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A Method for Identifying Congestion Using Non-Radial Models in Data Envelopment Analysis

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Abstract

Data envelopment Analysis) DEA) is a nonparametric method that aims to use scientific methods in order to investigate the performance of Decision Making Unit (DMU). One of the interesting subjects in DEA is estimation of congestion of DMUs. Congestion problem has an important role in economy because the congestion mostly occurs in the input which is itself cost based, thus by eliminating congestion, the cost reduces as well. Therefore, detecting and reducing congestion both have many benefits. Many methods have been proposed to detect congestion in DEA. In this paper, we are going to present a new method to identify congestion based on the definition of congestion. The proposed method in this paper has the following advantages compare with other methods; Firstly, it is less complicated than other methods. Secondly, our method is able to identify the congestion and its degree correctly.thirdly the new model identifies the congestion of all units. Finally, the proposed method in this paper has non-radial movement.

Keywords: Data envelopment analysis, Efficiency, Inefficiency, Congestion, non-Radial models

1. Introduction

Data envelopment analysis (DEA) as a non-parametric technique has been widely used to measure the relative efficiency of a set of similar decision making units (DMUs) which was introduced in the year 1987 by Charnes and Cooper et al. (CCR model) [1]. Banker, Charnes and Cooper developed a variable returns to scale that was called BCC model in 1984 [2]. Congestion is often involved in the real practice which describes the case whereby the decrease of one (or some) inputs will cause the maximum possible increase of one (or some) outputs without worsening any other input or output [3].

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Proceeding in reverse, congestion occurs when the increases in one or more inputs can be associated with the decreases in one or more outputs without improving any other input or output. Fare et al. proposed a model to analysis congestion under the framework of DEA, which is widely recognized today [4]. FGL (Fare, Grosskopf and Lovell) approach uses a radial Farrell measure and is based on the assumption of strong and weak disposability of input [5]. It has been employed by a lot of studies into congestion. Cooper et al. proposed an alternative approach to measure congestion [6]. This approach is extended by Brockett et al. and was known as BCSW model [7]. BCSW model is a slack-based model which can recognize the congestion and identify sources of congestion and estimate congestion amounts [8]. Wei and Yan [9] and Tone and Sahoo [10] rebuilt the production possibility set (PPS) and the corresponding DEA model to determine the congestion effect of the DMUs. Jahanshahloo and Khodabakhshi develop a model based on relaxed combinations of inputs to make good input combinations and measure the amount of input congestion in textile industry of China [11]. Khodabakhshi provided a one-model approach of input congestion based on input relaxation model [12]. Sueyoshi and Sekitani proposed a modified approach which is able to measure congestion under the occurrence of multiple solution [13]. Noura et al. presented a new method for measuring congestion [14]. Kao utilize Wei-Tone model to measure the congestion effect of Taiwan forests [15]. Jahanshahloo et al. [16] and Khodabakhshi et al. [17] proposed some methods for computing the congestion in DEA models with production trade-offs and weight restrictions. There exist some papers which reviewed congestion papers, as that of Khodabakhshi et al. [18]. Hajihosseini et al. proposed a new approach for measuring the congestion in DMUs by using common weights based on comparison of inputs [19]. Some advantages and disadvantages of the aforementioned methods are mentioned below. FGL and BCSW Methods identify the congestion of the decision making unit but are not able to recognize the congestion type. In addition, FGL method may recognize the congestion of the decision making unit when there is no congestion, but may not recognize the congestion when it exists. In addition, Tone and Sahoo method is one of the best methods to identify the congestion in DEA. The first advantage of this method is to identify congestion condition (strong or weak) in a decision making unit. In addition, this method can accurately identify congestion for the DMUs. It should be noted there are a lot of problems in the presented methods to identify the congestion problems. For example, Tone and Sahoo method considers all inputs and outputs of DMU positively which is rarely happen in real world.This paper unfolds as follows. Section 2 provides some theoretical considerations. In section 3, we are going to present a new method to identify congestion based on the definition of congestion. Section 4 is devoted to a numerical example with real data and finally section 5 presents some concluding remarks.

2. Preliminaries

Suppose that there exist n decision making units, DMU_j, $j = 1, \cdots, n$, and each DMU consumes m inputs to produce s outputs. The i th input and r th output for DMU_j are denoted by x_{ij} and y_{rj}, respectively for $i = 1, \cdots, m$ and $r = 1, \cdots, s$. We assume that all input and output values are non-negative, and at least one of each is non-zero. Let $DMUo = (xo, yo)$ be the unit under assessment. The production Possibility Set (PPS) with variable returns to scale (VRS) defined Banker et al. is as follows [2]

$$T_v = \left\{ (x, y) | x \geq \sum_{j=1}^{n} \lambda_j x_j, \;\; y \leq \sum_{j=1}^{n} \lambda_j y_j, \;\; \sum_{j=1}^{n} \lambda_j = 1, \;\; \lambda_j \geq 0, \;\; j = 1, \cdots, n \right\}.$$

The output-oriented BCC model for evaluating efficiency score of DMU_o in its envelopment form is as follows:

$$
\begin{aligned}
&\psi^* = \max \rho + \epsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_i^+) \\
s.t. \ &\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} && i = 1, \cdots, m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \rho y_{ro} && r = 1, \cdots, s \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \geq 0 && j = 1, \cdots, n \\
&s_i^- \geq 0 && i = 1, \cdots, m \\
&s_r^+ \geq 0 && r = 1, \cdots, s \qquad (2.1)
\end{aligned}
$$

Where ϵ is the non-Archimedean constant and variables s_i^- and s_r^+ are slacks. Now, we review three main methods for estimation of congestion in the DEA framework.

2.1. FGL model

Fare, Grosskopf and Lovell proposed the model to estimate congestion using the following model (called FGL model) [5]:

$$
\begin{aligned}
&\beta^* = \max \beta \\
s.t. \ &\sum_{j=1}^{n} \lambda_j x_{ij} = \tau x_{io} && i = 1, \cdots, m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \geq \beta y_{ro} && r = 1, \cdots, s \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \geq 0 && j = 1, \cdots, n \\
&0 \leq \tau \leq 1 && \qquad (2.2)
\end{aligned}
$$

Suppose that ψ^* and β^* represent the optimal objective values of model (2.1) and (2.2) respectively. It is clear that $\beta^* \leq \psi^*$, therefore $\frac{\beta^*}{\psi^*} \leq 1$. If $\frac{\beta^*}{\psi^*} < 1$ then DMU_o is congested, Otherwise, the congestion will not be recognized using FGL model.

2.2. CTT model

Cooper et al. proposed an alternative DEA approach, called CTT, to estimate the congestion [6]. First, solve model (2.2) to evaluate DMU_o. Suppose that $(\rho^*, s^{-*}, s^{+*}, \lambda^*)$ is the optimal solution for this model. Now, determine the benchmark of DMU_o on the strongly efficient frontier of T_v using the following formulation:

$$
\begin{aligned}
&\widehat{x_{io}} = x_{io} - s_i^{-*} && i = 1, \cdots, m \\
&\widehat{y_{ro}} = \rho^* y_{ro} + s_r^{+*} && r = 1, \cdots, s \qquad (2.3)
\end{aligned}
$$

It is clear that, if DMU_o is on the strongly efficient frontier of T_v, then $\widehat{x_0} = x_0$, $\widehat{y_0} = y_0$. Then, solve model (2.4) to determine the extent of the input congestion.

$$
\begin{aligned}
& \max \sum_{i=1}^{m} \delta_i^- \\
s.t.\ & \sum_{j=1}^{n} \lambda_j x_{ij} - \delta_i^- = \widehat{x_{i0}} && i = 1, \cdots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \widehat{y_{r0}} && r = 1, \cdots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 && j = 1, \cdots, n \\
& 0 \leq \delta_i^- \leq s_i^{-*} && i = 1, \cdots, m
\end{aligned} \tag{2.4}
$$

Thus, the extent of input congestion related to input x_{io} can be described as:

$$s_i^c = s_i^{-*} - \delta_i^{-*}, \qquad i = 1, \cdots, m. \tag{2.5}$$

2.3. TS model

Tone and Sahoo defined PPS (called P_{convex}) as follows [10]:

$$P_{convex} = \left\{ (x, y) | x = \sum_{j=1}^{n} \lambda_j x_j,\ \ y \leq \sum_{j=1}^{n} \lambda_j y_j,\ \ \sum_{j=1}^{n} \lambda_j = 1,\ \ \lambda_j \geq 0,\ j = 1, \cdots, n \right\} \tag{2.6}$$

Model (2.7) evaluates the efficiency score of DMU_o with respect to P_{convex}:

$$
\begin{aligned}
& \Phi^* = \max \Phi + \epsilon (\sum_{i=1}^{m} s_r^+) \\
s.t.\ & \sum_{j=1}^{n} \lambda_j x_{ij} = x_{io} && i = 1, \cdots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \Phi y_{ro} && r = 1, \cdots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 && j = 1, \cdots, n \\
& s_r^+ \geq 0 && r = 1, \cdots, m
\end{aligned} \tag{2.7}
$$

The benchmark of DMU_o on the strongly efficient frontier of P_{convex} is as follows:

$$
\begin{aligned}
& \widehat{x_{io}} = x_{io} && i = 1, \cdots, m \\
& \widehat{y_{ro}} = \Phi^* y_{ro} + s_r^{+*} && r = 1, \cdots, s
\end{aligned} \tag{2.8}
$$

Definition 2.1. *The unit $DMU_o = (x_o, y_o) \in P_{convex}$ is strongly efficient with respect to P_{convex}, if $\Phi^* = 0$.*

Definition 2.2. *Suppose that $DMU_o = (x_o, y_o) \in P_{convex}$ is strongly efficient with respect to P_{convex}. This DMU is strongly congested if there exists an activity $(\widehat{x_o}, \widehat{y_o}) \in P_{convex}$ such that $\widehat{x_o} = \alpha x_o$ $(0 < \alpha < 1)$ and $\widehat{y_o} \geq \beta y_o (\beta > 1)$.*

Definition 2.3. *The unit $DMU_o = (x_o, y_o) \in P_{convex}$ is (weakly) congested if it is strongly efficient with respect to P_{convex}. and there exists an activity in P_{convex} that uses less resources in one or more inputs for making more products in one or more outputs.*

Tone and Sahoo proposed the following method to determine the DMU's situation of strong and weak congestion [10].
Step 1: Solve model (2.1). Therefore, we have:

(a) If $\rho^* = 1$, $s^{-*} = 0$, $s^{+*} = 0$, then $DMU_o = (x_o, y_o)$ is BCC-efficient and not congested.

(b) If $\rho^* = 1$, $s^{-*} \neq 0$, $s^{+*} = 0$, then $DMU_o = (x_o, y_o)$ is BCC-inefficient.

(c) If $\rho^* = 1$, $s^{+*} \neq 0$ or $\rho^* > 1$, then $DMU_o = (x_o, y_o)$ displays congestion. Go to step 2.

Step 2: solve model (2.9):

$$
\begin{aligned}
& \bar{u} = \max u_0 & \\
s.t. \; & \sum_{i=1}^{m} u_r y_{ro} = 1 & i = 1, \cdots, m \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_o \leq 0 & j = 1, \cdots, n \\
& \sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} + u_o = 0 & \\
& u_r \geq 0 & r = 1, \cdots, s \\
& v_i \;\; free & i = 1, \cdots, m \\
& u_o \;\; free &
\end{aligned}
\tag{2.9}
$$

Suppose that $\bar{u}$ is the optimal value of model (2.9), also assume that $\bar{\rho} = 1 + \bar{u}$. If $\bar{\rho} < 0$ then DMU_o is strongly congested. If $\bar{\rho} \geq 0$ then DMU_o is weakly congested.

3. Proposed Model

The definition of congestion requires to be discussed in both decrement and increment of inputs. In other words, DMU_p has congestion, if and only if the following two conditions happen simultaneously

1- Decrement of the input leads to increment of the output for at least one component.

2- Increment of the input leads to decrement of the output for at least one component of output.

Suppose that there exist n decision-making units (DMU_j for $j = 1, \cdots, n$) and each DMU uses m inputs to produce s outputs. The ith input and rth output for DMU_j are specified by $x_{ij}(i = 1, \cdots, m)$ and $y_{rj}(r = 1, \cdots, m)$, respectively. Banker et al. defined PPS (called) with variable return to scale (VRS) as follows [2]:

$$T_v = \left\{ (x, y) | x \geq \sum_{j=1}^{n} \lambda_j x_j, \;\; y \leq \sum_{j=1}^{n} \lambda_j y_j, \;\; \sum_{j=1}^{n} \lambda_j = 1, \;\; \lambda_j \geq 0, \; j = 1, \cdots, n \right\}$$

Because the principle of disposability of inputs is incompatible with the definition of congestion, therefore, we eliminate this principle from PPS to define congestion.

$$T_N = \left\{ (x, y) | x = \sum_{j=1}^{n} \lambda_j x_j, \;\; y \leq \sum_{j=1}^{n} \lambda_j y_j, \;\; \sum_{j=1}^{n} \lambda_j = 1, \;\; \lambda_j \geq 0, \; j = 1, \cdots, n \right\}$$

DMU_p is congested, if and only if:

$$\forall (\bar{x}, \bar{y}) \big((\bar{x}, \bar{y}) \in T_N; \;\; \bar{x} \leq_{\neq} x_p \Longrightarrow \bar{y} \geq y_p \big)$$

$$\forall (\bar{\bar{x}}, \bar{\bar{y}}) \big((\bar{\bar{x}}, \bar{\bar{y}}) \in T_N; \;\; \bar{\bar{x}} \geq_{\neq} x_p \Longrightarrow \bar{y} \leq y_p \big)$$

We consider whether there is a possibility of increasing the outputs in DMU_p or not.

$$\begin{aligned}
& Z^* = \max \frac{1}{s} \sum_{r=1}^{s} \Phi_r \\
s.t. \; & \sum_{j=1}^{n} \lambda_j x_{ij} = t_i x_{ip} && i = 1, \cdots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \Phi_r y_{rp} && r = 1, \cdots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 && j = 1, \cdots, n \\
& \Phi_r \geq 1 \\
& t_i \geq 0
\end{aligned} \tag{3.1}$$

The constraints $t_i \geq 0$, $\Phi_r \geq 1$ are caused that increasing in outputs is considered with decreasing and increasing in inputs.

$$
\begin{aligned}
& Z^* = \max \frac{1}{m}\sum_{i=1}^{m} s_i \\
s.t. \;& \sum_{j=1}^{n} \lambda_j x_{ij} = s_i x_{ip} && i = 1, \cdots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \Phi_r^* y_{rp} && r = 1, \cdots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 && j = 1, \cdots, n \\
& s_i \geq 0 && (3.2)
\end{aligned}
$$

Now, by solving model (3.2), we understand whether there exists a point (x, y) in T_N such that at least one of the components of inputs and outputs of this unit be greater than the components of inputs and outputs of DMU_p.

Theorem 3.1. *Suppose that* (λ^*, Φ^*, t^*) *is an optimal solution for model* (3.1)*,* $(\hat{\lambda}, \hat{s})$ *is an optimal solution for model* (2.2) *and also* $\Phi^* \geq (1, \cdots, 1)$*,* $\hat{s} \leq (1, \cdots, 1)$ *then* DMU_p *is congested.*

3.1. Recognizing the strong and weak congestion

Suppose that (λ^*, Φ^*, t^*) is an optimal solution for model (3.1), $(\hat{\lambda}, \hat{s})$ is an optimal solution for model (3.2) and also $\Phi^* \geq (1, \cdots, 1)$, $\hat{s} \leq (1, \cdots, 1)$:
If $\Phi_r^* \geq 1$ for all $r = 1, \cdots, s$ and $\hat{s} < 1$ then DMU_p is strongly congested.
If Φ^* is not greater than 1 and $\hat{s}$ is not less than 1 then we must recognize DMU_p is strongly congested or weakly congested. For this purpose, we solve the following models:

$$
\begin{aligned}
& Z_1^* = \max \epsilon_1 \\
s.t. \;& \sum_{j=1}^{n} \lambda_j x_{ij} = t_i x_{ip} && i = 1, \cdots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \Phi_r y_{rp} && r = 1, \cdots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 && j = 1, \cdots, n \\
& \frac{1}{s}\sum_{r=1}^{s} \Phi_r = Z^{o*} \\
& \Phi_r \geq 1 + \epsilon_1 \\
& \lambda_j \geq 0 && j = 1, \cdots, n \\
& t_i \geq 0 && (3.3)
\end{aligned}
$$

Suppose that $(\lambda^*, t^*, \Phi^*, \epsilon_1^*)$ is an optimal solution for model (3.3), therefore there exist the following cases:
If $\epsilon_1^* = 0$, then all of the components of output of DMU_p cannot be increased. Since $\Phi^* \geq_{\neq} (1, \cdots, 1)$ in the optimal solution for model (3.1) and $\hat{s} \leq (1, \cdots, 1)$ in the optimal solution for model (3.2) then, DMU_p is weakly congested.
If $\epsilon_1^* = 0$ then $\Phi^* \geq 1 + \epsilon_1^* > 1 \; \forall \, r$.
That's mean, all of the components of output can be increased. now, we solve model (3.4) to determine whether all of the components of input of DMU_p can be decreased or not.

$$
\begin{aligned}
& Z_3^* = \max \epsilon_2 \\
s.t. \; & \sum_{j=1}^{n} \lambda_j x_{ij} = t_i x_{ip} && i = 1, \cdots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \Phi_r^* y_{rp} && r = 1, \cdots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \frac{1}{m} \sum_{r=1}^{s} t_i = Z_1^* \\
& t_i + \epsilon_2 \leq 1 \\
& \lambda_j \geq 0 && j = 1, \cdots, n \\
& t_i \geq 0 \\
& \epsilon_2 \geq 0
\end{aligned} \tag{3.4}
$$

If $(\lambda^*, t^*, \epsilon_2^*)$ is an optimal solution for model (3.4), then there exist two the following cases:
If $\epsilon_2^* = 0$, then all of the components of input of DMU_p cannot be decreased.
Since $\hat{s} \leq_{\neq} (1, \cdots, 1)$ in the optimal solution for model (3.2) then, DMU_p is weakly congested.
If $\epsilon_2^* > 0$, then $t_1^* \leq 1 - \epsilon_2^* < 1 \; \forall \, i$
And since, $\Phi_r^* > 1$ for all $r = 1, \cdots, s$ therefore DMU_p is strongly congested and the size of congestion in is equal to $c_1 = 1 - t_1^*$. As regards, in the optimal solution of model (3.4), $t_i^* < 1$ for all $i = 1, \cdots, m$, therefore, $C_i^* > 1$ for all $i = 1, \cdots, m$. note that $x_p > 0$, because if $x_p = 0$ then DMU_p is efficient with respect to T_v, hence this unit is not congested. We consider a case that (λ^*, Φ^*, t^*) is the optimal solution for model (3.1) in which $\Phi_r^* = 1$ for all $r = 1, \cdots, s$. As regards, the components of output of units can't be increased in T_v then DMU_p is not congested. Also in the optimal solution (λ^*, Φ^*, t^*) for model (2.1) and in the optimal solution $(\hat{\lambda}, s^*)$ for model (3.2), there exists I such that $\hat{s}_i < 1$ then DMU_p is not congested because, $\Phi_r \geq 1$ for all $r = 1, \cdots, s$ cause that by increasing of the ith component of input of DMU_p, all of the components of output of this unit are not decreased.

4. Numerical examples

To illustrate the algorithm process in this section, we apply the algorithm to real data. An example of 16 basic research institutes in the Chinese Academy of Science (CAS) in 2010 from G. L. Yang (2015) is considered in this example. The inputs and outputs in this study are presented in Table 1.

Table 1: The inputs and outputs

ID	variables	type of variables
1	**Staff:** Full-time equivalent of full-time research staff.	**Input**
2	**Res. Expend:** Amount of total income of each institute.	**Input**
3	**SCI Pub:** Number of international papers indexed by the Web of Science from Thompson Reuters.	**Output**
4	**High Pub:** Number of high-quality papers published in top research journals (e.g., journals with a top 15% impact factor)	**Output**
5	**Graduate Enroll:** Number of graduate student enrolment in 2009.	**Output**
6	**Extern. Fund:** Amount of external research funding from research contracts.	**Output**

The real data of two inputs and four outputs are shown in Table 2.

Table 2: Input-output data

DMU	Staff	Res. Expen	SCI	High	Grad. E	Exter. Fund.	Φ_o^*	η^*
DMU_1	252	117.945	436	133	184	31.558	1	0.2004
DMU_2	37	29.431	243	127	43	15.3041	1	0.2004
DMU_3	240	101.425	164	70	89	33.8365	1.1835	0.1485
DMU_4	356	368.483	810	276	247	183.8434	1	0.4200
DMU_5	310	195.862	200	55	111	12.9342	1.9684	0.3046
DMU_6	201	188.829	104	49	33	60.7366	1.6499	0.3910
DMU_7	157	131.301	113	49	45	72.5368	1.0437	0.3442
DMU_8	236	77.439	8	1	44	23.7015	1	0
DMU_9	805	396.905	371	118	89	216.9885	1	0.0878
DMU_{10}	886	411.539	607	216	168	88.5561	1	0
DMU_{11}	623	221.428	314	49	89	45.3597	1	0
DMU_{12}	560	264.341	261	79	131	41.1156	1.4478	0.1715
DMU_{13}	1344	900.509	627	168	346	645.4150	1	0
DMU_{14}	508	344.312	971	518	335	205.4528	1	0.3134
DMU_{15}	380	161.331	395	180	117	90.0373	1	0.1308
DMU_{16}	132	83.972	229	138	62	32.6111	1.3371	0.2374

First, we solve model (3.2). This model determines that DMU_3, DMU_5, DMU_8, DMU_9, DMU_{10}, DMU_{11}, DMU_{12}, DMU_{15}, DMU_{16} are strongly congested. Then, we solve model (3.3) and recognize that DMU_1 and DMU_4 are weakly congested. Finally, model (3.4) is solved and is determined that DMU_2 and DMU_3 are weakly congested.

5. Conclusion

There are several approaches for dealing with input congestion measurement in DEA. A more comprehensive definition of congesting in this paper was proposed by introducing two conditions for existence of congestion. Based on the first condition decrement of the input leads to increment of the output and in second condition increment of the input leads to decrement of the output. Models (3.1) and (3.2) were introduced based on the proposed definitions and model (3.3) and (3.4) should be solved in order to identify strong or weak congestion. According to numerical examples, for some DMUs, the decrement of the input was indicated, which led to increment of the output but increment of the input leads to increment of the output as well. Most of the congestion methods such as Cooper recognize this unit as congestion but based on the proposed definition in this paper, it doesn't have congestion because the proposed method in this paper has this advantage to identify congestion with both increasing and decreasing input. In addition, our proposed method can determine that congestion has occurred in which unit of input and to what extent. Moreover, this method can determine which output unit and to what extent will increase if congestion is eliminated. The proposed models in this paper are non-radial and have the ability of identifying strong or weak congestion.

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