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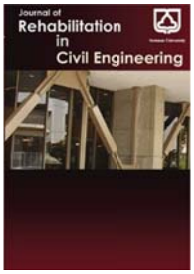

Strut-and-Tie Method for Prediction of Ultimate Shear Capacity of Shear-Strengthened RC deep beams with FRP

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ABSTRACT

The main objective of this study is to propose the Strut-and-Tie method (STM) to predict the shear capacity of simply supported RC deep beams shear-strengthened with carbon fiber reinforced polymers (CFRP). It is assumed that, the total carried shear force by shear-strengthened RC deep beam provided by three independent resistance, namely diagonal concrete strut due to Strut-and-tie mechanism, and the equivalent resisting force resulted by with web reinforcement and FRP layer. The STM approach is regressioned with 104 specimens shear-strengthened with different scheme which are modelled and analyzed through the Non Linear finite elements method and analyzed according under Push over load. For verifying of the accuracy of proposed method, it was used to determine the shear capacity of specimens which have been tested by other researchers. Obtained results were compared with experimental data, that this comparison indicate the proposed method is capable to predict the shear strength of strengthened deep beams with externally bonded (EB) CFRP with acceptable accuracy.

1. Introduction

Reinforced concrete deep beams have useful applications in many structures, such as tall buildings, foundations, offshore structures, and several others. Deep beams are identified as discontinuity regions where the strain distribution is significantly nonlinear and behave differently from shallow beams and generally their ultimate capacity is controlled by shear strength. The conventional design formulas not be useable for this type of RC beams. Some semi rational methods such as Strut-and-tie method have proposed to analysis and design of deep beams. Strut-and-tie modeling is the most rational and simple method for designing nonflexural members currently available. Specific strut-and-tie models need to be developed, whereas shallow beams are characterized by linear strain distribution and most of the applied load is transferred through a fairly uniform diagonal compression field. Design of nonflexural members using strut-and-tie modeling incorporates lower bound theory of plasticity assuming that both the concrete and the steel are perfectly plastic. The behavior and dimensional properties of steel are well known and the strength of members failing in tension can be predicted with some degree of certainty. The foundation of the method was laid by Ritter in 1899. Ritter's original goal was to explain that stirrups in reinforced concrete members provided more than dowel action in resisting shear. Mörsch (1909) expanded on Ritter's model by proposing that the diagonal compressive stresses in the concrete need not be discrete zones, but could be a continuous field. Foster, S.J et al (1998), Hwang et al (2002) and Brown et al (2007) proposed strut-and-tie model based on the softened strut-and-tie model, for determining the shear strength of discontinuity regions failing in diagonal compressions. The strut-and-tie provisions in ACI 318-02 were developed for the design of all forms of discontinuity regions and not specifically deep beams. Thus, it is not surprising that this study reveals that Appendix. A of ACI 318-05 provides conservative and scattered estimates of the strength of deep beams. The proposed compatibility is based on the strut-and-tie method, which considers the effects of compression softening, is shown to provide accurate estimates of the measured load-carrying capacities of reinforced concrete deep beams. Park et al (2007) proposed a different Strut-and -Tie method that the proposed method employs constitutive laws for cracked reinforced concrete, considers strain compatibility. Arabzadeh et al (2009) proposed a new method based on Strut-and-Tie Model (STM) to determine the shear capacity of simply supported RC deep beams and an efficiency factor for concrete with considering the effect of web reinforcements. they assumed that, the total carried shear force by RC deep beam provided by two independent resistance, namely diagonal concrete strut due to strut-and-tie mechanism and the equivalent resisting force resulted by web reinforcements, web reinforcing. Eom et al (2010) developed a secant stiffness method was developed for the inelastic design of strut-and-tie models (STMs). According to the design strategy intended by the engineer, struts and ties are classified as elastic and inelastic elements. An analytical method for predicting the shear strength of deep beams with respect to the force-transferring mechanisms is proposed by Lu et al (2013). The use of fiber-reinforced polymers (FRPs) can now be considered common practice in the field of strengthening and rehabilitation of reinforced concrete structures. The effectiveness of this technique is widely documented by theoretical and experimental researches and by applications on real structures. Numerous studies have been conducted in connection with the beams strengthened with FRP; For example D.I. Kachlakev et al (1999), J. Sim et al (2005) and Tersawy et al (2013) evaluated Effect parameters including strengthening pattern, angle of placement of fibers, the number of FRP layers and layer thickness. Khalifa (1999) and Omar Chaallal et al (2006) investigated combined effect of shear reinforcement and FRP layer. Adhikary et al (2004) and Yungon et al (2011) studied Shear Strengthening RC beams Using CFRP Laminates and Anchors, Maaddavwy et al (2009) reported that structural response of RC deep beams with opening was primary dependent on the degree of the interruption of the natural load path.

The objective of current study is investigation of the ability of the STM to analyze of RC deep beams strengthened in shear with externally bonded (EB) CFRP, research data to predict the shear capacity of RC deep beams with FRP are very limited; Godat (2013) proposed Strut- and- Tie Method for externally FRP Shear Strengthened Large scale RC Beams, In the method, externally bonded CFRP can act as additional tension ties. The tensile forces in the steel stirrups and the CFRP laminates are combined according to a proposed equation. Research data on shear strengthening of deep beams strengthened with FRP are very limited. Therefore, in current design guidelines such as the ACI code, CFRP-strengthened slender members can be analyzed with some accuracy, while FRP-strengthened deep beams are still being analyzed by approximate procedures that have been developed for slender members. At present study, it is beneficial for structural engineer to find a new method that estimate the loading capacity for RC deep beams strengthened with externally bonded FRP. Reinforced concrete beams strengthened in shear by externally bonded FRPs exhibit complex behavior, which makes it difficult to develop a robust predictive model that is appropriate for practical design work. For such beams, all existing models use a third term to account for the contribution of FRP to shear strength, as follows: Vr = Vc + Vs + V_{FRP}. Where, the shear resistant of Vr is the sum of the shear-strength contribution of the concrete (Vc), of the steel stirrups (Vs), and of the FRP (V_{FRP}). This equation is widely used to predict the shear strength of FRP-reinforced concrete beams because of its simplicity. However, the disadvantage of this equation is that the shear strength is based on the sum of the separate shear-strength contributions, with no recognition of existing interactions between the various components.

2. Numerical modeling program

In this paper, STM approach is calibrated with 104 test beams with a small shear span-to-depth ratio ($\frac{a}{d} = 0.89, 1.19, 1.34, 1.45$) representative of deep-beam behavior. Those specimens shear-strengthened with different scheme which are modelled through the finite elements method and analyzed according under statically push over load.

2.1. Finite Element Model

In this study, nonlinear analysis was utilized using the finite element analysis software ABAQUS. The concrete beam was modeled using C3D8R elements. For modeling reinforcing bars in three dimensional concrete elements, reinforcement bars are embedded as truss elements (T3D2). External FRP is modeled using S4R elements with orthotropic behavior.

2.2. Material properties

Reinforced concrete is a complicated material to be modelled. Among three crack models for reinforced concrete elements which ABAQUS software provides: (1) Smeared crack concrete model (2) Brittle crack concrete model, and (3) Concrete damaged plasticity model (CDP), in this paper, for modeling concrete, CPD model is used to model complete inelastic behavior of concrete in both tension and compression including damage characteristics. This model assumes that the main two failure mechanisms if concrete are tensile cracking and compressive crushing. In this model uniaxial tensile and compressive behavior is characterized by damaged plasticity.

For modeling steel reinforcing steel, an elastoplastic model is used to determine the behavior of steel in tension and compression.

Full bond between steel and concrete is assumed.

For modeling the FRP, it is considered as a linear elastic material until failure and the interaction between the concrete and the FRP is modeled without considering debonding. To check debonding, FRP strains were controlled during analysis and the effective strain at failure is evaluated by ACI 440.

2. Strut- and- Tie model basis

Fig. 1 shows a simply supported deep beam subjected to two point top loading. Two effects on the diagonal strut are resulted due to the applied load, V_u applied at a distance of ***a*** from the support. The main longitudinal reinforcement are placed at ***d*** effective depth of d from top face. It is assume that, the critical diagonal crack in a reinforced concrete deep beam occurred as a crack governed by shear rather than by bending.

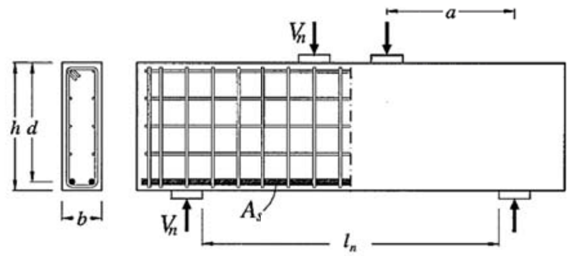


Fig.1. Geometry of concrete deep beam [Gaetano 2005]

The shear strength is predicted by STM due to the diagonal struts and shear force flows along the strut from loaded point to the support. The equilibrium of the applied forces leads to the following expressions (Fig.2).

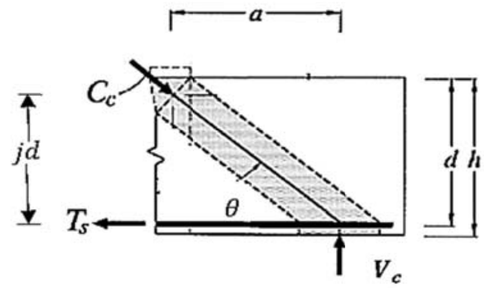


Fig.2. Equilibrium of strut without web reinforcement

$$T_S = C_C \cos\theta \quad (1)$$

$$V_C = C_C \sin\theta \quad (2)$$

Where: C_C is the compression force in the diagonal strut, θ is the angle between strut and longitudinal reinforcement, T_S is the tension force on longitudinal reinforcements (or ties) and V_C is the applied load on top of the deep beam. The inclined angle of the diagonal strut is given by

$$\theta = \tan^{-1}\left(\frac{jd}{a}\right) \quad (3)$$

$$jd = d - \frac{kd}{3} = \left(1 - \frac{k}{3}\right) d \quad (4)$$

Where: a is the shear span measured center-to center from load to support and jd is the distance of lever arm from the resultant compressive force to the center of the main tensile longitudinal reinforcements, kd is the depth of the compression zone that k is derived from the classical bending theory for a single reinforced section as;

$$\mathrm{k} = \sqrt{(\mathrm{n\rho})^2 + (2\mathrm{n\rho})} - \mathrm{n\rho} \quad (5)$$

Where: n is the modular ratio of elasticity, $\mathrm{n} = \frac{\mathrm{E_S}}{\mathrm{E_C}}$; $\mathrm{E_S}$, $\mathrm{E_C}$ are the steel and concrete elasticity module, consequently ρ is the longitudinal reinforcements ratio, $\rho = \frac{A_s}{b\mathrm{d}}$, A_s, is the area of main longitudinal reinforcements and b is the width of beam.

In this study it is assumed that the strut has a prismatic form with a uniform width and compressive stress in the strut can be computed as the force acting on the strut dividing by its cross-sectional area by following expression;

$$C_C = f'_{Ce}.A_{str} \quad (6)$$

$$A_{str} = b.a_s \quad (7)$$

Where: f'_{Ce} is the maximum strength of the softened concrete strut and A_{str} is the cross sectional area of strut, b is the width of beam and a_s is the uniform width of strut which can be estimated as:

$$a_s = min(l_b cos\theta + d_a sin\theta, l_p cos\theta + w_t sin\theta) \quad (8)$$

Where: l_p is depth of the bottom node, taken as twice the cover to the main reinforcements, w_t is width of the support bearing plate, l_b is the depth of the top node and d_a is the width of the loaded point bearing plate.

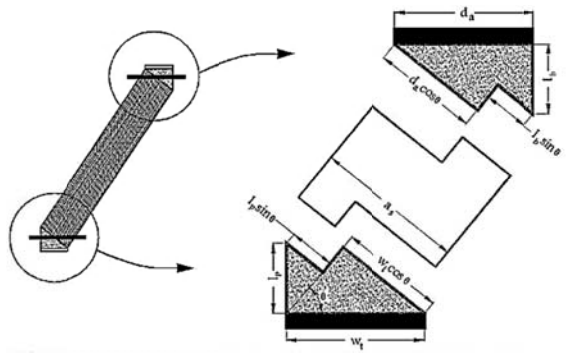


Fig.3. Prismatic strut geometry (Arabzadeh 2009)

Developing the proposed model by Arabzadeh et al' it can be assumed that the shear nominal strength $\mathrm{V_n}$ is resulted due to

$$\mathrm{V_n} = \mathrm{V_c} + \mathrm{V_s} + \mathrm{V_f} \quad (9)$$

Where, $\mathrm{V_c}$ is the shear strength provided by Strut-and-tie mechanism due to the diagonal compression strut, $\mathrm{V_s}$ is the shear strength resulted by web reinforcement, and $\mathrm{V_f}$ is the shear resistant given by FRP.

4. Web reinforcement and FRP Layer mechanism

According to the result of past experimental researches on deep beams, it is prove that, the effective resistance of web reinforcements and FRP layer are always after extensive cracking of web. With the onset and growth of cracks longitudinal and transverse, At first FRP layers act and by sustain additional strain of concrete prevent of opening crack and then web reinforcements act . If the splitting crack is assumed to open without shear slip along the crack, the force in the reinforcement bars and FRP Layer crossing the cracks must be calculated in uniaxial state. Otherwise, it can be assume that the shear slip the state of stress will be biaxial due to tension stress and shear stress applied in the reinforcement and FRP Layer. In current study, the shear slip along the splitting crack was not considered. In STM it is assumed that the diagonal cracks occur in fully state and there is no force due to aggregate interlocking.

It is consistent with the practices of Strut-and- tie modeling to assume no shear stress within strut. This model is a method of describing force in fully cracked RC structures undergoing plastic deformation.

A typical arrangement of web reinforcement is shown in Fig.4, The maximum equivalent resisting force perpendicular to splitting crack by web reinforcement $F_{PR(max)}$ can be written as:

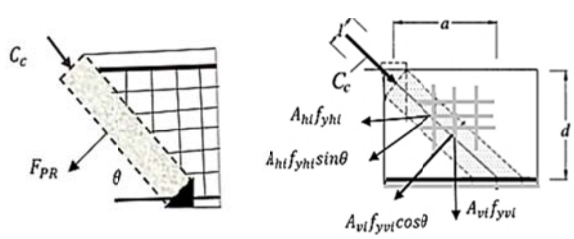


Fig.4. Resisting force perpendicular splitting crack

$$F_{PR(max)} = A_v f_{yv} cos\theta + A_h f_{yh} sin\theta \quad (10)$$

$$F_{PR(max)} = \rho_v \cdot (bL_S cos\theta) f_{yv} cos\theta + \rho_h \cdot (bL_s sin\theta) f_{yh} sin\theta \quad (11)$$

$$F_{PR(max)} = \rho_v f_{yv} bL_S cos^2\theta + \rho_h f_{yh} bL_s sin^2\theta \quad (12)$$

$$\rho_v = \frac{A_v}{ba}, \rho_h = \frac{A_h}{bh} \quad (13)$$

f_{yv}, f_{yh} are the tensile yield stress in the vertical and horizontal bars, respectively, A_h and A_v are the total area of horizontal and vertical web reinforcements crossing the crack, respectively and L_S is the length of strut.

$f_{yh} = f_{yv} = f_y$ in Eq. 12, it becomes

$$F_{PR(max)} = \rho_v f_y bL_S cos^2\theta + \rho_h f_y bL_s sin^2\theta \quad (14)$$

The new term ρ_{PR} is defined as "equivalent reinforcement ratio perpendicular to splitting crack" and can be computed as

$$\rho_{PR} = \rho_v cos^2\theta + \rho_h sin^2\theta \quad (15)$$

ρ_{PR}, is an equivalent ratio that be calculated in tangential-section area of strut.$\left(A_{str(t)} = bL_s\right)$, where $A_{str(t)}$ is the area of tangential-section of strut. A typical pattern of FRP layer is shown in Fig.5, The maximum equivalent resisting force perpendicular to splitting crack by FRP layer $F_{PF(max)}$ can be written as

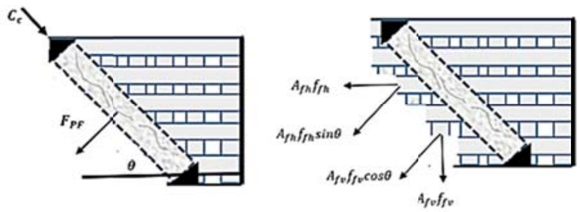


Fig.5. Resisting force perpendicular splitting crack by FRP layer

$F_{PF(max)} = A_{fv}f_{fv}.cos\theta + A_{fh}f_{fh}.sin\theta$ (16)

$F_{PF(max)} = \rho_v(bL_s cos\theta)f_{fv}.cos\theta + \rho_h(bL_s sin\theta)f_{fh}.sin\theta$ (17)

By assumption $f_{fh} = f_{fv} = f_f$ in Eq. 17, it becomes

$F_{DF} = bl_s f_f(\rho_v cos^2\theta + \rho_h sin^2\theta) = bl_s f_f \rho_{DF}$ (18)

f_{fv}, f_{fh} are the tensile yield stress in the vertical and horizontal FRP layer, respectively, A_{fh} and A_{fv} are the total area of horizontal and vertical FRP layer crossing the crack, respectively and L_S is the length of strut. The new term ρ_{DF} is defined as "equivalent FRP ratio perpendicular to splitting crack" and can be computed as

$\rho_{DF} = \rho_v cos^2\theta + \rho_h sin^2\theta$ (19)

5. Equilibrium of applied forces on strut

In Fig.6 equilibrium of the resulted internal forces along the concrete strut is shown. Three independent forces, namely C_c, F_{PR} and F_{PF} mobilized are expressed as a ratio of their respective capacity provided by the concrete section along the diagonal strut Act, the web reinforcement and the area of FRP layers. The total shear strength of beam is given by vertical equilibrium of C_c, F_{PR} and F_{PF}

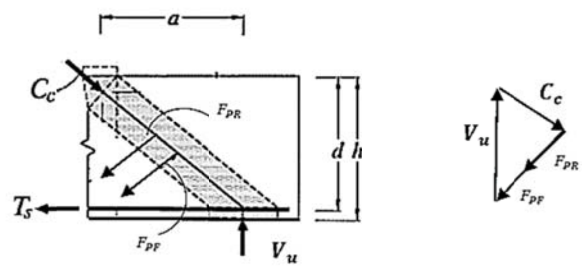


Fig.6. Equilibrium of strut reinforced by web reinforcement

$V_u = C_C sin\theta + F_{PR}cos\theta + F_{PF}cos\theta = V_c + V_s + V_f = C_C sin\theta + \beta\rho_{DR}f_y bl_s cos\theta + \alpha\rho_{DF}f_f bl_s cos\theta$ (20)

Where: $C_C sin\theta$ or V_c is the shear strength provided by the STM due to the diagonal concrete compressive strut, $F_{PR}cos\theta$ or V_s is the shear strength resulted by resisting mechanism of web reinforcements against concrete splitting, $F_{PF}cos\theta$ or V_f is the shear strength resulted by resisting FRP. Reinforcing bars or FRP sheet that located in the central region of strut have higher strain in comparison with the reinforcements or FRP sheet near the supports or loaded point. Therefore the term f_y is substituted by mean stress in web bars equals to βf_y and term f_f is substituted by mean stress in FRP sheet equals to αf_f .According to modeling results β and α is coefficient that depends on the equivalent perpendicular ratio and must be less than 1.0

Substituting Eq.19 equal to Eq. 2 gives

$$V_u = \upsilon_u f_c' A_{str} sin\theta \qquad (21)$$

Where:

$$\upsilon_u = \upsilon_C + \upsilon_R + \upsilon_F = \upsilon_C + \beta \rho_{DR} \frac{a}{a_s} . \frac{f_y}{f_c'} . \frac{1}{sin\theta} + \alpha \rho_{DF} \frac{a}{a_s} . \frac{f_f}{f_c'} . \frac{1}{sin\theta}$$

(22)

According to Eq.22, it is proved that after concrete diagonal cracking the reduction effect of concrete softening is reduced and the efficiency factor of concrete in the presence of web reinforcing and FRP layer can be substituted by υ_u , the difference between υ_u and υ_C just equals the $\upsilon_R + \upsilon_F$ provided by the web reinforcements and FRP layer. Therefore it can be assumed that the shear strength of deep beam is governed only by the diagonal compression strut, but to determine the strut force in Eq. 2 , the efficiency factor must be computed with considering the improvement effect of web reinforcements and FRP layer by Eq.22.

6. Solution procedure

The first term of Eq.22 presents the efficiency factor in the absence of web reinforcement and FRP. Cracked concrete subjected to high tensile strain in the direction normal to the compression is observed to be softer than concrete in a standard cylinder test. This phenomenon of strength and stiffness reduction is commonly referred to as compression softening. Applying this softening effect to the STM, it is recognized that the tensile straining perpendicular to the strut will reduce the capacity of the concrete strut to resist compressive stresses. The efficiency factor of concrete strength has offered numerous relationships; previous studies proved that, with increasing shear span-to-depth ratio measured efficiency factor for the strut concrete decreases and by increasing concrete strength, concrete becomes brittle and the efficiency factor of strut decreases. The numerical formulations developed in this paper relied on work previously carried out by Arabzadeh et al. Eq. 22 is a function of the two unknown parameters β and α which will be determined on the basis of obtained results from modeling and analysis of 104 deep beams shear-strengthened by CFRP under Push over statically load. According to the results of performed analysis all of specimens failed in shear or shear-flexural mode. Concrete compressive strength, Different scheme of FRP, various values of a/d and area of web reinforcing were assumed for modeling of deep beams.

The unknown factors are determined using regression and minimizing the residual errors finally as:

$$\beta = 0.357\rho_{DR}^{-0.45}$$

$$\alpha = \frac{0.085}{\zeta}\rho_{DF}^{-0.5}$$

Therefore, Eq.22 becomes

$$\upsilon_u = \frac{{f_c'}^{-0.3}}{0.7+0.15\left(\frac{a}{d}\right)^2} + 0.357{\rho_{DR}}^{0.55}\frac{a}{a_s}.\frac{f_y}{f_c'}.\frac{1}{sin\theta} + \frac{0.085}{\xi}{\rho_{DF}}^{0.5}\frac{a}{a_s}.\frac{f_f}{f_c'}.\frac{1}{sin\theta} \quad (23)$$

Where: in Eq.23, $\boldsymbol{\rho_{DR}}$ and $\boldsymbol{\rho_{DF}}$ are expressed as a percentage of equivalent reinforcement and FRP ratio perpendicular to splitting crack respectively. $\boldsymbol{\xi}$ is Proportional to the angle of the FRP layers and can be computed as

a. If Fibers placed horizontally or vertically, $\xi = 1$
b. If Fiber placed perpendicular to diagonal crack, $\xi = cos^2\theta$
c. If the Fiber is angle other than perpendicular to diagonal crack, equivalent horizontal and vertical of layers is calculated then $\xi = 1$

Substituting Eq.22 in Eq.21 gives;

$$V_u = \frac{{f_c'}^{0.7}}{0.7+0.15\left(\frac{a}{d}\right)^2}A_{str}.sin\theta + 0.357{\rho_{DR}}^{-0.45}A_{WP}f_y cos\theta + \frac{0.085}{\xi}{\rho_{DF}}^{-0.5}A_{WF}f_f cos\theta \quad (24)$$

Where: A_{WP} and A_{WF} are the equivalent area of perpendicular web reinforcements and FRP crossing strut respectively and can be computed as;

$$A_{WP} = A_V cos\theta + A_h sin\theta \quad (25)$$

$$A_{WF} = A_{fV} cos\theta + A_{fh} sin\theta \quad (26)$$

Where: A_V and A_h are the areas of vertical and horizontal reinforcement crossing strut respectively. A_{fV} And A_{fh} are the areas of vertical and horizontal FRP layer crossing strut respectively. ρ_{DR} and ρ_{DF} are expressed as a percentage of equivalent reinforcement and FRP ratio.

7. STM Expression

The proposed method has determined based on the behavior of CFRP shear-strengthened deep beams and result of shear-strengthened deep beams using the finite element method. To regression proposed method, 104 beams with different scheme of FRP which are modelled and analyzed through the Non Linear finite elements method and analyzed according under Push over load.

A summary of obtained results from nonlinear FEM (V_u) and the predictions for shear capacity (V_{STM}) by the proposed model are shown in the Table 1 and Fig 7, 8.

8. Evaluation of proposed model reliability

To evaluate the accuracy and reliability of proposed method, the shear capacity experimental specimens which have been tested by other researcher have been compared with result by Strut-and-Tie method.

The details of the experimental specimens and results of the comparison shown in Table 2

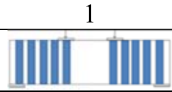
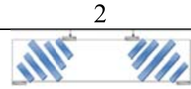
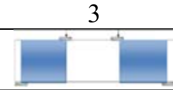
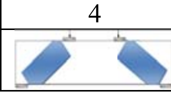
Correlation between the experimental results and predicted strength by proposed model is plotted for all series in Figs.9

Table1. A summary of these beams that have modeled in, and result of the application of proposed model for the beams

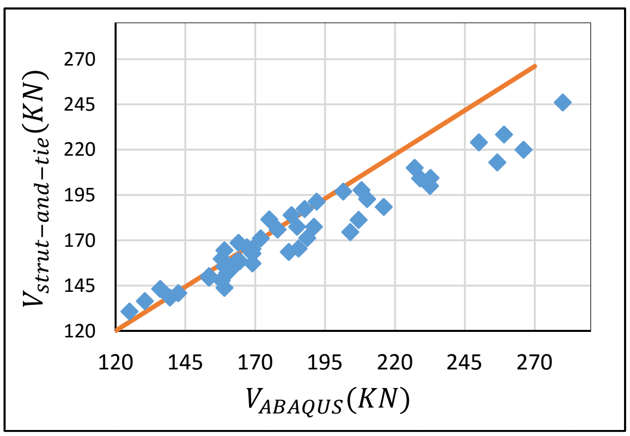
Beam	$\frac{a}{d}$	$\rho\%$	$\rho_{DR}\%$	$\rho_{DF}\%$	V_u	V_{STM}	$\frac{V_{STM}}{V_u}$	Beam	$\frac{a}{d}$	$\rho\%$	$\rho_{DR}\%$	$\rho_{DF}\%$	V_u	V_{STM}	$\frac{V_{STM}}{V_u}$
C50	0.89	2.5	0.182	-	179	172	0.96	C25	0.89	2.5	0.182	-	115	111	0.97
C50-1-1	0.89	2.5	0.182	0.208	183	183.7	1	C25-1-1	0.89	2.5	0.182	0.21	125	123	0.98
C50-2-1	0.89	2.5	0.182	0.4	208	198	0.95	C25-2-1	0.89	2.5	0.182	0.4	130	132	1.01
C50-3-1	0.89	2.5	0.182	0.75	201	197	0.98	C25-3-1	0.89	2.5	0.182	0.75	137	136	0.99
C50-4-1	0.89	2.5	0.182	0.66	250	217	0.87	C25-4-1	0.89	2.5	0.182	0.66	145	154.5	1.07
C50-1-2	0.89	2.5	0.182	0.347	187	187.2	1	C25-1-2	0.89	2.5	0.182	0.35	128	126	0.98
C50-2-2	0.89	2.5	0.182	0.667	232	205	0.88	C25-2-2	0.89	2.5	0.182	0.67	147	138.5	0.94
C50-3-2	0.89	2.5	0.182	1.25	229	205	0.89	C25-3-2	0.89	2.5	0.182	1.25	158	143	0.91
C50-4-2	0.89	2.5	0.182	1.1	259	230	0.89	C25-4-2	0.89	2.5	0.182	1.1	165	167	1.01
C50-1-3	0.89	2.5	0.182	0.55	192	191.5	1	C25-1-3	0.89	2.5	0.182	0.55	134	130	0.97
C50-2-3	0.89	2.5	0.182	1.06	250	213	0.85	C25-2-3	0.89	2.5	0.182	1.06	155	146	0.94
C50-3-3	0.89	2.5	0.182	2	276	216	0.78	C25-3-3	0.89	2.5	0.182	2	175	151.5	0.87
C50-4-3	0.89	2.5	0.182	1.76	299	246	0.82	C25-4-3	0.89	2.5	0.182	1.76	186	173.2	0.93
C50	1.19	1.85	0.228	-	135	133.4	0.99	C25	1.19	1.85	0.228	-	95	91	0.96
C50-1-1	1.19	1.85	0.228	0.296	149	148.7	1	C25-1-1	1.19	1.85	0.228	0.29	110	109.5	1
C50-2-1	1.19	1.85	0.228	0.276	169	162.7	0.96	C25-2-1	1.19	1.85	0.228	0.28	116	120.2	1.03
C50-3-1	1.19	1.85	0.228	0.75	169	165.3	0.98	C25-3-1	1.19	1.85	0.228	0.75	124	123	0.99
C50-4-1	1.19	1.85	0.228	0.59	178	176	0.99	C25-4-1	1.19	1.85	0.228	0.59	126	133.5	1.06
C50-1-2	1.19	1.85	0.228	0.49	153	157.2	1.02	C25-1-2	1.19	1.85	0.228	0.49	111	114.7	1.03
C50-2-2	1.19	1.85	0.228	0.48	188	172	0.9	C25-2-2	1.19	1.85	0.228	0.48	124	128.7	1.03
C50-3-2	1.19	1.85	0.228	1.25	204	175	0.86	C25-3-2	1.19	1.85	0.228	1.25	133	132	0.99
C50-4-2	1.19	1.85	0.228	0.98	216	188.5	0.87	C25-4-2	1.19	1.85	0.228	0.98	138	146	1.06
C50-1-3	1.19	1.85	0.228	0.79	153	163	1.06	C25-1-3	1.19	1.85	0.228	0.79	115	123.7	1.08
C50-2-3	1.19	1.85	0.228	0.765	207	181.3	0.88	C25-2-3	1.19	1.85	0.228	0.76	129	141	1.09
C50-3-3	1.19	1.85	0.228	2	232	194.5	0.84	C 25-3-3	1.19	1.85	0.228	2	146	145.5	1
C50-4-3	1.19	1.85	0.228	1.56	227	204	0.9	C 25-4-3	1.19	1.85	0.228	1.56	158	163.2	1.03
C50	1.34	1.46	0.245	-	121	119.5	0.99	C25	1.34	1.46	0.245	-	84	83.4	1
C50-1-1	1.34	1.46	0.245	0.23	138.	139.5	1	C25-1-1	1.34	1.46	0.245	0.23	96	98.5	1.02
C50-2-1	1.34	1.46	0.245	0.26	158	148.2	0.94	C25-2-1	1.34	1.46	0.245	0.26	107	112.2	1.04
C50-3-1	1.34	1.46	0.245	0.737	162	155.5	0.96	C25-3-1	1.34	1.46	0.245	0.74	117	119	1.01
C50-4-1	1.34	1.46	0.245	0.58	191	164.5	0.86	C25-4-1	1.34	1.46	0.245	0.58	119	128.5	1.08
C50-1-2	1.34	1.46	0.245	0.39	142	134.5	0.95	C25-1-2	1.34	1.46	0.245	0.39	101	103	1.02
C50-2-2	1.34	1.46	0.245	0.43	164	158.5	0.96	C25-2-2	1.34	1.46	0.245	0.43	114	122	1.07
C50-3-2	1.34	1.46	0.245	1.22	185	165.5	0.89	C25-3-2	1.34	1.46	0.245	1.22	127	129	1.02

C50-4-2	1.34	1.46	0.245	0.97	185	177.5	0.96	C25-4-2	1.34	1.46	0.245	0.97	128	141	1.1
C50-1-3	1.34	1.46	0.245	0.69	147	136.6	0.93	C25-1-3	1.34	1.46	0.245	0.69	106	108	1.02
C50-2-3	1.34	1.46	0.245	0.69	167	166.2	1	C25-2-3	1.34	1.46	0.245	0.69	110	130	1.1
C50-3-3	1.34	1.46	0.245	1.96	191	177.4	0.93	C25-3-3	1.34	1.46	0.245	1.96	134	141.5	1.05
C50-4-3	1.34	1.46	0.245	1.55	210	193	0.92	C25-4-3	1.34	1.46	0.245	1.55	145	156	1.08
C50	1. 45	1.46	0.257	-	117	111	0.95	C25	1.45	1.46	0.257	-	88	83.5	0.95
C50-1-1	1.45	1.46	0.257	0.225	132	131	0.99	C25-1-1	1.45	1.46	0.257	0.22	98	99	1.01
C50-2-1	1.45	1.46	0.257	0.247	142	142	1	C25-2-1	1.45	1.46	0.257	0.25	102	113	1.1
C50-3-1	1.45	1.46	0.257	0.696	158	148	0.94	C25-3-1	1.45	1.46	0.257	0.69	113	120	1.06
C50-4-1	1.45	1.46	0.257	0.59	159	156	0.98	C25-4-1	1.45	1.46	0.257	0.59	116	129	1.11
C50-1-2	1.45	1.46	0.257	0.376	140	136.5	0.98	C25-1-2	1.45	1.46	0.257	0.38	104	104	1
C50-2-2	1.45	1.46	0.257	0.417	153	149.5	0.98	C25-2-2	1.45	1.46	0.257	0.42	105	110	1.04
C50-3-2	1.45	1.46	0.257	1.16	163	158.5	0.97	C25-3-2	1.45	1.46	0.257	1.16	121	130	1.07
C50-4-2	1.45	1.46	0.257	0.98	164	168.6	1.03	C25-4-2	1.45	1.46	0.257	0.98	123	139	1.13
C50-1-3	1.45	1.46	0.257	0.6	145	143.1	0.99	C25-1-3	1.45	1.46	0.257	0.6	104	109	1.04
C50-2-3	1.45	1.46	0.257	0.66	158	159.8	1.01	C25-2-3	1.45	1.46	0.257	0.66	108	117	1.02
C50-3-3	1.45	1.46	0.257	1.85	172	171.1	0.99	C25-3-3	1.45	1.46	0.257	1.85	131	143	1.09
C50-4-3	1.45	1.46	0.257	1.46	175	181.5	1.04	C25-4-3	1.45	1.46	0.257	1.46	138	153	1.1

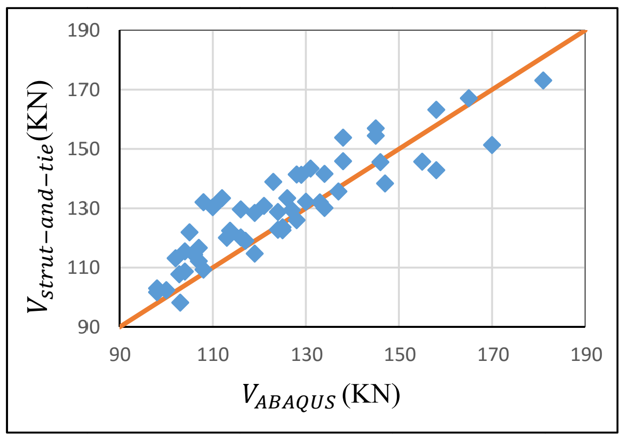
C50-1-1:
First number express compressive strength (50Mpa or 25Mpa)
Second number express FRP scheme;

Second number	1	2	3	4
FRP scheme				

Third number express number layer FRP

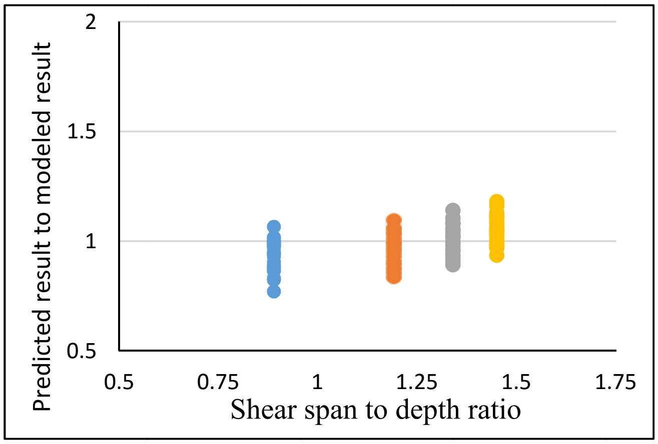


C50

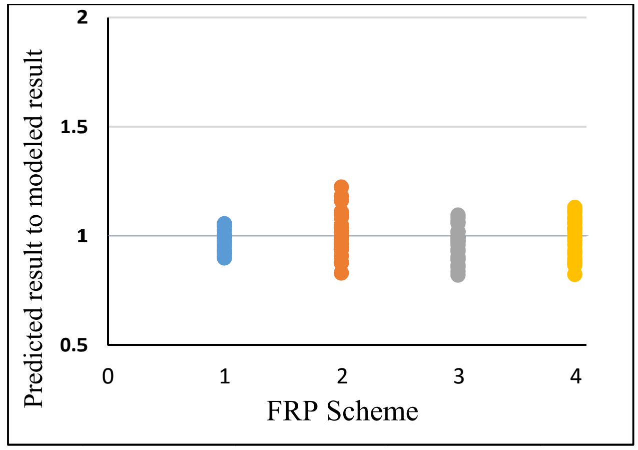


C25

Fig.7. Correlation between the modeling results and predicted strength by selected model



Effect of shear span-to-depth ratio

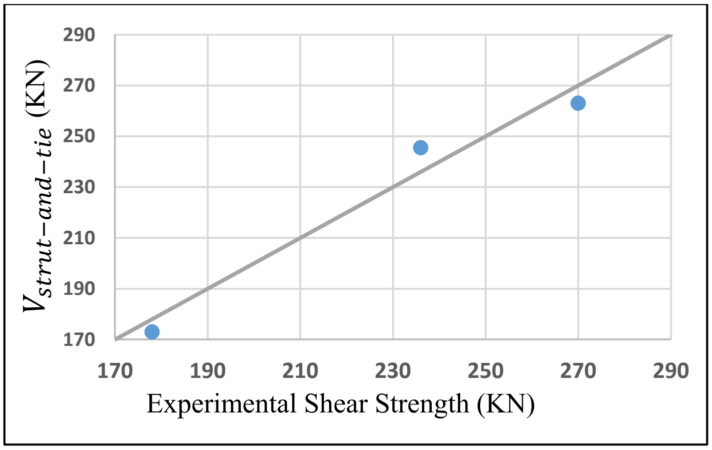


Effect of FRP scheme

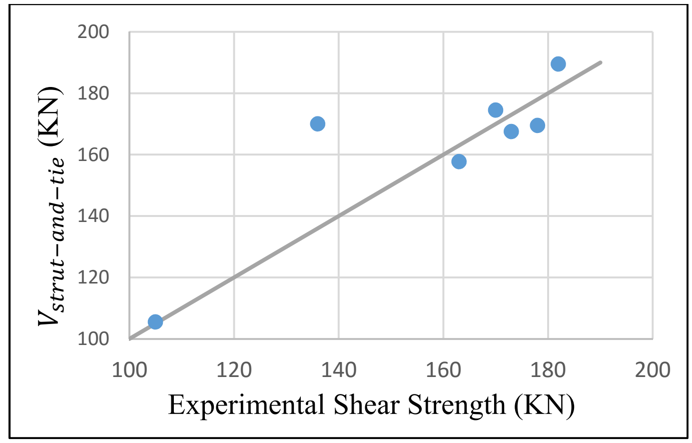
Fig.8. Effect of shear span-to-depth ratio and FRP scheme on the ratio of predicted shear strength by selected model to modeled shear strength

Table 2. Summary results of the application of the proposed model for the analysis of experimental specimens

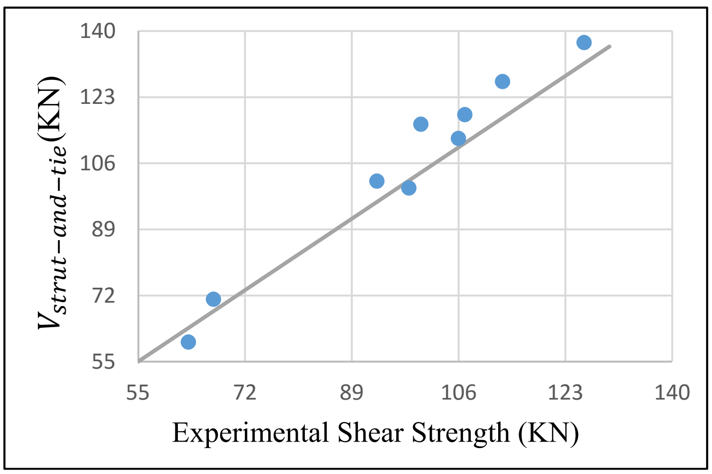
Refrence	f'_c	$\frac{a}{h}$	**Beam**	V_C	V_S	V_f	$V_u = V_C + V_S + V_f$	V_{exp}	$\frac{V_u}{V_{exp}}$
Arabzadeh, A et al (2008)	55	1.18	Without FRP	127.12	45.5	-	173	178	0.97
	55	1.18	Sp45/135	127.12	45.5	90.3	263	270	0.97
	55	1.18	W90/0	127.12	45.5	73	245.5	236	1.04
Sim, J et al (2005)	24	1.7	Without FRP	105.5	-	-	105.5	105	1
	24	1.7	CP90S	105.5	-	52	157.7	163	0.966
	24	1.7	CP90II	105.5	-	62	167.5	173	0.968
	24	1.7	CP45S	105.5	-	64	169.5	178	0.95
	24	1.7	CS90II	105.5	-	69	174.5	170	1.02
	24	1.7	CS45II	105.5	-	84	189.5	182	1.04
	24	1.7	CS90U	105.5	-	69	174.5	133	1.3
Tersawy. Et al (2013)	21	1.75	Without FRP	26	33.5	-	60	63	0.95
	21	1.75	B3V1L-21	26	33.5	39.6	99.6	98.1	1.01
	21	1.75	B4V2L-21	26	33.5	56	116	100	1.16
	21	1.75	B5INC1L-21	26	33.5	41.38	101.38	93	1.09
	21	1.75	B6INC2L-21	26	33.5	58.5	118.5	107	1.1
	35	1.75	Without FRP	37	33.5	-	71	67	1.06
	35	1.75	B7INC1L-35	37	33.5	41.4	112.38	106	1.06
	35	1.75	B8INC2L-35	37	33.5	58.5	127	113	1.128
	35	1.75	B9INC3L-35	37	33.5	71.6	137	124	1.1
Shin, S et al (1999)	52	1.5	MB 1/5-0	61	-	-	61	64	0.95
	52	1.5	MB 1/5-25	61	34.5	-	95.6	95.5	1
	52	1.5	MB 1/5-50	61	50.1	-	111.1	111	1
	52	1.5	MB 1/5-75	61	62.6	-	123.6	119	1.04
	52	1.5	MB 1/5-100	61	73	-	134	127	1.05



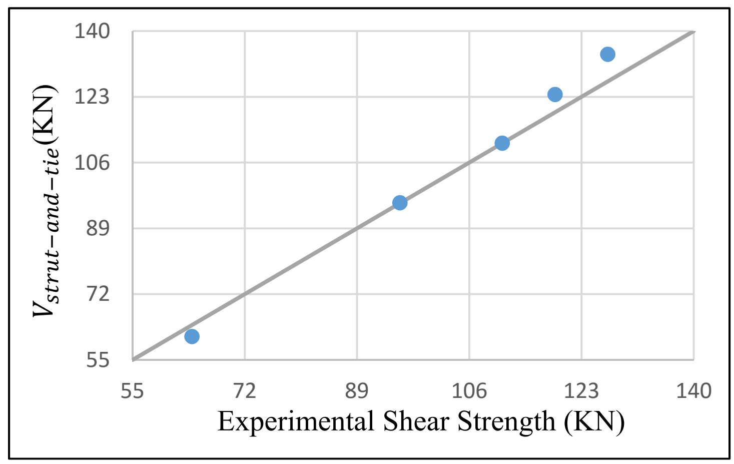
Arabzade, A



Sim, J



AL-Terzawy



Shin,S

Fig.9. Correlation between the experimental results and predicted strength by proposed model

According to Table 2 and Figs.9, it can be concluded that the proposed model is reliable and accurate. Table 3 summarizes the statistical results obtained from comparison:

Table 3. Summarizes statistical analysis of predicted shear strength-to- experimental ratio for proposed method

Refrence	STD	VAR	Mean	COR
Arabzade, A	0.0387	0.0015	0.995	0.98
Sim, J	0.125	0.0156	1.04	0.76
AL. Tersawy	0.062	0.00388	1.07	0.98
Shin, S	0.04	0.0016	1	0.99

STD: Standard diviation
VAR: The value of variation
Mean: The mean of $\left(\frac{V_{STM}}{V_u}\right)$ for specimens
COR: Correlation between experimental and predicted results

9. Final design expression

Based on the above findings (Table 2 and 3) it is obvious that in the proposed model, as the mean of the predicted shear strength-to-experimental ratio is equal to 1.0, it cannot be applied for the design and therefore must be modified. For this purpose, this formula is a modified form of Eq.24 by multiplying the coefficient 0.95, hence the new predicted design expression becomes;

$$V_u = \frac{{f'_c}^{0.7}}{0.73+0.158\left(\frac{a}{d}\right)^2} A_{str}.\sin\theta + 0.34{\rho_{DR}}^{-0.45} A_{WP} f_y \cos\theta + \frac{0.08}{\xi} {\rho_{DF}}^{-0.5} A_{WF} f_f \cos\theta \quad (27)$$

10. Proposed solution procedure

The algorithm starts with a selection of the vertical beam shear **V** and can be divided into 6 major steps as follows:

1. Determining the geometric properties of an alternative truss $(\theta, \frac{a}{d}, a_s, l_s, A_{str})$

2. The calculation of the strength provided by the concrete without considering reinforcement and FRP $\left(V_C = \frac{{f'_c}^{0.7}}{0.7+0.15\left(\frac{a}{d}\right)^2} A_{str}.\sin\theta\right)$

3.Determining the shear capacity provided by shear reinforcement and layers FRP ($V_s + V_f = V - V_c$). Determination of shear reinforcement arrangement and pattern installed FRP and also to determine the estimated value ρ_{DR} and ρ_{DF}.

4. Calculate the shear capacity of deep beams (V_u) by Eq.27

5. If V_u determined in step 4 is less than the required Shear capacity (V), iteration continues from step 3 by increasing the value of V_S, V_f

6. If $T_S > \frac{V_U}{\tan\theta}$ the failure of beam is governed by compression stress of concrete and diagonal cracking, otherwise, failure moment is dominant.

11. Conclusions

The Strut-and-tie method was implemented to predict the load capacity of CFRP shear-strengthened RC deep beam. The STM was calibrated by 104 beams with different scheme of FRP modelled through the Non Linear finite elements method and analyzed according under Push over load. After comparison with 24 specimens available in the literature, the following conclusions can be drawn,

1. The study demonstrated the ability of the STM to predict the capacity of FRP shear-strengthened deep beams.

2.The calculated capacities by the proposed method are both accurate and conservative with little scatter or trends for deep beams over a wide range in concrete strengths, values of a/d that ranged from 0.7 to 2, various combinations and amounts of web reinforcements, various amount and FRP scheme. The predictions by the proposed method are sufficiently conservative and accurate to conclude that it provides a safe and reliable means of calculating the capacity of deep beams.

3. consistent model to predict the shear capacity of CFRP shear-strengthened RC deep beams is obtained by superposing three independent factor in the shear resisting of the deep beams, namely diagonal concrete strut action due to strut-and-tie mechanism (STM), resisting equivalent force perpendicular to diagonal cracks resulted by web reinforcements and resisting equivalent force perpendicular to diagonal cracks resulted by CFRP sheet.

4. The carried shear strength by CFRP sheets was estimated based on their average stress. The average stress depends on the amount and pattern of FRP.

5. The FRP shear strengthening system was found more effective when the fibers were oriented in a direction perpendicular to the potential diagonal shear cracks.

6. According to the proposed model and experimental observations due to decreasing the inclination angle of the strut or increasing the span-to-depth ratio of a deep beam, the efficiency of horizontal web reinforcements and horizontal layer FRP are reduced because in this case diagonal cracks concrete to be horizontal position and in this case vertical reinforcement and vertical layer of FRP are placed perpendicular to crack and are more effective.

7. The design approach is based on the model proposed, the area of main reinforcement can be selected so that there are the possibility of simultaneous failure of flexural and shear beam reinforced.

8. The proposed STM analysis approach has very conservative to predict the experimental results.

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Notations

The following symbols are used in this paper:

$\mathbf{a}$ = the shear span, **mm**

$\mathbf{d}$= the width of the loaded point bearing plate, **mm**

$\mathbf{b}$ = the width of beam, **mm**

$\mathbf{L_S}$ = the length of strut **mm**

$\boldsymbol{C_C}$ = the compression force in the diagonal strut, **N**

$\boldsymbol{\theta}$ = the angle between strut and longitudinal reinforcements

$\boldsymbol{T_S}$ = the tension force on longitudinal reinforcements, **N**

$\mathbf{A_{str}}$ = the cross sectional area of strut, $\mathbf{mm^2}$

$\boldsymbol{a_s}$ = the uniform width of strut which can be estimated, **mm**

$\boldsymbol{lb}$ = depth of the top node, **mm**

$\boldsymbol{lp}$ = depth of the bottom node, **mm**

$\boldsymbol{wt}$ = width of the support bearing plate, **mm**

$\boldsymbol{F_{PR(max)}}$=the maximum equivalent resisting force perpendicular to splitting crack by web reinforcement, **N**

$\boldsymbol{f_{yv}}$, $\boldsymbol{f_{yh}}$ = the tensile yield stress in the vertical and horizontal bars, **MPa**

$\mathbf{A_h}$, $\mathbf{A_v}$ = the total area of horizontal and vertical web reinforcements crossing the crack, $\mathbf{mm^2}$

$\boldsymbol{\rho_h}, \boldsymbol{\rho_V}$ = horizontal and vertical web reinforcement ratio

$\boldsymbol{\rho_{PR}}$ =equivalent reinforcement ratio perpendicular to splitting crack

$\mathbf{F_{PF(max)}}$ =the maximum equivalent resisting force perpendicular to splitting crack by FRP layer, **N**

$\mathbf{f_{fv}}$, $\mathbf{f_{fh}}$ = the tensile yield stress in the vertical and horizontal FRP layer, **MPa**

$\mathbf{A_{fh}}$, $\mathbf{A_{fv}}$ = the total area of horizontal and vertical FRP layer crossing the crack, **mm**

$\boldsymbol{\rho_{hf}}, \boldsymbol{\rho_{Vf}}$ = horizontal and vertical FRP layer ratio

$\boldsymbol{\rho_{DF}}$ = equivalent FRP ratio perpendicular to splitting crack

$\mathbf{V_c}$ = the shear strength provided by the STM due to the diagonal concrete compression strut, **N**

$\mathbf{V_s}$ = the shear strength resulted by resisting mechanism of web reinforcements, **N**

$\mathbf{V_f}$ = the shear strength resulted by resisting FRP, **N**