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Effect of Viscous Dissipation on Seady Natural Convection Heat and Mass Transfer in a Vertical Channel with Variable Viscosity and Thermal Conductivity

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P A P E R I N F O

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A B S T R A C T

In this study, the effects of viscous dissipation and variable physical properties on the steady natural convection heat and mass transfer in a vertical channel are investigated. The viscosity and thermal conductivity are considered as the linear function of temperature. The governing equations are transformed into a set of coupled nonlinear ordinary differential equations and solved using Differential Transformation Method (DTM). Results obtained are compared with the exact solution when some of the flow conditions are relaxed. The current results have an excellent agreement with the exact solution which was obtained analytically. The influences of the flow parameters on the fluid temperature, concentration, and velocity are presented graphically and discussed. It is found that increasing viscous dissipation causes the increase in the fluid temperature, velocity, and skin friction on the surface of both channels. However, increasing the fluid viscosity retards the fluid motion and causes the decrease in the fluid temperature.

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1. Introduction

Natural convection with viscous dissipation has received a lot of attention because of its increasing practical application in areas of geological process, nuclear engineering, and heat exchangers, etc. Gebhart [1] investigated the importance of viscous dissipation in natural convection using perturbation technique for first time. It was shown that the viscous dissipation effect in natural convection is appreciable when the induced kinetic energy becomes appreciable as compared to the amount of transferred heat. Gebhart and Mollendorf [2] investigated the effect of viscous dissipation in the external natural convection with a class of similar boundary layer solutions. They considered the wide ranges of dissipation and Prandtl numbers. Soundalgekar [3] studied the effect of viscous dissipation on the unsteady free convective flow on an infinite vertical porous plate with the constant suction. Soundalgekar [3] used an approximate solution for the fluctuating parts of velocity, transient velocity, and temperature profiles. Soundalgekar [4] extended the study conducted by Soundalgekar [3] with variable suction at the plate with approximate solutions for the fluctuating parts of the velocity, transient velocity, and temperature profiles. The results of this study revealed that greater dissipative heat causes a drop in the mean temperature of water and an increase in the mean velocity. Mahajan and Gebhart [5] investigated the viscous dissipation effects in the buoyancy induced flows with no restrictions on Prandtl number. They concluded that the viscous dissipation effects are smaller than the pressure effects for all values of Prandtl number. Pantokratoras [6] presented a similarity solution for the effect of viscous dissipation in the natural convection along a heated vertical plate. It was observed that the viscous dissipation has a strong influence on the results as it assists the upward flow and opposes the downward flow. Viscous dissipation effect on the natural convection flow along a vertical wavy surface has been investigated by Parveen et al. [7] numerically. Their results showed that increasing the viscous dissipation increases

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the skin friction coefficient and decreases the heat transfer rate.

In all the aforementioned studies, physical properties of fluid such as viscosity and thermal conductivity have been assumed constant. However, to predict accurately the flow behavior of fluid, it is very important to take into account the variations of viscosity and thermal conductivity with temperature. Meanwhile, major industrial problems involving fluid flows require variation of fluid properties with temperature. Raja et al. [8] investigated the effects of variable physical properties and viscous dissipation on a free convective flow over a vertical plate using Runge-Kutta Gill method coupled with a shooting technique. They concluded that the viscous dissipation is significantly greater in the fluids with higher viscosities. Kairi et al. [9] studied the effect of viscous dissipation on the natural convection in a non-Darcian porous medium with the variable viscosity using a local non-similarity method. The above work reveals that increasing viscous dissipation lead to increase in both fluid temperature and velocity. Atul et al. [10] studied a free convection flow with the variable viscosity through horizontal channel using perturbation method. Their work shows that the fluid temperature increases with increasing the viscous dissipation. Singh [11] studied numerically the effects of variable fluid properties and viscous dissipation on the mixed convection fluid flow around a vertical plate. It was concluded that increasing viscous dissipation increases the fluid temperature and velocity. Rudraiah et al. [12] studied the effects of variable viscosity and viscous dissipation on Oberbeck Magnetoconvection in a chiral fluid using both perturbation and numerical methods. It was observed that increasing the fluid viscosity decreases the flow velocity. Mahanti and Gaur [13] investigated the effects of linearly varying viscosity and thermal conductivity on the steady free convective flow of a viscous incompressible fluid along an isothermal vertical plate in the presence of heat sink. They used the Runge-Kutta fourth order method with shooting technique to carry out their investigation. In the study of Choudhury and Hazarika [14], the effects of variable viscosity and thermal conductivity on the free convective oscillatory flow of a viscous incompressible and electrically conducting fluid are investigated using explicit finite difference method. It was observed that the Nusselt number decreases with increasing the thermal conductivity of the fluid. Uwanta and Hamza [15] investigated the hydromagnetic flow of reactive viscous fluid in a vertical channel with the thermal diffusion and temperature dependent properties using implicit finite difference scheme for both unsteady and steady states. Manjunatha and Gireesha [16] studied the effects of variable viscosity and thermal conductivity on MHD flow and heat transfer of a dusty fluid. They concluded that increase in the fluid viscosity decreases the velocity whereas increase in the thermal conductivity increases both fluid temperature and velocity.

A lot of techniques have been used to solve the nonlinear and coupled equations. Some of these techniques are Runge-kutta shooting, finite difference, finite element, perturbation, Homotopy perturbation, Adomian decomposition, He-Laplace, and differential transformation methods. The differential transformation method has been proven to be accurate and more efficient which requires less computational effort in comparison to other methods mentioned above. The differential transformation method (DTM) is a strong mathematical tool to solve the problems with linear and nonlinear differential equations which requires significantly less computational resources since it does not need any auxiliary parameter, initial guess, and small parameter like “p" in Homotopy perturbation method (HPM). Zhou [17] introduced the differential transformation method to solve the linear and nonlinear initial value problems in an electrical circuit theory for the first time. Chen and Ho [18] also developed the DTM for partial differential equations and developed a closed form series solutions for the linear and nonlinear initial value problems. Umavathi and Shekar [19] investigated the combined effects of variable viscosity and thermal conductivity on the free convection flow of a viscous fluid in a vertical channel using the DTM. The efficiency of the DTM was compared with the Runge-Kutta method and the results revealed that both methods agree to the order of10-6. Hatami et al. [20] carried out a comparison between the DTM and HPM for the Newtonian and Non-Newtonian fluid flows. They concluded that the DTM is very effective and can achieve more suitable results than HPM in some areas of equations in engineering and science problems. Oke [21] investigated the convergence of DTM for the ordinary differential equation.

Jha and Ajibade [22] investigated the free convection heat and mass transfer in a vertical channel with the Dufour effect. The steady state of the problem is independent of Dufour effect. The fluid viscosity and thermal conductivity were assumed constant and viscous heating was not taken into account. The assumptions made by Jha and Ajibade [22] are used in the current study in order to present the realistic conditions in which the fluid flow is correctly predicted. Indeed, the study of Jha and Ajibade [22] is reformulated to capture a real problem in which the fluid viscosity and thermal conductivity can vary with temperature change and the viscous heating near the walls is also considered. The governing equations of the flow fields are solved by the DTM. The effects of different physical parameters on the velocity, temperature, concentration, skin friction, and heat transfer rate are presented graphically and discussed.

2. Mathematical Formulation

In this study, a steady flow of an incompressible fluid with viscous dissipation between two vertical parallel plates positioned at $y'=0$ and $y'=h$ with the uniform temperatures of T_1 and T_0 on the hot and cold walls is considered. The flow is assumed to be in the x' -direction which is taken vertically upward along the vertical plates

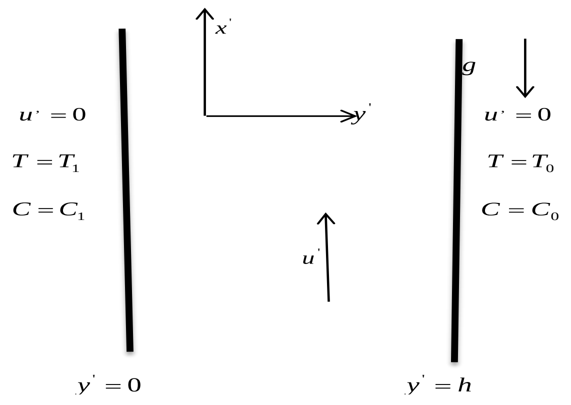

Figure 1. Schematic diagram of fluid flow

and the y' -axis is taken normal to the plates as shown in figure 1. The plates have infinite lengths and as a result, the velocity, temperature, and concentration fields are only the function of y'.

All fluid properties are considered constant except the viscosity and thermal conductivity that can be varied linearly with the temperature. The viscous dissipation, heat generation, chemical reaction, and the temperature dependent viscosity and thermal conductivity are incorporated into the momentum, temperature, and concentration equations. In the absence of slip and boundary jump, the steady natural convection fully developed heat and mass transfer in a vertical channel under the Boussinesq's approximation is governed by the following equations:

$$\frac{1}{\rho_0}\frac{d}{dy'}\left(\mu\frac{du'}{dy'}\right)+g\beta(T'-T_0')+g\overline{\beta}(C'-C_0')=0 \tag{1}$$

$$\frac{1}{\rho_0 c_p}\frac{d}{dy'}\left(k\frac{dT'}{dy'}\right)+\frac{Q_0}{\rho_0 c_p}(T'-T_0')+\frac{\mu}{\rho_0 c_p}\left(\frac{du'}{dy'}\right)^2=0 \tag{2}$$

$$D_m\frac{d^2C'}{dy'^2}-K_r'(C'-C_0')=0 \tag{3}$$

The boundary conditions for these equations are:

$$\begin{aligned} &u'=0, T'=T_1', C'=C_1' \qquad \text{at } y'=0\\ &u'=0, T'=T_0', C'=C_0' \qquad \text{at } y'=h \end{aligned} \tag{4}$$

where Q_0 is the internal heat generation constant which may be either positive (heat source) or negative (heat sink). g is the acceleration due to gravity. c_p, ρ_0, μ, and κ indicate the specific heat at constant pressure, the constant density, viscosity, and thermal conductivity of the fluid, respectively. κ_r' is the chemical reaction constant. The following linear relationships are used for the temperature variable viscosity and thermal conductivity [12, 15]:

$$\mu=\mu_0[1-a(T'-T_0{}')] \quad \text{and} \quad k=k_0[1+b(T'-T_0{}')] \tag{5}$$

In the above relationships, the cases of a<0 and a>0 are considered for gasses and liquids, respectively. In addition, the case of b>0 is considered for fluids such as water and gases, while the case of b<0 can be used for fluids such as lubricating oils. From equation (5), the viscosity and thermal conductivity can be written in the following form:

$$\mu=\mu_0(1-\lambda\theta) \quad \text{and} \quad K=k_0(1+\gamma\theta) \tag{6}$$

In this study, the viscosity and thermal conductivity are varied in the ranges of $-0.7\le\lambda\le 0$, $0\le\gamma\le 6$ for the air, $0\le\lambda\le 0.6$, $0\le\gamma\le 0.12$ for the water, and $0\le\lambda\le 3$, $-0.1\le\gamma\le 0$ for the lubricating oil [16].

The following non-dimensional parameters are used:

$$\begin{aligned} &u=\frac{u'}{u_0}, y=\frac{y'}{h}, \theta=\frac{T'-T_0'}{T_1'-T_0'}, C=\frac{C'-C_0'}{C_1'-C_0'},\\ &\lambda=a(T_1'-T_0')\\ &K_c=\frac{K_r^*h^2}{v_0}, Pr=\frac{\mu c_p}{k}, Sc=\frac{v_0}{D},\\ &Q=\frac{Q_0h^2}{v_0k_0}, \; N=\frac{\overline{\beta}(C_1'-C_0')}{\beta(T_1'-T_0')}\\ &u_0=\frac{g\beta h^2(T_1'-T_0')}{v_0}, Pr_0=\frac{\mu_0 c_p}{k_0},\\ &Pr=\frac{\mu c_p}{k}=\frac{Pr_0(1-\lambda\theta)}{(1+\gamma\theta)} \quad \gamma=b(T_1'-T_0'),\\ &Ec=\frac{u_0{}^2}{c_p(T_1'-T_0')} \end{aligned} \tag{7}$$

By using the above equations, Eqs (1) to (4) can be rewritten in the following non-dimensional forms:

$$(1-\lambda\theta)\frac{d^2u}{dy^2}-\lambda\frac{d\theta}{dy}\frac{du}{dy}+\theta+NC=0 \tag{8}$$

$$(1+\gamma\theta)\frac{d^2\theta}{dy^2}+\gamma\left(\frac{d\theta}{dy}\right)^2+Q\theta+\mathrm{Pr}.Ec(1+\gamma\theta)\left(\frac{du}{dy}\right)^2=0 \tag{9}$$

$$\frac{1}{Sc}\frac{d^2C}{dy^2}-K_cC=0 \tag{10}$$

The non-dimensional forms of the boundary conditions are:

$$\begin{aligned} &u=0, \theta=1, C=1, \qquad at \quad y=0\\ &u=0, \theta=0, \; C=0, \qquad at \quad y=1 \end{aligned} \tag{11}$$

where Pr, Ec, N, and Sc are the Prandtl number, Eckert number, buoyancy parameter, and Schmidt number, respectively. Kc and Q are the chemical reaction parameter and the heat source/ heat sink parameter, respectively. Furthermore, the Prandtl number is a function of viscosity and thermal conductivity. As the Prandtl number is varied, both fluid viscosity and thermal conductivity are varied across the boundary layer [14]. From Eq. (9), the viscosity parameter is removed.

3. Differential Transformation Method

The transformation of the K^{th} derivative of a function can be defined by:

$$F(K)=\frac{1}{k!}\left[\frac{d^kf(\eta)}{dn^k}\right]_{\eta=\eta_0} \tag{12}$$

where $f(\eta)$ and $F(K)$ are the original the transformed functions, respectively. The differential inverse transform of $F(K)$ is given by:

Table 1. The operations of differential transformation method [19]

Original function	Transformed Function
1. $f(y) = g(y) \pm h(y)$	$F(k) = G(k) \pm H(k)$
2. $f(y) = \lambda g(y)$	$F(k) = \lambda G(k)$
3. $f(y) = \frac{d^n g(y)}{dy^n}$	$F(k) = (k+1)(k+2)\ldots\ldots(k+r)G(k+r)$
4. $f(y) = g(y)h(y)$	$F(k) = \sum_{r=0}^{k} G(r)H(k-r)$
5. $f(y) = g(y)\frac{dh(y)}{dy}$	$F(k) = \sum_{r=0}^{k} (k-r+1)H(k-r+1)G(r)$
6. $f(y) = g(y)\frac{d^2h(y)}{dy^2}$.	$F(k) = \sum_{r=0}^{k} (k-r+1)(k-r+2)H(k-r+2)G(r)$

$$f(\eta) = \sum_{k=0}^{\infty} F(K)(\eta - \eta_0)^k \tag{13}$$

The concept of differential transformation is derived from a Taylor series expansion and in real applications; the function $f(\eta)$ is expressed by a finite series as follows:

$$f(\eta) = \sum_{k=0}^{m} F(K)(\eta - \eta_0)^k \tag{14}$$

where the value of *m* is determined by the convergence of the series coefficient. The operations used by the DTM for different functions are listed in Table 1.

4. Solution with differential transformation method

By using the differential transforms of Eqs (10) to (13), the following equations can be achieved:

$$\overline{U}(k+2) = \frac{1}{(k+1)(k+2)}[\lambda \textstyle\sum_{r=0}^{k}(k-r+1)(r+1)\overline{U}(k-r+1)\overline{\theta}(r+1) + \lambda^2 \sum_{r=0}^{k}\sum_{s=0}^{r}(k-r+1)(r-s+1)\overline{U}(k-r+1)\overline{\theta}(r-s+1) - \overline{\theta}(k) - \lambda\sum_{r=0}^{k}\overline{\theta}(k-r)\overline{\theta}(r) - N\overline{C}(k) - \lambda N\sum_{r=0}^{k}\overline{\theta}(k-r)\overline{C}(r)] \tag{15}$$

$$\overline{\theta}(k+2) = \frac{1}{(k+1)(k+2)}[-\gamma \textstyle\sum_{r=0}^{k}(k-r+1)(r+1)\overline{\theta}(k-r+1)\overline{\theta}(r+1) + \gamma^2\sum_{r=0}^{k}\sum_{s=0}^{r}(k-r+1)(r-s+1)\overline{\theta}(k-r+1)\overline{\theta}(r-s+1)\overline{\theta}(s) - Q\overline{\theta}(k) + Q\gamma\sum_{r=0}^{k}\overline{\theta}(k-r)\overline{\theta}(r) - Pr.Ec\sum_{r=0}^{k}(k-r+1)(r+1)\overline{U}(k-r+1)\overline{U}(r+1)] \tag{16}$$

$$\overline{C}(k+2) = \frac{1}{(k+1)(k+2)}\left[Sc.Kc\overline{C}(k)\right] \tag{17}$$

where $\overline{U}(k)$, $\overline{\theta}(k)$, and $\overline{C}(k)$ are the differential transforms of $U(y)$, $\theta(y)$, and C(y), respectively. The transformed boundary conditions are:

$$\overline{U}(0) = 0, \overline{U}(1) = a$$
$$\overline{\theta}(0) = 1, \overline{\theta}(1) = b\ \overline{C}(0) = 1, \overline{C}(1) = c \tag{18}$$

where a, b, and c are the constants computed from the boundary conditions in equation (11). The above equations for the temperature, velocity, and concentration are solved and the results obtained are presented graphically in figures 2 to 13, and numerically in tables 2 and 8 for different parameters.

5. Solution with exact method

By considering the zero values for the viscosity parameter (λ) and thermal conductivity parameter (γ) Eqs (8) to (10) are reduced into the following form:

$$\frac{d^2U}{dy^2} + \theta + NC = 0 \tag{19}$$

$$\frac{d^2\theta}{dy^2} + Q\theta = 0 \tag{20}$$

$$\frac{d^2C}{dy^2} - Sc.KcC = 0 \tag{21}$$

The following relationships can be achieved by solving Eqs (19) to (21) with the corresponding boundary condition of Eq (11):

$$\theta(y) = A_2 e^{i\sqrt{Q}y} + A_1 e^{-i\sqrt{Q}y} \tag{22}$$

$$C(y) = A_4 e^{\sqrt{ScKc}y} + A_3 e^{-\sqrt{ScKc}y} \tag{23}$$

$$U(y) = A_6 y + A_5 + \frac{A_2 e^{i\sqrt{Q}y}}{Q} + \frac{A_1 e^{-\sqrt{Q}y}}{Q} - \frac{NA_4 e^{\sqrt{ScKc}y}}{ScKc} - \frac{NA_3 e^{-\sqrt{ScKc}y}}{ScKc} \tag{24}$$

$$\frac{d\theta}{dy} = A_2 i\sqrt{Q}e^{i\sqrt{Q}y} - A_1 i\sqrt{Q}e^{-i\sqrt{Q}y} \tag{25}$$

$$\frac{dU}{dy} = A_6 + \frac{A_2 i e^{i\sqrt{Q}y}}{\sqrt{Q}} - \frac{A_1 i e^{-i\sqrt{Q}y}}{\sqrt{Q}} - \frac{NA_4 e^{\sqrt{ScKc}y}}{\sqrt{ScKc}} + \frac{NA_3 e^{-\sqrt{ScKc}y}}{\sqrt{ScKc}} \quad (26)$$

where:

$$A_1 = \frac{e^{i\sqrt{Q}}}{e^{i\sqrt{Q}} - e^{-i\sqrt{Q}}}, A_2 = 1 - A_1,$$

$$A_3 = \frac{e^{\sqrt{ScKc}}}{e^{\sqrt{ScKc}} - e^{-i\sqrt{ScKc}}}, A_4 = 1 - A_3$$

The dimensionless skin frictions for the hot plate ($y = 0$) and the cold plate ($y = 1$) are given by:

$$\tau_0 = (1 - \lambda\theta)\left(\frac{du}{dy}\right)_{y=0} \; and \quad \tau_1 = -(1 - \lambda\theta)\left(\frac{du}{dy}\right)_{y=1} \quad (27)$$

The Nusselt numbers for the hot plate ($y = 0$) and the cold plate ($y = 1$) can be calculated by:

$$Nu_0 = -(1 + \gamma\theta)\left(\frac{d\theta}{dy}\right)_{y=0} \; and \quad Nu_1 = -(1 + \gamma\theta)\left(\frac{d\theta}{dy}\right)_{y=1} \quad (28)$$

The volumetric flow rate is given by:

$$V_m = \int_0^1 u(y)dy \quad (29)$$

The results for the volumetric flow rate, skin friction, and heat transfer are presented in tables 5 to 8.

6. Validation and convergence of differential transformation method

In order to verify the accuracy of the present method (DTM), the results for the skin friction at both boundary walls for different values of Buoyancy ratio parameter (N) are compared with the results reported by Jha and Ajibade [22] for the free convection heat and mass transfer in a vertical channel. The verification is conducted in the absence of variable viscosity, thermal conductivity, heat source/sink, viscous dissipation, and chemical reaction and the Dufour effect is also considered. The results of this comparison are shown in table 2. It can be seen that there is excellent agreement between the current results with those reported by Jha and Ajibade [22]. As a result, the DTM can be used in this study.

The convergence of the DTM was established by Oke [21]. It is shown that the solution obtained through DTM can converge to exact solution when the problem is linear [21]. In this Study, The comparison of the DTM with the exact and numerical methods is carried out on the temperature, velocity, and concentration fields for different parameters in the boundary walls. Based on the data presented in table 3, the results achieved by the DTM shows a strong convergence with the results obtained by the exact method when viscosity and thermal conductivity parameters as well as Eckert number are zero. The data in table 4 also reveal a strong convergence to five decimal places of the DTM with the numerical method for different parameters. The Dsolve/Numeric for BVP from Maple application package is used to obtain the numerical solution.

7. Results and discussion

In this study, the effects of viscous dissipation on the free convection heat and mass transfer through a vertical channel with temperature dependent viscosity and thermal conductivity are investigated. The nonlinear coupled governing equations have been solved by DTM to obtain the results for temperature, velocity, and concentration fields. In addition, the results of exact and numerical methods are also compared with the results achieved by DTM. The temperature, velocity, and concentration fields are presented graphically for different parameters. The values of Pr=0.71 (air) and Sc= 0.94 (carbon dioxide) are considered for the Prandtl and Schmidt numbers. Note that the binary mixture of CO_2 in air is considered as the working fluid.

The effects of viscous dissipation, Ec, on the temperature and velocity profiles for the temperature-dependent viscosity and thermal conductivity are investigated in figures 2 and 3. When the viscous heating is important, a transversal temperature gradient develops near the walls to dissipate the heat through the boundaries. This temperature increase leads to a decrease in the viscosity and accordingly, an increase in the flow velocity can be observed. This velocity increase produces an increase in the velocity gradients near the walls, and a further increase in the initial temperature occurs. It is evidently clear that both fluid temperature and velocity increase with increasing the Eckert number in the channel. Physically this trend is true since the viscous dissipation produces heat as a result of hindrance caused between the fluid particles which can increase the initial temperature of fluid. The increase in the temperature strengthens the buoyancy force, which causes an increase in fluid velocity [6].

Figures 4 and 5 show the effects of variable viscosity (λ) on the velocity and temperature profiles. It can be seen that both fluid temperature and velocity decrease in the channel with increasing the viscosity ($\lambda < 0$). This is physically true since boosting the viscosity can create impedance in the free flow of fluid particles and as a result, the fluid velocity is decreased. This consequently diminishes the viscous dissipation heating across the fluid sections and accordingly, the fluid temperature decreases with increasing the viscosity [19].

The effects of thermal conductivity parameter (γ) on the velocity and temperature profiles are shown in figures 6 and 7. As shown in these figures, increasing the thermal conductivity leads to an increase in both fluid temperature

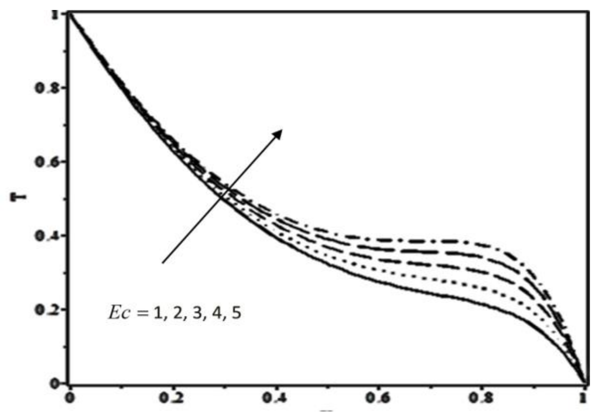

Figure 2. The temperature profile for binary mixture of carbon dioxide in air at different values of Eckert number ($Q = -10, N = 1, Kc = 0.7, \lambda = -0.1$ and $\gamma = 0.5$)

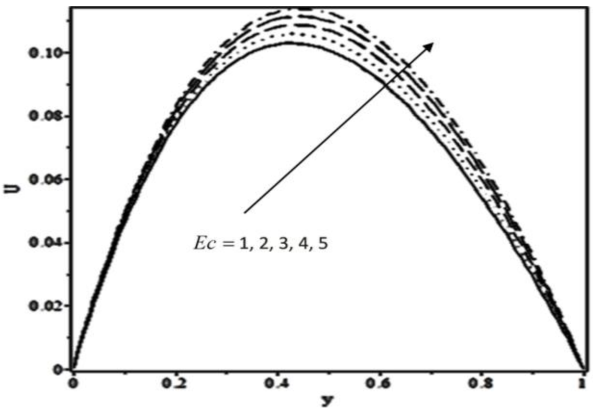

Figure 3. The velocity profile for binary mixture of carbon dioxide in air at different values of Eckert number ($Q = -10, Kc = 0.7, N = 1, \lambda = -0.1$ and$\gamma = 0.5$)

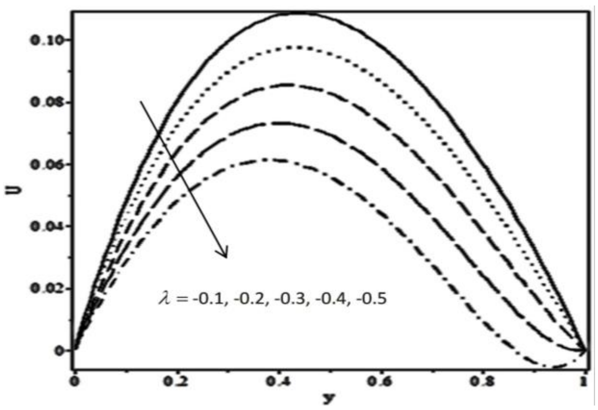

Figure 4. The velocity profile for binary mixture of carbon dioxide in air at different values of viscosity parameter ($Q = -10, Kc = 0.7, N = 1, Ec = 0.2$ and $\gamma = 0.5$)

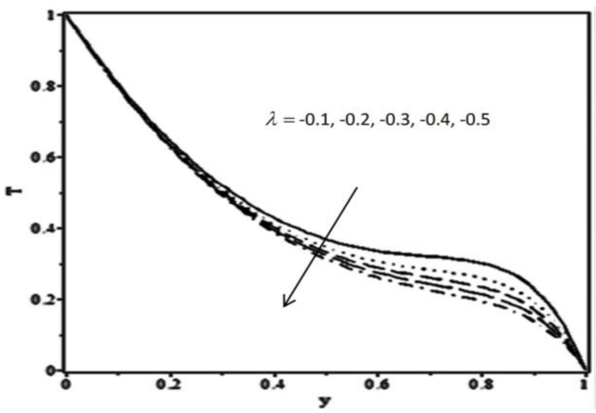

Figure 5. The temperature profile for binary mixture of carbon dioxide in air at different values of viscosity parameter ($Q = -10, Kc = 0.7, N = 1, Ec = 3$ and$\gamma = 0.5$

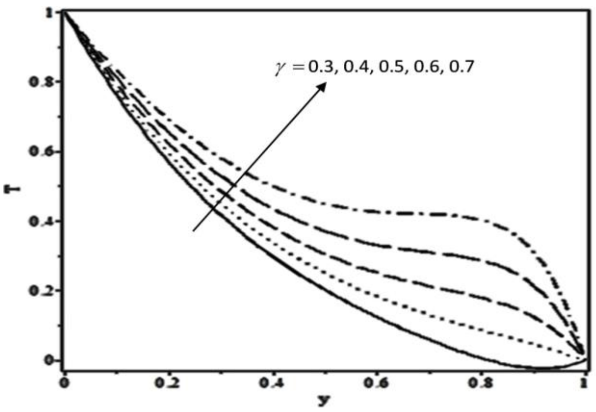

Figure 6. The temperature profile for binary mixture of carbon dioxide in air at different values of thermal conductivity parameter ($Q = -10, N = 1, Kc = 0.7, \lambda = -0.1$ and $Ec = 0.2$)

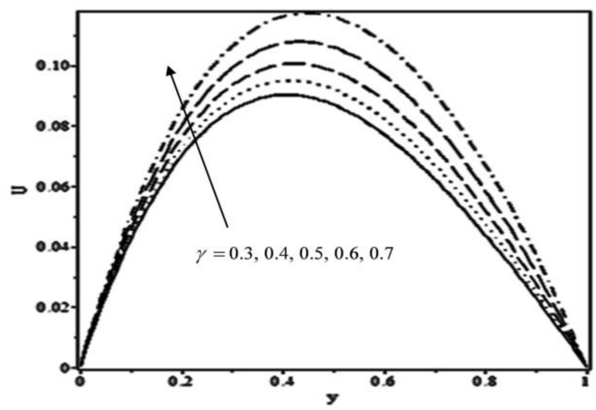

Figure 7. The velocity profile for binary mixture of carbon dioxide in air at different values of thermal conductivity parameter ($Q = -10, N = 1, Kc = 0.7, \lambda = -0.1$ and$Ec = 0.2$)

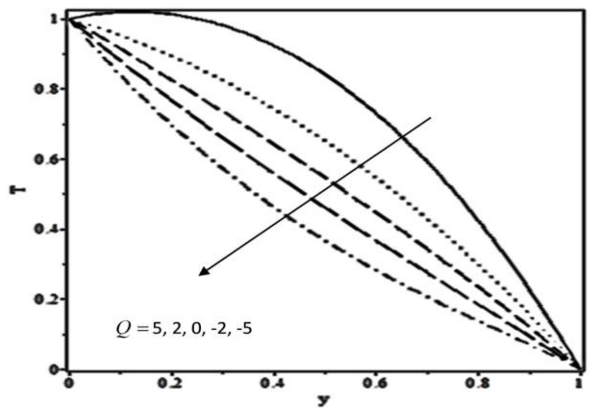

Figure 8. The temperature profile for binary mixture of carbon dioxide in air at different values of heat source/heat sink parameter ($\gamma = 0.5, N = 1, Kc = 0.7, \lambda = -0.1$ and $Ec = 0.2$)

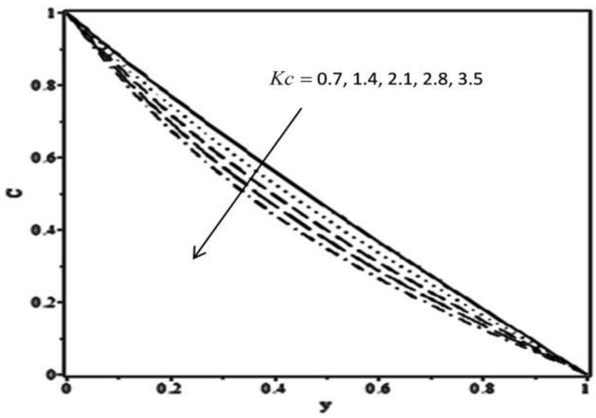

Figure 11. The concentration profile for binary mixture of carbon dioxide in air at different values of chemical reaction parameter

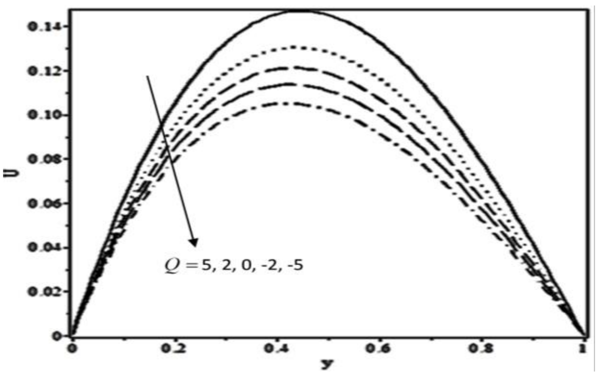

Figure 9. The velocity profile for binary mixture of carbon dioxide in air at different values of heat source/heat sink parameter ($\gamma = 0.5, N = 1, Kc = 0.7, \lambda = -0.1$ and $Ec = 0.2$)

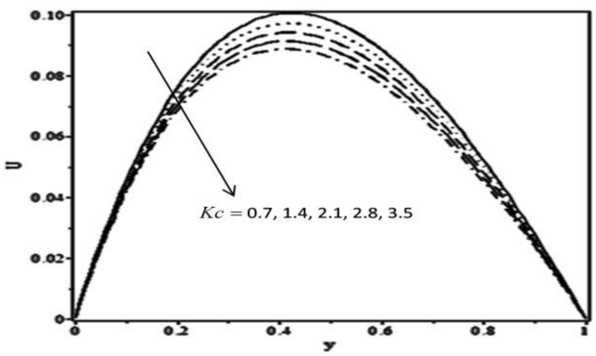

Figure 12. The velocity profile for binary mixture of carbon dioxide in air at different values of chemical reaction parameter ($\gamma = 0.5, N = 1, Q = -10, \lambda = -0.1$ and $Ec = 0.2$)

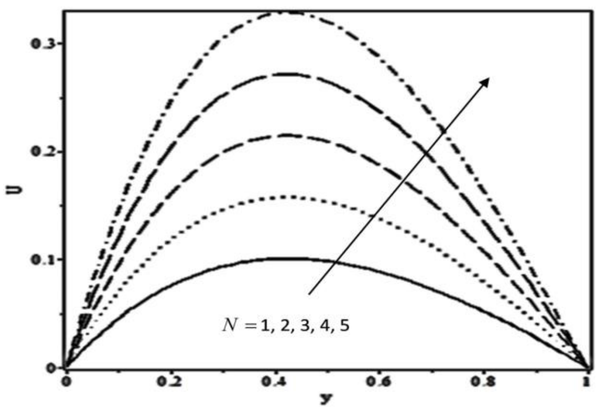

Figure 10. The velocity profile for binary mixture of carbon dioxide in air at different values of Buoyancy ratio parameter ($\gamma = 0.5, Q = -10, Kc = 0.7, \lambda = -0.1$ and $Ec = 0.2$)

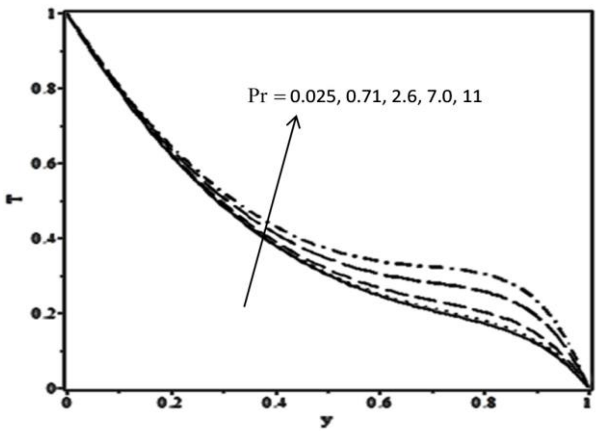

Figure 13. The temperature profile at different values of Prandtl number ($\gamma = 0.5, Ec = 0.2, N = 1, \lambda = -0.1$ and $Q = -10$)

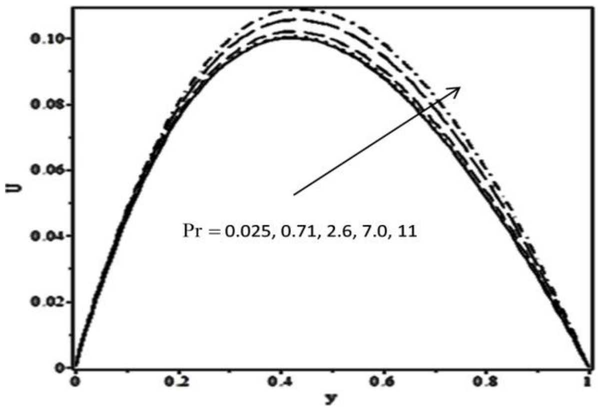

Figure 14. The velocity profile at different values of Prandtl number ($\gamma = 0.5, Kc = 0.7, N = 1, Ec = 0.2, \lambda = -0.1$ and $Q = -10$)

and velocity in the channel. This is expected since the thermal flux received by the fluid is increased as a result of increase in thermal conductivity. This strengthens the convection currents and as a result, the fluid velocity increases with increasing the thermal conductivity [15].

The effects of heat source/sink parameter (Q) on the velocity and temperature profiles are illustrated in figures 8 and 9. It is observed that the fluid temperature and velocity are increased in the channel with increasing the heat source parameter $(Q > 0)$. This is physically true since increasing the heat source parameter amplifies the applied temperature, which causes an increase in the fluid temperature. In addition, the convection current is strengthened in the channel as the heat source parameter increases and this leads to an increase in fluid velocity. Furthermore, increasing heat sink $(Q < 0)$ decreases both fluid temperature and velocity [13].

The influences of Buoyancy ratio parameter (N) and chemical reaction parameter (K_C) on the velocity and temperature profiles are illustrated in figures 10 to 12. It can be seen that an increase in the buoyancy ratio increases the fluid velocity, while increasing the chemical reaction parameter decreases both fluid velocity and temperature. In general, increasing the Buoyancy ratio parameter increases the buoyancy due to the mass transfer and hence, an increase in fluid velocity can be observed (See figure 10). In addition, increasing the chemical reaction causes a decrease in the fluid concentration (See figure 11) and this weakens the convection due to the mass transfer and decreases the fluid velocity as shown in figure 12.

The effects of Prandtl number (Pr) on the temperature and velocity profiles are illustrated in figures 13 and 14. As shown in these figures both fluid temperature and velocity increase with increasing the Prandtl number. This is physically true since the thermal diffusivity of the working fluid decreases with increasing the Prandtl number. This is therefore a hindrance to the diffusion of heat generated by the viscous dissipation at every section of fluid in the channel, which leads to heat accumulation and accordingly, increase in the temperature. In addition, the increase in the temperature achieved by the strengthening of the convection current causes an increase in the fluid velocity.

Table 5 represents the skin friction, heat transfer rate, and volumetric flow rate for different values of Eckert number (Ec). Increasing the viscous dissipation leads to an increase in the volumetric flow rate as well as the skin friction on the surface of both walls of the channel. This is the transitive effect of increase in the temperature achieved by the thermal accumulation that characterizes increasing the viscous dissipation and the fluid velocity. The consequent increase in the velocity leads to the increase in the volumetric flow rate and skin friction. It is also observed that amplifying the viscous dissipation causes a decrease in the heat transfer on the heated plate and an increase in the heat transfer on the cold plate. This is linked to the increase in the temperature in the channel, which affects the temperature gradients on the channel plates.

Table 6 shows the skin friction, heat transfer rate, and the volumetric flow rate for different values of viscosity parameter $(\lambda < 0)$. With increasing the viscosity of the fluid, the volumetric flow rate and the skin friction on the surface of both plates are decreased. However, the heat transfer rate increases on the heated wall $(y = 0)$, while it decreases on the cold wall $(y = 1)$ with increasing the fluid viscosity. This is physically true since an increase in the fluid viscosity causes a decrease in the velocity, which can decrease the volumetric flow rate in the channel. The decrease in the fluid velocity caused by increasing the viscosity is also responsible for the reduction in the skin friction on the walls.

Table 7 shows the skin friction, heat transfer rate, and volumetric flow rate for different values of thermal conductivity parameter (γ). With increasing the thermal conductivity of fluid, volumetric flow rate, the skin friction, and the heat transfer on both plates are increased. This is due to the physical fact that an increase in the thermal conductivity enhances the thermodynamics in the channel and this increases heat flux into the channel. The increase in temperature due to increase in thermal conductivity also strengths the convection current and increases the velocity. As a result, the mass flux and skin friction are increased on both plates.

Table 8 shows the skin friction, heat transfer rate, and volumetric flow rate for different values of heat source/sink parameter (Q). With increasing the heat source parameter $(Q > 0)$, the volumetric flow rate and the skin friction are increased on both plates. It is also observed that the heat transfer rate decreases on the heated wall $(y = 0)$, while it increases on the cold wall $(y = 1)$. In addition, increasing the heat sink parameter $(Q < 0)$ decreases the volumetric flow rate and the skin friction on the plates which, is due to the decrease in the velocity caused by increasing the heat sink. The heat transfer rate increases on the heated wall $(y = 0)$, while it decreases on the cold wall $(y = 1)$ with increasing the heat sink parameter.

Table 2. The comparison between the skin frictions of the present method (DTM) and those provided by Jha and Ajibade [22] at $\lambda = \gamma = Q = Ec = Kc = 0$ and different values of buoyancy parameter (N)

N	Jha and Ajibade [22] , $d_f = 0$		Present study	
	τ_0	τ_1	τ_0	τ_1
0	0.33333333	0.16666667	0.33333333	0.16666667
0.25	0.41666667	0.20833333	0.41666667	0.20833333
0.5	0.50000000	0.25000000	0.50000000	0.25000000
0.75	0.58333333	0.29166667	0.58333333	0.29166667
1.0	0.66666667	0.33333333	0.66666667	0.33333333

Table 3. The comparison between the skin frictions of the DTM method with those achieved by the exact solution at$y = 0, = \gamma = Ec = 0, Kc = 0.7, N = 1$, and $Sc = 0.94$

Q	DTM	EXACT
0.1	0.655147833313604	0.655147833313541
0.2	0.657435061832565	0.657435061832500
0.3	0.659767268997745	0.659767268997679
0.4	0.662145861842601	0.662145861842533
0.5	0.664572307295862	0.664572307295786

Table 4. The comparison between the velocities of the DTM method with those achieved by the numerical method at$\lambda = -0.1, \gamma = 0.5, Ec = 0.2, Sc = 0.94, \Pr = 0.71, N = 1, Q = -0.5$, and $Kc = 0.7$

y	DTM	NUMERICAL
0	0.000000000000000	0.000000000000000
0.25	0.100388349549281	0.100388337919645
0.5	0.116587533290355	0.116586744834451
0.75	0.074151747280852	0.074148882742424
1.0	0.000000000000000	0.000000000000000

Table 5. The mass flow rates, skin frictions, and Nusselt numbers for different values of Eckert number (Ec) at$Sc = 0.94, N = 1, Q = -0.5, Kc = 0.7, \lambda = -0.1$, and $\gamma = 0.5$

Ec	V_m	τ_0	τ_1	Nu_0	Nu_1
0.1	0.07757924	0.66364140	0.32158317	1.41895531	1.15982965
0.2	0.07760062	0.66377439	0.32167404	1.41420817	1.16239981
0.3	0.07762213	0.66390844	0.32176270	1.40944297	1.16507948
0.4	0.07764357	0.66404270	0.32184699	1.40466610	1.16781815
0.5	0.07766472	0.66417628	0.32192474	1.39988409	1.17056471

Table 6. The mass flow rates, skin frictions, and Nusselt numbers for different values of viscosity parameter (λ) at$Sc = 0.94, N = 1, Q = -0.5, Kc = 0.7, \gamma = 0.5, Ec = 0.2$, and $\Pr = 0.71$

λ	V_m	τ_0	τ_1	Nu_0	Nu_1
-0.1	0.07760062	0.66377439	0.32167404	1.41420817	1.16239981
-0.3	0.06796781	0.64527341	0.30107759	1.41686210	1.16118637
-0.5	0.05823279	0.59375553	0.27633396	1.41900036	1.16017950
-0.7	0.04839651	0.56909664	0.24786360	1.42072362	1.15905503
-0.9	0.03843201	0.38922403	0.21598969	1.42218034	1.15695493

Table 7. The mass flow rates, skin frictions, and Nusselt numbers for different values of conductivity parameter (γ) at $Sc = 0.94, N = 1, Q = -0.5, Kc = 0.7, \lambda = -0.1, Ec = 0.2$, and $\Pr = 0.71$

γ	V_m	τ_0	τ_1	Nu_0	Nu_1
0.1	0.07519457	0.65068569	0.30815899	1.20617979	0.97297861
0.3	0.07653434	0.65808657	0.31552162	1.30694111	1.06954100
0.5	0.07760062	0.66377439	0.32167404	1.41420817	1.16239981
0.7	0.07837943	0.66776749	0.32644243	1.53711823	1.24572372
0.9	0.07885946	0.67006983	0.32962353	1.68603319	1.31381662

Table 8. The mass flow rates, skin frictions, and Nusselt numbers for different values of heat source/heat sink parameter (Q) at $Sc = 0.94, N = 1, Kc = 0.7, \lambda = -0.1, \gamma = 0.5, Ec = 0.2$, and $\Pr = 0.71$

Q	V_m	τ_0	τ_1	Nu_0	Nu_1
0.5	0.08039954	0.67957648	0.33665198	1.11543975	1.32982654
0.3	0.07981779	0.67629796	0.33352590	1.17687699	1.29429215
0.1	0.07924708	0.67307884	0.33046553	1.23746401	1.25979947
0.0	0.07896584	0.67149146	0.32895985	1.26744114	1.24293816
-0.1	0.07868735	0.66991881	0.32747041	1.29720879	1.22633066
-0.3	0.07813855	0.66681748	0.32454007	1.35612027	0.32454007
-0.5	0.07760062	0.66377439	0.32167404	1.41420817	1.16239981

Conclusion

In this study, the influences of viscous dissipation on the natural convection through a vertical channel were investigated. The effects of chemical reaction, heat source/sink, and variations in the thermo-physical properties of fluid, such as temperature dependent viscosity and thermal conductivity, were taken into account. The governing equations for temperature, velocity, and concentration fields were solved analytically using the DTM. The results were verified with the results from the exact and numerical methods and an excellent agreement was found. The main results are summarized as follows:

1. Increasing the viscous dissipation causes an increase in both fluid temperature and velocity in the channel.
2. Increasing the fluid viscosity decreases the fluid velocity and the temperature in the channel.
3. Skin friction increases with increasing the thermal conductivity of the working fluid.
4. The volumetric flow rate can be controlled effectively using the variations in the viscosity and the thermal conductivity of the working fluid.
5. Increasing the heat source parameter causes an increase in the skin friction on the plates, while an inverse trend can be observed with increasing the heat sink parameter.
6. The results achieved by this study are of great importance to science and technology especially in polymer industries. Furthermore, the viscous heating can provide basis for further investigation in magma, poiseuille, lava, and Coquette flows in tubes and channels.

Nomenclature

c_p	specific heat at constant pressure [$w.k/kg.s$]
D_m	mass diffusivity
Ec	Eckert number
g	acceleration due to gravity [m/s^2]
h	distance between the walls [m]
Kc	chemical reaction parameter
k	thermal conductivity [$W/m.k$]
K_0	thermal conductivity at $T=T_0$ [$W/m.k$]
N	buoyancy ratio parameter
Nu_0	Nusselt number at y=0
Nu_1	Nusselt number at y=1
Pr	Prandtl number
Q	heat source parameter
Sc	Schmidt number
T'	Temperature [K]
T'_1, T'_0	hot wall/ cold wall temperature [K]
u'	dimensional velocity component
u	dimensionless velocity component
x'	vertical coordinate [m]
y'	horizontal coordinate [m]

Greek Symbols

μ	viscosity of the fluid [$kg.m^{-1}.s^{-1}$]
μ_0	viscosity at $T=T_0$ [$kg.m^{-1}.s^{-1}$]
β	thermal expansion coefficient of fluid [K^{-1}]
$\bar{\beta}$	volume expansion coefficient
ρ	Density [$kg.m^{-3}$]
θ	dimensionless temperature
λ	viscosity parameter

γ thermal conductivity parameter
υ_0 kinematic viscosity of fluid [$m^2.s^{-1}$]
τ_0 skin friction at y=0
τ_1 skin friction at y=1

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