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Entropy generation in hydromagnetic and thermal boundary layer flow due to radial stretching sheet with Newtonian heating

Sanatan Das[1*], Rabindra Nath Jana[2] and Oluwole Daniel Makinde[3]

[1*] *Department of Mathematics, University of Gour Banga, Malda 732 103, India*
[2] *Department of Applied Mathematics, Vidyasagar University, Midnapore 721 102, India*
[3] *Faculty of Military Science, Stellenbosch University, Private Bag X2, Saldanha 7395, South Africa*

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ABSTRACT

The entropy generation during hydromagnetic boundary layer flow of a viscous incompressible electrically conducting fluid due to radial stretching sheet with Newtonian heating in the presence of a transverse magnetic field and the thermal radiation has been analyzed. The governing equations are then solved numerically by using the fourth order Runge-Kutta method with shooting technique. The effects of the pertinent parameters on the fluid velocity, temperature, entropy generation number, Bejan number, as well as the shear stress at the surface of the sheet are discussed graphically and quantitatively. It is examined that because the presence of a magnetic field, the entropy generation in a thermal system can be controlled and reduced.

1. Introduction

The boundary layer flow of a continuously stretching sheet has attracted considerable attention in recent years due to its numerous applications in industry. It occurs frequently in manufacturing involving the hot metal rolling, wire drawing, glass-fiber production, paper production, drawing of plastic films, and metal spinning, as well as metal and polymer extrusion processes. The boundary layer flow and the heat transfer over a continuous stretching surface have been discussed by a significant number of researchers [1-7] due to its wide applications in industrial and engineering processes. However, less attention has been paid to the boundary layer flow over a radial stretching sheet. Hayat et al. [8] have studied an axisymmetric flow and heat transfer of a second grade fluid past a stretching sheet. Sahoo and Sharma [9] have investigated the MHD flow and heat transfer from a continuous surface in a uniform free stream of a non-Newtonian fluid. Sajid et al. [10] have presented an unsteady axisymmetric flow and heat transfer over a radial stretching sheet. Ariel [11] has presented the extended homotopy perturbation method and computation of flow past a stretching sheet. Sahoo [12] has examined the effects of the partial slip on axisymmetric flow of an electrically conducting viscoelastic fluid past a stretching sheet. Effects of slip, viscous dissipation and Joule heating on the MHD flow and heat transfer of a second grade fluid past a radial stretching sheet has been investigated by Sahoo [13].

* Corresponding author: S. Das, Department of mathematics, University of Gour Banga, Malda, India. Email: tutusanasd@yahoo.co.in

The second law of thermodynamics is more reliable than the first law of thermodynamics due to the limitation of the efficiency of the first law in heat transfer engineering systems (Oztop and Al-Salem [14]). In order to access the best design of the thermal systems, one can employ the second law of thermodynamics by minimizing the irreversibility. Entropy generation is a criterion of the destruction of the available system work. The evaluation of the entropy generation is carried out to improve system performance. Heat transfer, mass transfer, viscous dissipation, etc. can be used as sources of entropy generation. Entropy generation can be used as a quantitative measure of irreversibilities that are associated with a process, because of this fact that the greater the entropy generation indicates the greater the extent of irreversibilities. In many engineering and industrial processes, entropy production, destroys the available energy in the system. It is therefore imperative to determine the rate of entropy generation in a system, in order to optimize energy in the system for efficient operation in the system. According to the second law of thermodynamics, all the flows and heat transfer processes undergo changes that are irreversible. These irreversible changes are mostly caused by the energy losses during the processes. Although measures can be taken to reduce these irreversible effects, it is impossible to recover all of the lost energy. This process causes the entropy of the system to increase. Due to this, the entropy generation rate is used as a standard metric to study the irreversibile effects. This method was proposed by Bejan [15,16].

In recent years, many papers have been published on the applications and entropy generation rates of the second law of thermodynamics. Odat et al. [17] have explored the entropy generation effects in the laminar flow past a flat plate under the influence of the magnetic field and found that the entropy generation rate increases with the magnetic field intensity. This study has revealed that the magnetic field is one of the causes that are responsible for the entropy production in the systems. Saouli and Aiboud-Saouli [18] investigated the second law analysis of laminar falling liquid film along an inclined heated plate. Esfahani and Jafarian [19] have presented the entropy generation analysis of a flat plate boundary layer with different solution techniques. An irreversibility analysis for gravity driven non-Newtonian liquid film along an inclined isothermal plate has been presented by Makinde [20]. Arikoglu et al. [21] have examined the effect of slip on entropy generation in a single rotating disk in MHD flow. Aiboud and Saouli [22] have illustrated the application of the second law analysis of thermodynamics to viscoelastic magneto-hydrodynamic flow over a stretching surface analytically by using Kummer's functions. Makinde [23] has conducted a thermodynamic analysis on a gravity-driven liquid film along an inclined heated plate. He assumed viscosity to be a variable quantity and considered the convective cooling effect. Makinde [24] has studied a variable viscosity boundary layer flow over a flat plate under the effects of thermal radiation and Newtonian heating, as well as explored entropy generation effects in this flow. Makinde [25] has examined the entropy generation on magnetohydrodynamic flow and heat transfer over a flat plate with a convective boundary condition. The effect of viscous dissipation and thermal radiation on entropy generation in Blasius flow has been displayed numerically by Butt et al. [26]. Rashidi et al. [27] have analyzed the entropy generation in steady MHD flow due to a rotating porous disk in a nanofluid. The results showed that as the thermal radiation parameter increases, the generated entropy decreases. Butt and Ali [28] have illustrated the effects of magnetic field on entropy generation in the flow and heat transfer due to a radially stretching surface. Butt and Ali [29,30] have carried out the entropy analysis of flow and heat transfer caused by a moving surface. Recently, Butt and Ali [31] have presented the entropy analysis of magnetohydrodynamic flow and heat transfer over a convectively heated radially stretching surface.

The aim of the present paper is to explore the entropy generation in MHD boundary layer flow of a viscous incompressible electrically conducting fluid due to a radial stretching sheet in the presence of a transverse magnetic field by taking the convective boundary condition into account. The partial differential equations governing the flow are reduced to nonlinear ordinary differential equations, which are solved numerically by shooting technique by using fourth order Runge-Kutta method. A systematic study of the effects of the various pertinent parameters on the flow and heat transfer characteristic is carried out with the help of graphs and table.

2. Mathematical formulation

Consider a steady two-dimensional boundary layer flow due to the stretching of the sheet along the radial direction with the velocity $U(r) = ar$, where $a(> 0)$ is a constant, as shown in Fig. 1. The sheet is located in the plane $z = 0$ and the fluid is confined to the region $z > 0$. The flow is due to the stretching of the sheet. A uniform magnetic field of strength B_0 is applied perpendicularly to the sheet, i.e. in the z-direction. The lower surface of the sheet is heated by convection from a hot fluid at temperature T_f

which provides a heat transfer coefficient h_f while the temperature of the ambient cold fluid is T_∞. The cold fluid on the upper side of the sheet is assumed to be an electrically conducting Newtonian fluid with constant fluid property. There is no external electric field. Also, it is assumed that the magnetic Reynolds number is small enough so that the induced magnetic field can be neglected.

An order-of-magnitude analysis of the momentum equation (normal to the sheet) by using the usual boundary layer approximations shows that the pressure gradient is constant. Thus neglecting the pressure gradient, the continuity, momentum and energy equations in a viscous MHD incompressible boundary layer flow can be written, respectively, as follows

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2}-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial z}, \tag{3}$$

Where u and w are the velocity components along the r and z -directions, respectively, T the temperature of the fluid, σ the electric conductivity of the fluid, ν the kinematic viscosity, ρ the fluid density, k the thermal conductivity, c_p the specific heat at constant pressure and q_r the radiative heat flux.

The appropriate boundary conditions are

$$u=U(r)=ar,\ w=0,$$
$$-k\left(\frac{\partial T}{\partial z}\right)=h_f(T_f-T)\ \text{at}\ z=0, \tag{4}$$
$$u\to 0,\ T\to T_\infty\ \text{as}\ z\to\infty,$$

Where k is the thermal conductivity.

For an optically thick fluid, in addition to emission there is also self absorbed and usually the absorption coefficient is large and dependent on the wavelength so that we can apply Rosseland approximation for radiative flux. The Rosseland approximation [32] applies to optically thick medium and taking into account the model proposed by Magyari and Pantokratoras [33], the net radiation heat flux q_r is given by

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{5}$$

Where σ^* is the Stefan-Boltzmann constant and k^* the Rosseland mean absorption coefficient. The Rosseland mean absorption coefficients for the optically thick media are expressed as function of the thermodynamic properties of the media. It is assumed that the temperature difference between the fluid temperature and the free stream temperature T_∞ is small, so that the term T^4 may be expressed as a linear function of temperature. This is done by expanding T^4 in a Taylor series about a free stream temperature T_∞ as follows:

$$T^4=T_\infty^4+3T_\infty^3(T-T_\infty) \tag{6}$$
$$+6T_\infty^2(T-T_\infty)^2+\cdots$$

Neglecting higher-order terms in the equation (6) beyond the first order in $(T-T_\infty)$ we get

$$T^4\approx 4T_\infty^3T-3T_\infty^4. \tag{7}$$

The use of the equation (5) and (7), the equation (3) becomes

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z^2}+\frac{16\sigma^* T_\infty^3}{3k^*\rho c_p}\frac{\partial^2 T}{\partial z^2}, \tag{8}$$

The following similarity variables are introduced (Butt and Ali [28]):

$$\eta=z\sqrt{\frac{a}{\nu}},\ u=arf'(\eta),$$
$$w=-2\sqrt{a\nu}\,f(\eta),\ \theta=\frac{T-T_\infty}{T_f-T_\infty}, \tag{9}$$

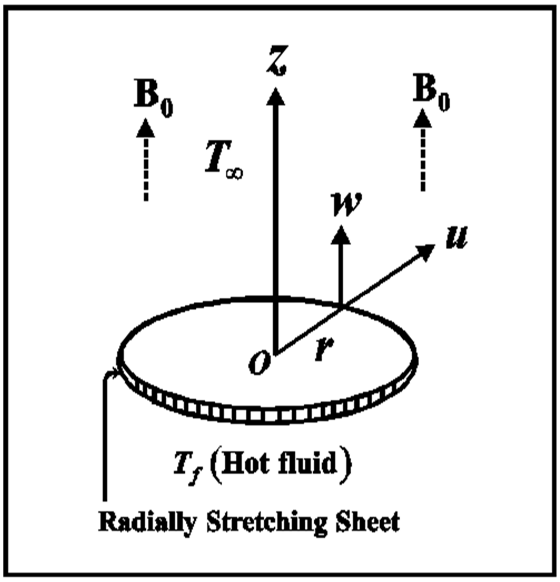

Figure 1. Geometry of the problem

where η is the independent similarity variable, $f(\eta)$ the dimensionless stream function and $\theta(\eta)$ the dimensionless temperature.

The use of equation (9), equations (2) and (8) reduce to

$$f''' + 2ff'' - f'^2 - M^2 f' = 0, \tag{10}$$

$$(1+R)\theta'' + 2Pr f\theta' = 0, \tag{11}$$

Where $M^2 = \dfrac{\sigma B_0^2}{\rho a}$ is the magnetic parameter, $R = \dfrac{16\sigma^* T_\infty^3}{3kk^*}$ the radiation parameter, and $Pr = \dfrac{\mu c_p}{k}$ the Prandtl number which measures the ratio of momentum diffusivity to the thermal diffusivity.

The thermal radiation is quite significant and the quality of the final product can be controlled by the control of cooling rate via the radiation parameter. In polymer industry, the thermal radiation effect may play an important role in the control of heat transfer process if the process is directed in a thermally controlled environment.

The corresponding boundary conditions are

$$\begin{aligned} &f(0) = 0,\ f'(0) = 1, \\ &\theta'(0) = -\mathrm{B}i[1-\theta(0)], \\ &f'(\infty) \to 0,\ \theta(\infty) \to 0, \end{aligned} \tag{12}$$

Where $\mathrm{B}i = \dfrac{h_f}{k}\sqrt{\dfrac{\nu}{a}}$ is the surface convection parameter or so-called Biot number. When $\mathrm{B}i \to \infty$, the convective boundary condition reduces to a uniform surface temperature boundary condition. For uniformly heated ($\mathrm{B}i \to \infty$) radial stretching sheet and in the absence of thermal radiation ($R = 0$) the present problem reduces to the Butt and Ali [28].

3. Numerical method for solution

The governing non-linear ordinary differential equations (10) and (11) cannot be solved analytically. This set of equations with the boundary conditions (12) must be solved numerically by applying most efficient fourth-order Runge-Kutta integration scheme with shooting algorithm. Equations (10) and (11) and boundary conditions (12) are reduced to a set of simultaneous first order differential equations by setting $y_1 = f$, $y_2 = f'$, $y_3 = f''$, $y_4 = \theta$ and $y_5 = \theta'$ as follows

$$\begin{aligned} &y_1' = y_2, \\ &y_2' = y_3, \\ &y_3' = y_2^2 + M^2 y_2 - 2y_1 y_3, \\ &y_4' = y_5, \\ &y_5' = -\frac{2Pr}{1+R} y_1 y_5, \end{aligned} \tag{13}$$

With the boundary conditions

$$\begin{aligned} &y_1(0) = 0,\ y_2(0) = 1,\ y_3(0) = b, \\ &y_4(0) = c,\ y_5(0) = -\mathrm{B}i(1-c), \end{aligned} \tag{14}$$

Where b and c are determined such that $y_1(\infty) = 0$ and $y_4(\infty) = 0$. The essence of this method is to reduce the boundary value problem to an initial value problem and then to use the shooting numerical technique to guess b and c until the boundary conditions $y_1(\infty) = 0$ and $y_4(\infty) = 0$ are satisfied. It is important to note that the *infinity* ($= \eta_{max}$) in the above equations represents the boundary layer thickness. We compare the calculated values for $y_1 (= f')$ and $y_4 (= \theta)$ at $\eta_{max} = 15$ with the given boundary condition $f'(15) = 0$ and $\theta(15) = 0$ and adjust the estimated values, $f'(0)$ and $\theta(0)$, to give a better approximation for the solution. The accuracy of the assumed missing initial condition is checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. The step size 0.01 is used to obtain the numerical solution with seven-decimal place accuracy as the criterion of convergence. The numerical computations are done by MATLAB built-in routine. The method is adequately explained in literature and it has second order convergence, unconditionally stable. It gives accurate result for boundary layer equations. In the present study, a uniform grid is used which is concentrated toward the wall.

4. Results and discussion

In order to gain a clear physical insight of the problem, we have discussed the effects of different values of magnetic parameter M^2, radiation parameter R, Prandtl number Pr and the Biot number on the velocity, temperature and shear stress at the surface of the sheet. The default values of the other parameters are mentioned in the description of the respected figures. Fig.2 shows that the fluid velocity $f'(\eta)$ decreases with an increase in magnetic parameter M^2. A drag-like Lorentz force is created by the application of the transverse magnetic field of the electrically conducting fluid. This force has the tendency to slow down the fluid flow. Fig.3 shows that the fluid temperature $\theta(\eta)$ increases by increasing the magnetic parameter M^2 in the boundary layer region and the thermal boundary layer thickness increases as the magnetic field becomes stronger. It is important to notice that large resistances on the fluid particles, which cause heat to be generated in the fluid, as the transverse applied magnetic field increases. It can be observed from Fig.4 that the fluid temperature increases by increasing values of radiation parameter R. The increase in radiation parameter means the release of heat energy from the flow region and so the fluid temperature decreases. A decrease in the values of R for given k and T_∞ means a decrease in the Rosseland radiation absorptivity k^*. Since the divergence of the radiative heat flux $\frac{\partial q_r}{\partial y}$ increases, k^* decreases which in turn causes to increase the rate of radiative heat transfer to the fluid and hence the fluid temperature increases. In the presence of thermal radiation, the thermal boundary layer always found to be increased. This means that the thermal boundary layer increases and there is more uniform temperature distribution across the boundary layer.

The effect of Prandtl number Pr on the heat transfer process is shown in the Fig.5. This figure shows that an increase in Prandtl number number Pr results in a decrease in the temperature distribution because, the thermal boundary layer thickness decreases by increasing the Prandtl number Pr. In summary, an increase in the Prandtl number means slow rate of the thermal diffusion. The graph also shows that the wall temperature decreases, as the values of Prandtl number Pr increase. It is also found that the thermal boundary layer thickness reduces as Pr increases. Fig.6 demonstrates the effects of the Biot number Bi on fluid temperature. It can be seen that the temperature profiles within the boundary layer increase by increasing in Biot number, leading to an increase in the thermal boundary layer thickness. Biot number is the ratio of the hot fluid side convection resistance to the cold fluid side convection resistance of a surface. For fixed cold fluid properties, Biot number Bi is directly proportional to the heat transfer coefficient h_f associated with the hot fluid. The thermal resistance on the hot fluid side is inversely proportional to h_f. Thus, as Bi increases, the hot fluid side convection resistance decreases and consequently, the surface temperature increases. It is also noticed that for large values of Bi, i.e. $Bi \to \infty$, the temperature profile attains its maximum value 1; thus the convective boundary condition becomes the prescribed surface temperature case. The thermal boundary layer thickness increases as Bi increases.

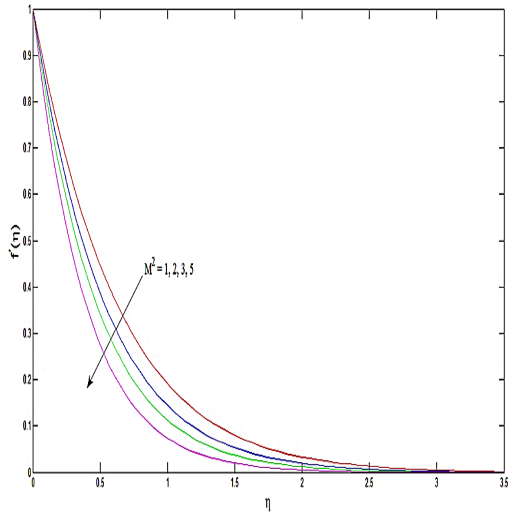

Figure 2. Velocity profiles for different M^2

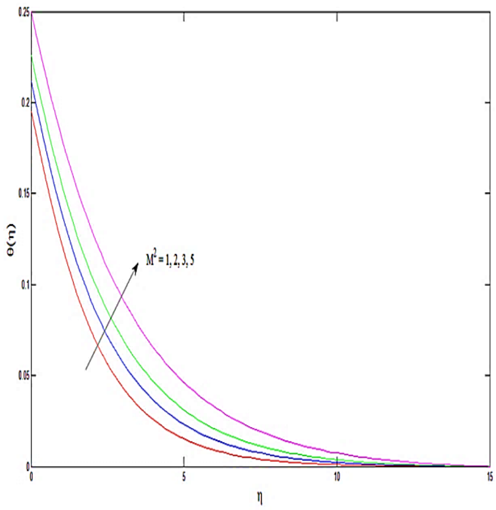

Figure 3. Temperature profiles for different M^2 when $M^2 = 1$, $R = 0.5$ and $Pr = 0.72$

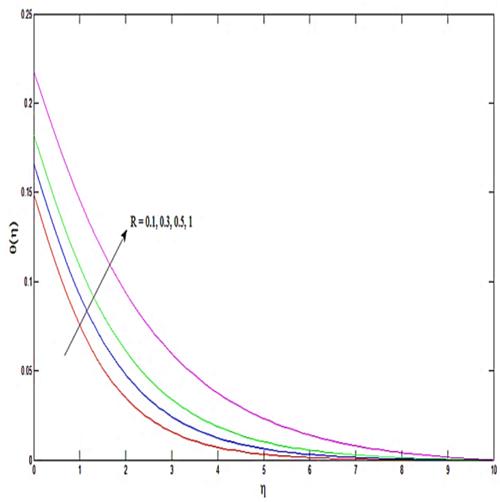

Figure 4. Temperature profiles for different R when $M^2 = 1$, $Bi = 0.1$ and $Pr = 0.72$

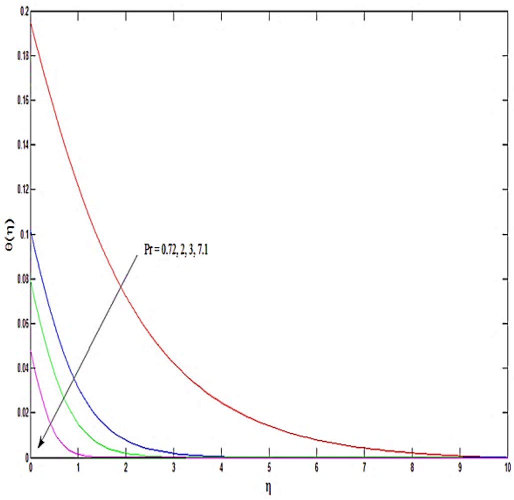

Figure 5. Temperature profiles for different Pr when $M^2 = 1$, $R = 0.5$ and $Bi = 0.1$

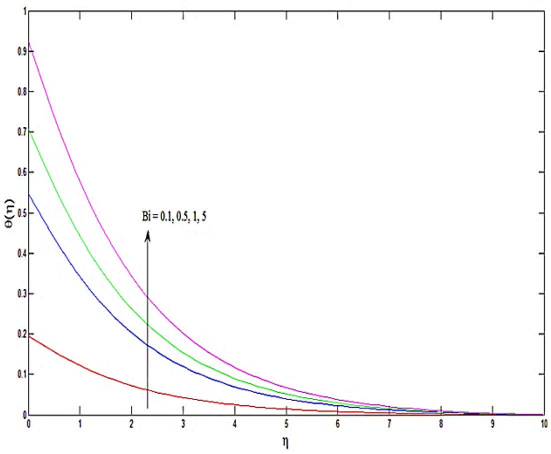

Figure 6. Temperature profiles for different Bi when $M^2 = 1$, $R = 0.5$ and $Pr = 0.72$

At the engineering point of view, the shear stress at the surface of the sheet is an important characteristic in the heat transfer studies, since it is directly related to the heat transfer coefficients. The increased shear stress is generally a disadvantage in the technical applications, while the increased heat transfer can be exploited in some applications such as heat exchangers, but should be avoided in other applications such as a gas turbine, for instance. The numerical values of the surface temperature $\theta(0)$ and the shear stress $-f''(0)$ at the surface of the sheet $\eta = 0$ are entered in the Table 1 for several values of M^2, R, Pr and Bi. It can be seen from the Table 1 that the surface temperature $\theta(0)$ increases by increasing values of either M^2 or Bi or R while it decreases by increasing in Pr. This is because of the fact that the Biot number Bi is directly proportional to the heat transfer coefficient associated with hot fluid. The thermal resistance on the hot fluid side is inversely proportional to h_f. Thus, as Bi increases, the hot fluid side convection resistance decrease; as a result, the surface temperature $\theta(0)$ increases. The shear stress $-f''(0)$ increases by increasing values of M^2.

4.1. Entropy generation

Non-equilibrium conditions happen due to the exchange of the energy and momentum within the fluid and at solid boundaries that result in entropy generation. According to Woods [34], the local volumetric rate of entropy generation for a viscous incompressible conducting fluid in the presence of a magnetic field is given by

$$E_G = \frac{k}{T_\infty^2}\left(\frac{\partial T}{\partial z}\right)^2 + \frac{\mu}{T_\infty}\left(\frac{\partial u}{\partial z}\right)^2 + \frac{\sigma B_0^2}{T_\infty}u^2. \tag{15}$$

The first term in equation (15) is the irreversibility due to the heat transfer, the second term is the entropy generation due to viscous dissipation and the third term is a local entropy generation due to the effect of magnetic field (Joule
The dimensionless entropy generation number may be defined by the following relationship:

$$N_S = \frac{T_\infty^2 r^2 E_G}{k(T_f - T_\infty)^2}. \tag{16}$$

The use of (9), the entropy generation number in dimensionless form is

$$N_S = Re\left[\theta'^2 \quad \frac{Br}{\Omega}\left(f''^2 + M^2 f'^2\right)\right], \tag{17}$$

Where $Re = \frac{Ur}{\nu}$ is the Reynolds number, $Br = \frac{\mu U^2}{k(T_f - T_\infty)}$ is the Brinkmann number and $\Omega = \frac{T_f - T_\infty}{T_\infty}$ the non-dimensional temperature difference.

The entropy generation number N_S can be written as a summation of the entropy generation due to heat transfer denoted by N_S and the entropy generation due to fluid friction with magnetic field denoted by N_S given as

$$N_1 = Re\theta'^2, \qquad N_2 = \frac{Br\,Re}{\Omega}\left(f''^2 + M^2 f'^2\right). \tag{18}$$

In order to obtain an idea of whether entropy generation due to the heat transfer dominates over entropy generation due to the fluid friction and the magnetic field, or vice versa, the Bejan number $\mathrm{B}e$ is defined to be the ratio of entropy generation due to the heat transfer to the entropy generation number [35].

$$\mathrm{B}e = \frac{\text{entropy generation due to heat transfer}}{\text{entropy generation number}} = \frac{N_1}{\mathrm{N}_s} = \frac{1}{1+\Phi}, \tag{19}$$

Where $\Phi = \frac{N_2}{N_1}$ is the irreversibility ratio. Heat transfer dominates for $0 \le \Phi < 1$ and fluid friction with magnetic effects dominates when $\Phi > 1$. The contribution of both heat transfer and fluid friction to entropy generation is equal when $\Phi = 1$. The Bejan number $\mathrm{B}e$ takes the values between 0 and 1 (see Cimpean et al. [36]). The value of $\mathrm{B}e = 1$ is the limit at which the heat transfer irreversibility dominates, $\mathrm{B}e = 0$ is the opposite limit at which the irreversibility is dominated by the combined effects of fluid friction and magnetic field and $\mathrm{B}e = 0.5$ is the case in which the heat transfer and fluid friction with magnetic field entropy production rates are equal. Further, the behavior of the Bejan number $\mathrm{B}e$ is studied for the optimum values of the parameters at which the entropy generation takes its minimum.

The influences of the different governing parameters on entropy generation and Bejan number are presented in Figs. 7-14. It can be seen from Fig.7 that the entropy generation number N_S increases by increasing in magnetic parameter M^2. An increase in the magnetic field intensity causes an increase in the entropy generation. It reveals that the magnetic field is a source of entropy generation in addition to the fluid friction and the heat transfer. Also, it can be seen that the entropy generation effects are prominent on the surface of the stretching sheet and in the region close to it. However, in the free stream region the entropy effects are negligible. This implies that in order to control the entropy which is generated in boundary layer flow, the value of the magnetic parameter should be reduced, which is an issue of interest in nuclear-MHD propulsion. Fig.8 shows that the entropy generation number N_S increases by increasing Reynolds number $\mathrm{R}e$ due to higher heat transfer rates at the sheet surface. An increase in Reynolds number $\mathrm{R}e$, the entropy effects due to heat transfer entropy effects become prominent and fluid friction and magnetic field are lessen near the stretching sheet surface. However, as the distance increases from the sheet surface, these effects are negligible. Fig.9 illustrates the effects of the Biot number $\mathrm{B}i$ on the entropy generation. Increasing $\mathrm{B}i$ enhances the entropy generation. It means that the convective surface boundary condition acts as a strong source of irreversibility. Therefore, the entropy can be minimized by reducing the convection through the boundary. The closeness of the curves in Fig.9 can be attributed that the entropy effects are dominated by the entropy effects due to fluid friction and magnetic field. It can be observed from Fig.10 that entropy generation number increases by increasing the group parameter $\mathrm{B}r\Omega^{-1}$ due to the viscous heating effects. An increase in the values of the group parameter $\mathrm{B}r\Omega^{-1}$ due to the combined effects of viscous heating and temperature difference yields a higher entropy generation number.

Fig.11 shows that the Bejan number $\mathrm{B}e$ increases by increasing the values of magnetic parameter M^2. For large values of M^2, the entropy effects due to fluid friction and magnetic field are fully dominated by heat transfer entropy effects near the sheet surface. Fig.12 illustrates the effects of the radiation parameter R on the Bejan number $\mathrm{B}e$. By increasing values of R, the entropy effects due to heat transfer become strong and hence Bejan number $\mathrm{B}e$ increases. This is due to the fact that the effect of the radiation parameter R is to enhance the temperature significantly in the flow region. The increase in radiation parameter means the generation of heat energy in the flow region. An increase in Biot number $\mathrm{B}i$ leads to increase the Bejan number $\mathrm{B}e$ as shown in Fig.13. Also, an increase in the values of the Biot number results in an increase in the dominant effect of heat transfer irreversibility at the sheet surface. This means that the sheet surface acts as a strong source of irreversibility. Fig.14 reveals that the Bejan number $\mathrm{B}e$ decreases by increasing group parameter $\mathrm{B}r\Omega^{-1}$. This is quite true as higher values of $\mathrm{B}r\Omega^{-1}$, which increase the magnitude of fluid friction with magnetic field irreversibility N_2 but has no effect on the heat transfer irreversibility N_1, increases the values of Φ leading to lower Bejan number. The group parameter is an important dimensionless number for irreversibility analysis. It determines the relative importance of viscous effects to that of temperature gradient entropy generation. The graphs of the Bejan number are useful to obtain an idea on whether heat transfer irreversibility dominates fluid friction irreversibility or vice versa.

5. Conclusion

Entropy generation analysis in hydromagnetic boundary layer flow of a viscous incompressible electrically conducting fluid due to a radially stretching sheet with Newtonian heating in the presence of a transverse magnetic field has been carried out. The velocity and temperature profiles are obtained numerically and used to compute the entropy generation number. The effects of the pertinent parameters on velocity and temperature profiles are presented graphically.

The influences of the same parameters and the dimensionless group parameter on the entropy generation rate and Bejan number are also discussed.

From the results the following conclusions could be drawn:

- The magnetic field retards the fluid velocity while it causes to increase the fluid temperature.
- The fluid temperature decreases by increasing the values of the radiation parameter, leading to a decrease in the thermal boundary layer thickness.
- It is also found that the thermal boundary layer thickness reduces as Prandtl number increases.
- The fluid temperature increases by increasing values of Biot number, leading to an increase in the thermal boundary layer thickness.
- The surface of the sheet acts as strong source of entropy and the heat transfer irreversibility.

Table 1. The shear stress $f''(0)$ and the temperature $\theta(0)$ at the sheet $\eta = 0$.

M^2	R	Pr	Bi	$-f''(0)$	$\theta(0)$
0.5		0.72	0.1	1.57758	0.08345
1		0.72	0.1	1.70391	0.08358
2		0.72	0.1	1.94118	0.08384
1	0.1				0.08018
1	0.5				0.08283
1	1				0.08474
		0.72			0.08358
		2			0.07279
		3			0.06597
			0.1		0.08358
			1		0.47701
			5		0.82016

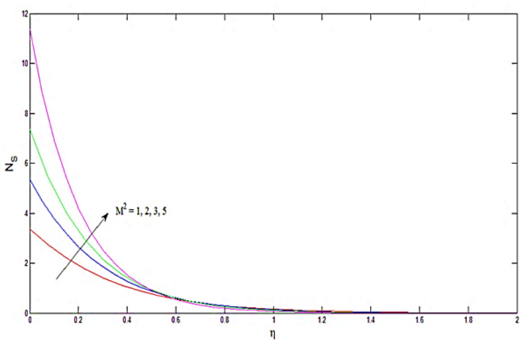

Figure 7. N_S for different M^2 when $Bi = 0.1$, $Re = 1$ and $Br\Omega^{-1} = 1$

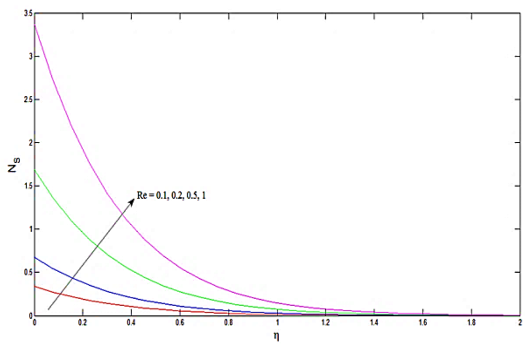

Figure 8. N_S for different Re when $M^2 = 5$, $Bi = 0.1$ and $Br\Omega^{-1} = 1$

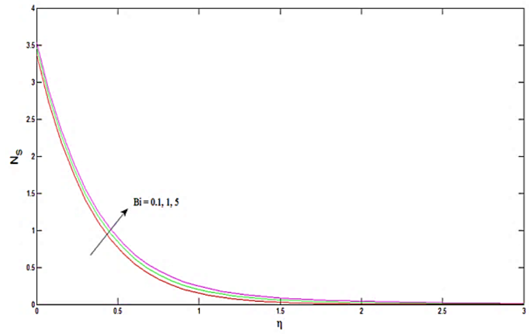

Figure 9. N_S for different Bi when $M^2 = 1$, $Re = 1$ and $Br\Omega^{-1} = 1$

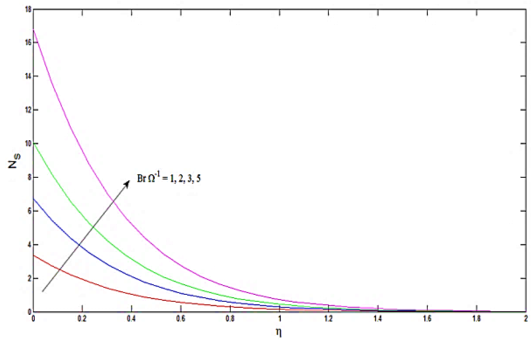

Figure 10. N_S for different $Br\Omega^{-1}$ when $M^2 = 1$, $Bi = 0.1$ and $R = 0.5$

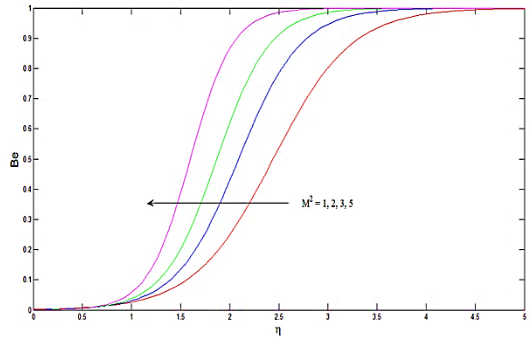

Figure 11. Bejan number Be for different M^2 when $Bi = 0.1$, $R = 0.5$ and $Br\Omega^{-1} = 1$

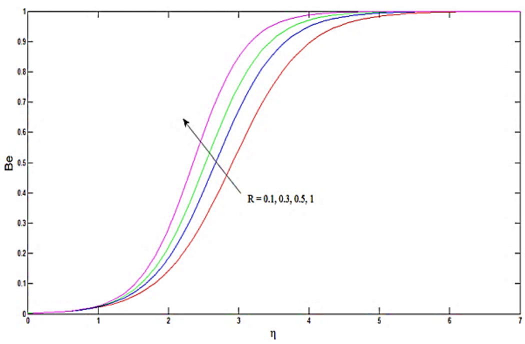

Figure 12. Bejan number Be for different R when $M^2 = 1$, $Bi = 0.1$ and $Br\Omega^{-1} = 1$

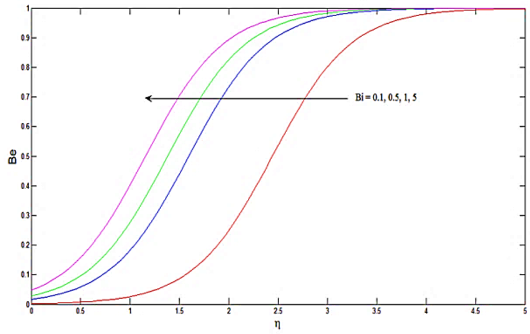

Figure 13. Bejan number Be for different Bi when $M^2 = 1$, $R = 0.5$ and $Br\Omega^{-1} = 1$

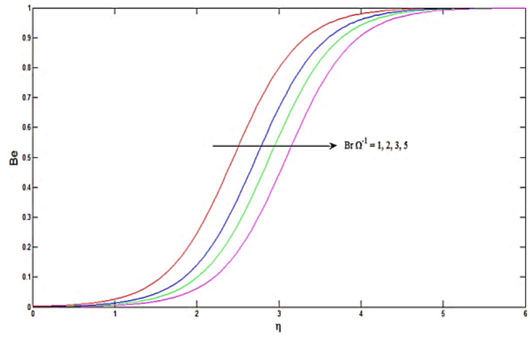

Figure 14. Bejan number Be for different $Br\Omega^{-1}$ when $M^2 = 1$ and $Bi = 0.1$

- The optimum design and efficient performance of a flow system or a thermally designed system can be improved by choosing the appropriate values of the physical parameters. This will be enabled to reduce the effects of entropy generated within the system.

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Nomenclature

a	a constant
B_0	applied magnetic field
Be	Bejan number
Bi	Biot number
Br	Brinkmann number
c_p	specific heat at constant pressure
E_G	volumetric rate of entropy generation
f	non-dimensional steam function
f'	first order derivative with respect to η
f''	second order derivative with respect to η
f'''	third order derivative with respect to η
h_f	heat transfer coefficient
k	thermal conductivity
k^*	Rosseland mean absorption coefficient

M^2	magnetic parameter
N_1	entropy generation due to heat transfer
N_2	entropy generation due to fluid friction and magnetic field
N_S	entropy generation number
Pr	Prandtl number
q_r	radiative heat flux
R	radiation parameter
Re	Reynolds number
T	fluid temperature
T_f	hot fluid temperature
T_∞	free stream temperature
u, w	velocity components in r and z -directions
U	Stretching velocity of the sheet in radial direction
	Greek symbols
η	similarity variable
ν	kinematic viscosity
Φ	irreversibility distribution ratio
ρ	the fluid density
σ^*	Stefan-Boltzman constant
Ω	non-dimensional temperature difference
θ	non-dimensional temperature
θ'	first order derivative with respect to η
θ''	second order derivative with respect to η

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