)$ from Equations (25)–(27), we obtain the following equation:

$$\begin{aligned}&D^8 v^*(y) - \mathfrak{I}_{11}D^6 v^*(y) + \\ &\mathfrak{I}_{22}D^4 v^*(y) - \mathfrak{I}_{33}D^2 v^*(y) + \\ &\mathfrak{I}_{44}v^*(y) = 0\end{aligned} \tag{28}$$

where

$$\begin{aligned}&\mathfrak{I}_{11} = \tfrac{1}{(c_{54}c_{61}c_{42} - c_{66}c_{54}a_1)}[c_{42}(c_{61}c_{55} + \\ &c_{62}c_{54} + c_{64}c_{51}) + c_{63}c_{54}c_{61} - \\ &a_1(c_{54}c_{67} - c_{55}c_{66} - c_{64}c_{53}) - \\ &c_{41}c_{66}c_{54}]\end{aligned}$$

$$\begin{aligned}&\mathfrak{I}_{22} = \tfrac{1}{(c_{54}c_{61}c_{42} - c_{66}c_{54}a_1)}[c_{42}(c_{54}c_{63} + \\ &c_{62}c_{55} + c_{64}c_{52} + c_{65}c_{51}) + \\ &c_{43}(c_{61}c_{55} + c_{62}c_{54} + c_{64}c_{51}) - \\ &a_1(c_{55}x_{67} - c_{65}c_{53}) - \mathfrak{I}_{55}]\end{aligned}$$

$$\begin{aligned}&\mathfrak{I}_{33} = \tfrac{1}{(c_{54}c_{61}c_{42} - c_{66}c_{54}a_1)}[c_{43}(c_{54}c_{63} + \\ &c_{62}c_{55} + c_{64}c_{52} + c_{65}c_{51}) + \\ &c_{42}(c_{55}c_{63} + c_{65}c_{52}) - c_{41}(c_{55}c_{67} - \\ &c_{65}c_{53})]\end{aligned}$$

$$\begin{aligned}&\mathfrak{I}_{44} = \tfrac{1}{(c_{54}c_{61}c_{42} - c_{66}c_{54}a_1)}[c_{43}(c_{55}c_{63} + \\ &c_{65}c_{52})]\end{aligned}$$

$$\mathfrak{I}_{55} = c_{41}(c_{54}c_{67} - c_{55}c_{66} - c_{64}c_{53})$$

The solution of Equation (28) is obtained as:

$$v^*(y) = \sum_{j=1} R_j(a, \omega)\, e^{-k_j y} \tag{29}$$

where $k_j^2 (j = 1,2,3,4)$ are the roots of the equation

$$k^8 - \Im_{11}k^6 + \Im_{22}k^4 - \Im_{33}k^2 + \Im_{44} = 0 \quad (30)$$

It can be shown that $u^*(y)$ and $\theta^*(y)$ satisfy the same equation as follows:

$$[D^8 - \Im_{11}D^6 + \Im_{22}D^4 - \Im_{33}D^2 + \Im_{44}]\{u^*(y), \theta^*(y)\} = 0 \quad (31)$$

Thus, the solutions are:

$$u^*(y) = \sum_{j=1}^{4} R'_j(a,\omega)\, e^{-k_j y} \quad (32)$$

$$\theta^*(y) = \sum_{j=1}^{4} R''_j(a,\omega)\, e^{-k_j y} \quad (33)$$

Substituting Equations (29), (32). and (33) into Equations (25) and (26), we obtain:

$$R'_j(a,\omega) = p_j R_j(a,\omega)$$
$$j = 1(1)4$$
$$R''_j(a,\omega) = q_j R_j(a,\omega)$$
$$j = 1(1)4$$

The mass concentration is given by:

$$C^*(y) = \sum_{j=1}^{4} \beta_j R_j(a,\omega)\, e^{-k_j y} \quad (34)$$

where

$$\beta_j = \frac{1}{1+\tau^1\omega}\left[\frac{\{a_1 k_j^2 - (a^2+\omega^2)\}p_j}{ia} - a_2 k_j - (1+\tau_1\omega)q_j\right], j = 1(1)4 \quad (35)$$

Furthermore, the chemical potential is given by:

$$P^*(y) = \sum_{j=1}^{4} \alpha_j R_j(a,\omega)\, e^{-k_j y} \quad (36)$$

where

$$\alpha_j = -iap_j + k_j + \alpha_3\beta_j - \alpha_1 q_j$$
$$j = 1(1)4 \quad (37)$$

Therefore, by substituting Equations (29) and (32)–(34) into Equations (11)–(13), the stress components are given by:

$$\sigma^*_{xx}(y) = \sum_{j=1}^{4}\left[iap_j - (a_2 - a_1)k_j - q_j - \beta_j\right] R_j(a,\omega) e^{-k_j y} \quad (38)$$

$$\sigma^*_{yy}(y) = \sum_{j=1}^{4}\left[ia(a_2 - a_1)p_j - k_j - q_j - \beta_j\right] R_j(a,\omega) e^{-k_j y} \quad (39)$$

$$\sigma^*_{xy}(y) = \left(\frac{1+a_1-a_2}{2}\right)\sum_{j=1}^{4}\left[ia - k_j p_j\right] R_j(a,\omega) e^{-k_j y} \quad (40)$$

4. Boundary Conditions

The problem is to solve subjected to the following boundary conditions. The bounding plane $y = 0$ is subjected to a thermal loading as follows:

$$\theta(x,y,t) = n(x,t) \text{on} y = 0 \quad (41)$$

The mechanical boundary conditions on the bounding plane $y = 0$ are given by:

$$\sigma_{yy}(x,y,t) = -p(x,t) \text{on} y = 0 \quad (42)$$

$$\sigma_{xy}(x,y,t) = 0 \text{on} y = 0 \quad (43)$$

The bounding plane $y = 0$ is subjected to a chemical loading as follows:

$$P(x,y,t) = g(x,t) \text{on} y = 0 \quad (44)$$

Employing the normal mode analysis on the boundary conditions, we derive the following equations from Equations (41)–(44):

$$\sum_{j=1}^{4} q_j R_j(a,\omega) = n^* \quad (45)$$

$$\sum_{j=1}^{4}\left[ia(a_2 - a_1)p_j - k_j - q_j - \beta_j\right] R_j(a,\omega) = -p^* \quad (46)$$

$$\sum_{j=1}^{4}\left[ia - k_j p_j\right] R_j(a,\omega) = 0 \quad (47)$$

$$\sum_{j=1}^{4} \alpha_j R_j(a,\omega) = g^* \quad (48)$$

5. Numerical Results and Discussions

The aim of this section is to present the analytical numerical results obtained in the preceding sections. For the numerical computations, we have considered a copper-like material. Since $\omega$is a complex time constant, we have $\omega = \omega_0 + i\xi$ and $e^{\omega t} = e^{\omega_0 t}(cos\,\xi\, t + i\, sin\,\xi\, t)$.

The values of the material constants are given as follows [27, 34, 35]:

$C_{11} = 1.628 \times 10^{11} Nm^{-2}$

$C_{12} = 0.632 \times 10^{11} Nm^{-2}$

$C_{13} = 0.508 \times 10^{11} Nm^{-2}$

$C_{33} = 0.627 \times 10^{11} Nm^{-2}$

$C_{44} = 0.770 \times 10^{11} Nm^{-2}$

$\alpha_t = 1.78 \times 10^{-5} K^{-1}$

$\alpha_c = 1.98 \times 10^{-4} m^3 Kg^{-1}$

$\theta_0 = 293K$

$\omega_0 = -0.5$

$\xi = 1$

$P^* = -4$

$n^* = 5.5$

$g^* = 1.4$

$a = 1$

$\beta_1 = 5.5889 \times 10^6 Nm^{-2}\, deg^{-1}$

$\beta_2 = 6.72737 \times 10^7 Nm^{-2}\, deg^{-1}$,

$K = 405\, Wm^{-1}\, deg^{-1}$,

$D = 0.95 \times 10^{-8} kgsm^{-3}$

$c = 0.9 \times 10^4\, m^2 s^{-2} K^{-1}$

$d = 0.6 \times 10^6\, m^5 Kg^{-1} s^{-2}$

The hypothetical values of the dimensionless relaxation time parameters are**:**

$\alpha_0 = 0.01$

$\tau_1 = 0.02$

$\alpha_1 = 0.1$

$\tau^1 = 0.2$

Figs. 1–7 have been plotted to study the effect of thermodiffusion on the thermophysical quantities when $x = 0.2$ and $t = 0.4$ for the thermoelastic diffusion model with two relaxation times (GLD) and the thermoelastic diffusion model with one relaxation time (LSD). In these figures, the continuous lines correspond to a thermodiffusive medium (WD) and the dotted lines correspond to a without thermodiffusive medium (WOD) for the GL model of heat transfer and the LS model of heat transfer.

Fig. 1 depicts the variation of the displacement component u with respect to $y$for $x = 0.2$ at time $t = 0.4$. The figure shows that the displacement attains its maximum magnitude at $y = 0$; as$y$ increases, the magnitude of $u$decreases and reaches zero for both the LS and GL models. In addition, the magnitude of $u$is larger for the GL model than the LS model. Furthermore, as seen in the figure, the magnitude of $u$corresponding to the GLD and LSD models is greater than that of the GL and LS models.

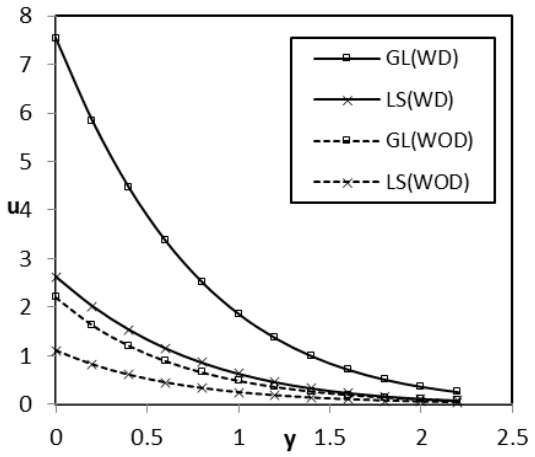

Fig. 1. Variation of u with respect to y for $t = 0.4$ and $x = 0.2$

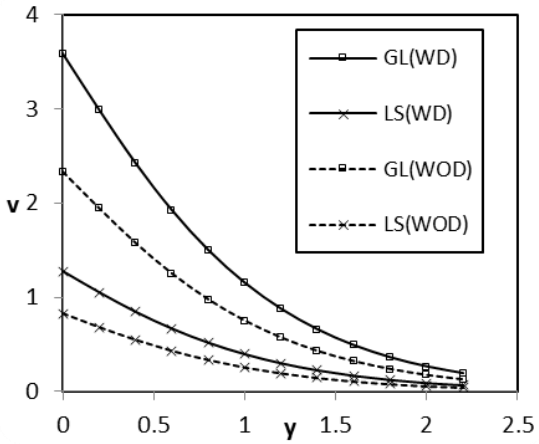

Fig. 2. Variation of *v* with respect to *y* for *t*=0.4 and *x*=0.2.

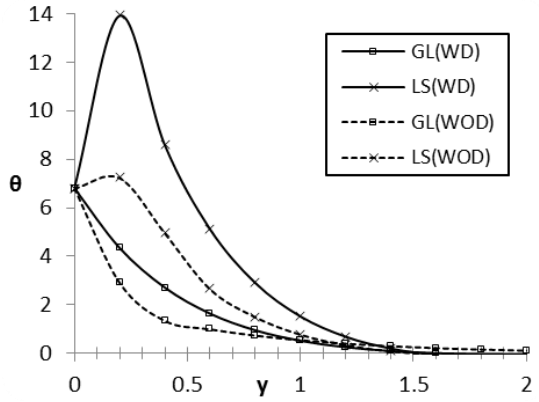

Fig. 3. Variation of θ with respect to y for *t*=0.4 and *x*=0.2

Fig. 2 shows the variation of the displacement component $v$with respect to y when $x = 0.2$ and $t = 0.4$ for both models. From the figure, we can observe that the vertical displacement v attains its maximum magnitude on the plane $y = 0$; as$y$increases, the magnitude of v decreases. Furthermore, due the presence of thermodiffusion, the magnitude of v is greater for the GLD and LSD models than for the GL and LS models. For both the diffusive medium and the elastic medium, the magnitude of $v$for the GL model is greater than that of the LS model.

Fig. 3 depicts the variation of the temperature θ with respect to y for both the LS and GL models for WD and WOD when $t = 0.4$ and $x = 0.2$. The figure shows that the magnitude of θ on the plane $y = 0$ satisfies the thermal boundary condition of our problem as given in Equation (41). Furthermore, in the LS model, the magnitude of θ increases for the interval $0 < y < 0.2$ to attain its maximum value at $y = 0.2$ in the diffusive medium, and then decreases sharply as $y$increases. However, in the GL model, the smoothness in the profile of θ is revealed. The presence of thermodiffusion has a tendency to increase the magnitude of the profile of the temperature field.

Fig. 4 shows the variation of the stress component $\sigma_{xy}$with respect to distance y when $t = 0.4$ and $x = 0.2$ for the thermoelastic diffusion model with one relaxation time (LSD) and the model corresponding to two relaxation times (GLD) against the LS and GL heat transfer models. It is observed that σ_{xy} has a value of zero on the plane$y = 0$ for both models, which satisfies the mechanical boundary condition of the problem given in Equation (43). The figure also shows that the magnitude of σ_{xy} is greater for the diffusive medium than for the elastic medium. The decay of the magnitude of σ_{xy} is also faster for the elastic medium (WOD) than the diffusive medium (WD).

Fig. 5 depicts the variation of the stress component σ_{yy} against distance y for $x = 0.2$ and $t = 0.4$. As shown in the figure, σ_{yy} attains its maximum magnitude on the plane $y = 0$ where the pressure is given and the magnitude of σ_{yy} decreases as y increases. The decay of σ_{yy} is faster for the elastic medium (WOD) than for the diffusive medium (WOD).

Fig. 6 shows the variation of the chemical potential Pagainst the distance $y$for $t = 0.4$ and $x = 0.2$for both the LS and GL models in the presence and absence of thermodiffusion. As shown in the figure, for the LS model, the chemical potential increases for the interval $0 < x < 0.2$ and then decreases sharply as$y$ increases. However, the decrease of $P$is slower in the GL model than in the LS model. Furthermore, because of the presence of thermodiffusion (GLD and LSD), the magnitude of Pbecomes less in the elastic medium (WOD) than in the diffusive medium (WOD) for both the GL and LS heat transfer models.

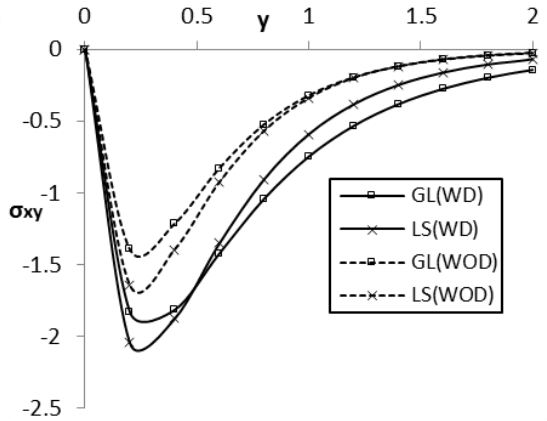

Fig. 4. Variation of σ_{xy} with respect to y for $t = 0.4$ and $x = 0.2$

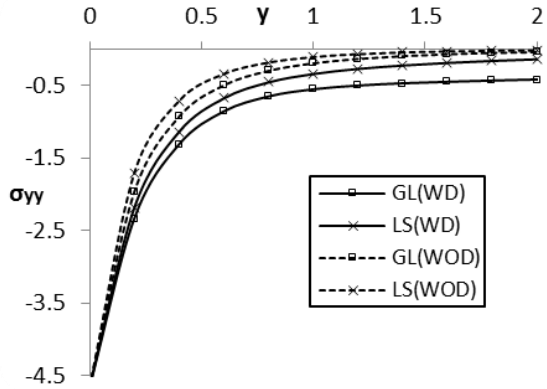

Fig. 5. Variation of σ_{yy} with respect to y for $t = 0.4$ and $x = 0.2$

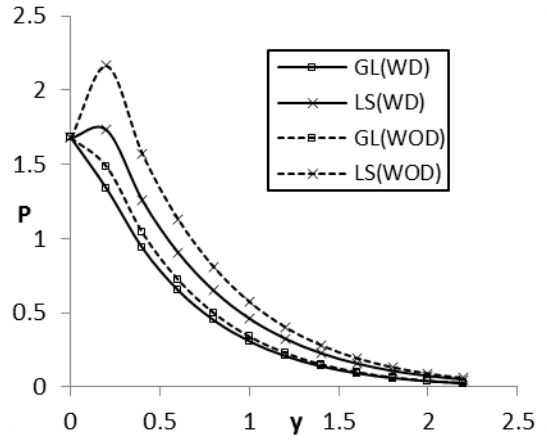

Fig. 6. Variation of P with respect to y for $t = 0.4$ and $x = 0.2$

Fig. 7 shows the variation of the mass concentration C for $t = 0.4$ and $x = 0.2$. The presence of thermodiffusion has a tendency to decrease the magnitude of the profile of C for both the LS and GL models.

Figs. 8 and 9 depict the variation of the shearing stress σ_{xy} and temperature θ for the LS model in a diffusive medium (WD) for different values of $y$and t when $x = 0.2$. It can be observed that as time increases, the magnitudes of the profiles of the shearing stress and the temperature distribution also increase, which supports the physical fact.

6. Conclusion

In the present analysis, the classical Fick's diffusion law is replaced by a generalized expression that involves two relaxation times. It allows a delayed response between the relative mass flux vector and the potential gradient. A two-dimensional transversely isotropic thermodiffusive medium has been considered in the context of the LSD model and the GLD model of generalized thermoelasticity. All the figures exhibit the different peculiarities that occur during the propagation of waves. The conclusions may be summarized as follows.

- The presence of thermodiffusion has a tendency to increase the magnitude of the profile of the displacement components, temperature, stress components, chemical potential, and the mass concentration within the medium.
- The magnitude of the displacement components are greater for the GL model than the LS model of generalized thermoelasticity.
- As time increases, the magnitudes of the stress component and temperature also increase, which occurs in the real situation.

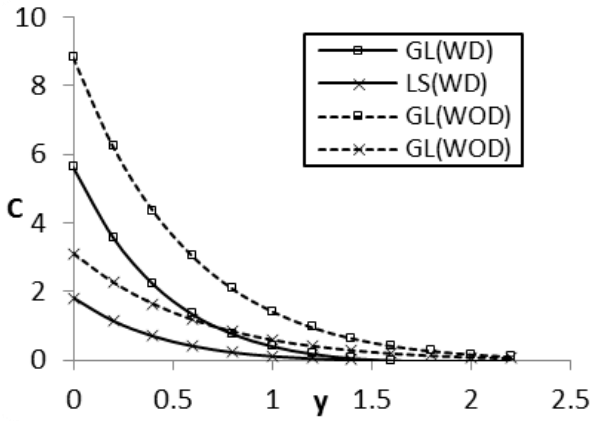

Fig. 7. Variation of C with respect to y for t=0.4 and x=0.2

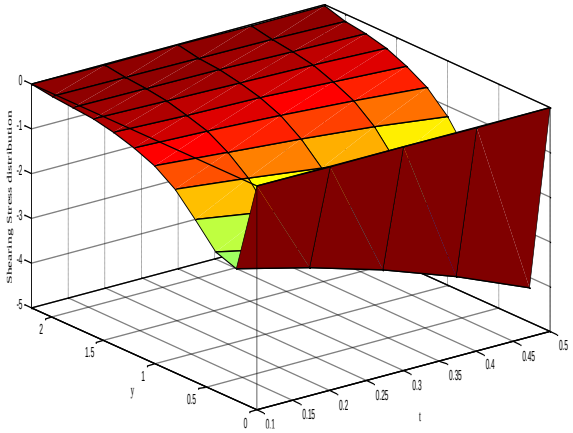

Fig. 8. Profile of σ_{xy} with respect to y and t when $x = 0.2$

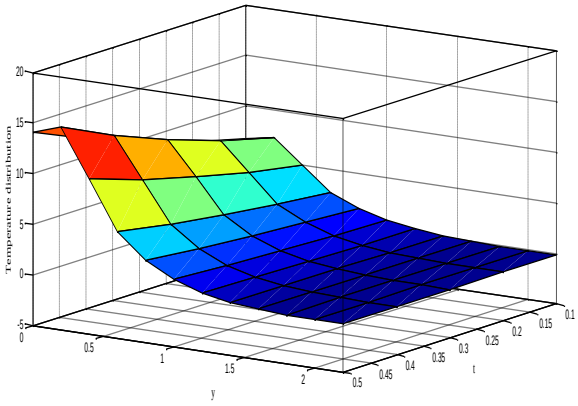

Fig. 9. Profile of θ with respect to y and t when $x = 0.2$

Acknowledgement

We are grateful to Prof. S. C. Bose of the Department of Applied Mathematics, University of Calcutta for his valuable suggestions and guidance during the preparation of this paper.

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