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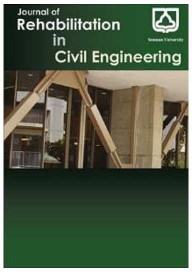

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An Effective Approach for Damage Identification in Beam-Like Structures Based on Modal Flexibility Curvature and Particle Swarm Optimization

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ABSTRACT

In this paper, a computationally simple approach for damage localization and quantification in beam-like structures is proposed. This method is in consonance with applying modal flexibility curvature (MFC) and particle swarm optimization (PSO) algorithm. Analytical studies in the literature have revealed that changes in the modal flexibility curvature can be considered as a sensitive and suitable criterion for identifying damage in the beam-like structures. Modal flexibility curvature can be calculated utilizing central difference approximation, based on entries of the modal flexibility matrix. The PSO algorithm, as a powerful optimization tool, is employed in order to minimize the error function which is formulated as an error function between the measured modal flexibility curvatures of the damaged structure and those calculated from the analytical structure. To demonstrate the efficiency of the method, two beam-like structures under different damage scenarios are studied. In addition, the robustness of presented method is investigated only when the first several modal data are available. It is observed that the proposed approach is able to localize and quantify various damage cases only by a few lower vibrational modes and also, it is low-sensitive to measurement noise.

1. Introduction

In recent years, the problem of health monitoring in beam-like structures has become an important research issue in the different areas of mechanical, aerospace and civil engineering [1-8, 11, 15, 18]. Different approaches for damage localization and

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quantification has been developed which are confirming to the static and/or dynamic responses of the beam-like structural system. Abdo [1] conducted a research for localizing of single and multiple damage patterns in beam-like structures employing displacement curvatures derived from static responses. Despite the simplicity of static loading-response-based damage identification approaches, they have drawbacks, such as susceptibility to the loading cases, and also presenting less information about the damage characteristics compared with other damage identification methods which are based on dynamic and vibration theory. Therefore, vibration-based techniques are deliberated as more promising tools for damage identification in beam-like structures. Such methods have received a considerable attention in the past decades. Pandey et al. [2] explored the curvature of mode shape vectors to reveal damage in beam-like structures. According to the obtained results from this method, if the absolute difference between the mode shape curvatures of the damaged and undamaged beams are plotted, not only does a peak value appear at the damaged element(s), but also some small peaks at different undamaged locations for the higher modes are observed. To overcome this drawback, Wahab and De Roeck [3] proposed a damage indicator, called curvature damage factor (CDF), to apply only the first few mode shape curvatures for damage prognosis. Lin and Cheng [4] used a frequency change index and mode shape curvature to detect crack in beams. Lu et al. [5] presented a two-step approach pursuant to mode shape curvature and response sensitivity analysis for both single and multiple crack identification in different types of beam-like structures. The impacts of the statistical errors on damage detection by means of the structural flexibility and mode shape curvature in beam-like structures are evaluated by Tomaszewska [6]. Lu et al. [7] developed a procedure to identify multiple damage patterns with modal flexibility curvature and relative frequency change in the beam-like structures. Their studies demonstrated that changes in the modal flexibility curvature can be considered as a sensitive and suitable criterion for damage identification in the beams. Hence, seeking the way to increase the capabilities of the damage identification approach based on modal flexibility curvature in the beam-like structures is a promising topic.

Recently, some researchers have explored damage identifications of the different kinds of structures applying soft computing techniques such as artificial neural networks [8-10], conventional optimization algorithms [11] and evolutionary optimization algorithms [12-20]. The utilization of an artificial neural network needs data-training phase requiring large amount of data while it can be contemplated as a drawback of these methods [14]. Unlike the traditional optimization algorithms, the evolutionary algorithms are not sensitive to the initial guess of the solution and only use objective function’s value (instead of its derivatives) for seeking the solution domain for global minimization. Another important feature of the evolutionary methods is seeking the solution domain by means of multiple point routes rather than only a single point route. Therefore, model updating-based approaches via evolutionary optimization algorithms can be efficiently used for finite element model updating and damage detection by following a minimization strategy through defining an objective function to minimize the errors between the experimental data and those related to the analytical model. Sahin et al.

[8] presented a damage detection method using artificial neural networks in consonance with changes in natural frequencies and curvature mode shapes for localization and quantification of damage in beam-like structures. Lee [11] identified the crack locations and quantities in beams using the Newton–Raphson method and the singular value decomposition strategy. A new damage model with a single transverse edge crack, in arbitrary position of beam element, is developed by Mehrjoo et al. [15]. They evaluated parametric model of the cracked Euler–Bernoulli beam element within an optimization procedure to identify the depth and location of the crack(s) in beams.

The aim of this paper is to develop an effective vibration-based method for damage localization and quantification in beam-like structures by means of the modal data. The proposed approach utilizes modal flexibility curvature indicator (MFCI) and particle swarm optimization (PSO) algorithm to perform the finite element model updating using the data acquired from a damaged beam-like structure. Confirming to the existing rich literature review, changes in the MFC can be considered as a sensitive criterion for damage identification in the beam-like structures due to this fact that damage can result in changes in the MFC [7]. The PSO algorithm is a global search strategy that was firstly introduced by Kennedy and Eberhart [21] which simulates social behavior of animals such as bird flocking, fish schooling and insect swarming in nature. Fast convergence and suitable performance are the major advantages of the PSO algorithm in comparison to the other optimization algorithms [22]. Hence, the algorithm is employed to minimize the error between the measured modal flexibility curvatures (from the damaged structure) and those calculated from the baseline structure through the optimization process.

The rest of the paper is organized as follows. In Section 2, a concise description of the mathematical model of proposed approach containing modeling of damage and modal flexibility curvature indicator is presented. A brief summary of the PSO algorithm is provided in Section 3. Section 4 presents the numerical studies to illustrate the efficiency and robustness of the proposed approach utilizing both exact and noisy modal parameters. The paper ends with conclusion remarks presented in Section 5.

2. Mathematical Model of the Proposed Approach

2.1. Damage Modeling

In general, when damage occurs in a structure, the physical properties of structure will change. In this paper, it is assumed that damage changes the stiffness of the structure. Therefore, this change can be simulated by decreasing one of the parameters that have contribution in the stiffness of the beam-like elements, such as the modulus of elasticity (E), cross sectional area (A), moment of inertia (I) and so on. Moreover, it is assumed that no change would occur before and after damage in the mass matrix, which is acceptable in most real applications. The stiffness changes are small and would not cause a change in the connectivity of the structure. The proposed approach would be feasible to a lightly damped structure where changes in stiffness would not affect significantly the damping property of the structure. These assumptions are common in linear damage simulation and have been intensively employed in the related literature [17-20]. In the present study, damage is simulated by a relative reduction of the elasticity modulus of the damaged element as:

$$E_i^d = (1 - x_i) E_i \tag{1}$$

where, E_i and E_i^d are the modulus of elasticity for the ith element in the undamaged and damaged cases, respectively, and $x_i \in [0,1]$ is a damage index to represent the damage severity of the ith element. If the damage index is zero for an element, it will be concluded that the element is healthy. However, damage index of 1 will be returned if an element is completely damaged.

2.2. Modal Flexibility Curvature Indicator

Selection of the suitable objective function is a key step in model updating-based structural damage identification procedure, since it can influence the performance of the proposed approach in terms of converging to false-positive/negative results. In general, the objective function should reflect the amount of error between the behavior of the analytical and monitored models. To form a suitable objective function, damage-sensitive parameters should be selected. Among vibrational characteristics, the natural frequencies as well as the mode shape vectors and their derivatives can be contemplated as sensitive parameters to damage occurrence. The dynamic characteristics equation for a multi-degrees of freedom un-damped system is as follows:

$$[M][\Psi][\Lambda] = [K][\Psi] \tag{2}$$

in which, $[K]$ and $[M]$ are the stiffness and mass matrices, respectively, and $[\Psi]$ is the non-mass-normalized mode shape matrix obtained from output-only modal analysis, and $[\Lambda] = diag\left\{\omega_1^2, \omega_2^2, ..., \omega_j^2, ..., \omega_{dof}^2\right\}$ is the diagonal matrix of the jth squared natural frequency and *dof* stands for the number of degrees of freedom. When the non-mass normalized mode shape $[\Psi]$ is scaled to the mass-normalized mode shape ($[\Phi]$), the stiffness matrix $[K]$ can be written in modal form as:

$$[K] = [\Phi]^{-T} [\Lambda] [\Phi]^{-1} \tag{3}$$

The first reported idea of the flexibility approach was presented by Pandey and Biswas [23], in which the flexibility matrix $[MF]$ were basically defined as the inverse of the stiffness matrix:

$$[MF] = [\Phi][\Lambda]^{-1}[\Phi]^T = \sum_{j=1}^{dof} \omega_j^{-2} \phi_j \phi_j^T \tag{4}$$

where, $[\Phi] = \left\{\phi_1 \quad \phi_2 \quad ... \quad \phi_j \quad ... \quad \phi_{dof}\right\}^T$ is the mass-normalized mode shapes matrix, ϕ_j is the jth mode shape. As mentioned in [23], the local damage not only affects the local structural stiffness, but also it can change the local flexibility. This claim can be obviously proved if Eq. (3) and Eq. (4) are inspected. Therefore, these characteristics can become indicators for structural damage detection. Generally, the structural damages result in reduction in the stiffness which can be interpreted as increase in the flexibility of the damaged elements. Taking into the account the global format of the flexibility matrix, its diagonal members can implicitly return valuable information about change in the physical properties of the monitored structure [18]. Therefore, in this study, the modal flexibility curvature is calculated by contemplating the diagonal elements of the modal flexibility matrix applying the central differential equation [7]:

$$MFC_k^{dn} = \frac{MF_{k-1,k-1}^{dn} - 2MF_{k,k}^{dn} + MF_{k+1,k+1}^{dn}}{L_e^2} \quad (5)$$
$$k = 2,...,dof - 1$$

where, $MF_{k,k}{}^{dn}$ and $MCF_k{}^{dn}$ are diagonal term of the numerical modal flexibility matrix (corresponding to the kth degree of freedom) and the kth term of the numerical modal flexibility curvature vector in the damaged state, respectively; and L_e is the length of the elements. Similarly, the experimental modal flexibility curvature (MFC^{de}) in the damaged state can be calculated by considering the diagonal elements of the experimental modal flexibility matrix (MF^{de}) using the central differential equation:

$$MFC_k^{de} = \frac{MF_{k-1,k-1}^{de} - 2MF_{k,k}^{de} + MF_{k+1,k+1}^{de}}{L_e^2} \quad (6)$$
$$k = 2,...,dof - 1$$

The difference between the experimental and numerical (or analytical) modal flexibility curvature can then be applied to form an objective function for damage quantification:

$$MFCI\left(x_1, x_2, ..., x_N\right) = \sum_{k=2}^{dof-1} \left\| \frac{MFC_k^{de} - MFC_k^{dn}}{MFC_k^{de}} \right\|$$
$$subjected\ to: 0 \le x_i \le 1 \quad (7)$$

where, $MFCI$ is the proposed objective function which should be minimized, || || is the sign of the Frobenius norm, and N is the number of the elements in the finite element model of the studied structure.

3. Particle Swarm Optimization (PSO) Algorithm

The optimization problem can be easily defined as a logical and step-by-step procedure to find an argument X whose relevant cost $F(X)$ is optimum, and it has been broadly applied in many various areas such as pattern recognition, scheduling, building design, optimal vibration control, structural damage identification, structural shape and size optimization, industrial planning, resource allocation, and so on. Unlike the traditional optimization techniques, metaheuristic techniques attempt to mimic some characteristics of natural phenomena or social behavior. These optimization techniques can increase the overall computational efficiency and capability in solving complex and ill-posed inverse problems. Among evolutionary algorithms, particle swarm optimization (PSO) technique generally outperforms in term of solution accuracy and convergence rate. In this paper, PSO algorithm is employedfor solving structural damage detection problem. PSO is a global search strategy that was first introduced by Kennedy and Eberhart [21] and it simulates social behavior of animals (like bird flocking, fish schooling and insect swarming) in nature. In comparison with genetic algorithm (GA), PSO does not require the operators like crossover and mutation to manipulate the individuals, and this is the main reason of fast performance of PSO in comparison with GA.

Similar to the other evolutionary algorithms, PSO begins with generating a random population of individuals (called a swarm). Any individual of a population is called a particle. This algorithm utilizes swarm intelligence to achieve the goal of the optimization. Besides individual intelligence, it also develops some social behavior and harmonizes the particles' movement towards the best position to find the swarm's best position with some random perturbation. In this algorithm, the particles move through the solution space and each particle has a velocity which is dynamically adjusted according to both their own experience and the population's experience.

In an N dimensional optimization problem, ith particle in the PSO algorithm is defined by three vectors namely current position vector of the ith particle $P_i=\{p_{i1},p_{i2},\ldots,p_{iN}\}$, velocity vector of the ith particle $Vi=\{v_{i1},v_{i2},\ldots,v_{iN}\}$ and the obtained best position vector of the ith particle $P_i^{best}=\{p_{i1}^{best},p_{i2}^{best},\ldots,p_{iN}^{best}\}$. In the damage identification problem, N represents the number of structural elements (the number of the variables) and the values of each variable describe the position of the particle which is a possible solution of the optimization problem. The particle's new position can be calculated in consonance with the current position of particle modified by its current velocity and the distance between the current position and its previous best position, and also the best global position among all the particles in the swarm. Therefore, the updating of particle position and velocity can be mathematically defined as follow:

$$V_i[t+1]=wV_i[t]+C_c r_c\left(P_i^{best}[t]-P_i[t]\right)+C_s r_s\left(P^{gbest}[t]-P_i[t]\right) \tag{8}$$

$$P_i[t+1]=P_t[t]+V_i[t+1] \tag{9}$$

where, $P^{gbest}=\{p_1^{gbest},p_2^{gbest},\ldots,p_N^{gbest}\}$ is the best previously visited global position among all the particles in the swarm; C_c and C_s are two positive constants to control the information flowing and social learning factors, respectively; r_c and r_s are random numbers uniformly distributed between 0 and 1; t represents the number of current iteration; and w is inertia weight introduced by Shi and Eberhart [23] to modify the original version of PSO to control the effects of the local and global searches during the evolution process. The appropriate selection of w will provide a balance between global and local searches during the evolution process, and this results in a condition in which less iteration (on average) is required to find an optimal solution. This parameter reduces linearly from a maximum value w_{max} to a minimum value w_{min}, that has been introduced by Shi and Eberhart [24] and it can be defined as follow:

$$w=w_{max}-t\times\left(w_{max}-w_{min}\right)/t_{max} \tag{10}$$

in which, t_{max} is the maximum allowable iterations. From Eq. (10), it is obvious that the particles use a relatively large inertia weight during the initial exploration though they use relatively low inertia weight as the iterations successfully go forward. The main purpose of this modification is to avoid premature convergence in the early search stages. Moreover, improving the rate of convergence to the global optimal solution during the latter search stages can be listed as another important benefits of this modification. Furthermore, in order to control any change in the particle's velocities (the step length of the algorithm), the velocity vector V_i is limited to a minimum value V_{min} and a maximum value V_{max}.

In this algorithm the continuation of the optimization steps will hopefully cause the particles to converge to the global minimum point of the objective function and finally the algorithm stops if the maximum number of iteration is reached.

The Flowchart of the proposed approach for structural damage localization and quantification using PSO algorithm is portrayed in Fig. 1.

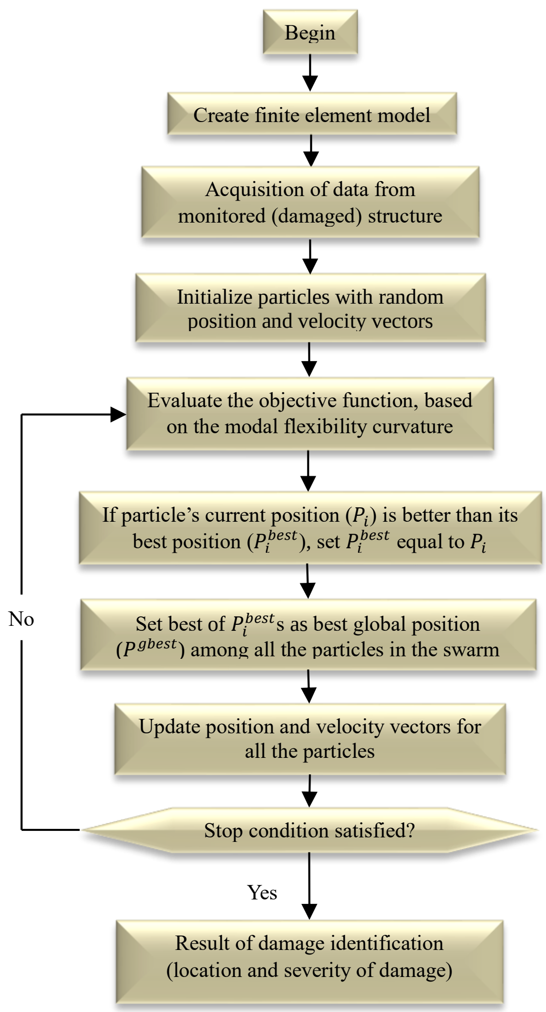


Fig. 1. Flowchart of PSO algorithm and its interaction with the proposed method.

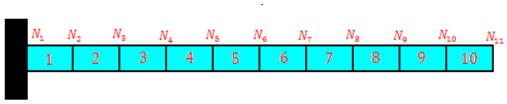


Fig. 2. A ten-element cantilever beam.

4. Numerical Examples

In order to demonstrate the performance of the proposed method for structural damage identification, a cantilever beam and a simply supported beam are considered as the test examples. Moreover, the effectiveness of the proposed approach is investigated by considering both exact and noisy modal parameters (i.e., frequencies and mode shapes). In all the examples, first, first two and first three vibrational modes are utilized for damage localization and quantification.

The proper control of local exploitation and global exploration is the important feature of PSO algorithm which depends on the selected parameters. Therefore, selection of PSO parameters can considerably influence the convergence and performance of the optimization procedure. In this study, the PSO's parameters were selected as follows applying trial-and-error approach: cognition learning factor C_c is 2; social learning factor C_s is 2; minimum of the inertia weight $w_{\min}$ is 0.4; maximum of the inertia weight $w_{\max}$ is 0.9; population size is 100; and maximum number of iterations is 1000. It was observed that these values are suitable for the presented numerical examples in this study. Furthermore, in order to deliberate the stochastic nature of the optimization process, the mean values of ten independent runs are contemplated as the damage detection results for each damage case.

4.1. A Cantilever Beam

The first numerical example for validation of the proposed identification approach is a cantilever beam with 5 *m* length and rectangular cross section with an area equal to 0.0250 m^2. The finite element model of this beam consisting of 10 two-dimensional elements and 20 degrees of freedom (two degrees of freedom in each free node) is shown in Fig. 2. The mass density, the elasticity modulus and the moment of inertia for this example are considered as 7590 kg/m^3, 120 *GPa* and 4.0×10^{-6} m^4, respectively. It is assumed that all elements of the beam have uniform cross sections and the length of each of these elements is 0.5 *m*.

In this example, two different damage scenarios were simulated as follows:

Scenario-A: 20% reduction in the elasticity modulus of element 5.

Scenario-B: 30% and 15% reductions in the elasticity modulus of elements 3 and 7, respectively.

Here, both exact and noisy modal parameters (frequencies and mode shapes) are fed to the method as input data.

4.1.1. Damage Identification without Noise Effect

The damage identification results for the damage scenarios of the cantilever beam employing exact modal parameters (i.e., there is no measurement error) are depicted in Fig. 3. In this figure, *M* represents number of the vibrational mode(s) which are used for damage identification procedure. It is observed that the location and severity of damage are identified accurately employing the proposed approach.

The convergence curves of the PSO algorithm for the damage scenarios of the cantilever beam (under exact modal parameters) are illustrated in Fig. 4 for all the studied states. A complete convergence to the global extremums can be seen in all states. Therefore, it is concluded that the proposed approach is a robust and effective method for localizing and quantifying of damage in the cantilever beam when exact (i.e., noise free) modal parameters are employed. Another point that can be concluded from the convergence curves is that, by increasing the number of the employed modal data (i.e., *M*), the objective function can converge to a number which is very close to zero. In noisy states, this issue is of vital importance: In such cases, the obtained results are considered as the global extremums if the associated objective function's value is very close to zero.

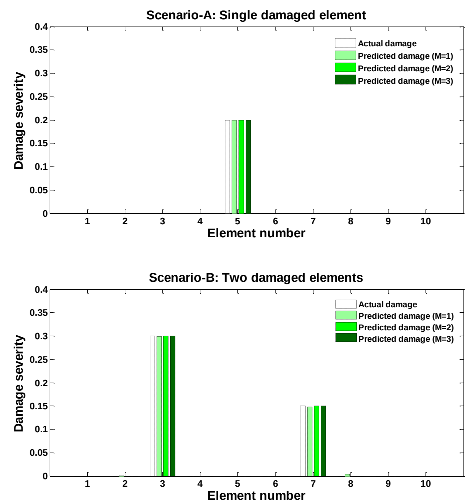


Fig. 3. Damage identification results for the cantilever beam under exact modal parameters.

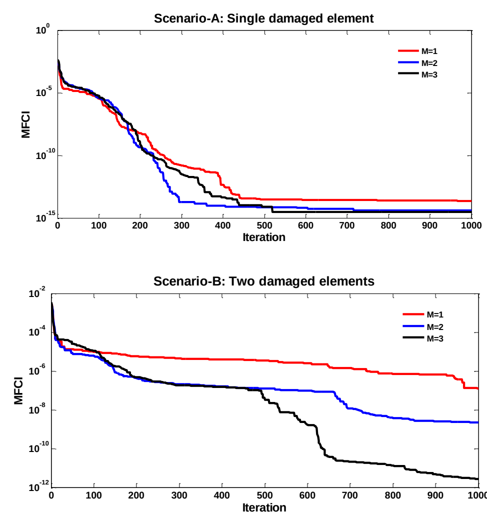


Fig. 4. The convergence history of the PSO algorithm for the cantilever beam under exact modal parameters.

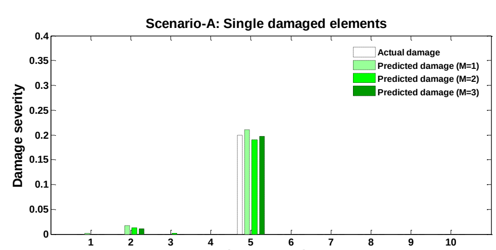


Fig. 5. Damage identification results for the cantilever beam under noisy modal parameters.

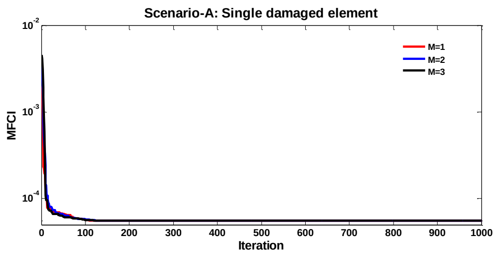


Fig. 6. The convergence history of the PSO algorithm for the cantilever beam under noisy modal parameters.

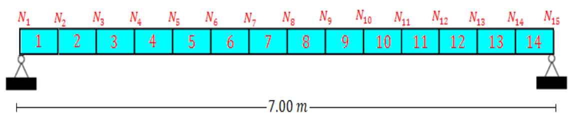


Fig. 7. Fourteen-element simply supported beam.

4.1.2. Damage Identification with Noise Effect

In order to evaluate the noise effects on the performance of the proposed method, a normally distributed random error is added to the modal parameters. The noise was simulated applying the approach proposed by Ghodrati Amiri et al. [18] and the noise level was 1.5%. The damage identification results for single damage scenario (Scenario-A) of the cantilever beam under noisy modal parameters are portrayed in Fig. 5. As it is evident the location and severity of the structural damage can be identified reasonably accurate using the proposed approach. The convergence curve of the PSO algorithm for this case is depicted in Fig. 6. It can be seen that the convergence performance of the method for damage identification in the cantilever beam is quiet acceptable. In this example, it was illustrated that the proposed approach is a robust and effective method for localizing and quantifying of the damage in the cantilever beam even when modal parameters are contaminated with noise.

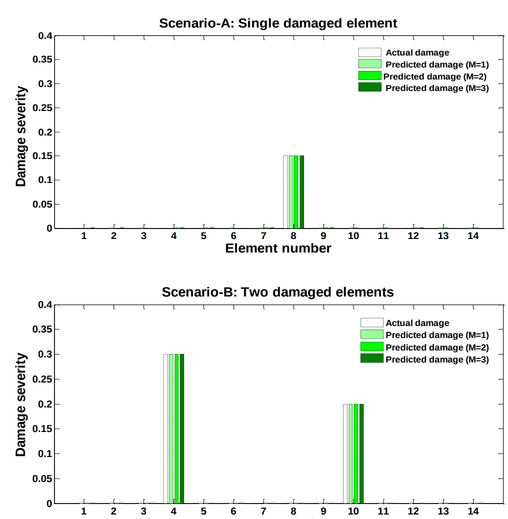
4.2. A Simply Supported Beam

A simply supported bam with below mentioned property is contemplated as the second numerical example for evaluating the efficiency of the proposed identification approach. The length and cross section of this beam are 7 *m* and 0.0500 m^2, respectively. Its finite element model consists of 14 two-dimensional elements and 28 degrees of freedom (see Fig. 7). The mass density, elasticity modulus and moment of inertia for this beam are equal to 2500 kg/m^3, 32 *GPa* and 1.667×10^{-4} m^4, respectively.

In general, the identification problem has two unknown parameters, location and severity of damage in the elements. However, the identified damage severities can also return information about the damage existence (or location). Elements with damage severity of zero (or very close to zero) are considered as healthy elements. In this example, three different damage scenarios are simulated:

Scenario-A: 15% reduction in the elasticity modulus of element 8.

Scenario-B: 30% and 20% reduction in the elasticity modulus of elements 4 and 10, respectively.



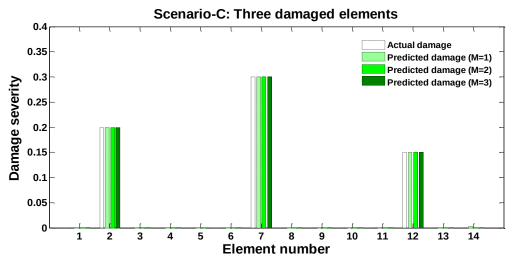


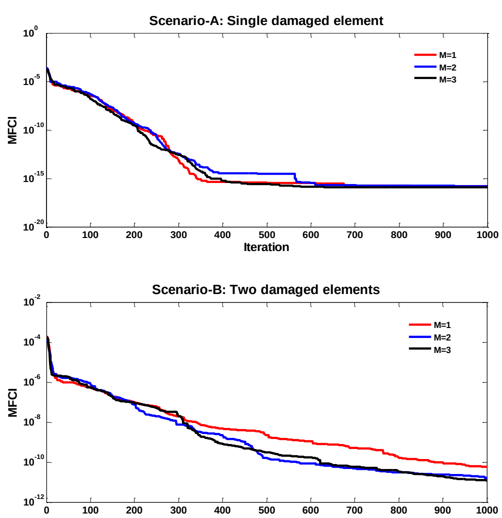
Fig. 8. Damage identification results for scenario-A of the simply supported beam under exact modal parameters.

Scenario-C: 20%, 30% and 15% reduction in the elasticity modulus of elements 2, 7 and 12, respectively.

It is note-worthy to mention that the damage in the damaged elements was simulated by defining reduction in the modulus of elasticity. Moreover, in all the simulated damage scenarios, the damage severities of the damaged elements are considered smaller than 40% to make sure that the behavior of the damaged structure is in the linear range.

4.2.1. Damage identification without noise effect

Fig. 8 shows the obtained results for damage identification of the simply supported beam



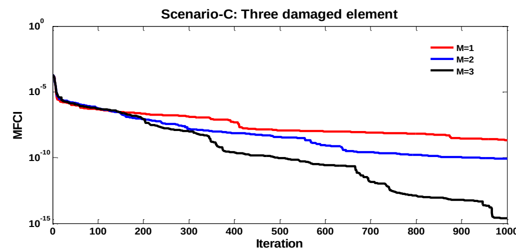


Fig. 9. The convergence history of the PSO algorithm for the simply supported beam under exact modal parameters.

for Scenarios-A to C, when the noise free modal data are employed. It is observed that the proposed method identifies the location and extent of damage in all the elements with high level of accuracy, without any error. Moreover, it is noticed that the method has acceptable level of sensitivity to different level of damages (small, moderate and severe) and also, their combinations (see Scenario-C). Fig. 9 shows the mean of the convergence curves obtained in ten independent runs. Similar to the previous example, it is concluded that the PSO algorithm is able to find the optimal solution with fast speed and high level of accuracy. Moreover, it is observed that by increasing *M*, the objective function converges to a number which is so close to zero.

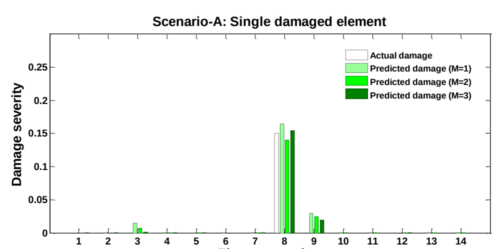


Fig. 10. Damage identification results for Scenario-A of the simply supported beam under noisy modal parameters.

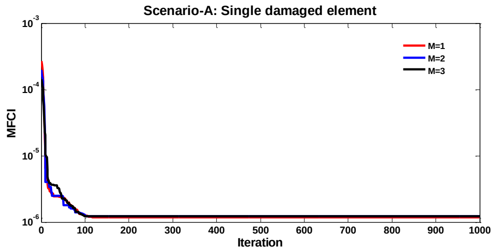


Fig. 11. The convergence history of the PSO algorithm for Scenario-A of the simply supported beam under noisy modal parameters.

4.2.2. Damage identification with noise effect

In this section, the noise effect on the performance of the proposed method for structural damage identification in the simply supported beam is investigated. The damage identification results for single damage scenario (Scenario-A) in the noisy state (with 1% noise in the modal data) are demonstrated in Fig. 10. It is observed that location and severity of the structural damage are correctly identified using the proposed approach. The convergence curve of the PSO algorithm for this damage scenario is shown in Fig. 11. As it can be observed, the method converges to the optimal solution after ~100 iterations and this an support this claim that PSO algorithm is well-suited for the proposed objective function, even if the noisy data are fed in the method as the input data.

5. Conclusions

A model updating-based approach to identify damages in beam-like structures applying modal flexibility curvature and optimization algorithm was proposed. The PSO algorithm was used to minimize the difference between the measured modal flexibility curvatures of the monitored structure and those calculated from the analytical structure (with unknown damage severities). To demonstrate the applicability of the method, two numerical examples of beams with different damage patterns are studied. For each damage scenario, the mean values of ten independent runs was reported to consider existing the uncertainties in the evolutionary optimization algorithms. The results indicated that changes in the modal flexibility curvature can be considered as a sensitive and suitable criterion for damage prognosis in the beam-like structures. Moreover, it was concluded that the proposed approach is able to localize and quantify different levels of damages only by a few lower vibrational modes and also, it has low level of sensitivity to unavoidable noises in input data.

REFERENCES

[1] Abdo, M.A.B. (2012). “Parametric study of using only static response in structural damage detection”. Engineering Structures, Vol. 34, pp. 124-131.

[2] Pandey, A.K., Biswas, M., Samman, M.M. (1991). “Damage detection from changes in curvature mode shapes”. Journal of Sound and Vibration, Vol. 145, pp. 321-333.

[3] Abdel Wahab, M.M., DeRoeck, G. (1999). “Damage detection in bridges using modal curvatures: application to a real damage scenario”. Journal of Sound and Vibration, Vol. 226, No. 2, pp. 217-235.

[4] Lin, R.J., Cheng, F.P. (2008). “Multiple crack identification of a free-free beam with uniform material property variation and varied noised frequency”. Engineering Structures, Vol. 30, pp. 909-929.

[5] Lu, X.B., Liu, J.K., Lu, Z.R. (2013). “A two-step approach for crack identification in beam”. Journal of Sound and Vibration, Vol. 332, pp. 282-293.

[6] Tomaszewska, A. (2010). “Influence of statistical errors on damage detection based on structural flexibility and mode shape curvature”. Computers & Structures, Vol. 88, pp. 154-164.

[7] Lu, Q., Ren, G., Zhao, Y. (2002). “Multiple damage location with flexibility

curvature and relative frequency change for beam structures". Journal of Sound and Vibration, Vol. 253, No. 5, pp. 1101-1114.

[8] Wong, F.S., Thint, M.P., Tung, A.T. (1997). "On-line detection of structural damage using neural networks". Civil Engineering and Environmental Systems, Vol. 14, pp. 167–197.

[9] Sahin, M., Shenoi, R.A. (2003). "Quantification and localization of damage in beam-like structures by using artificial neural networks with experimental validation". Engineering Structures, Vol. 25, pp. 1785-1802.

[10] Vinayak, H.K., Kumar, A., Agarwal, P., Thakkar, S.K. (2010). "Neural network-based damage detection from transfer function changes". Journal of Earthquake Engineering, Vol. 14, pp. 771–787.

[11] Lee, J. (2009). "Identification of multiple cracks in a beam using vibration amplitudes". Journal of Sound and vibration, Vol. 326, pp. 205-212.

[12] Lopes, P.S., Jorge, A.B., Cunha Jr, S.S. (2010). "Detection of holes in a plate using global optimization and parameter identification techniques". Inverse Problems in Science and Engineering, Vol. 18, No. 4, pp. 439-463.

[13] Meruane, V., Heylen, W. (2011). "An hybrid real genetic algorithm to detect structural damage using modal properties". Mechanical Systems and Signal Processing, Vol. 25, pp. 1559-1573.

[14] Na, C., Kim, S.P., Kwak, H.G. (2011). "Structural damage evaluation using genetic algorithm". Journal of Sound Vibration, Vol. 330, pp. 2772-2783.

[15] Mehrjoo, M., Khaji, N., Ghafory-Ashtiany, M. (2013). "Application of genetic algorithm in crack detection of beam-like structures using a new cracked Euler–Bernoulli beam element". Applied Soft Computing, Vol. 13, pp. 867-880.

[16] Zare Hosseinzadeh, A., Bagheri, A., Ghodrati Amiri, G. (2013). "Two-stage method for damage localization and quantification in high-rise shear frames based on the first mode shape slope". International journal of optimization in civil engineering, Vol. 3, No. 4, pp. 653-672.

[17] Tabrizian, Z., Ghodrati Amiri, G., Hossein Ali Beigy, M. (2014). "Charged system search algorithm utilized for structural damage detection". Shock and Vibration, Vol. 2014, 13 pages, Article ID: 194753.

[18] Ghodrati Amiri, G., Zare Hosseinzadeh, A., Seyed Razzaghi, S.A. (2015). "Generalized flexibility based model updating approach via democratic particle swarm optimization algorithm for structural damage prognosis". International Journal of Optimization in Civil Engineering, Vol. 5, No. 4, pp. 44-65

[19] Zare Hosseinzadeh, A., Ghodrati Amiri, G., Seyed Razzaghi, S.A. (2017). "Model-based identification of damage from sparse sensor measurements using Neumann series expansion". Inverse Problems in Science and Engineering, Vol. 25, No. 2, pp. 239-259.

[20] Kaveh, A., Hoseini Vaez, S.R., Hosseini, P. (2019). "Enhanced vibrating particles system algorithm for damage identification of truss structures". Scientia Iranica, Vol. 26, No. 1, pp. 246-256.

[21] Kennedy, J., Eberhart, R. (1995). "Particle swarm optimization". In: Proceedings of the IEEE international conference on neural networks, Vol. 4, pp. 1942-1948.

[22] Golbon-Haghighi, M.H., Saeidi-Manesh, H., Zhang, G., Zhang, Y. (2018). "Pattern synthesis for the cylindrical polarimetric phased array radar (CPPAR)". Progress in Electromagnetics Research M, Vol. 66, pp. 87-98.

[23] Shi, Y., Eberhart, R.C. (1998). "A modified particle swarm optimizer". Proceedings IEEE International Conference on Evolutionary Computation, pp. 69-73.

[24] Shi, Y., Eberhart, R.C. (1999). "Empirical study of particle swarm optimization". Proceedings of the Congress on Evolutionary Computation, Vol. 3, pp. 1945-1950.