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Analytical Approach for Thermo-electro-mechanical Vibration of Piezoelectric Nanoplates Resting on Elastic Foundations based on Nonlocal Theory

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ABSTRACT

In the present work, thermo-electro vibration of the piezoelectric nanoplates resting on the elastic foundations using nonlocal elasticity theory are considered. In-plane and transverse displacements of the nanoplate have been approximated by six different modified shear deformation plate theories considering transverse shear deformation effects and rotary inertia. Moreover, two new distributions of transverse shear stress along the thickness of the nanoplate were introduced for the first time. The equations of motion were derived by implementing Hamilton's principle and solved using analytical method for various boundary conditions including SSSS, CSSS, CSCS, CCSS and CCCC. Based on a comparison with the previously published results, the accuracy of the results was confirmed. Finally, the effects of different parameters such as boundary conditions, variations of the thickness to length ratio, aspect ratio, increasing temperature, external voltage, foundation coefficients and length scale on the natural frequency of the plate were shown and discussed in details.

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1. Introduction

Piezoelectric materials such as ZnO, ZnS, PZT, etc. are widely used as actuators and sensors due to their electromechanical coupling effects. In recent years, piezoelectric nanostructures such as nanohelices, nanowires, nanorings, etc., have been in the spotlight thanks for desires in having small systems in comparison with the macro-scaled system and are used as essential components in different industries as nano-generators, chemical sensors, light-emitting diodes, etc. [1-3]. Structures with the small dimensions between a few nanometers and 100 nanometers (nanostructures) do not treat the same as structures with macro scale. In fact, the behavior of the structure is dependent on the length scale and, conventional continuum model cannot be used on such scale. Unlike the classical theories which do not consider size effect, other developed theories such as strain gradient [4], couple stress [5] and nonlocal theory [6] are sensitive to length scale in material behavior. Nonlocal theory of Eringen as one of the most

applied theories could capture the nonlocal effects and is beneficial for nanostructures. In local classical theories, there is a one-to-one relation between stress and strain of each point i.e. the state of stress in every point is a function of the strain at that point; while in nonlocal theory, the stress of a reference point is dependent on strains of the whole domain.

Nanostructures held attentions due to their unique properties. Recently, nanostructures such as nanobeams and nanoplates are widely used in nano-electromechanical (NEM) devices. Dynamic behavior of nanoplates based on classic plate theory (CPT) has been discussed using Eringen model [7-9]. Pradhan and Phadikar [10] utilized the nonlocal theory based on first-order shear deformation theory (FSDT) so as to predict the vibrational behavior of nanoplates. Aghababaei and Reddy [11] employed third-order shear deformation plate theory (TSDT) for bending and free vibration of nanoplates. Khorshidi and Asgari [12] studied the free vibration analysis of functionally graded rectangular nanoplates based on nonlocal

exponential shear deformation theory. Alibeigloo [13] used the three-dimensional theory of elasticity for analyzing the free vibration of nanoplates employing nonlocal continuum mechanics. Extensive studies on piezoelectric structures can be found in open literature [14-16]. Khorshidi and Pagoli [17] presented an analytical solution for vibrating circular plates coupled with piezoelectric layers. Dynamic response of piezoelectric nanobeam according to Timoshenko beam theory and nonlocal theory has been evaluated by Ke and Wang [18]. Hung and Yu illustrated the efficacy of the surface piezoelectricity on the electromechanical response of piezoelectric ring [19]. Jandaghian and Rahmani [20] studied the size-dependent vibration analysis of functionally graded piezoelectric plates subjected to thermo-electro-mechanical loading with Kirchhoff plate theory. Nikoei and Hassani [21] employed an isogeometric analysis approach for free vibration of laminated composite plates integrated with piezoelectric using Reissner-Mindlin theory.

The elastic foundation was used to model a rather soft material in contact with the plate surface. Elastic mediums could be simulated with various models. First of all, Winkler presented the simplest foundation model because He did not consider the interaction between lateral springs. For improving interaction between springs, different foundation models were proposed later such as Pasternak [22], Vlasov [23] and Filonenko-Borodich [24] models. Pasternak is a useful two-parameter foundation model which captures the interaction of adjacent springs by joining ends of springs to a shear layer. Many studies are dedicated to the dynamic behavior of plates resting Pasternak foundation [25, 26]. Moradi et al. [27, 28] analyzed the nanocomposite plates and sandwich plates reinforced by wavy carbon nanotubes resting on elastic foundation. They used a mesh-free method and first-order shear deformation theory in their analysis.

Classical plate theory was proposed by Kirchhoff [29, 30]. He assumed that straight lines normal to the midplane before deformation, remain straight and normal to the midplane after deformation. Since Kirchhoff theory ignores the transverse shear deformation effects, it is not proper for moderately thick plates. Therefore, it is devoted to the thin plates. First-order shear deformation theory [31, 32] incorporates the shear deformation effects with a constant transverse shear deformation distribution along the thickness of the plate. Thus, it violates the stress-free conditions at the bottom and top of the plate and needs a shear correction factor to compensate this error. In order to get more accurate results and avoid using the shear

correction factor, higher-order shear deformation theories (HSDT) have been developed. Reddy [33] employed a parabolic transverse shear stress distribution along the thickness of the plate. His model did not need shear correction factor because of satisfying free stress conditions at the bottom and top of the plate. Various distributions of transverse shear stress through the thickness of the plate can be found in the literature [34-37]. For example, Sayyad and Ghugal [38, 39] presented exponential and trigonometric shear deformation theories for bending and free vibration analysis of moderately thick plates. Different methods such as analytical, numerical and semi-analytical methods have been utilized for solving the equations of motion according to literature. Ke et al. [40] investigated the vibration of the Mindlin piezoelectric nanoplate for different boundary conditions using the differential quadrature method. Khorshidi et al. [41] presented the Navier solution for vibrating piezoelectric nanoplate. They considered only fully simply-supported boundary condition in their analytical solution.

In the present study, the vibration analysis of piezoelectric nanoplates under external voltage and temperature resting on a Pasternak foundation for five different boundary conditions has been illustrated. Dynamic analysis was carried out for six types of different modified shear deformation theory and nonlocal elasticity theory. Governing equations were reduced to a single equation using a simple method and analytical solution for this single equation was obtained for five different boundary conditions. In the result section, efficacy of different variables such as foundation coefficients, nonlocal parameter, aspect ratio, temperature rising and external voltage on fundamental frequency was discussed in detail.

2. Nonlocal elasticity theory for piezoelectric nanoplates

As previously mentioned, the stress at each point in the nonlocal theory is related to the strain field in whole domain of the body. This theory has been developed by Zenkur [42] for capturing thermal effects. Based on this theory, stress and electric displacement at a reference point are expressed as:

$$\sigma_{jk}(x) = \int_V \alpha(|x - x'|, \tau) (C_{ijkl} \varepsilon_{kl}(x') - e_{kij} E_k(x') - \lambda_{ij} \Delta T) dx' \quad (1)$$

$$D_i(x) = \int_V \alpha(|x - x'|, \tau) (e_{ikl} \varepsilon_{kl}(x') - \kappa_{kij} E_k(x') - p_i \Delta T) dx' \quad (2)$$

$$\sigma_{ij,j} = \rho \ddot{u}_i, D_{i,j} = 0, E_i = -\phi_{,i} \quad (3)$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (4)$$

In the above equations, σ_{ij} , D_i , ε_{ij} , u_i and E_k ($i, j, k = 1, 2, 3$) denote the components of stress field, electric displacement, strain, displacement and electric field respectively. C_{ijkl} are elastic constants; e_{ikl} and κ_{kij} are piezoelectric and dielectric constants; λ_{ij} and p_i are thermal moduli and pyroelectric constants respectively. $\alpha(|x - x'|, \tau)$ represents the Kernel function in nonlocal theory such that $\alpha(|x - x'|, \tau)$ is Euclidean distance and $\tau = e_0 a / L$, where e_0 , a and L denote the material constant, internal and external characteristic length respectively. Since the integral form of nonlocal elasticity theory is complicated, Eringen [6] obtained a differential form of nonlocal theory in the following equations.

$$(1 - (e_0 a)^2 \nabla^2) \sigma_{ij} = C_{ijkl} \varepsilon_{ij} - e_{kij} E_k - \lambda_{ij} \Delta T \quad (5)$$

$$(1 - (e_0 a)^2 \nabla^2) D_i = e_{ikl} \varepsilon_{kl} + \kappa_{ik} E_k + \tilde{p}_i \Delta T \quad (6)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ represent the Laplace operator. Under the assumption of plane stress conditions, following constitutive relations would be derived by expanding (5) and (6):

$$\begin{aligned} (1 - (e_0 a)^2 \nabla^2) \sigma_{11} &= \tilde{c}_{11} \varepsilon_{11} + \tilde{c}_{12} \varepsilon_{22} - \tilde{e}_{31} E_3 - \lambda_{11} \Delta T \\ (1 - (e_0 a)^2 \nabla^2) \sigma_{22} &= \tilde{c}_{12} \varepsilon_{11} + \tilde{c}_{11} \varepsilon_{22} - \tilde{e}_{31} E_3 - \lambda_{11} \Delta T \\ (1 - (e_0 a)^2 \nabla^2) \sigma_{13} &= 2\tilde{c}_{44} \varepsilon_{13} - \tilde{e}_{15} E_1 \\ (1 - (e_0 a)^2 \nabla^2) \sigma_{23} &= 2\tilde{c}_{44} \varepsilon_{23} - \tilde{e}_{15} E_2 \\ (1 - (e_0 a)^2 \nabla^2) \sigma_{12} &= 2\tilde{c}_{66} \varepsilon_{12} \end{aligned} \quad (7)$$

$$\begin{aligned} (1 - (e_0 a)^2 \nabla^2) D_1 &= 2\tilde{e}_{15} \varepsilon_{13} + \tilde{\kappa}_{11} E_1 + \tilde{p}_1 \Delta T \\ (1 - (e_0 a)^2 \nabla^2) D_2 &= 2\tilde{e}_{15} \varepsilon_{23} + \tilde{\kappa}_{11} E_2 + \tilde{p}_1 \Delta T \\ (1 - (e_0 a)^2 \nabla^2) D_3 &= \tilde{e}_{31} \varepsilon_{11} + \tilde{e}_{31} \varepsilon_{22} + \tilde{\kappa}_{33} E_3 + \tilde{p}_3 \Delta T \end{aligned} \quad (8)$$

where

$$\begin{aligned} \tilde{c}_{11} &= c_{11} - \frac{c_{13}^2}{c_{33}}, \tilde{c}_{12} = c_{12} - \frac{c_{13}^2}{c_{33}}, \tilde{c}_{66} = c_{66}, \tilde{c}_{44} = c_{44} \\ \tilde{e}_{31} &= e_{31} - \frac{c_{13} e_{33}}{c_{33}}, \tilde{e}_{15} = e_{15}, \tilde{\kappa}_{11} = \kappa_{11}, \\ \tilde{\kappa}_{33} &= \kappa_{33} + \frac{e_{33}^2}{c_{33}} \\ \tilde{\lambda}_{11} &= \lambda_{11} - \frac{c_{13} \lambda_{33}}{c_{33}}, \tilde{\lambda}_{33} = \lambda_{33}, \tilde{p}_1 = p_1, \tilde{p}_3 = p_3 + \frac{e_{33} \lambda_{33}}{c_{33}} \end{aligned} \quad (9)$$

3. Modified shear deformation theory

Generally, the modified shear deformation theories are displacement-based ones which are recognized to be strong methods for upgrading the accuracy of the results [43]. Against the classical plate theory, modified shear deformation theories consider both rotary inertia and shear deformation effects. Fig. 1 not only shows the distribution of transverse shear stress along the thickness of the plate, but also verifies that these theories satisfy the tangential traction free boundary condition. Thus, a shear correction factor is not required in such theories.

According to former studies, modified shear deformation theories have been developed using hyperbolic, exponential, polynomial and trigonometric functions along the thickness of the plate. Based on the modified shear deformation theory, the displacement field could be expressed in the following form:

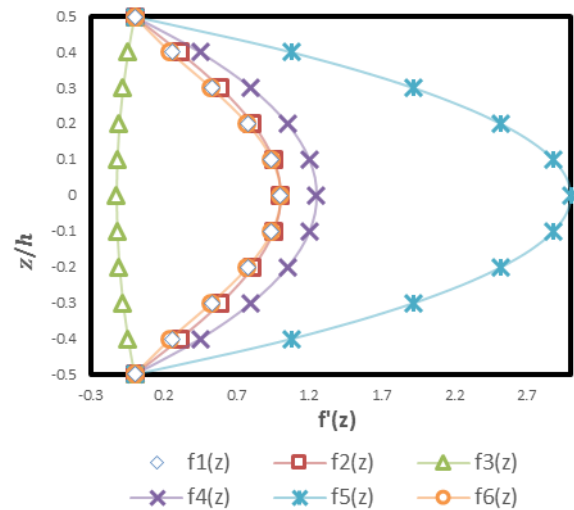


Fig. 1. Distribution of transverse shear stress through thickness of plate

$$\begin{aligned}
u_1(x, y, z, t) &= u(x, y, t) - z \frac{\partial w(x, y, t)}{\partial x} + f_i(z) \xi(x, y, t) \\
u_2(x, y, z, t) &= v(x, y, t) - z \frac{\partial w(x, y, t)}{\partial y} + f_i(z) \psi(x, y, t) \\
u_3(x, y, z, t) &= w(x, y, t)
\end{aligned} \quad (10)$$

where u_1 , u_2 and u_3 represent displacements of an arbitrary point along x , y and z axis, respectively; w is the out-plane displacement of the mid-plane in the nanoplate (on the z -direction); ξ and ψ are rotation functions in xoz - and yoz - plane; u and v denote in-plane displacements of midplane surface of the plate along x -axis and y -axis, respectively. Based on this theory, displacements u_1 and u_2 include two parts; First part is the same as classical plate theory; second part which is considered to capture shear deformations. Depending on choosing $f_i(z)$ ($i=1,6$), these shear deformation theories are varied according to Table 1. In fact, the function $f_i(z)$ specifies the distribution of transverse shear stress along thickness coordinate. The last two functions $f_5(z)$ and $f_6(z)$ in Table 1 are suggested for the first time in this article. It is worth mentioning that the in-plane displacement components could be neglected (u and $v=0$) in the analysis of transverse vibration due to the homogeneity of the structure [12].

4. Free Vibration Analysis of Piezoelectric Nanoplates

Consider a rectangular piezoelectric nanoplate with length L_1 along x - axis ($0 < x < L_1$), width L_2 along y - axis ($0 < y < L_2$) and thickness h in z -direction ($-h/2 < z < h/2$) as shown in Fig. 2. Nanoplate is subjected to external voltage V_0 , temperature rising ΔT and is rested on an elastic foundation. Wang [3] approximated the electric potential as a combination of cosine and linear variation in order to satisfy the Maxwell equation:

$$\tilde{\phi}(x, y, z, t) = -\cos(\gamma z) \phi(x, y, t) + \frac{2V_0}{h} z \quad (11)$$

where $\gamma = \pi/h$ and $\phi(x, y, t)$ is electric potential in midplane of nanoplate. By neglecting the in-plane displacements and substituting displacement field (10) in (4) and electric potential (11) in (3), linear strains and electric field could be found as below:

$$\begin{aligned}
\varepsilon_{11} &= -z \frac{\partial^2 w}{\partial x^2} + f_i(z) \frac{\partial \xi}{\partial x} \\
\varepsilon_{22} &= -z \frac{\partial^2 w}{\partial y^2} + f_i(z) \frac{\partial \psi}{\partial y}
\end{aligned} \quad (12)$$

$$\begin{aligned}
\varepsilon_{12} &= \frac{1}{2} \left(-2z \frac{\partial^2 w}{\partial x \partial y} + f_i(z) \left(\frac{\partial \psi}{\partial x} + \frac{\partial \xi}{\partial y} \right) \right) \\
\varepsilon_{13} &= \frac{1}{2} \xi \frac{\partial f_i(z)}{\partial z}, \quad \varepsilon_{23} = \frac{1}{2} \psi \frac{\partial f_i(z)}{\partial z} \\
E_1 &= \cos(\gamma z) \frac{\partial \phi}{\partial x}, \quad E_2 = \cos(\gamma z) \frac{\partial \phi}{\partial y}, \\
E_3 &= -\gamma \sin(\gamma z) \phi - \frac{2V_0}{h}
\end{aligned} \quad (13)$$

Strain energy (U), kinetic energy (T) and work done by external forces (W_f) in piezoelectric nanoplate could be obtained as follow:

$$\begin{aligned}
U &= \frac{1}{2} \int_A \int_{-h/2}^{h/2} (\sigma_{11} \varepsilon_{11} + \sigma_{22} \varepsilon_{22} + 2\sigma_{12} \varepsilon_{12} + 2\sigma_{13} \varepsilon_{13} + 2\sigma_{23} \varepsilon_{23} - D_1 E_1 - D_2 E_2 - D_3 E_3) dz dA
\end{aligned} \quad (14)$$

$$\begin{aligned}
T &= \frac{1}{2} \rho \int_A \int_{-h/2}^{h/2} \left(\left(\frac{\partial u_1}{\partial t} \right)^2 + \left(\frac{\partial u_2}{\partial t} \right)^2 + \left(\frac{\partial u_3}{\partial t} \right)^2 \right) dz dA
\end{aligned} \quad (15)$$

$$\begin{aligned}
W_f &= \frac{1}{2} \int_A (F_x \frac{\partial^2 w}{\partial x^2} + F_y \frac{\partial^2 w}{\partial y^2} + k_w w - k_p \nabla^2 w) dA \\
F_x &= F_{Px} + F_{Tx} + F_{Ex}, \quad F_y = F_{Py} + F_{Ty} + F_{Ey}
\end{aligned} \quad (16)$$

Table 1. Different shear deformation theories

Theory	$f_i(z)$
$f_1(z)$ exponential [38]	$ze^{-2(\frac{z}{h})^2}$
$f_2(z)$ trigonometric [39]	$\frac{h}{\pi} \sin\left(\frac{\pi z}{h}\right)$
$f_3(z)$ hyperbolic [35]	$h \sinh\left(\frac{z}{h}\right) - z \cosh\left(\frac{1}{2}\right)$
$f_4(z)$ parabolic [36]	$z\left(\frac{5}{4} - \frac{5z^2}{3h^2}\right)$
$f_5(z)$ 1 st suggestion	$z\left(\frac{3}{h} - \frac{4z^2}{h^3}\right)$
$f_6(z)$ 2 nd suggestion	$z\left(\frac{1}{h} - \frac{2z^2}{h^3} + \frac{8z^4}{5h^5}\right)$

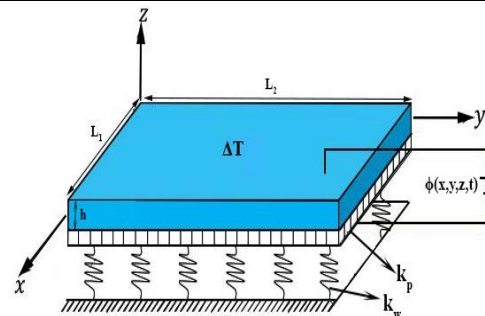


Fig. 2. Geometry of piezoelectric nanoplate subjected to the thermo-electro loads resting on elastic foundation

where A is the mid-plane of nanoplate (at $z=0$); k_s and k_p are spring stiffness and Pasternak's modulus respectively. (F_{Px}, F_{Py}) , (F_{Tx}, F_{Ty}) and (F_{Ex}, F_{Ey}) are normal forces created by axial force P , temperature rising ΔT and external electric voltage V_0 given as:

$$F_{Px} = F_{Py} = P \quad (17)$$

$$F_{Tx} = F_{Ty} = \tilde{\lambda}_{11} h \Delta T \quad (18)$$

$$F_{Ex} = F_{Ey} = -2\tilde{e}_{31} V_0 \quad (19)$$

Hence, Hamilton's principle would be transformed to:

$$\int_0^t (\delta T + \delta W_f - \delta U) dt = 0 \quad (20)$$

By incorporating the Eqs. (14) through (19) into Eq. (20) and integrating by parts, the following equations would be obtained [41]:

$$L_1 \xi + L_2 \psi + L_3 \phi = L_4 w \quad (21)$$

$$L_5 \xi + L_6 \psi + L_7 \phi = L_8 w \quad (22)$$

$$L_9 \xi + L_{10} \psi + L_{11} \phi = L_{12} w \quad (23)$$

$$L_{13} \xi + L_{14} \psi + L_{15} \phi = L_{16} w \quad (24)$$

where the operator L_i is defined by

$$L_1 = L_6 = (A_{17} + A_{18}) \frac{\partial^2}{\partial x \partial y} \quad (25)$$

$$L_3 = L_{10} = (B_5 + B_7) \frac{\partial}{\partial y} \quad (26)$$

$$L_2 = A_{16} \frac{\partial^2}{\partial y^2} + A_{18} \frac{\partial^2}{\partial x^2} - A_{19} - I_6 (1 - (e_0 a)^2 \nabla^2) \frac{\partial^2}{\partial t^2} \quad (27)$$

$$L_4 = L_{14} = (A_{13} + 2A_{15}) \frac{\partial^3}{\partial x^2 \partial y} + A_{11} \frac{\partial^3}{\partial y^3} - I_5 (1 - (e_0 a)^2 \nabla^2) \frac{\partial^3}{\partial t^2 \partial y} \quad (28)$$

$$L_5 = A_{18} \frac{\partial^2}{\partial y^2} + A_{16} \frac{\partial^2}{\partial x^2} - A_{19} - I_6 (1 - (e_0 a)^2 \nabla^2) \frac{\partial^2}{\partial t^2} \quad (29)$$

$$L_7 = L_9 = (B_5 + B_7) \frac{\partial}{\partial x} \quad (30)$$

$$L_{12} = L_{15} = B_3 \nabla^2 \quad (31)$$

$$L_{11} = B_8 \nabla^2 - B_9 \quad (32)$$

$$L_8 = L_{13} = (A_{13} + 2A_{15}) \frac{\partial^3}{\partial y^2 \partial x} + A_{11} \frac{\partial^3}{\partial x^3} - \quad (33)$$

$$I_5 (1 - (e_0 a)^2 \nabla^2) \frac{\partial^3}{\partial t^2 \partial x} \\ L_{16} = A_{10} \frac{\partial^4}{\partial x^4} + (2A_{12} + 4A_{14}) \frac{\partial^4}{\partial y^2 \partial x^2} + \\ A_{10} \frac{\partial^4}{\partial y^4} - k_w + k_p \nabla^2 - (1 - (e_0 a)^2 \nabla^2) (I_3 \frac{\partial^4}{\partial t^2 \partial x^2} + I_3 \frac{\partial^4}{\partial t^2 \partial y^2} - F_x \frac{\partial^2}{\partial x^2} - F_y \frac{\partial^2}{\partial y^2}) \quad (34)$$

Moreover, A_i and B_i are given as:

$$\{A_{10}, A_{12}, A_{14}\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \{\tilde{c}_{11}, \tilde{c}_{12}, \tilde{c}_{66}\} z^2 dz \quad (35)$$

$$\{A_{11}, A_{13}, A_{15}\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \{\tilde{c}_{11}, \tilde{c}_{12}, \tilde{c}_{66}\} z f(z) dz \quad (36)$$

$$\{A_{16}, A_{17}, A_{18}\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \{\tilde{c}_{11}, \tilde{c}_{12}, \tilde{c}_{66}\} f(z)^2 dz \quad (37)$$

$$A_{19} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{c}_{44} \left(\frac{\partial f(z)}{\partial z} \right)^2 dz \quad (38)$$

$$\{I_3, I_5, I_6\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho \{z^2, z f(z), f(z)^2\} dz \quad (39)$$

$$\{B_3, B_5\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \{z, f(z)\} \tilde{e}_{31} \gamma \sin(\gamma z) dz \quad (40)$$

$$B_7 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{e}_{15} \frac{\partial f(z)}{\partial z} \cos(\gamma z) dz \quad (41)$$

$$B_8 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{\kappa}_{11} \cos^2(\gamma z) dz \quad (42)$$

$$B_9 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{\kappa}_{33} (\gamma \sin(\gamma z))^2 dz \quad (43)$$

Then, ϕ , ψ and ξ can be obtained in terms of w according to Cramer method in solving systems of linear equations into Eqs. (21) through (23).

$$\Gamma_1 \xi = \Gamma_2 w, \Gamma_1 \psi = \Gamma_3 w, \Gamma_1 \phi = \Gamma_4 w \quad (44)$$

where the operator Γ_i is defined as:

$$\Gamma_1 = L_{11} L_2 L_5 - L_{10} L_3 L_5 - L_1 L_{11} L_6 + L_1 L_{10} L_7 + L_3 L_6 L_9 - L_2 L_7 L_9 \quad (45)$$

$$\Gamma_2 = L_{12}L_3L_6 - L_{11}L_4L_6 - L_{12}L_2L_7 + L_{10}L_4L_7 + L_{11}L_2L_8 - L_{10}L_3L_8 \quad (46)$$

$$\Gamma_3 = -L_{12}L_3L_5 + L_{11}L_4L_5 + L_1L_{12}L_7 - L_1L_{11}L_8 - L_4L_7L_9 + L_3L_8L_9 \quad (47)$$

$$\Gamma_4 = L_{12}L_2L_5 - L_{10}L_4L_5 - L_1L_{12}L_6 + L_1L_{10}L_8 + L_4L_6L_9 - L_2L_8L_9 \quad (48)$$

Finally, by taking the operator Γ_1 from both sides of Eq. (24) and using Eqs. (44) through (48), transverse displacement equation of piezoelectric nanoplate can be obtained as follows:

$$L_{13}\Gamma_2w + L_{14}\Gamma_3w + L_{15}\Gamma_4w = L_{16}\Gamma_1w \quad (49)$$

5. Solution Procedure

Analytical solution for Eq. (49) can be applied to different boundary conditions according to Table 2. The solution that satisfies appropriate boundary conditions can be expressed in the following form:

$$w(x, y, t) = W_{mn}X_m(x)Y_n(y)e^{i\omega t} \quad (50)$$

where ω is the natural frequency associated with (mth, nth) mode. Eigenfunctions $X_m(x)$ and $Y_n(y)$ which are listed in Table 2 are chosen so that, satisfy at least geometric boundary conditions. By substituting Eq. (50) into Eq. (49) and using orthogonal properties of trigonometric functions, partial differential equation (49) would be converted to an algebraic equation and the solution will be found.

6. Numerical results

In this section, numerical results for vibration analysis of piezoelectric nanoplate subjected to thermo-electro loads resting on the elastic foundation under different boundary conditions including, SSSS, CSSS, CSCS, CCSS, CCCC are illustrated. For all calculations, length of nanoplate (L_1) is considered 50nm and thickness (h) is taken as 5nm, otherwise they are specified. The results obtained from the modified shear deformation theories are so similar to each other and it is not possible to show this slight difference between them through the figures; consequently, it is necessary to choose one theory for interpreting the results.

Accordingly, all figures were plotted based on $f_6(z)$. Material properties of PZT4 used in this section are given as [40]:

$$\begin{aligned} E_0 &= 10^9, E_1 = 10^{-9}, E_2 = 10^5, E_3 = 10^{-4} \\ c_{11} &= 132E_0, c_{12} = 71E_0, c_{13} = 73E_0 \\ c_{33} &= 115E_0, c_{44} = 26E_0, c_{66} = 30.5E_0, \rho = 7500, \\ e_{31} &= -4.1, e_{15} = 10.5, e_{33} = 14.1, h = 5E_1 \\ L_1 &= 50E_1, \kappa_{11} = 5.841E_1, \kappa_{33} = 7.124E_1 \\ \lambda_{11} &= 4.738E_2, \lambda_{33} = 4.529E_2, p_1 = p_3 = 0.25E_3 \end{aligned}$$

Moreover, the following parameters were used for illustrating results:

$$\begin{aligned} \mu &= \frac{e_0 a}{L_1}, \Omega = \omega L_1 \sqrt{\frac{\rho}{c_{11}}}, D = \frac{c_{11} h^3}{12} \\ K_w &= \frac{L_1^4 k_w}{D}, K_p = \frac{L_1^2 k_p}{D}, S = \frac{h}{L_1} \end{aligned}$$

Table 2. Admissible functions for various boundary conditions [44]

Boundary conditions		functions X(x) and Y(y)	
	At $x = 0, x = L_1$	At $y = 0, y = L_2$	
SSSS	$X_m(0) = X''_m(0) = 0$	$Y_n(0) = Y''_n(0) = 0$	$X_m(x)$
	$X_m(L_1) = X''_m(L_1) = 0$	$Y_n(L_2) = Y''_n(L_2) = 0$	$Y_n(y)$
CSSS	$X_m(0) = X'_m(0) = 0$	$Y_n(0) = Y''_n(0) = 0$	$\sin(\frac{m\pi x}{L_1})$
	$X_m(L_1) = X''_m(L_1) = 0$	$Y_n(L_2) = Y''_n(L_2) = 0$	$\sin(\frac{n\pi y}{L_2})$
CSCS	$X_m(0) = X'_m(0) = 0$	$Y_n(0) = Y'_n(0) = 0$	$\sin(\frac{m\pi x}{L_1})\cos(\frac{m\pi x}{L_1} - 1)$
	$X_m(L_1) = X''_m(L_1) = 0$	$Y_n(L_2) = Y''_n(L_2) = 0$	$\sin(\frac{n\pi y}{L_2})\cos(\frac{n\pi y}{L_2} - 1)$
CCSS	$X_m(0) = X'_m(0) = 0$	$Y_n(0) = Y''_n(0) = 0$	$\sin^2(\frac{m\pi x}{L_1})$
	$X_m(L_1) = X'_m(L_1) = 0$	$Y_n(L_2) = Y''_n(L_2) = 0$	$\sin^2(\frac{n\pi y}{L_2})$
CCCC	$X_m(0) = X''_m(0) = 0$	$Y_n(0) = Y''_n(0) = 0$	$\sin^2(\frac{m\pi x}{L_1})$
	$X_m(L_1) = X'_m(L_1) = 0$	$Y_n(L_2) = Y'_n(L_2) = 0$	$\sin^2(\frac{n\pi y}{L_2})$

For verification of this model, results obtained from simply supported square nanoplate have been compared with corresponding ones in the open literature. Table 3 indicates an excellent agreement between the present work and the former studies results. According to this table, there is a bit different among the results obtained from various shear deformation theories. These differences are due to the fact that, function $f(z)$ have different expansions through the thickness in various theories. It is worth to mention that every extra power in the expansion of function $f(z)$ through the thickness of the structure includes additional unknown variables in those theories. Additionally, physical interpretation of these unknown variables are difficult [46]. Thus, it is better to use such distributions that are simpler with acceptable accuracy. Although two new proposed theories are simpler than other modified shear deformation theory, they are nearly identical in accuracy. The expansion of the functions $f(z)$ through the thickness of the structure for various distribution mentioned in Table 1 are given as follow:

$$\begin{aligned} f_1(z) &= z - \frac{2z^3}{h^2} + \frac{2z^5}{h^4} - \frac{4z^7}{3h^6} + \dots \\ f_2(z) &= z - \frac{\pi^2 z^3}{6h^2} + \frac{\pi^4 z^5}{120h^4} - \frac{\pi^6 z^7}{504h^6} + \dots \\ f_3(z) &= \left(1 - \cosh\left(\frac{1}{2}\right)\right)z + \frac{z^3}{6h^2} + \frac{z^5}{120h^4} + \frac{z^7}{5040h^6} + \dots \\ f_4(z) &= \frac{5z}{4} - \frac{5z^3}{3h^2} \\ f_5(z) &= \frac{3z}{h} - \frac{4z^3}{h^3} \\ f_6(z) &= \frac{z}{h} - \frac{2z^3}{h^3} + \frac{8z^5}{5h^5} \end{aligned}$$

Fundamental frequencies of the simply supported piezoelectric nanoplate are presented in Table 4 based on different theories for various values of aspect ratios and thickness ratios. According to this table, the results obtained based on $f_5(z)$ are equal to the results obtained by $f_4(z)$ up to seven decimal places due to the fact that both of these distributions are third-order polynomial along the thickness of the plate. Moreover, it is seen that

the fundamental frequencies of the structure based on various $f_i(z)$ are a little different and can be sorted in the order of $f_6(z) > f_1(z) > f_2(z) > f_3(z) > f_4(z) = f_5(z)$. In fact, by decreasing the thickness of the structure, the difference among these theories declines since shear deformation effects could be neglected in the thin plates. The results in this table have been extracted for the values of $\mu=0.1$ and $\Delta T=V_0=K_w=K_p=0$.

Fig. 3 shows the variation of the first four dimensionless frequencies of simply-supported nanoplate with the nonlocal parameter. This figure depicts that nonlocal parameter is more highlighted in higher modes because wavelength gets smaller by increasing the number of modes. Thus, the nonlocal parameter is more significant for smaller wavelengths. If nonlocal model views as atoms connected to each other by springs, in the case of local elasticity, the stiffness of these springs takes an infinite value. Thus, frequencies and stiffness of structure decreased with the increase in the nonlocal parameter, as Fig. 4 emphasizes on this matter.

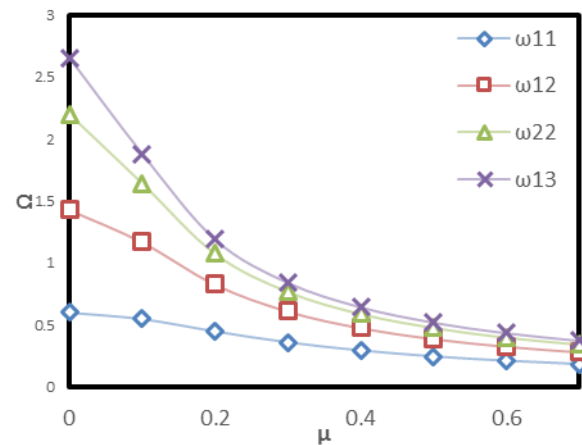


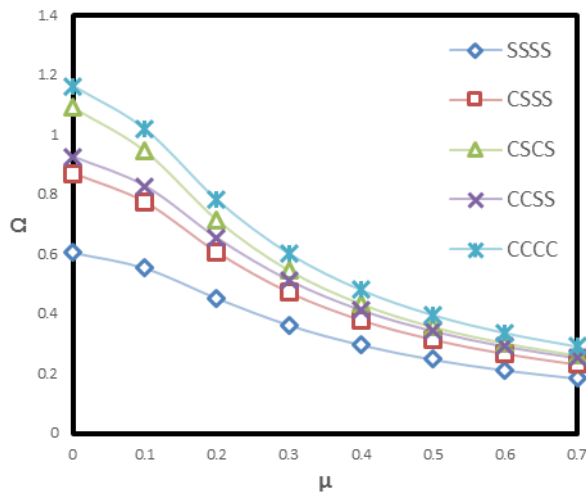
Fig. 3. Variation of first four dimensionless frequencies of SSSS nanoplate with nonlocal parameter

Table 3. Comparison of fundamental dimensionless frequency of SSSS square piezoelectric nanoplate with varying nonlocal parameter

e_0a/L_1	exponential	trigonometric	hyperbolic	parabolic	1 st suggestion	2 nd suggestion	[40]	[45]
0	0.60590	0.60582	0.60580	0.60580	0.60580	0.60593	0.6068	0.6290
0.1	0.55371	0.55364	0.55361	0.55361	0.55361	0.55374	0.5545	0.5748
0.2	0.45292	0.45286	0.45285	0.45285	0.45285	0.45295	0.4536	0.4702
0.3	0.36362	0.36357	0.36356	0.36356	0.36356	0.36364	0.3641	0.3775
0.4	0.29712	0.29709	0.29708	0.29707	0.29707	0.29714	0.2976	0.3085
0.5	0.24871	0.24868	0.24867	0.24867	0.24867	0.24872	0.2491	0.2582

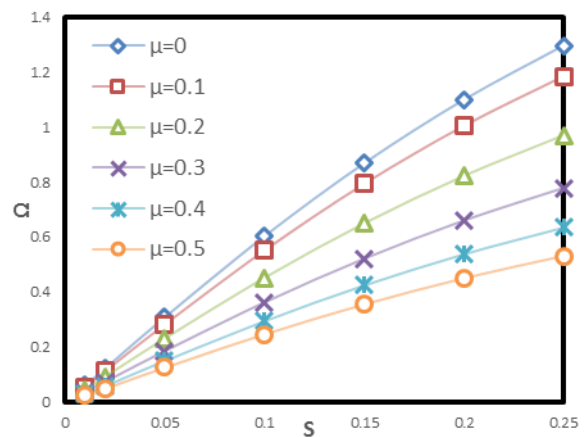
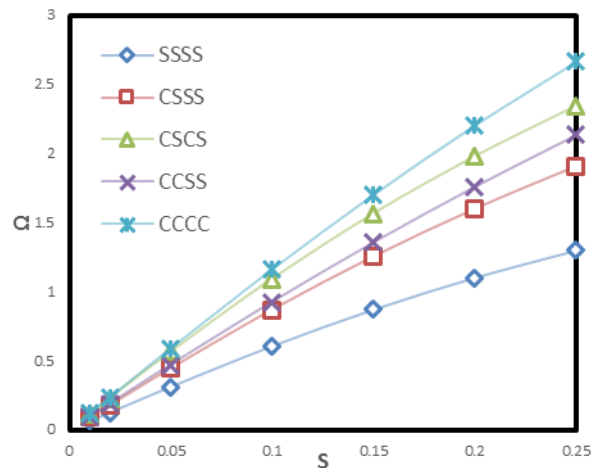
Table 4. Comparison fundamental frequency (GHz) of SSSS piezoelectric nanoplate with various theories

S	$\frac{L_1}{L_2}$	$f_1(z)$	$f_2(z)$	$f_3(z)$	$f_4(z)$	$f_5(z)$	$f_6(z)$
0.01	0.5	0.3993743	0.3993740	0.3993739	0.3993739	0.3993739	0.3993745
	1	0.6188323	0.6188315	0.6188312	0.6188312	0.6188312	0.6188327
	2.5	1.8720008	1.8719913	1.8719888	1.8719887	1.8719887	1.8720048
0.05	0.5	1.9852550	1.9852150	1.9852033	1.9852030	1.9852030	1.9852717
	1	3.0644812	3.0643795	3.0643506	3.0643500	3.0643500	3.0645238
	2.5	9.0365220	9.0354308	9.0351154	9.0351089	9.0351089	9.0369784
0.1	0.5	3.9011997	3.9008939	3.9008005	3.9007993	3.9007993	3.9013267
	1	5.9565367	5.9557790	5.9555485	5.9555453	5.9555453	5.9568515
	2.5	16.471277	16.464190	16.461682	16.461695	16.461695	16.474162
0.2	0.5	7.3288709	7.3267856	7.3260518	7.3260551	7.3260551	7.3297205
	1	10.823112	10.818284	10.816484	10.816503	10.816503	10.825062
	2.5	25.831823	25.796501	25.778487	25.779217	25.779217	25.845245

**Fig. 4.** Variation of dimensionless fundamental frequency with nonlocal parameter for various boundary conditions

Variation of the first four dimensionless frequencies of simply-supported nanoplate with thickness ratio is displayed in Fig. 5. As the thickness of the plate increases, frequency rises because increasing the thickness would improve the strain energy and rigidity of the structure as shown in this figure. Moreover, this figure reveals that nonlocal effect is insignificant for thin plates. As boundary condition gets stronger support, the rigidity of structure rises and frequency of vibration increases. Thus, frequency parameter is lowest in SSSS and highest in CCCC just as Fig. 6 approves this. The effect of aspect ratio on fundamental frequency parameter for SSSS, CSCS and CCCC nanoplates is plotted in Figs. 7-9. For a constant length of nanoplate, the width of nanoplate gets smaller by increasing the aspect ratio. On the other hand, it is obvious that the dynamic behavior of nanoplate is considerably dependent on the dimensions of nanoplate. Hence, as these figures depict, the

nonlocal effect is more notable for higher aspect ratios. This behavior was observed for other two boundary conditions.

**Fig. 5.** Variation of first four dimensionless frequencies of SSSS nanoplate with length to thickness ratio**Fig. 6.** Variation of dimensionless fundamental frequency with thickness ratio for various boundary conditions

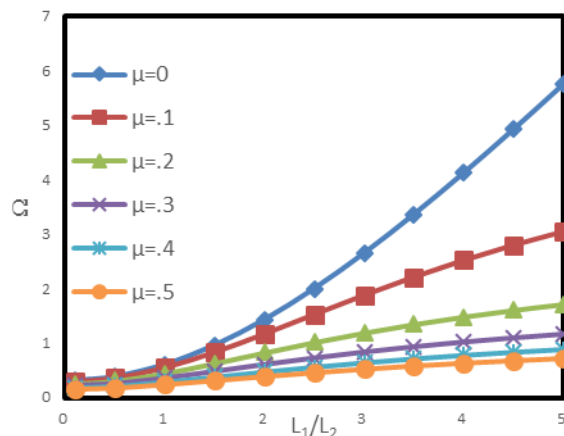


Fig. 7. Variation of fundamental dimensionless frequencies of SSSS nanoplate with aspect ratio

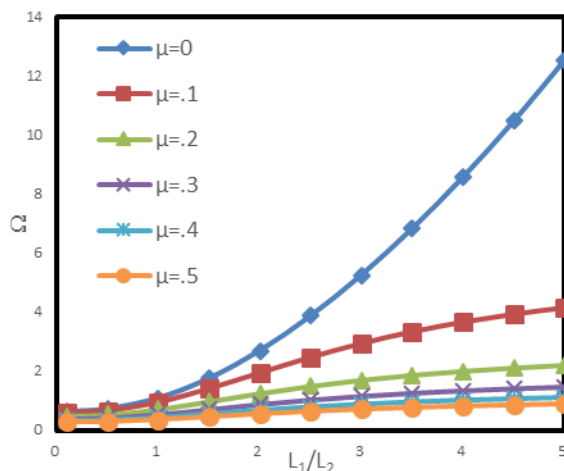


Fig. 8. Variation of fundamental dimensionless frequencies of CSCS nanoplate with aspect ratio

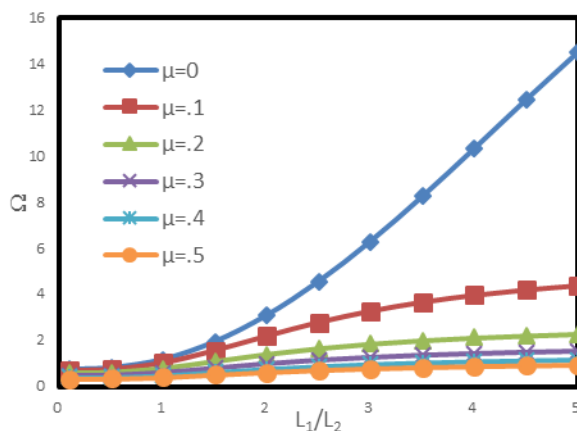


Fig. 9. Variation of fundamental dimensionless frequencies of CCCC nanoplate with aspect ratio

Figs. 10-11 show the effect of external voltage on dimensionless fundamental frequency for a piezoelectric nanoplate. It can be seen from the Fig. 9 that fundamental frequency is quite dependent on

the external voltage. Exerting negative and positive voltage creates the compressive ($P > 0$) and tensile ($P < 0$) forces, respectively. Compressive force decreases or weakens the stiffness and tensile force, increases or improves the stiffness so fundamental frequencies decrease by increasing the external voltage. Additionally, it can be understood from Fig. 10 that decreasing the frequency is more prominent in larger values of the nonlocal parameters.

It is observed from Figs. 11-12 that changing temperature would not have much effect on fundamental frequency for all sets of boundary condition and increasing the temperature, causes a slight reduction in the stiffness and natural frequency of piezoelectric nanoplate. It is seen from Figs. 10-13 that external voltage has the most effect on the fundamental frequency; while changing the temperature has not much influence on fundamental frequency. This is due to the fact that the coefficient of electrical load is much more than the coefficient of thermal load according to the Eqs. (18) and (19), i.e. $2\tilde{e}_{31} \gg \tilde{\lambda}_{11}h$.

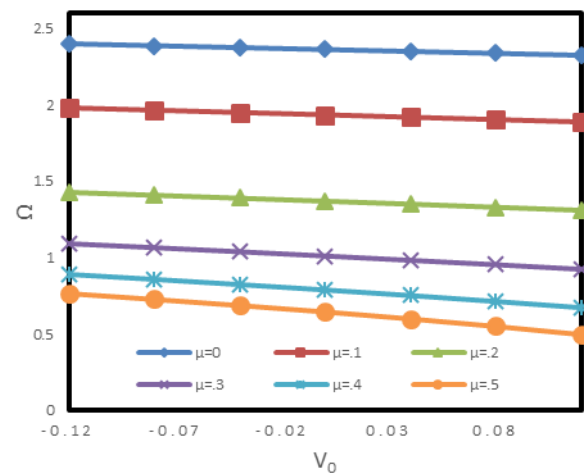


Fig. 10. Variation of fundamental frequency of SSSS nanoplate with external voltage ($L_1=50\text{nm}$, $L_2=25\text{nm}$, $h=10\text{nm}$)

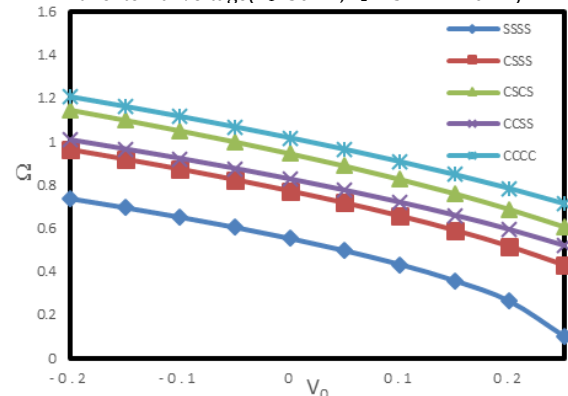


Fig. 11. Variation of dimensionless fundamental frequency of nanoplate with external voltage for various boundary conditions

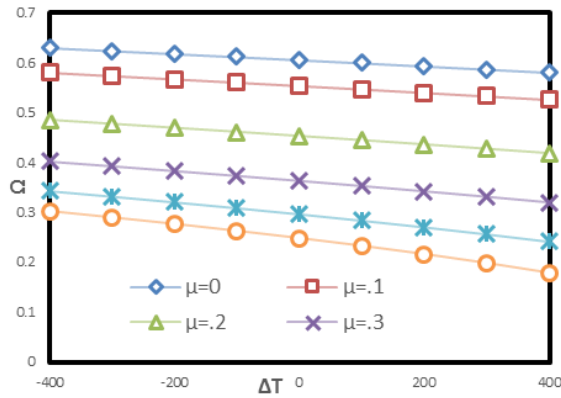


Fig. 12. Variation of dimensionless fundamental frequency of SSSS nanoplate with increasing temperature ($S=0.2$)

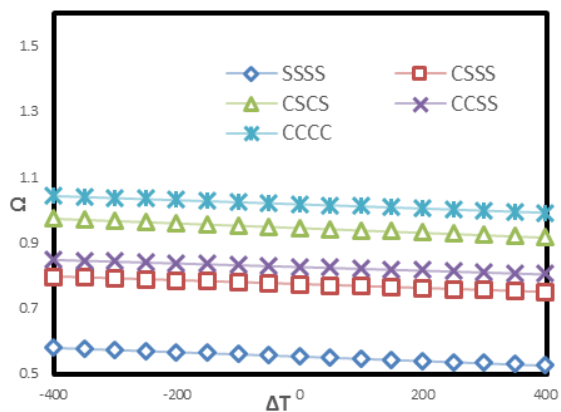


Fig. 13. Variation of dimensionless fundamental frequency of nanoplate with increasing temperature ($S=0.2$, $\mu=0$)

Fig. 14 shows the effects of Winkler and shearing layer coefficient on fundamental frequency. As it could be seen, foundation enhances the frequency of the structure by increasing the strain energy and rigidity of nanoplate. Furthermore, it is observed that fundamental frequency is considerably dependent on shearing layer coefficient than Winkler parameter for all sets boundary conditions. In order to show the effect of foundation parameters on the frequency parameter, the variation of the dimensionless fundamental frequency of the structure versus foundation parameters is plotted in Figs. 15-16. In addition, it is seen that nonlocal effects are more prominent in larger values of foundation parameters.

7. Conclusions

The free vibration analysis of piezoelectric nanoplates subjected to electrical-thermal loads resting on the elastic foundation using nonlocal elasticity theory based on the various modified shear deformation theories was studied. Two new distributions of shear stress along thickness were

introduced for the first time in this article. Governing equations were derived using Hamilton's principle. The system of governing equations was converted to a single partial differential equation using a simple approach which is the equation of transverse vibration of nanoplate. The transverse vibration equation was solved for five different boundary conditions including SSSS, CSSS, CSCS, CCSS, CCCC, and effects of different parameters such as thickness to length ratio, aspect ratio, increasing temperature, external voltage, foundation coefficients and length scale on natural frequencies were illustrated in detail. The numerical results show that:

- Two new modified shear deformation theories have acceptable accuracy like other modified shear deformation theories.

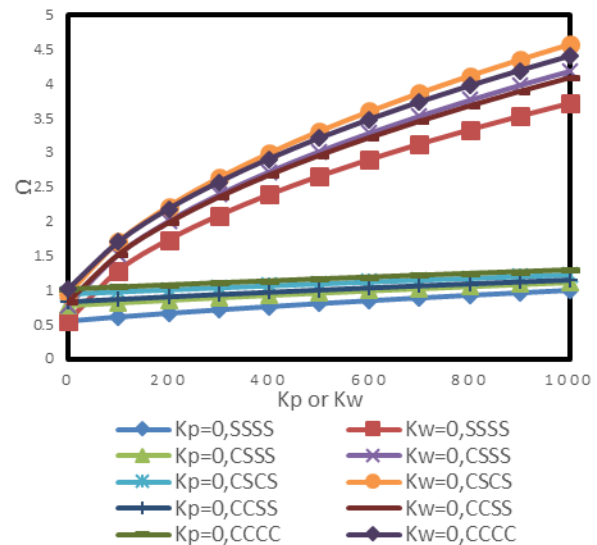


Fig. 14. Variation of dimensionless fundamental frequency of nanoplate with increasing foundation parameter ($S=0.2$, $\mu=0.1$)

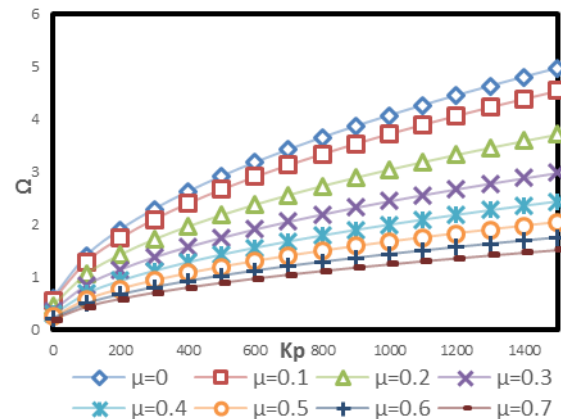


Fig. 15. Variation of dimensionless fundamental frequency of SSSS nanoplate with increasing shearing layer coefficient

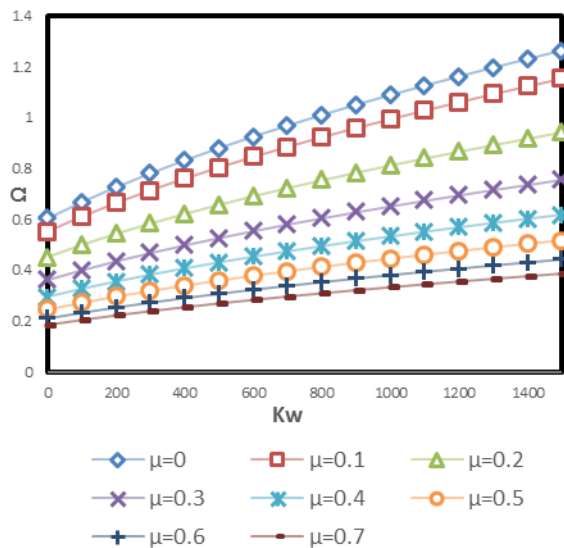


Fig. 16. Variation of dimensionless fundamental frequency of SSSS nanoplate with Winkler coefficient

- Two new proposed theories satisfy free stress conditions at the top and bottom of the plate automatically and they do not need any shear correction factor.
- Nonlocal effects reduced the natural frequencies of structure and this reduction is more prominent in higher modes.
- As aspect ratio increases, natural frequencies increase and nonlocality has significant effects at higher aspect ratio.
- Stiffness of structure and natural frequencies increase as nanoplate gets thicker and nonlocal effects are more noticeable in thicker nanoplates
- As boundary condition get stronger support, rigidity of structure rises and frequency of vibration increases. Thus, frequency parameter is lowest in SSSS and is highest in CCCC.
- Although thermal effects would not have much effects on frequencies of nanoplate, external voltage influences largely on them.
- Foundation enhances the frequency of structure by increasing the strain energy and rigidity of nanoplate. Moreover, frequency is more dependent to shearing layer coefficient than Winkler parameter.
- Nonlocality effects are more prominent in larger values of foundation parameters.

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