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Presenting a Differential Equation Strengthened by the Cross Braces for Roof System

Nobakht Bakhtiari*[a], Nohali Agayof[b]

[a]Assistant prof. of Islamic Azad University, Marageh Branch, Marageh, Iran
[b]Faculty of Civil Engineering, Azerbaijan Architecture and Construction University, Baku, Azerbaijan

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Abstract

Plate girders structures is a major industry, much used in the systems covered by these beams, reduce deflection and decrease the maximum bending moment can be considered as a development methodology that way, by the cross beams or braced beams element perpendicular to the girders is provided. In this paper, the effect of stiffness and distance elements development (braced beams) are studied. Differential equation of degree four governing the behavior of the partial structures a quadratic relationship is determined by solving the following mathematical transformations and the fundamental equations are derived. The resulting equations are solved by a practical problem and the answer to the problem. The modeling results are compared with the SAP2000 program shows that highly accurate implementation and compliance. It is thus stiffness changing elements within the cross can be different amounts of stress, bending moment and improved cover and roof system and regulation forces to be calculated. In the proposed method by changing several parameters affecting. Calculate the amount of stress and reduced bending moments and an optional stress adjustment achieved. Other factors, including the backing, boundary conditions, a subsidiary of loading and shooting the junction depth of the main beam are effective in regulating stress.

Keywords: Girders, Strengthened, Brace, Stiffness.
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1. INTRODUCTION

Plate girders used in building construction may be economical for spans p to about 100 m. larger building spans may require used of a truss. Web Thin flexural members, influenced by the flexural

*Nobakht Bakhtiari
Email address: Nobakht.bakhtyari@gmail.com (Nobakht Bakhtiari*)

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behavior of stiffness and place the cross beam and the ability to side. Two - dimensional structure of the beam for plate industrial floor systems. We can add elements and new components perpendicular to the bearing element a two-dimensional structure, the entire structure for increased stiffness. These members can be either horizontal or vertical, existing structures located between the structure and behavior to transforms space. The creation of three-dimensional structures of the upper connections in two directions perpendicular to communicate and connect.

Extensive research on the slender web plates, was started an in the past several decades has. (since 1965). Sadovsky a method to determine is based on the principle of stability energy to measure the degree of distortion and deformation effects of errors in his research focus is on. However, some aspects of the issue is still unresolved[1], Hu and Cui, Simple analytical formulas designed to compare the ultimate strength of reinforced and non-paid cards[2], Lee and Paik[3] a method for the treatment of reinforced plastic plate under different loads are not final until reaching their final strength. [4]

Kovesdi et al. (2012) carried out research into the effect of cross section on the life of these beams and provided simplifications for uniform loads [5]. In 2013, Hassanein et al. Proposed a new relationship to increase the shear buckling resistance of the wings by interacting with the wings by rigid coupling and showed that this resistance should be corrected in normal relations [6]. According to Bassler and Porter's theory, which form the basis of the AISC and BS5950 Regulations, respectively, shear strength of beam sheets can be plotted in two buckling and post-buckling sections and what's important is that the ultimate shear capacity and post-buckling capacity increases with increasing die thickness. But the implementation of this method of reinforcing and refining the sheets of a roof covering is not without its drawbacks and is associated with specific fractures. subsequent research has shown that corrugated sheets can be used to increase the capacity of sheet beams [7]. It can also be used to strengthen and increase the strength of girders, used prestressed method and reduced the rise of the existing loads and increased loads, on the other hand, despite the high strength of the girders, Instability against high deformation and general and local buckling, one of the disadvantages of this type of beam is especially for the purpose of improvement one recent study to address this problem is that of Lee et al. [8]. In another study by Granath, using finite element analysis results, a method for increasing the strength of non-hardening girders has been proposed [9]. Also Graciano, by incorporating more and more complete parameters, provides relationships to determine the shear strength of existing sheets with horizontal or longitudinal hardeners. which was very complex and compared to the Makovich and Hojdin method, there has been a lengthy process of calculations to estimate the shear strength of existing girders. [10, 11]

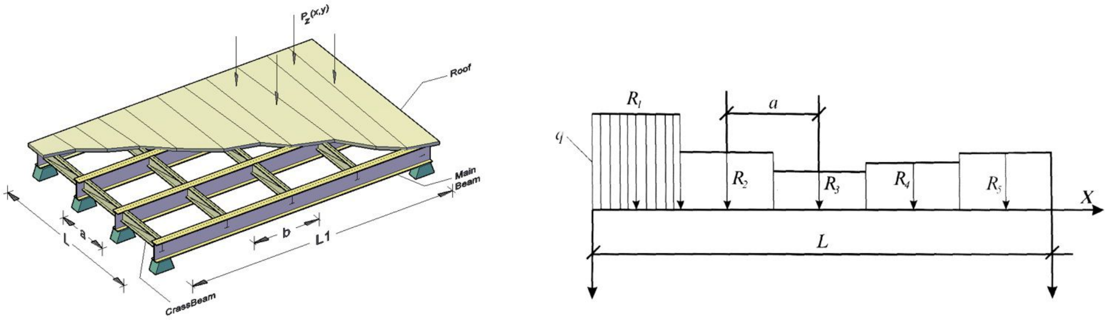
Change the behavior of a two-dimensional structure to a space structure, which leads to increased stiffness in the whole space, deflection of structure is reduced. His one of the key points in the linear elastic behavior of the steel structure of the elements in the project resistant Structures be paired with a particular perspective is investigated in this paper.

2. THE DIFFERENTIAL EQUATION GOVERNING

Transverse beam envelopes (inhibitory) girder or trusses perpendicular to each other are located a distance equal to the original figure 1. We assume Profile stiffness and load them equal reaction forces exerted by the main beams images onto the transverse beams, as Figure 2 shows a transverse beam at the crossing point, optionally, be deformation differential equation can be written [12]:

$$Z_i = +\beta \frac{QL^3}{EI_i} - \alpha \frac{R_j L^3}{EI_j} \tag{2.1}$$

Z_i: deflection of the girder or truss, Q: loads the main beam, R_j: reaction transverse beam response of the main beam, E: modulus of elasticity of materials



a) b)

Figure 1: Girders in roof systems, cross beam reactions on the main beam, a) main girders, b) Secondary girders

Ii: Moment of inertia for main beam, α and β are coefficients related to the profile of times coefficients can be pre-set according to the type of load and rest. If the primary beam is not less than 5 digits it will still be photo single recipe can be substituted with massive loads [13], ie q=R_j/a q: a massive load of intensity.
Thus, for the exist beams (main beams) in order to have i shot at the intersection of the main beam transverse nodes j and, optionally, a set of a.b from equation (2.2) obtained:

$$Z_i = +\beta_i \frac{QL^3}{EI_i} - \frac{L^3}{EI_j} \sum_{j=1}^{m} \alpha_j . R_j \tag{2.2}$$

In the above equation m number of beams or trusses to roof system is added to the cross. Placement and simplify:

$$q = \frac{d^2 M}{dx^2} = EI_j \frac{d^4 z}{dx^4} \quad , \qquad R_j = aq = aE\dot{I}_j \frac{d^4 z}{dx^4}$$

$$Z_i = +\beta_i \frac{QL^3}{EI_i} - \frac{a\ L^3}{EI_j} \sum_{j=1}^{m} \alpha_j EI_j \left(\frac{d^4 z}{dx^4} \right) \tag{2.3}$$

3. TRANSFORMATIONS MATHEMATICS

Equation (2.3) is rather difficult and complicated mathematical transformations can be used to solve it. If we assume that both the number of beams (or trusses) and high and stiffness in both directions are equal. Then solving the problem would be more easily and solve differential equation of degree 4 will continue as follows:
From figure 1 for each node in the intersection of the beams, as $q.a$.b will load. In the general case q(x, y), once node at the intersection of the incoming beam is split into two parts to. The R-value for the main beams parallel to the x-axis and y-axis is parallel to the transverse beam for R will be

$$R + \overline{R} = q\,(x, y)\,ab$$

$$aEI_x \frac{\partial^4 \omega}{\partial x^4} + bEI_y \frac{\partial^4 \omega}{\partial y^4} = q\,(x, y)\,ab \tag{3.1}$$

Considering the size of the system created in two directions x and y, and simulate a structural steady, consistent performance space, three-dimensional can skip the grid ceiling system as an integrated suite written as [14, 15]:

$$\omega = f \sin\frac{\pi x}{l}\ .\sin\frac{\pi y}{l_1} \tag{3.2}$$

ω: deflection of any part of the roof f: deflection in the middle of the roof l_1 and l: length of main and cross beams. Placement relation (3.1) in equation (3.2) ordinary differential equations can be obtained from the results of its solution equivalent to the solution of the differential equation (2.3) will be. But before continuing resolution, the following is required to apply mathematical transformations. For the general case, equation (3.2) can be written as:

$$\omega = \sum X_n\left(x\right) Y_n(y) \tag{3.3}$$

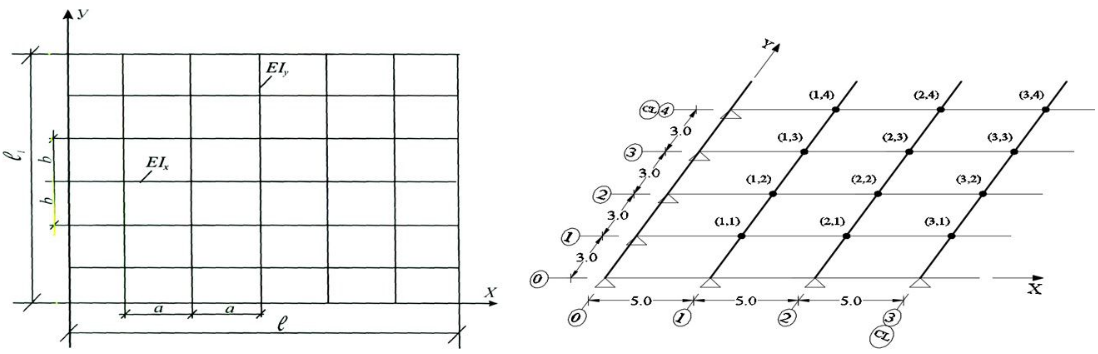


Figure 2: Grid ceiling system with the main beams, secondary beams added

Derivates X_n (x) with respect to the constant factor change of the properties of iterative and can be written as follows:

$$\frac{d^4 X_n\left(x\right)}{dx^4} = \eta_n^4 X_n\left(x\right) \tag{3.4}$$

Using orthogonal properties of these relationships make it possible to the right side of equation (3.1) to form a series of related mathematical functions X_n (x), wrote as follows:

$$q\left(x, y\right) ab = b \sum P_n\left(y\right) X_n(x) \tag{3.5}$$

P_n (y) function arguments can be y. To determine the expression P_n (y) can be related to (3.5), X_n (x) multiplied by the parties and following the integration.

$$\int_0^l q\left(x, y\right) ab X_n(x) dx = b \sum P_n\left(y\right) \int_0^l X_n(x) X_n(x) dx$$

Here for X_n (x) using orthogonal properties considered property and can write

$$P_n\left(y\right) = a \frac{\int_0^l q\left(x, y\right) X_n\left(x\right) dx}{\int_0^l X_n^2(x) dx} \tag{3.6}$$

Placement relation (3.3) in equation (3.1) and taking into account the relationship (3.4) relationship be equation:

$$\sum X_n\left(x\right)\left[bEI_y Y_n^{IV}\left(y\right) + \eta_n^4 a EI_x Y_n\left(y\right) - P_n\left(y\right) b\right] = 0 \tag{3.7}$$

$$E\dot{I}_y Y_n^{IV}\left(y\right) = P_n\left(y\right) - k_n Y_n\left(y\right) k_n = EI_x \eta_n^4 \frac{a}{b} \tag{3.8}$$

Equation (3.7) is the reaction beam on elastic foundation. Where the parameter Y_n can be determined from the condition of an anchor. Thus, instead of a complicated equation (2.3) can be ordinary differential equations (3.4) and (3.7) used to determine the values of X_n (x) and Y_n (y) by letting them in equation (3.3), ω values will be calculated.
The values obtained for two and three times differentiable with respect to x and y and the bending moment and shear force are calculated in both directions - and stress levels than a two-dimensional roof, can be obtained and comes with a choice of a transverse beams provide distances and stiffness adjusting stress applied to the main beam is .

4. Application

Equation (3.4) will include: $\lambda^4 - \eta_n^4 = 0$ the roots of the equation $\lambda_1 = \eta_n$; $\lambda_2 = -\eta_n$; $\lambda_3 = i\eta_n$; $\lambda_4 = -i\eta_n$
$e^{\eta_n \ x}$; $e^{-\eta_n}$; $e^{i\eta_n{}^x}$; $e^{-i\eta_n{}^x}$
Private solutions:
Will combine private answer as the general solution of the equation will be as follows.

$$X_n(x) = A\ ch\ \eta_n{}^x + B\ sh\ \eta_n{}^x + Ccos\eta_n{}^x + Dsin\eta_n{}^x \tag{4.1}$$

The integral equation (3.7) and applying the boundary conditions, the coefficients of equation (3.7) to determine and apply mathematical transformations and integration:

$$\frac{d^4 Y_n(y)}{dy^4} + 4\beta_n{}^4 Y_n(y) = \frac{P_n(y)}{EI_y} \tag{4.2}$$

$$\beta_n{}^4 = \frac{k_n}{4EI_y} \ ; \ \beta_n = \sqrt[4]{\frac{a}{48}\frac{EI_x}{EI_y}} \cdot \frac{n\pi}{l} \tag{4.3}$$

To solve equation (4.2) used hyperbolic functions simplify the integration of optional requirements are as follows:

$$\overline{A}_{0n} = -\frac{aq}{n\pi\beta_n{}^4 EI_y} \tag{4.4}$$

$$\overline{B}_{0n} = -\frac{aq}{n\pi\beta_n{}^4 EI_y} \cdot \frac{sin2\beta_n l_1 - 2chs\beta_n l_1}{cos2\beta_n l_1 - ch2\beta_n l_1}$$

$$\overline{C}_{0n} = -\frac{aq}{n\pi\beta_n{}^4 EI_y} \cdot \frac{sh2\beta_n l_1 - 2shc\beta_n l_1}{cos2\beta_n l_1 - ch2\beta_n l_1}$$

Finally, the general solution of the equation will be deflection by the following:

$$\omega = \sum_{n=1} sin\frac{n\pi}{l}x\left(\frac{aq}{n\pi^4\beta_n{}^4 EI_y} + \overline{A}_{0n}chc\beta_n y + \overline{B}_{0n}chs\beta_n y + \overline{C}_{0n}shc\beta_n y\right) \tag{4.5}$$

Here

$$chc\beta_n y = ch\beta_n y.cos\beta_n y$$

$$chs\beta_n y = ch\beta_n y.sin\beta_n y$$

$$shc\beta_n y = sh\beta_n y.cos\beta_n y$$

$$shs\beta_n y = sh\beta_n y.sin\beta_n y$$

Using the *relations* M_x=-EI_x ω''_x, Q_x=-EI_x ω'''_x *and* M_y =-EI_y ω''_y, Q_y=-EIy ω'''_y can be shear and moments applied to determine the values of the reaction of rely on the confluence of the main and cross beams are also calculated.

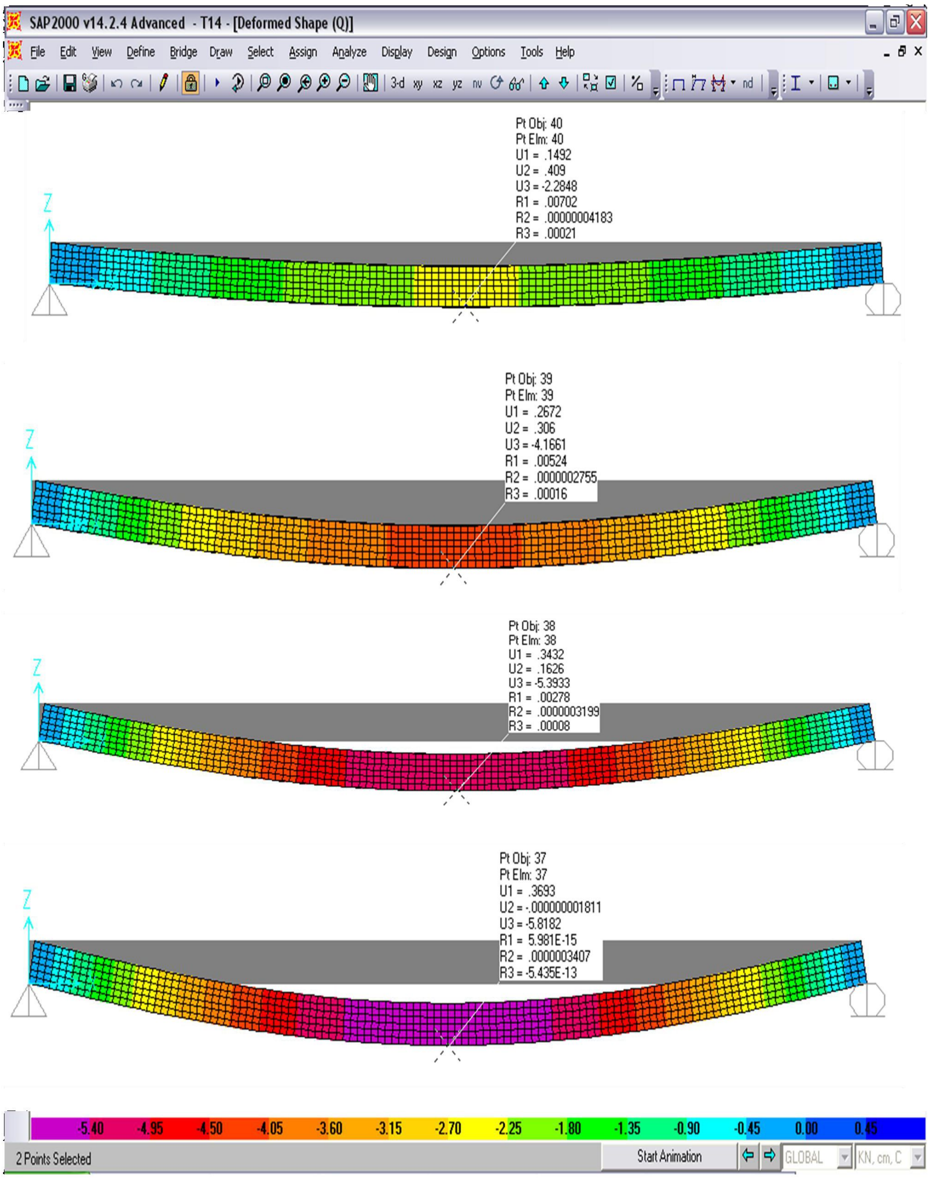


Figure 3: results computed by sap 2000 for deflections

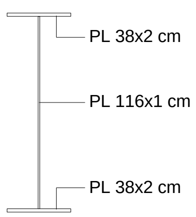
5. Problem Solving

To plan and cross section shown in Figure 2, and cross section girder in Figure 2 to regulate stresses, beams with a length of 30 m in the x beam transverse (strengthening) of length L = 24 m in the y considered are to be masts Existing Beam distance of 3 meters in length direction and 5 meters in the transverse direction considered .Flexural stiffness in the two directions is assumed to be equal to the load of the roof covering q = 5 KN \ m^2. Reduce deflection and bending moment roof, state without transverse beams and cross beams, compared to. For various nodes, using the Excel program to skip the relations given in this paper were calculated using the computer program as well as the modeling Sap2000 [16], The results are shown in Table 1.

Table 1: Calculate the deflection and moment at the junction of the main and secondary beams

$\beta_n = \sqrt[4]{\frac{a}{48}\frac{EI_x}{EI_y}} \cdot \frac{n\pi}{l}$ ***Equation (17)***	$\beta_1 = 0{,}0842$ $\beta_3 = 0{,}2526$ $\beta_5 = 0.421$ $\beta_1^{\,2} = 0.00709$ $\beta_3^{\,2} = 0.0638$ $\beta_5^{\,2} = 0.1772$ $\beta_1^{\,4} = 5 \times 10^{-5}$ $\beta_3^{\,4} = 4.07 \times 10^{-3}$ $\beta_5^{\,4} = 3.1414 \times 10^{-2}$		
Equation (19)	$\bar{A}_{01} = -\frac{159155}{\boldsymbol{EI_y}}$	$\bar{A}_{03} = -\frac{652}{\boldsymbol{EI_y}}$	$\bar{A}_{05} = -\frac{51}{\boldsymbol{EI_y}}$
	$\bar{B}_{01} = -\frac{42097}{EI_y}$	$\bar{B}_{03} = -\frac{70}{\boldsymbol{EI_y}}$	$\bar{C}_{05} = \frac{51}{\boldsymbol{EI_y}}$
	$\bar{C}_{01} = \frac{173298}{\boldsymbol{EI_y}}$	$\bar{C}_{03} = \frac{649}{\boldsymbol{EI_y}}$	$\bar{C}_{05} = \frac{51}{\boldsymbol{EI_y}}$

***Sections Properties*:**



$I_x = 653100$ cm^4
$E = 2{,}1 \times 10^4$ KN /cm²

$\omega_{1,1} = \omega_{1,5} = 1.25$ cm
Equation (20)

	1	2	3	4	5
1	1.25	2.10	2.4	2.1	1.25
2	2.20	3.85	4.41	3.85	2.2
3	2.94	4.99	5.11	4.99	2.96
4	3.13	5.32	5.85	5.32	3.13
5	2.94	4.99	5.11	4.99	2.2
6	2.2	3.85	4.41	3.85	2.2
7	1.25	2.10	2.4	2.10	1.25

$M_{x(1,1)} = M_{x(1,5)} = 224$ KN. m
Equation (21)

	1	2	3	4	5
1	224	292	314	292	224
2	391	543	282	543	391
3	489	707.5	766	707.5	482
4	579	754	824	754	579
5	486	707.5	766	707.5	482
6	391	543	585	543	391
7	224	292	314	292	284

Table 1 shows the maximum deflection ω and maximum moment M_x at node (4,3) in the middle of the roof system is given.
in simple case $\omega = \frac{5ql^4}{384EI_y} = 11.42$ cm and $M_{max} = \frac{ql^2}{8} = 1675.8$ KN.m

Table 2: Comparing the proposed method with the results from SAP2000

	ω			M_x			M_y			Calculation method`
node	1	2	3	1	2	3	1	2	3	
1	*1.25*	*2.10*	*2.40*	*224*	*292*	*314*	*345*	*545*	*612*	*Analytical*
	1.19	*2.00*	*2.28*	*218*	*290*	*308*	*336*	*537*	*610*	*SAP 2000*
2	*2.20*	*3.85*	*4.41*	*391*	*543*	*585*	*537*	*903*	*1024*	*Analytical*
	2.16	*3.65*	*4.17*	*390*	*537*	*578*	*531*	*897*	*1018*	*SAP 2000*
3	*2.94*	*4.99*	*5.11*	*489*	*708*	*766*	*634*	*1045*	*1265*	*Analytical*
	2.78	*4.71*	*5.39*	*483*	*701*	*762*	*625*	*1038*	*1259*	*SAP 2000*
4	*3.13*	*5.32*	*5.85*	*579*	*754*	*824*	*666*	*1160*	*1335*	*Analytical*
	3.00	*5.08*	*5.82*	*574*	*752*	*821*	*661*	*1156*	*1331*	*SAP 2000*

The results computed by SAP 2000 for deflections shows in figure 3 (U3=5.81 cm).
The table 2 the computer program SAP2000 analysis results with the results of the method presented in this paper, has been compared. The results show excellent compatibility with each other to. It's shows the maximum deflection ω and maximum moment M_x at node (4,3) in the middle of the roof system is given by at the digits that are not strength towards strengthening the state has declined by 50%.
In Figure 4, the amount of deflection with other parameters constant It is calculated, its shown by reducing the distance between cross beams, of their effectiveness in reducing deflection, low.

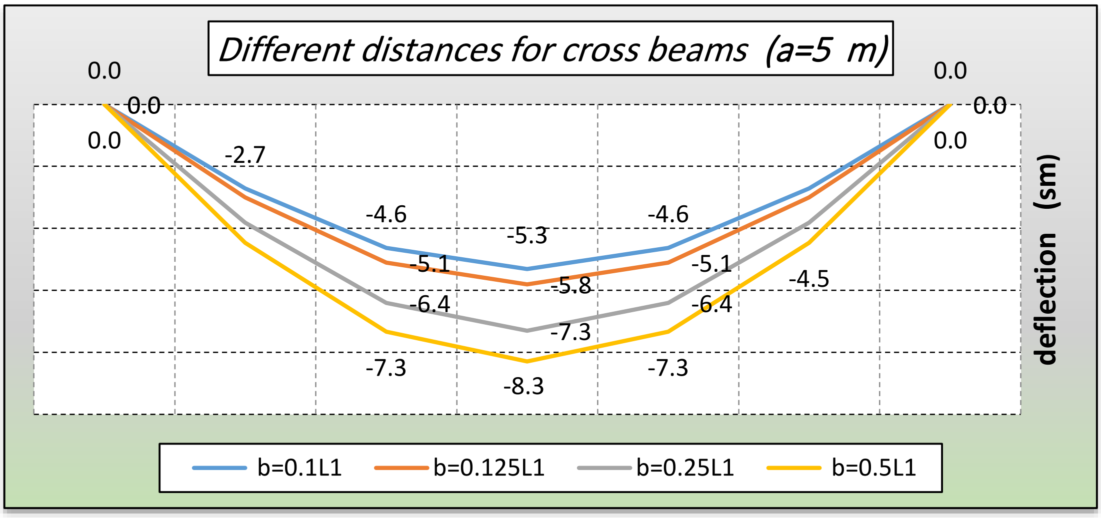


Figure 4: The effect of stiffness transverse element

6. DISCUSSION AND CONCLUSION

With the increasing number of cross beams and reduce the distance between them decreases and their impact on economic development does not comply. The optimal distance for the cross beams, can be determined for each type of roof system.

In the proposed method by changing several parameters affecting. Calculate the amount of stress and reduced bending moments and an optional stress adjustment achieved. Other factors, including the backing, boundary conditions, a subsidiary of loading and shooting the junction depth of the main beam are effective in regulating stress. Other results can be referred to as follows:

1. The determination of the differential equation governing the behavior of the main beams and cross beams at the intersection, additional roof load bearing system of ordinary differential equations into providing analytical solution, accurate results are obtained by solving equations the results were compared with the results of computer detailed analysis, precisely coincide
2. With constant load and execute the appropriate technology, we can reduce the amount of bending moment and shear force exerted
3. Conversion from two-dimensional to three-dimensional structural behavior and the constant bending moment and shear force we can increase the load on the roof structure
4. Beam ceiling plan, the physical and geometrical characteristics of beams and trusses secondary reinforcement in tension adjust have a direct impact.

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