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A nonlinear multiobjective model for the product portfolio optimization: An integer programming

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Abstract

Optimization of the product portfolio has been recognized as a critical problem in industry, management, economy and so on. It aims at the selection of an optimal mix of the products to offer in the target market. As a probability function, reliability is an essential objective of the problem which linear models often fail to evaluate it. Here, we develop a multiobjective integer nonlinear constraint model for the problem. Our model provides opportunities to consider the knowledge transferring cost and the environmental effects, as nowadays important concerns of the world, in addition to the classical factors operational cost and reliability. Also, the model is designed in a way to simultaneously optimize the input materials and the products. Although being to some extent complicated, the model can be efficiently solved by the metaheuristic algorithms. Finally, we make some numerical experiments on a simulated test problem.

Keywords: Product portfolio optimization, Nonlinear programming, Multiobjective optimization, Reliability, Metaheuristic algorithm.
2010 MSC: Primary 90C30; Secondary 65K05.

1. Introduction

Rising competition in different market segments, rapidly changing the technologies and shortened product lifecycles made the companies and industries to offer a set of optimized products in order to meet changing the customer needs [16]. So, as an important scenario for investing, allocating resources and ensuring strategic fit on the products, portfolio management has attracted especial

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attentions [16]. Companies can benefit from considering several products families, instead of optimizing each product separately, and offer a portfolio with product variety in order to achieve the competitive differentiation and responding increasingly requests of the customers [21]. Nevertheless, too wide products range may confuse the customers and lead to complexity which has negative effects on the profitability, costs, new products development time and customers satisfaction [21]. Hence, the suggested product portfolio should be planned carefully to address customers requirements of the target market [9].

Because of the manufacturers concerns about the high failure rates and associated losses of the product portfolios [19], product portfolio optimization has been traditionally dealt with maximization of the profit or minimization of the cost. As example, Jiao *et al.* [9] suggested a model with the objective of maximizing the expected value of the shared surplus of the product portfolio in the sense of simultaneously accounting the customers benefit of purchasing a product in a less price and the producers benefit of selling a product in a higher price [14]. Jiao and Zhang [10] considered customer preference and choice probabilistically to maximize value of the shared surplus. Muller [13] dealt with a value–based portfolio optimization in order to maximize profit of the generated revenue and operational cost. Seifert *et al.* [20] used a linear programming model to maximize the total profit of the product portfolio. Azari–Takami *et al.* [2] proposed a profit maximization model for the problem by simultaneously considering both of the production and supply rates. Sadeghi *et al.* [18] suggested a multiobjective model in order to maximize the market share and to minimize the operational cost of the product portfolio as well. In another effort, Mangun and Thurston [12] proposed a nonlinear model to maximize the total portfolio utility with respect to the cost and reliability of the products. Also, Relich [17] developed a constraint satisfaction problem by taking into account the reliability in selecting products in the portfolio. All of the reviewed models have been efficiently solved using the metaheuristic algorithms.

As seen in the recent studies on the product portfolio optimization reviewed above, environmental effects has been ignored in the suggested models. Nowadays, considering product effects on the environment during its lifecycle is an important issue. In addition, due to the fact that knowledge is a powerful tool for the organizations to achieve competitive advantages [11], decreasing the knowledge transferring cost in the product portfolio planning should be taken care of. As another important engineering characteristic, reliability has been often disregard in recent product portfolio models. Although being a costly factor, generally reliability is a crucial element which can decrease the warranty cost and increase the customers satisfaction [5, 22]. Motivated by these, here we deal with a product portfolio optimization model in order to simultaneously consider all the four important objectives operational cost, knowledge transferring cost, environmental effects and reliability. In addition, it is worth noting that our model can be efficiently solved by metaheuristic algorithms.

The remainder of this work is organized as follows. In Section 2, after a detailed discussion on the problem specifications, we suggest a multiobjective nonlinear model for the product portfolio optimization. Using a metaheuristic algorithm, we made some numerical experiments on a simulated test problem in Section 3. Finally, concluding remarks are provided in Section 4.

2. A multiobjective nonlinear binary model for the product portfolio optimization

Here, we describe specifications, objectives and constraints of our model in details. To proceed, at first consider a manufacturing company which produces p different products $P_1,\ldots,P_p$ where each product is a combination of m different materials $M_1,\ldots,M_m$. The material M_i $(i = 1,\ldots,m)$ can be provided as one of the four types 'new', 'reused', 'remanufactured' or 'recycled', respectively indexed by $k = 1, 2, 3, 4$ and abbreviated by 'N', 'Ru', 'Rm' and 'Rc'. Preparation process of each material type consists of at most eight stages 'providing the raw material', 'manufacturing', 'assembling', 'collecting', 'disassembling', 'remanufacturing', 'recycling', and 'disposing', respectively indexed by $n = 1, \dots, 8$. For example, if the material is of the type new, then its preparation process begins with disposing the used material, replacing as a raw material, enduring the manufacturing process and assembling, respectively; then, it should be disassembled at the end of its life. Figure 1 shows the detailed preparation process of all the four material types.

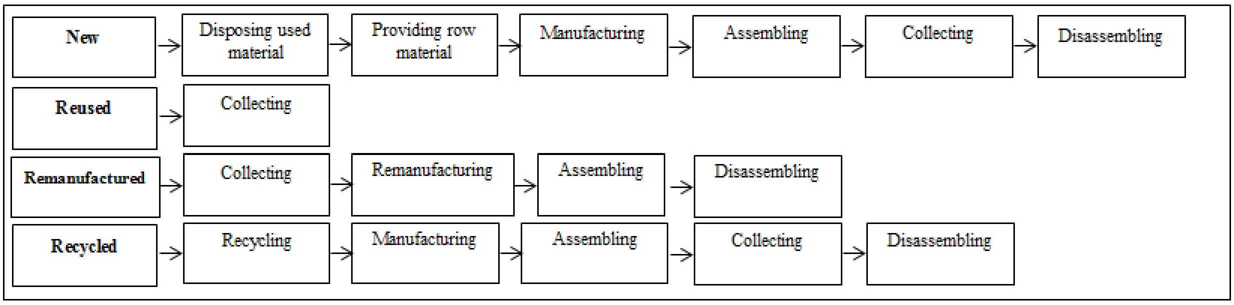

Figure 1: Preparation process of different material types

The company needs to decide which products should be included in the portfolio to achieve reasonable levels of the operational cost, knowledge transferring cost, environmental effects and reliability. So, a multiobjective approach can be employed to model the problem. Note that using the reliability function makes the model to be nonlinear. Here, we apply the Weibull distribution to determine the reliability as well [15].

To present our model, we need to consider the following two preliminary assumptions: (i) the company can meet all the market demands, and (ii) the costs of the product return is linearly added to all the materials. In what follows, we list the parameters of our multiobjective nonlinear product portfolio optimization model:

- θ_i : the characteristic life of the i–th material;
- h : the average time of diary usage of the j–th product;
- d : the average time of annual usage of the j–th product;
- b_i : the slope of the Weibull distribution of the i–th material;
- K : the portfolio capacity;
- T_j : the return time of the j–th product, bounded by a positive constant T;
- B : a large positive number;
- C_{ni} : the operational cost of the i–th material at the n–th preparation step, bounded by a positive constant C_{max};
- En_{ni} : the environmental effects of the i–th material at the n–th preparation step, bounded by a positive constant En_{max};
- Kc_{ni} : the knowledge transferring cost of the i–th material at the n–th preparation step, bounded by a positive constant Kc_{max};

– Re : the reliability function which is determined based on the Weibull distribution, bounded by a positive constant Re_{max}.

In addition, we consider the following binary decision variables for our model:

$$y_{kij} = \begin{cases} 1, & \text{if the } i\text{–th material of the } j\text{–th product is of the type } k, \\ 0, & \text{otherwise;} \end{cases}$$

$$y_j = \begin{cases} 1, & \text{if the portfolio contains the } j\text{–th product,} \\ 0, & \text{otherwise;} \end{cases}$$

and define the normalized decision elements by

$$z_1 = \frac{C}{C_{\max}},\ z_2 = \frac{En}{En_{\max}},\ z_3 = \frac{Kc}{Kc_{\max}},\ z_4 = \frac{Re}{Re_{\max}},$$

where, based on Figure 1, we have

$$C = \sum_{j=1}^{p}\sum_{i=1}^{m}(y_{1ij}(C_{8i} + \sum_{n=1}^{5} C_{ni}) + y_{2ij}\, C_{4i} + y_{3ij}\sum_{n=3}^{6} C_{ni} + y_{4ij}(C_{7i} + \sum_{n=2}^{5} C_{ni})), \tag{2.1}$$

$$En = \sum_{j=1}^{p}\sum_{i=1}^{m}(y_{1ij}(En_{8i} + \sum_{n=1}^{5} En_{ni}) + y_{2ij}\, En_{4i} + y_{3ij}\sum_{n=3}^{6} En_{ni} + y_{4ij}(En_{7i} + \sum_{n=2}^{5} En_{ni})), \tag{2.2}$$

$$Kc = \sum_{j=1}^{p}\sum_{i=1}^{m}(y_{1ij}(Kc_{8i} + \sum_{n=1}^{5} Kc_{ni}) + y_{2ij}\, Kc_{4i} + y_{3ij}\sum_{n=3}^{6} Kc_{ni} + y_{4ij}(Kc_{7i} + \sum_{n=2}^{5} Kc_{ni})), \tag{2.3}$$

$$Re = \prod_{j=1}^{p}\prod_{i=1}^{m}((y_{1ij} + y_{4ij})\{\exp -(\frac{h \times d \times T_j}{\theta_i})^b\} + (y_{2ij} + y_{3ij})\{\exp -(\frac{h \times d \times T_j}{\theta_i})\}). \tag{2.4}$$

In (2.1), note that the costs of assembling and disassembling are a functions of the consumed time. Also, the cost of collecting is a function of the distances and the consumed time for the products collection, and the cost of recycling is a function of the required energy. In (2.2), the SimaPro software (as a lifecycle assessment program) can be applied to estimate the environmental effects [8, 12]. It considers the three media 'solid waste', 'air pollution' and 'waste water' for all the eight stages of the preparation process for each material. The knowledge transferring cost in (2.3) is a function of three indicators including the documentation time, the support time and the time spent in participating in the meetings at all the eight stages of the preparation process [1]. As an important element of the objective function given by (2.4), reliability is determined as a nonlinear function (against the linear functions which cannot manage the uncertainty [23]) because the breakdown information related to the materials is not observed and so, it should be estimated [7]. Since failure rate is not constant, the best reliability probability function is the Wiebull distribution [6]. According to the bathtub curve, the slope of the Weibull distribution is considered as 1 for reused and remanufactured materials, because they are in the useful stage and undergo minimal repair or refurbishment, while it is considered less than 1 for the new and recycled materials. Also, θ_i is the mean time to failure of the i–th material.

Now, with these preliminaries we are in a position to state our product portfolio optimization model as follows:

$$\min \; z = (z_1, z_2, z_3, -z_4),$$

$$s.t. \sum_{i=1}^{m} \sum_{k=1}^{4} y_{kij} \le y_j B, \; j = 1, \ldots, p, \tag{2.5}$$

$$y_1 + \cdots + y_p \le K, \tag{2.6}$$

$$y_{1ij} + y_{2ij} + y_{3ij} + y_{4ij} = y_j, \; i = 1, \ldots, m, \; j = 1, \ldots, p, \tag{2.7}$$

$$y_{kij}, y_j, \in \{0, 1\}, \; \forall k \; \forall i \; \forall j.$$

Constraint (2.5) ensures that if the j–th product is not placed in the portfolio, then the corresponding materials will not used. Also, constraint (2.6) is a capacity condition and controls the maximum number of the products that could be placed in the portfolio. In addition, constraint (2.7) limits the i–th material of the j–th product to be only one of the new, reused, remanufactured or recycled material. As a final note, we can simply choose B in the interval $[4m, +\infty)$.

Although the model seem to be to some extent complicated in the sense of illustrating an NP–hard problem, there are many studies reporting promising results of the metaheuristic algorithms for solving such problems (see [3] and the references therein). The interest in these strategies remains particularly vivid for several motivations: the high flexibility that makes it possible to reuse the softwares, and the good performances that allow to efficiently address some large–scale and complicated problems. Here, we employe the metaheuristic algorithm suggested in [9] to solve the problem.

3. Numerical experiments

Here, we simulate a numerical product portfolio optimization model. In this context, consider a manufacturing company which produces fifteen different products $P_1,\ldots,P_{15}$ where each product is a combination of eight different materials $M_1,\ldots,M_8$. Suppose that the company determined the practical upper bounds of the operational cost, knowledge transferring cost, environmental effects, reliability, the portfolio capacity, the return time of the products, and other necessary information, as given in Appendix 1.

As mentioned before, the problem was solved by the metaheuristic algorithm of [9] with weighted sum scalarization of the objective function elements [4]. The generated solution has been illustrated in Table 1. As seen, six products P_1, P_6, P_8, P_{13}, P_{14} and P_{15} as well as their material types were determined as the optimal products to be placed in the portfolio. More exactly, it can be observed that product P_1 is made of the two materials a_1 and a_2 which a_1 is of the type recycled and a_2 is of the type new. Similar detailed results can be stated for the other five products.

4. Conclusions and future works

We have developed a nonlinear multiobjective model to optimally determine the products which should be included in a portfolio, together with their material types. In contrast to the recent studies in this guideline, we have considered the new challenging factors of the real world in the

Table 1: The generated solution

Constituent products	P_1	P_6	P_8	P_{13}	P_{14}	P_{15}
Material Type	$M_1(Rc)$	$M_2(N)$	$M_1(Rm)$	$M_3(Rm)$	$M_2(Ru)$	$M_1(Rc)$
	$M_2(N)$	$M_4(Rc)$	$M_2(Rm)$	$M_4(Rm)$	$M_4(Ru)$	$M_2(Rc)$
		$M_7(N)$	$M_4(Rm)$	$M_6(Rc)$	$M_5(Rm)$	$M_3(N)$
		$M_8(Rc)$	$M_5(Rm)$	$M_7(Ru)$	$M_6(Ru)$	$M_5(Ru)$
			$M_6(N)$		$M_7(Rc)$	$M_6(Ru)$
			$M_7(Rm)$			$M_7(Rm)$
			$M_8(N)$			$M_8(N)$

sense that we have embedded the environmental effects and the knowledge transferring cost in our model, in addition to the classical factors operational cost and reliability. The model can be efficiently solved by the metaheuristic algorithms. A numerical simulation study has been also carried out.

The model can help the companies which intend to produce green products, i.e. products with the least environmental effects. Obviously, when a company only uses the new materials in the production process, extensive costs are imposed on the system. However, by entering reused, remanufactured or recycled materials in the product cycle and considering a closed–loop supply chain, company can benefit while reducing the environmental effects.

As future studies, it would be interesting to consider customer groups to suggest the portfolio of products according to their needs. Allowing uncertainty in the costs, considering the aspects of product design such as scheduling and logistics, and evaluating performance of different metaheuristic algorithms on the model are relevant issues that can be investigated as well.

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Appendix 1: Test Problem specifications

Table 2: Test problem data

Item	**Acceptable upper bounds**
Operational cost ($C_{\max}$)	700 $
Environmental effects ($En_{\max}$)	800 mPt
Knowledge transferring cost ($Kc_{\max}$)	150 $
Reliability ($Re_{\max}$)	1
Portfolio capacity (K)	7
Return time of products (T)	21900 H

Table 3: Materials/Products

Product	**Material**							
	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
P_1	1	1	0	0	0	0	0	0
P_2	1	1	0	0	0	1	0	0
P_3	1	1	0	0	1	0	0	0
P_4	0	1	0	1	1	1	1	1
P_5	0	1	0	0	1	0	0	0
P_6	0	1	0	1	0	0	1	1
P_7	0	0	1	0	0	1	1	0
P_8	1	1	0	1	1	1	1	1
P_9	0	1	0	1	0	1	1	0
P_{10}	1	1	0	0	0	0	1	0
P_{11}	0	0	1	1	1	1	1	1
P_{12}	1	1	1	0	1	1	1	0
P_{13}	0	0	1	1	0	1	1	0
P_{14}	0	1	0	1	1	1	1	0
P_{15}	1	1	1	0	1	1	1	1

Table 4: Operational costs

Parameter	**Preparation process**	**Material**							
		M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
C_{1i}	Providing the raw material	2	20	23	15	12	3	4	8
C_{2i}	Manufacturing	4	25	27	30	15	5	6	12
C_{3i}	Assembling	1.5	8	9	10	6	2.5	3	5
C_{4i}	Collecting	1.5	3	3	3	2	1	1.5	2
C_{5i}	Disassembling	1	2.5	2.5	5	3	1.5	1.5	3
C_{6i}	Remanufacturing	2	15	15	17	10	3	3.5	8
C_{7i}	Recycling	1	7	7	10	5	1.5	2	4
C_{8i}	Disposing	1	3	3	5	2	1	1.5	2

Table 5: Environmental effects

Parameter	**Preparation process**	**Material**							
		M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
En_{1i}	Providing the raw material	8	10	10	9	10	9	9	8
En_{2i}	Manufacturing	9	18	18	17	13	10	11	13
En_{3i}	Assembling	10	12	13	10	15	13	12	15
En_{4i}	Collecting	10	12	10	15	10	12	10	12
En_{5i}	Disassembling	8	10	10	12	10	10	12	12
En_{6i}	Remanufacturing	12	13	14	18	15	16	15	18
En_{7i}	Recycling	8	10	10	12	10	12	10	12
En_{8i}	Disposing	8	17	15	15	13	10	12	15

Table 6: Knowledge transferring costs

Parameter	**Preparation process**	**Material**							
		M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
Kc_{1i}	Providing the raw material	0.5	2	2	2	1	1	1	1
Kc_{2i}	Manufacturing	1.5	3	3.5	5	2.5	3	3	2.5
Kc_{3i}	Assembling	1	2.5	2.5	4	1.5	1.5	2	1.5
Kc_{4i}	Collecting	1	1	1	1	1	1	1	1
Kc_{5i}	Disassembling	0.25	1.5	1.5	2	1	0.5	0.5	0.5
Kc_{6i}	Remanufacturing	0.75	1.5	2	3	1	1	1.5	1
Kc_{7i}	Recycling	0.25	0.5	0.5	1	0.5	0.5	0.5	0.5
Kc_{8i}	Disposing	0.25	0.5	0.5	1	0.5	0.5	0.5	1

Table 7: Characteristic life of the materials

Material	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
θ_i	20000	40000	40000	30000	16000	9000	8760	17520

Table 8: Reliability parameters

Product	h	d
P_1	10	8760
P_2	10	8760
P_3	10	8760
P_4	10	8760
P_5	10	8760
P_6	10	8760
P_7	10	8760
P_8	10	8760
P_9	10	8760
P_{10}	10	8760
P_{11}	10	8760
P_{12}	10	8760
P_{13}	10	8760
P_{14}	10	8760
P_{15}	10	8760