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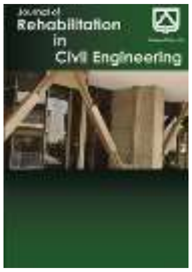

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Damage Detection of Structures Using Modal Strain Energy with Guyan Reduction Method

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ABSTRACT

The subject of structural health monitoring and damage identification of structures at the earliest possible stage has been a noteworthy topic for researchers in the last years. Modal strain energy (MSE) based index is one of the efficient methods which are commonly used for detecting damage in structures. It is also more effective and economical to employ some methods for reducing the degrees of freedom in large-scale structures having a large number of degrees of freedom. The purpose of this study is to identify structural damage via an index based on MSE and reconstructed mode shapes. The Guyan reduction method (GRM) is utilized here to reconstruct the mode shapes. Therefore, in the first step by employing GRM, mode shapes in slave degrees of freedom are estimated by those of master degrees of freedom. In the second step, the modal strain energy based index (MSEBI) is used to find the location of damaged elements. In order to assess the efficiency of the method, two standard examples are considered. Damage is identified with considering complete mode shapes and reconstructed mode shapes, and the results are compared together. The outcomes show that the combination of MSE and GRM can be useful for the structural damage detection, when considering the noise.

1. Introduction

One of the most essential stages of structural maintenance is to accurately identify the locations and severity of the damage. The damage may occur due to several reasons such as the lack of correct principles of construction, lack of specific regulations in structural design, the high age of structures and the lack of proper maintenance.

Structural damage will change some properties such as the stiffness and ductility of structures, which leads to change vibrational frequencies, mode shapes,

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damping ratio, and so on [1-4]. By considering this law, damage in the structure can be identified through a damage identification method that can prevent spreading damage in the structure, and it can also increase the structural lifetime. Many approaches for damage identification in steel, composite and beam-like structures have been developed in recent years [5-10]. Some researchers [11-12] have been applied damage identification to reinforced concrete structures.

During recent years, modal strain energy has been commonly utilized to identify damage in structures by many researchers. Shi et al. [13] used the change of MSE to identify the damage in structural elements. The results showed that the proposed method is effective for damage localization. Liu et al. [14] conducted a practical study for detecting the damage in the wind turbines using the improved strain energy method. The experimental results indicated that the presented approach could properly identify the damage location for different damage cases. Entezami and Shariatmadar [15] utilized a damage detection technique in structural systems using the improved sensitivity of modal strain energy and the Tikhonov regularization method. Pradeep et al. (2014) also addressed the modal strain energy in the identification of the honeycomb sandwich structure [16]. Esfandiari (2014) presented a model updating method to obtain parameters of the structures using the frequency domain representation of the strain data. The results showed that the method is appropriate for updating the structural models [17]. Seyedpoor and Yazdanpanah [18] could identify the multiple damage locations in structures using the change of strain energy based on static noisy data. They showed the effectiveness of the proposed indicator for finding the damage site by numerical examples. Moradipour et al. (2015) used an improved MSE procedure to detect damage location in 2D structures [19]. They mathematically developed the MSE method, and then a beam and a two-dimensional frame were used to indicate the efficiency of the method. The results illustrated that the method is a reliable approach to detect damage considering five modes of the structure. Shan & Zhou (2015) applied a model updating method to update a cable-suspension bridge prototype based on a surrogate model and the substructure technique [20]. Seyedi et al. (2015) presented an energy-based damage detection algorithm based on modal data. The outcomes showed the damaged areas detected by the proposed method coincide well with the damage scenarios [21]. Yazdanpanah et al. (2015) presented a new damage index based on mode shape data to detect damage in beam structures. The outcomes illustrated the proposed indicator had a good performance in detecting the damage [22]. Yan et al. [23] performed research regarding damage identification based on an appropriate algebraic algorithm of element modal strain energy sensitivity. They considered the noise effects on their study, and the outcomes showed that the mentioned method is suitable in a one-step procedure. Li et al. [24] developed an improved MSE method for damage detection in offshore platform structures. Wu et al. [25] were presented a novel MSE-based damage detection method in beam structures. They performed numerical and experimental analyses to illustrate the advantages of the proposed algorithm.

Yazdanpanah et al. (2018) [26] presented a technique for beam-column structures considering axial load effects. They used the Guyan method to update the structural models. The outcomes illustrated the locations of damage cases can be well determined in low axial loads. Gomes et al. (2018) proposed a method to locate sensor placement for structures considering Fisher information matrix and mode shape interpolation [27]. The outcomes illustrated that the presented approach is capable to distribute a reduced number of sensors on a structure. Khatir et al. (2018) proposed a new method for crack identification using vibration analysis based on model reduction. The outcomes demonstrated that the POD-RBF combined with the Cuckoo search algorithm is a reliable method for detecting damage in CFRP beams [28]. Khatir and Wahab (2018) investigated crack detection and quantification using model updating method based on POD-RBF approach combined with Jaya algorithm. THE Outcomes illustrate high stability for noise levels up to 4% [29].

Khatir et al. (2019) presented a two-stage technique for damage identification in beam-like structures. They proposed a new damage index based on normalized MSE index to locate the damaged elements. The results show that the presented method can be used to identify properly both damage location and severity in beam-like structures [30]. Damage detection in carbon fiber reinforced polymer plates via reduced mode shapes and GA-ANN methods investigated by [31]. They achieved the reduced mode shape by Fisher information matrix. The outcomes illustrated the proposed method is efficient for estimating delamination position in plates-like structures. Daneshvar et al. (2020) proposed a new and effective approach for detecting, locating, and quantifying beam-like structures based on Rayleigh- Ritz approach. The result showed that the presented method is a reliable tool for damage identification in the beam-like structures [32]. Lale Arefi et al. (2020) introduced a modified modal strain energy based index for damage detection of structures using improved reduction system method. The results showed the proposed index is reliable to identify the location of damage accurately [33]. Lale Arefi and Gholizad (2020) used the modal strain energy method with the System Equivalent Reduction Expansion Process method in truss structures. The results showed the presented method is effective for detecting the damage in truss structures [34].

It should be noted that there are no researches related to damage identification, which combines the Guyan reduction method (GRM) with the modal strain energy. This paper uses the modal strain energy based index to locate damaged elements when a limited number of sensors are installed on the structure. The Guyan method is merely used to decrease the degrees of freedom of the structures. Therefore, the current paper presents an efficient damage detection method based on combining the Guyan reduction method and the MSE method.

2. Modal Strain Energy Based Index Method (MSE Method)

An efficient indicator based on the MSE for damage identification of structures was presented by Seyedpoor [35]. In this method, the modal strain energy of *e*th element in *i*th mode of the structure can be expressed by mse_i^e as

$$\{mse_i^e\} = \frac{1}{2}\{\varphi_i^e\}^{\mathrm{T}}[k^e]\{\varphi_i^e\}, \qquad i = 1, \ldots, ndf,\ e = 1, \ldots, nte \tag{1}$$

where k^e is the stiffness matrix of *e*th element of the structure, ${\varphi^e}_i$ is the vector of corresponding nodal displacements of element *e* in *i*th mode, *ndf* represents the total number of active degrees of freedom and *nte* is the total number of elements. The total modal strain energy of *i*th mode of the structure can also be determined by the summation of MSE for all elements as given below:

$$mse_i = \sum_{e=1}^{nte} mse_i^e \,, \qquad i = 1, \dots, ndf \tag{2}$$

The normalized MSE of elements with respect to the total MSE of the structure can be defined as:

$$nmse_i^e = \frac{mse_i^e}{mse_i} \tag{3}$$

where $nmse^e{}_i$ is the normalized MSE of *e*th element in the *i*th mode of the structure. The mean of Eq. (3) for the first *nm* modes can be chosen as an effective parameter as:

$$mnmse^e = \frac{\sum_{i=1}^{nm} nmse_i^e}{nm} \,, \qquad e = 1, \dots, nte \tag{4}$$

By determining the efficient parameter $mnmse^e$ for each element of healthy structures ($mnmse^h$) and damaged structures ($mnmse^d$), the modal strain energy based index (MSEBI) can be determined as:

$$MSEBI^e = \max\left[0, \frac{(mnmse^e)^d - (mnmse^e)^h}{(mnmse^e)^h}\right], \qquad e = 1, \dots, nte \tag{5}$$

3. Guyan Reduction Method

The Guyan reduction method (GRM) is a technique for reducing the number of degrees of freedom by disregarding the inertial conditions of the equilibrium equations [36]. In this method, the displacement ($\vec{x}$) and force ($\vec{f}$) vectors are divided into sub vectors, and the mass $[M]$ and stiffness $[K]$ matrices are split into the measured (master) and unmeasured (slave) coordinates. Suppose that no force is exerted to the slave degrees of freedom, so [10]:

$$\begin{bmatrix} [M_{mm}] & [M_{ms}] \\ [M_{sm}] & [M_{ss}] \end{bmatrix} \begin{Bmatrix} \ddot{\vec{x}}_m \\ \ddot{\vec{x}}_s \end{Bmatrix} + \begin{bmatrix} [K_{mm}] & [K_{ms}] \\ [K_{sm}] & [K_{ss}] \end{bmatrix} \begin{Bmatrix} \vec{x}_m \\ \vec{x}_s \end{Bmatrix} = \begin{Bmatrix} \vec{f}_m \\ \vec{0} \end{Bmatrix} \tag{6}$$

where the symbols *m* and *s* denote the master and slave coordinates, respectively. By ignoring the inertia terms, the second equation can be expressed as:

$$K_{sm}\,\vec{x}_m + K_{ss}\vec{x}_s = 0 \tag{7}$$

This equation may be used to remove the slave coordinate to leave the following equation:

$$\begin{Bmatrix} \vec{x}_m \\ \vec{x}_s \end{Bmatrix} = \begin{bmatrix} [I] \\ -[K_{ss}]^{-1}[K_{sm}] \end{bmatrix} \vec{x}_m = [T_s]\vec{x}_m \tag{8}$$

The matrix $[T_s]$ refers to the static transformation between the full state vector and the master coordinates, and the $[I]$ is the identity matrix.

Accordingly, Eq. (8) can also be employed to expand the mode shapes as:

$$\begin{bmatrix} \Phi_m \\ \Phi_s \end{bmatrix} = \begin{bmatrix} [I] \\ -[K_{ss}]^{-1}[K_{sm}] \end{bmatrix} [\Phi_m] = [T_s][\Phi_m] \tag{9}$$

It should be mentioned that the masters' degrees of freedom would not be changed as seen by the upper partition of the equation while, the eliminated DOFs are estimated as:

$$[\Phi_s] = \left[-[K_{ss}]^{-1}[K_{sm}]\right][\Phi_m] \tag{10}$$

Though this approach is particularly based on the static stiffness of the system, the mode shape expansion may be accurate or not. Certainly, the Guyan reduction method will produce admissible results when there are enough degrees of freedom to express the mass inertia of the system. On the other hand, it will never generate exact results because of the approximate inherent formulation of the transformation matrix.

The flowchart of two-step method proposed here for identifying structural damaged elements is shown in Fig. 1.

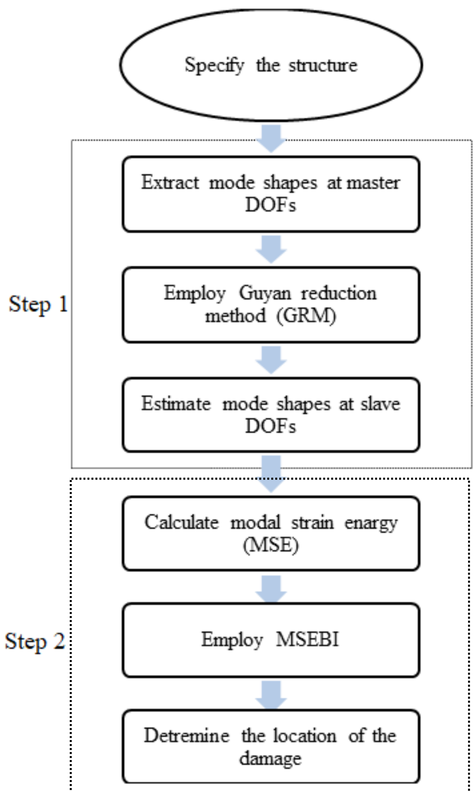

Fig. 1. Flowchart of two-step proposed method.

4. Numerical Examples

In order to assess the performance of the proposed method for locating the damaged elements, two numerical examples are considered. The first example is a 31-bar planar truss, and the second one is a 47-bar planar truss of a power line tower.

4.1. Thirty One-Bar Planar Truss

The 31-bar planar truss shown in Fig. 2 is considered as the first example [37]. The structure consists of 14 nodes, 31 elements and 25 active degrees of freedom after eliminating the constrained degrees of freedom at supports. The elasticity modulus and density of the material for all members are 70GPa and 2770kg/m^3, respectively. In order to simulate damage at each element of the structure, the modulus of elasticity of each element is reduced from the original value by damage ratio. Three different damage cases are listed in Table 1, and the proposed method is tested for each case.

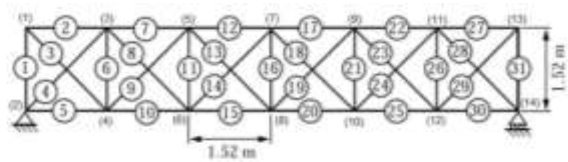

Fig. 2. Planar truss with 31 elements [37].

Table 1. Different damage cases for 31-bar planar truss.

Damage case	Element Number	Damage ratio
Case 1	11	0.25
Case 2	25	0.15
	16	0.30
Case 3	6	0.10
	15	0.05
	26	0.10

In order to evaluate the competence of the method of reducing degrees of freedom for damage detection using the modal strain energy, three different states are applied. The first method named here as ADOF (All Degrees of Freedom) considering the sensors can be placed at all active degrees of freedom. In other words, the response (mode shape) is available at all degrees of freedom. The second and third states named as GRM1 and GRM2 respectively, are based on the Guyan reduction method. However, the placement of sensors is different between the second and third ones to evaluate the effect of sensors' location for identifying damage. Seven sensors are used here for the current example. The master degrees of freedom in the truss are given in Table 2.

To assess the effect of the mode number on the efficiency of the damage detection method, the first 5 and 7 modes of the structure are considered. Values of MSEBI for three damage cases considering 5 and 7 modes are in Figs. 3 and 4, respectively, without considering any noise. As can be observed in the figures, for damage case 1, identifying the location of damage for the state ADOF is more accurate than two other states. For the case 2, all three states can correctly identify the exact location of the damage without error. In the case 3, the identification by the ADOF is better than other ones. In general, for the noiseless condition, it can be said that the identification using the ADOF provides a better performance as compared with those of two other states, and the location of master DOFs in the GRM1 can lead to a better result than the GRM2.

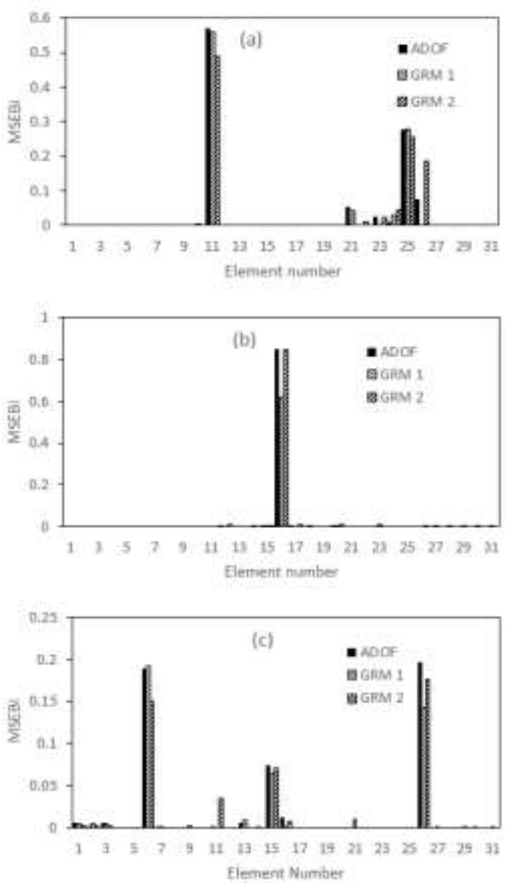

Fig. 3. MSEBI values for different elements of the 31-bar truss considering 5 modes for (a) damage case 1, (b) damage case 2, (c) damage case 3, without considering noise

Table 2. The positions of master degrees of freedom for the 31-bar planar truss

State	Master Degrees of Freedom (node number/direction)						
ADOF	All FEM DOFs						
GRM 1	3y	4y	6x	7y	9y	11x	12y
GRM 2	4x	4y	5y	7y	8y	10y	12x

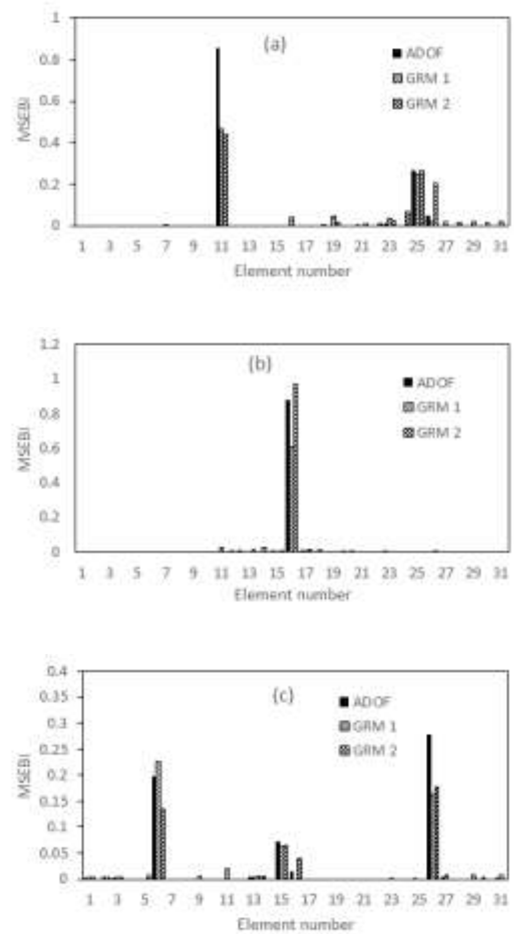

Fig. 4. MSEBI values for different elements of the 31-bar truss considering 7 modes for (a) damage case 1, (b) damage case 2, (c) damage case 3, without considering noise

In order to consider the noise effect on the performance of the proposed method, noise level 3% is applied to mode shapes as [38]:

$$\varphi_i^{\text{noisy}} = \varphi_i(1 + n\beta_i) \tag{11}$$

where φ_i^{noisy} is the i*th* noisy mode shape of the structure, φ_i is i*th* mode shape of structure, n is the noise level, and β_i is a random value between [-1 1].

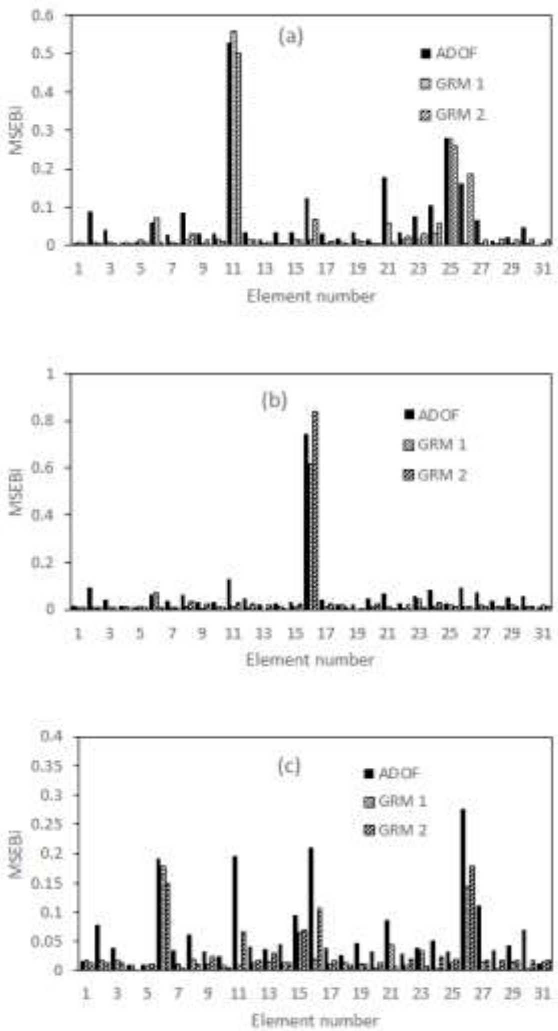

Fig. 5. MSEBI mean values for different elements of the 31-bar truss considering 5 modes for (a) damage case 1, (b) damage case 2, (c) damage case 3, contaminating 3% noise

The outcomes of MSEBI values considering 5 and 7 modes for three damage cases are shown in Fig. 5 and Fig. 6, respectively. In these cases, the values calculated for each element are the average of 100 different runs. Fig. 5 and Fig. 6 indicate that the state of reducing degrees of freedom using the GRM1 and GRM2 has a better performance as compared to that of the ADOF state. It can be interpreted that because the sensors are located on all degrees of freedom; therefore, the responses of all the sensors are contaminated by noise. Hence, ADOF cannot accurately recognize the structural damage compared to the GRM1 and GRM2, which sensors are located on master degrees of freedom. Also, in the GRM1, sensors are placed on better positions than the GRM2 indicating the importance of the placement of sensors on structures to detect the exact locations of damage.

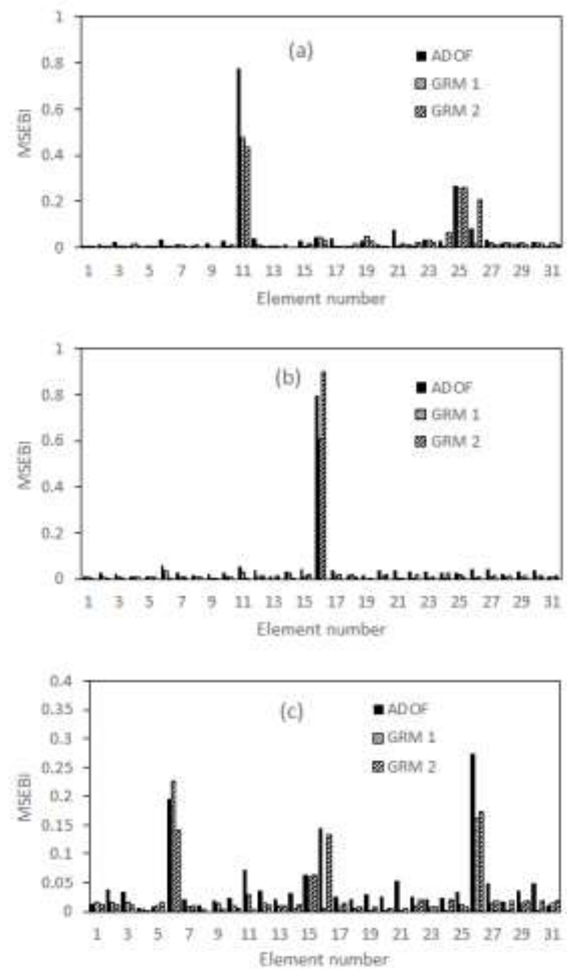

Fig. 6. MSEBI mean values for different elements of the 31-bar truss considering 7 modes for (a) damage case 1, (b) damage case 2, (c) damage case 3, contaminating 3% noise

It can also be noticed from Fig. 3 to Fig. 6 that with considering more modes of the structure, better identification results can be achieved. It can be said that the results will be more accurately if a higher number of modes are considered.

Table 3. The identified elements for the 31-bar truss without considering noise

State	Damage case	5 modes	7 modes
ADOF	Case 1	11, 21, 25, 26	11, 25
	Case 2	16	11, 16
	Case 3	6, 15, 26	6, 15, 26
GRM1	Case 1	11, 12, 24, 25	11, 25
	Case 2	16	16
	Case 3	6, 15, 26	6, 15, 26
GRM26	Case 1	11, 25, 26	11, 25, 26
	Case 2	16	16
	Case 3	6, 15, 26	6, 15, 26

Table 4. The identified elements for the 31-bar truss with considering 3% noise

State	Damage case	5 modes	7 modes
ADOF	Case 1	2, 6, 8, 11, 16, 21, 23, 24, 25, 26, 27, 30	11, 25, 26
	Case 2	2, 11, 16, 24, 26	16
	Case 3	2, 6, 8, 11, 15, 16, 21, 26, 27, 30	6, 11, 15, 16, 26
GRM1	Case 1	6, 11, 21, 25	11, 25
	Case 2	16	16
	Case 3	6, 15, 26	6, 15, 26
GRM2	Case 1	11, 16, 24, 25, 26	11, 25, 26
	Case 2	16	16
	Case 3	6, 11, 15, 16, 26	6, 15, 16, 26

Table 3 and Table 4 show the identified elements for 31-bar truss without considering noise and with considering 3% noise, respectively. The elements whose MSEBI values exceed 0.05 are selected as damaged elements.

4.2. Forty-Seven-Bar Planar Power Line Tower

For the second example, the 47-bar planar power line tower shown in Fig.7 is selected [37] to verify the accuracy of the method. The structure has 22 nodes, 47 elements and 40 active degrees of freedom. The elasticity modulus, and material density of for all members are 30,000ksi and 0.3 lb/in3, respectively. Three different damage scenarios are considered, as shown in Table 5.

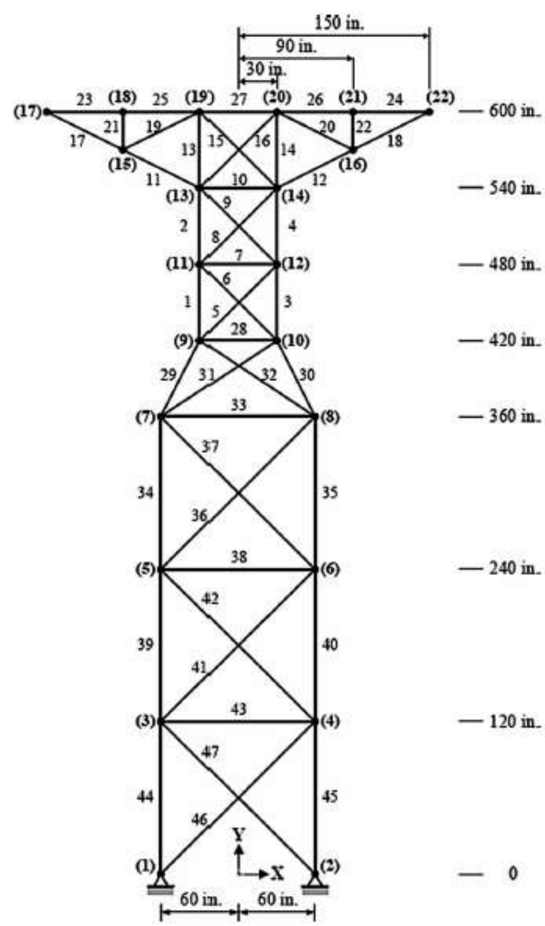

Fig. 7. Forty-seven-bar planar power line tower [37]

Table 5. Different damage cases for the 47-bar planar power line tower

Damage case	Element Number	Damage ratio
Case1	27	0.30
Case2	10	0.30
	30	0.30
Case 3	3	0.15
	30	0.10
	47	0.05

In order to assess the proposed method for damage identification, two states are studied. The first state named ADOF considering the sensors are available at all degrees of freedom. The second state is based on the Guyan reduction method named here as GRM. The number of master degrees of freedom in which sensors can be placed is twelve. The placement of master degrees of freedom in 47-bar planar truss is given in Table 6.

Table 6. The placement of master degrees of freedom in the 47-bar planar power line tower

State	**Master Degrees of Freedom (node number/direction)**						
ADOF	All FEM DOFs						
GRM	2x	3x	3y	4x	4y	5y	7x
	10x	11x	14x	16x	18x	20x	22x

In order to study the effect of the mode number on damage identification performance, 8 and 12 modes of the structure are considered. The MSEBI values for three damage cases without considering noise using 8 and 12 modes are shown in Fig. 8 and Fig. 9, respectively. As observed in the figures, damage identification in the state ADOF has better results via both 8 and 12 modes. It can be seen that, the Guyan method is led to detecting more damaged elements because of its approximate property.

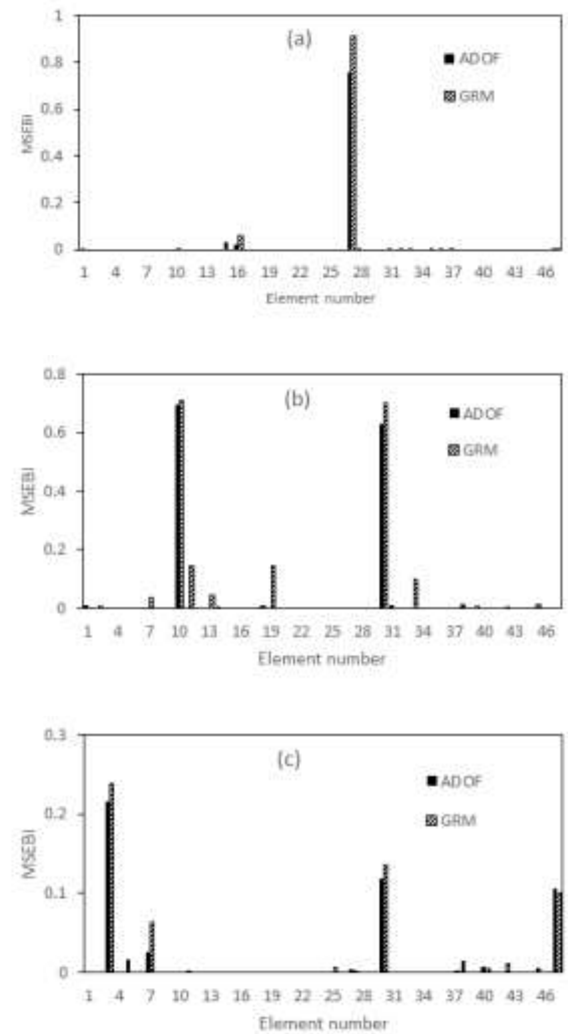

Fig. 8. MSEBI values for different elements of the 47-bar planar truss considering 8 modes for (a) damage case 1, (b) damage case 2, (c) damage case 3, without considering noise.

To assess the performance of the method, the noise effect is applied to mode shapes. By considering the noise effect, as shown in Fig. 10 and Fig.11, it is observed that the MSEBI employing Guyan reduction method has a better performance than MSEBI in detecting the structural damage. Also, in damage case 1, the method using both states correctly identify the damage locations considering the first 8 modes. However, the elements 7, 10, 21, 22 and 28 for the ADOF and the elements 7, 12, 17 and 23 for the GRM are wrongly detected. However, the MSEBI indicator of the elements based on GRM has lower values than ADOF.

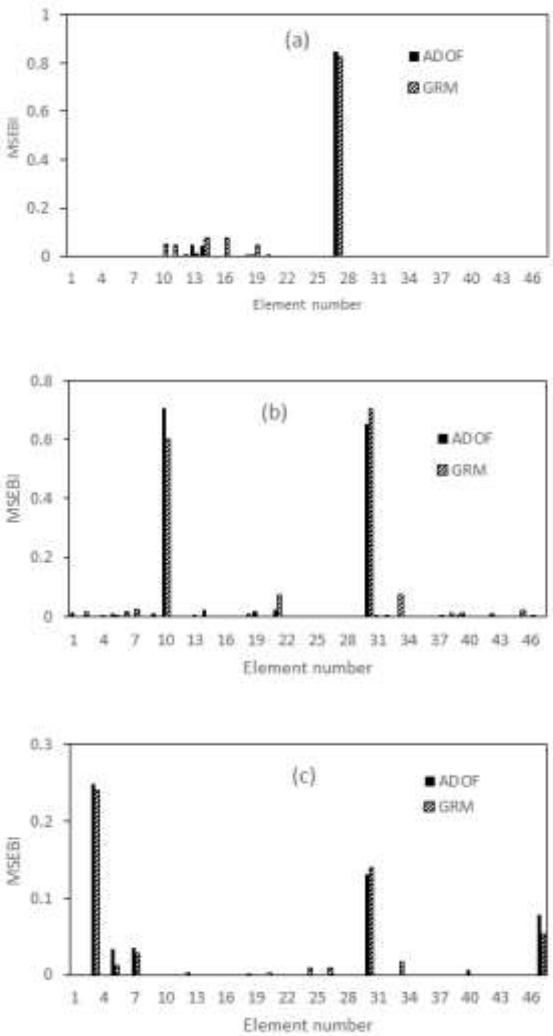

Fig. 9. MSEBI values for different elements of the 47-bar planar truss considering 12 modes for (a) damage case 1, (b) damage case 2, (c) damage case 3, without considering noise.

Although in damage case 2, for both states the locations of damage are accurately identified (elements 10 and 30), the elements 7, 21, 22 and 28 for ADOF and the elements 7, 12, 12 and 19 for GRM are incorrectly identified. The results demonstrate that the GRM state leads to a better outcome for detecting damaged elements. In case 3, the damage location in the two states is correctly recognized which are elements 3, 30 and 47, however the ADOF and GRM wrongly detect elements 7, 15, 16, 17, 19, 20, 23, 24, 28, 38 and 17, 12, 17, 20 respectively, as damaged elements. However, the number of the wrong elements of GRM state is fewer than the ADOF state. It can be said that the GRM has a better performance as compared with that of ADOF for identifying structural damage.

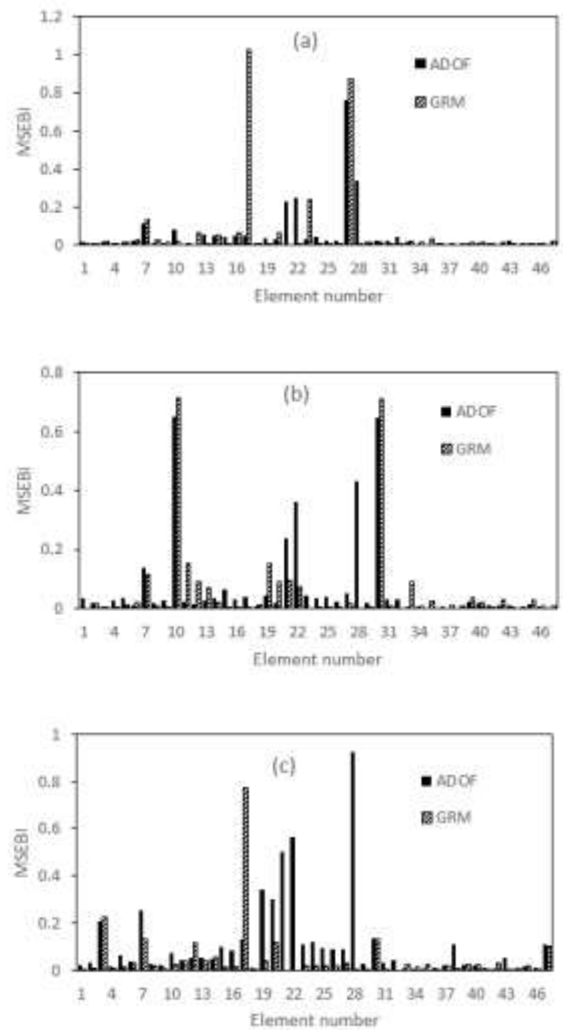

Fig. 10. MSEBI mean values for different elements of the 47-bar planar truss considering 8 modes for (a) damage case 1, (b) damage case 2, (c) damage case 3, contaminating 3% noise

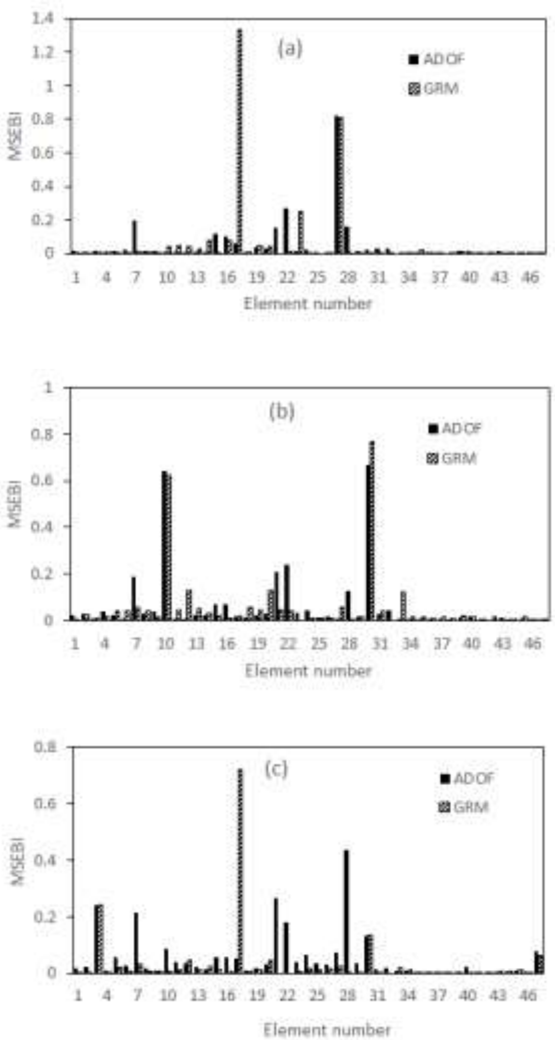

Fig. 11. MSEBI mean values for different elements of the 47-bar planar truss considering 12 modes for (a) damage case 1, (b) damage case 2, (c) damage case 3, contaminating 3% noise

The results from Figs. 8 to Fig.11 show that the number of modes is also effective for improving the damage detection in the truss. By increasing the number of modes, the method can better identify the damaged elements and better results are provided by the damage index to find the exact damage location, especially for GRM.

Table 7. The identified elements for the 47-bar planar truss without considering noise

State	Damage case	8 modes	12 modes
ADOF	Case 1	27	27
	Case 2	10, 30	10, 30
	Case 3	3, 30, 47	3, 30, 47
GRM	Case 1	16, 27	16, 27
	Case 2	10,11, 19, 30, 33	10, 30, 33
	Case 3	3, 7, 30, 47	3, 30, 47

Table 8. The identified elements for the 47-bar planar truss with considering 3% noise

State	Damage case	8 modes	12 modes
ADOF	Case 1	7, 10, 21, 22, 27, 28	7, 15, 16, 21, 22, 27
	Case 2	7, 10, 21, 22 , 28, 30	7, 10, 14, 15, 21, 22, 28, 30
	Case 3	3, 7, 15, 16, 17, 19, 20, 23, 24, 28, , 30, 38, 47	3, 7, 21, 22, 28, 30, 47
GRM	Case 1	7, 12, 17, 23, 27	17, 23, 27
	Case 2	7, 10, 11, 12, 19, 30	10, 21, 22, 30
	Case 3	3, 17, 12, 17, 20, 30, 47	3, 17, 30, 47

Table 7 and Table 8 show the identified elements for the 47-bar planar power line tower without considering noise and with considering 3% noise, respectively. The elements whose MSEBI values exceed 0.05 are selected as damaged elements.

5. Conclusions

In this study, a method has been introduced to detect structural damage via modal strain energy based index (MSEBI) and Guyan reduction method (GRM) for reconstructing mode shapes. Two numerical examples selected from the literature have been considered to test the effectiveness of the proposed method. The examples were analyzed and evaluated by considering noise effect and without considering measurement noise. Moreover, the effect of the number of modes on the efficiency of the method on detecting the damage is investigated. The results showed that in damage cases without considering noise, two states, considering complete mode shapes and GRM with MSEBI can detect damage location accurately. Even though, the MSEBI with GRM performed well regarding fewer master degrees of freedom, however, the MSEBI without GRM considering all degrees of freedom as master degrees of freedom had better outcomes than the GRM based method. In addition, in damage cases contaminating noise level 3%, the GRM based method had a better performance as compared with another one due to considering fewer numbers of degrees of freedom. In other words, the fewer number of sensors leads to less noise in master degrees of freedom, and better responses can be obtained. Also, the outcomes showed that increasing the mode number to 7 led to better results, especially for GRM1 in 31-bar planar truss. Finally, it is useful to determine the damage location in structures using the combination of the modal strain energy index and the Guyan method.

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