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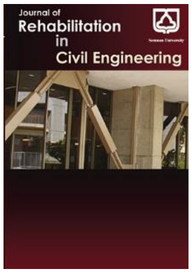

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Gaussian Process Regression Model for Damage Localization in Plates Based on Modal Data

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ABSTRACT

The applications of plate like structures in different fields of engineering are increasing. In this paper, a new damage detection method investigated based on Gaussian process regression model (GPR). GPR is an efficient learning machines which has been used in different fields of engineering. To identify damage, mode shaped and natural frequencies of damaged structures used to train GPR. Finite element modelling of numerical examples and Gaussian process regression (GPR) model are carried out within the MATLAB environment. To show the effectiveness of presented approach, a two-fixed supported plate and a cantilever plate was studied. In other work, a comparative study has been done using a cantilever plates. The natural frequencies were contaminated with noise in above mentioned numerical examples. Results reveal that the proposed method works well using the only first mode data which may be noisy. In other word, GPR can be trained using limited sample numbers for training.

1. Introduction

The applications of plate-structures in different fields of engineering such as aerospace, mechanical and civil are increasing. Identification of early damage in this kind of structures is important. Between damage detection methods, vibrational based methods are more efficient methods [1-4]. Most of this methods were focused on one-dimensional elements and a few damage localization methods have been proposed on plate structures [5]. Hu et al. [6] presented an approach of non-destructive identification of damage in plate like structures based on modal strain energy method (MSEM) and experimental modal analysis (EMA). In this study, a non-destructive damage identification method investigated in plate structures. In other work, Xiang et al. [7] reported a two-phase damage localization

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approach in thin pallet like structures. Recently, Rucevskis et al. [8] studied a new damage identification method in plate-like structures using mode shape curvature. Artificial intelligent based structural health monitoring and damage detection is increasing. Saeed et al. [9] used neural network and adaptive neuro fuzzy interface system(ANFIS) to identify damage in curvilinear beam elements. Also, Kourehli [10-12] proposed new damage identification algorithms based on sparse measured modal data and static responses using different optimization algorithms and artificial neural network. In other work, Kourehli [13] used sparse modal data to train least squares support vector machine (LSSVM). To condense mass and stiffness matrices, iterative improved reduction systems has been used. The results revealed satisfactory performance of presented method. Recently, Ghadimi et al. [14] presented a beam like structures crack detection method using extreme learning machine (ELM). In this study, natural frequencies and modal strain energy have been used as input data. Also, Xu [15] used LSSVM to identify impact for an aluminium plate structure. In other work, Fu et al. [16] integrate the principal component analysis (PCA) and SVM to identify the location of impact on a clamped aluminium plate structure. In other work, Naderpour et al. [17] proposed a structural modal identification method based on peak picking method using wavelet packet transform. Also, applications of GPR in different fields of engineering has been reported [18-19].

The main goal of this work is to detect and identify damage in plate-like structures based on the first two modal data using Gaussian process regression (GPR). Two examples consist of two-fixed supported and cantilever plates were presented. The results reveal the efficiency of the GPR model in predicting damage in plate structures.

2. The damage detection method

To identify damage in plate structures, first eigenvalue and eigenvector of damaged plates obtained solving the characteristic equation. Then this modal data used as input of GPR.

2.1. Damage determination

In this paper, damage was determined as a reduction in elastic modulus of damaged finite element as follows:

$$E_j^d = E_j^{ud}(1-d_j) \tag{1}$$

where, E_j^d and E_j^{ud} are modulus of elasticity of the *j*th element in damaged and undamaged stats, respectively; and d_j is the damage severity at the *j*th element.

2.2. Gaussian process regression (GPR) model

A Gaussian process is a stochastic process specified by its mean $\mu(x)$ and covariance functions $k(x, x')$ as follows [20]

$$\mu(x) = E[f(x)] \tag{2}$$

$$k(x.x') = E[(f(x) - \mu(x)[(f(x') - \mu(x')] \tag{3}$$

In vector form, GPR model is equivalent to:

$$P(y|f;\ X) \sim N(y|H\beta + f; \sigma^2 I) \tag{4}$$

Where, y, f and X are the response, latent and observation variables. Also, H is a set of basis function and β is estimated coefficient from data and σ^2 is the error variance.

3. Examples

In this section, two numerical examples consist of two-fixed supported and cantilever plates were studied. Also, two different damage scenarios considered. The examples are simulated by MATLAB R2018b [21].

Also, the flow chart of proposed method is shown in Fig.1.

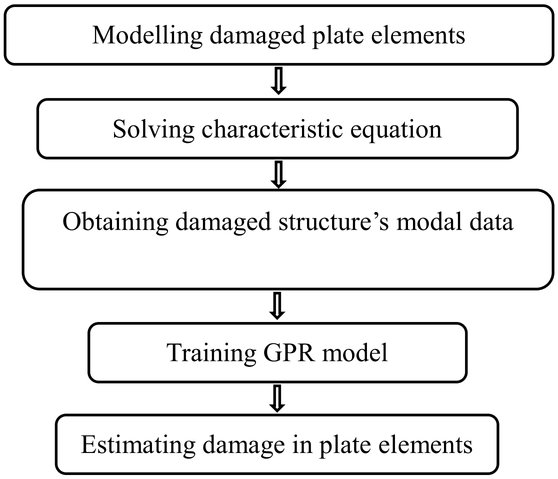


Fig. 1. Flow chart of proposed method.

3.1. Two-fixed supported plate

First example is a two-fixed supported plate as showed in Fig. 2. The studied concrete plate’s modulus of elasticity is 20 GPa, Poisson’s ratio of μ=0.2, mass density of ρ=2400 kg/m^3 and thickness is t=15cm. Also, two damage scenarios are presented in Table 1.

Table 1. Two damage scenarios for two-fixed supported plate.

Damaged element No.	14	6, 12, 14
Severity of damage	0.2	0.2, 0.2, 0.3

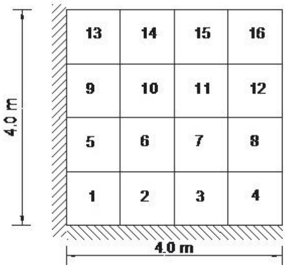


Fig. 2. The two-fixed supported plate with element numbers.

For training purpose different damage cases has been generated. Damage severities of zero and ten percent for element numbered 1,3,5,7,9,11,13,15 respectively and zero, ten percent and twenty percent for elements numbered 2,4,6,8,10,12,14,16, respectively, were considered. For training GPR, 200 and 400 random combinations of damage states have been assigned. Also, the 5% noise has been considered in frequencies using the following equation:

$$\omega_a^{mesured} = \omega_a\left(1+\varphi\; rand\left[-1\;\; 1\right]\right) \qquad (5)$$

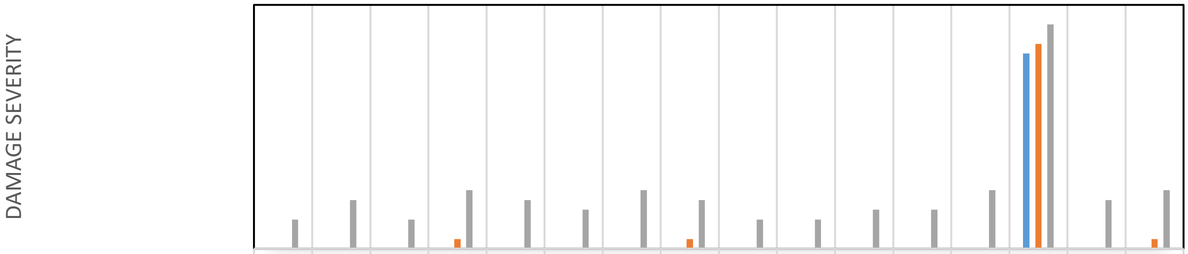
where $\omega_a^{mesured}$ and ω_a are the noisy and noise free frequencies of ath mode, respectively and φ is the noise level.

Table 2 shows the GPR performance in different stages. In which, P is the number of modes used to train GPR. In this study, 10% of data used to test model’s performance. According to Table 2, the Mean Square Error (MSE) values in the case of noisy data are more than noise free case. Also, the MSE values are low and the output data are close to target data.

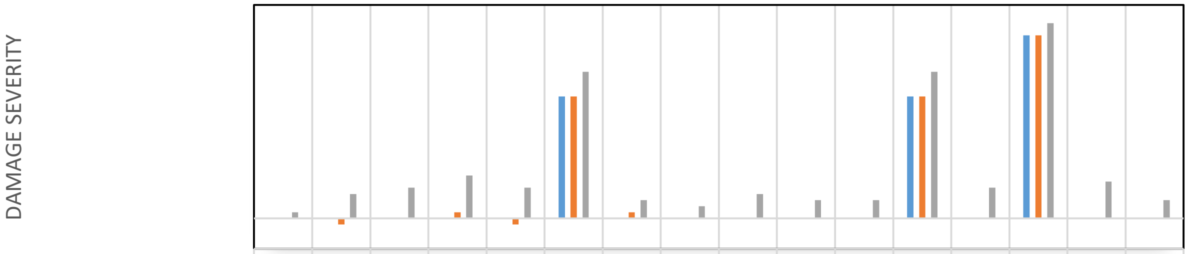
Figs. 3 and 4 show that the proposed method gives completely accurate results in the noise-free state using first mode data. While, in the noisy case, some damage values have been incorrectly detected. Also, using more data for training will increase the accuracy of the proposed method. Generally, the results show high accuracy of the GPR for predicting damage in plate like structures based on first mode data using only 200 samples which may be noisy.

Table 2. MSE values for two-fixed supported plate.

		Samples	Noise Free	5 % Noise
P=1	Training	180	1.800e-04	3.027e-04
	Testing	20	3.256e-04	4.578e-04
P=1	Training	360	1.706e-04	3.874e-04
	Testing	40	2.281e-04	5.075e-04
P=2	Training	180	2.086e-04	3.005e-04
	Testing	20	2.281e-04	3.834e-04
P=2	Training	360	1.609e-04	3.967e-04
	Testing	40	2.180e-04	6.179e-04

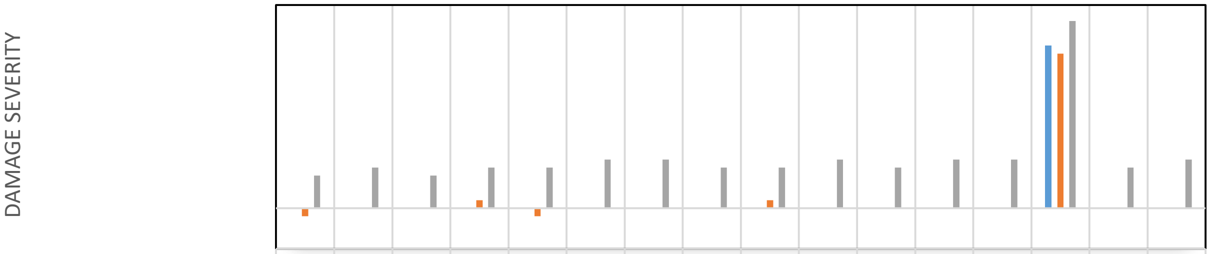


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
■ Reduction factors 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2	0	0
■ Predicted (Noise free)	0	0	0	0.01	0	0	0	0.01	0	0	0	0	0	0.21	0	0.01
■ Predicted (5% Noise)	0.03	0.05	0.03	0.06	0.05	0.04	0.06	0.05	0.03	0.03	0.04	0.04	0.06	0.23	0.05	0.06

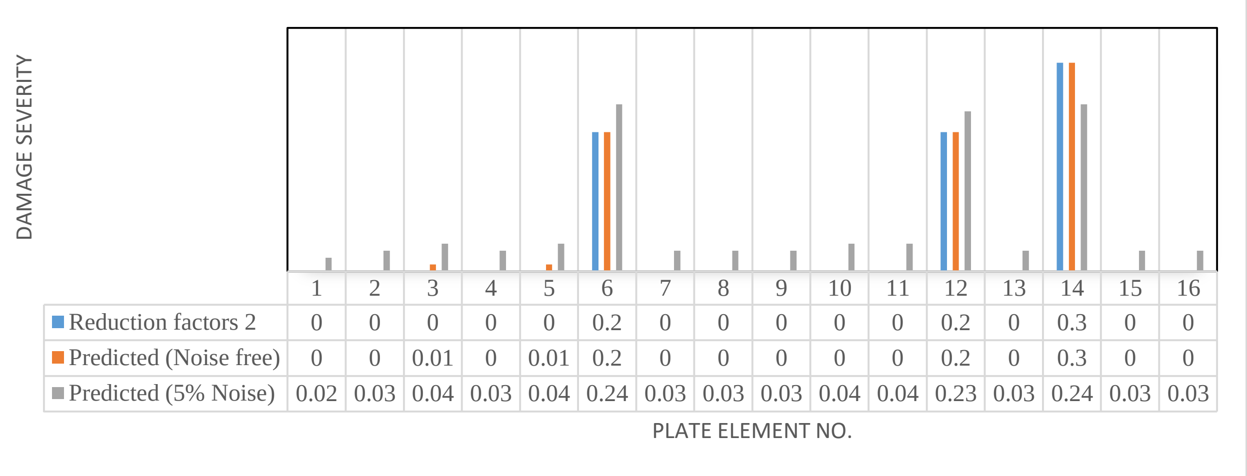


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
■ Reduction factors 2	0	0	0	0	0	0.2	0	0	0	0	0	0.2	0	0.3	0	0
■ Predicted (Noise free)	0	-0.01	0	0.01	-0.01	0.2	0.01	0	0	0	0	0.2	0	0.3	0	0
■ Predicted (5% Noise)	0.01	0.04	0.05	0.07	0.05	0.24	0.03	0.02	0.04	0.03	0.03	0.24	0.05	0.32	0.06	0.03

Fig. 3. Identified damage in the two-fixed supported plate using 200 sample and first mode shape and frequency.

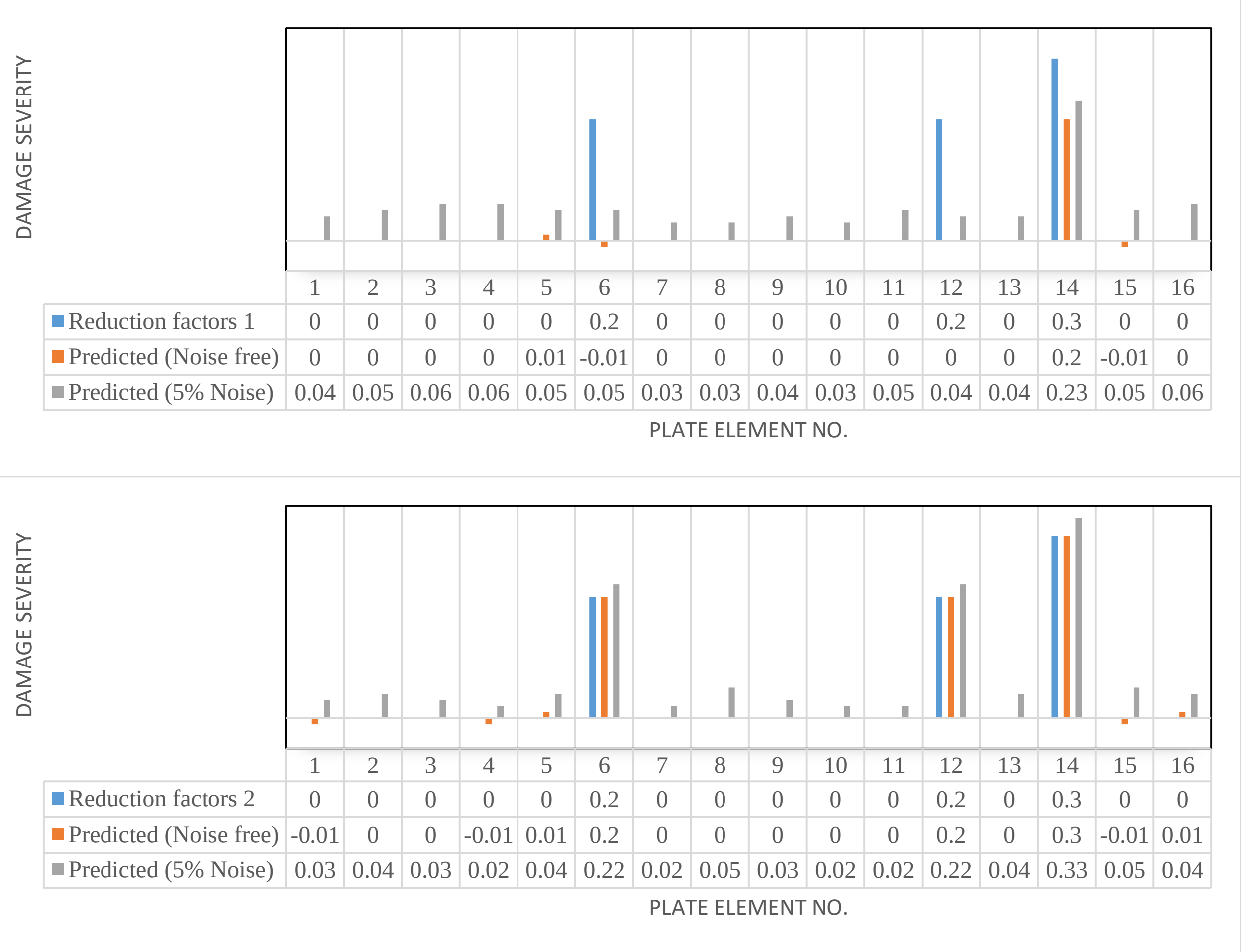


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
■ Reduction factors 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2	0	0
■ Predicted (Noise free)	-0.01	0	0	0.01	-0.01	0	0	0	0.01	0	0	0	0	0.19	0	0
■ Predicted (5% Noise)	0.04	0.05	0.04	0.05	0.05	0.06	0.06	0.05	0.05	0.06	0.05	0.06	0.06	0.23	0.05	0.06



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
🟦 Reduction factors 2	0	0	0	0	0	0.2	0	0	0	0	0	0.2	0	0.3	0	0
🟧 Predicted (Noise free)	0	0	0.01	0	0.01	0.2	0	0	0	0	0	0.2	0	0.3	0	0
◼ Predicted (5% Noise)	0.02	0.03	0.04	0.03	0.04	0.24	0.03	0.03	0.03	0.04	0.04	0.23	0.03	0.24	0.03	0.03

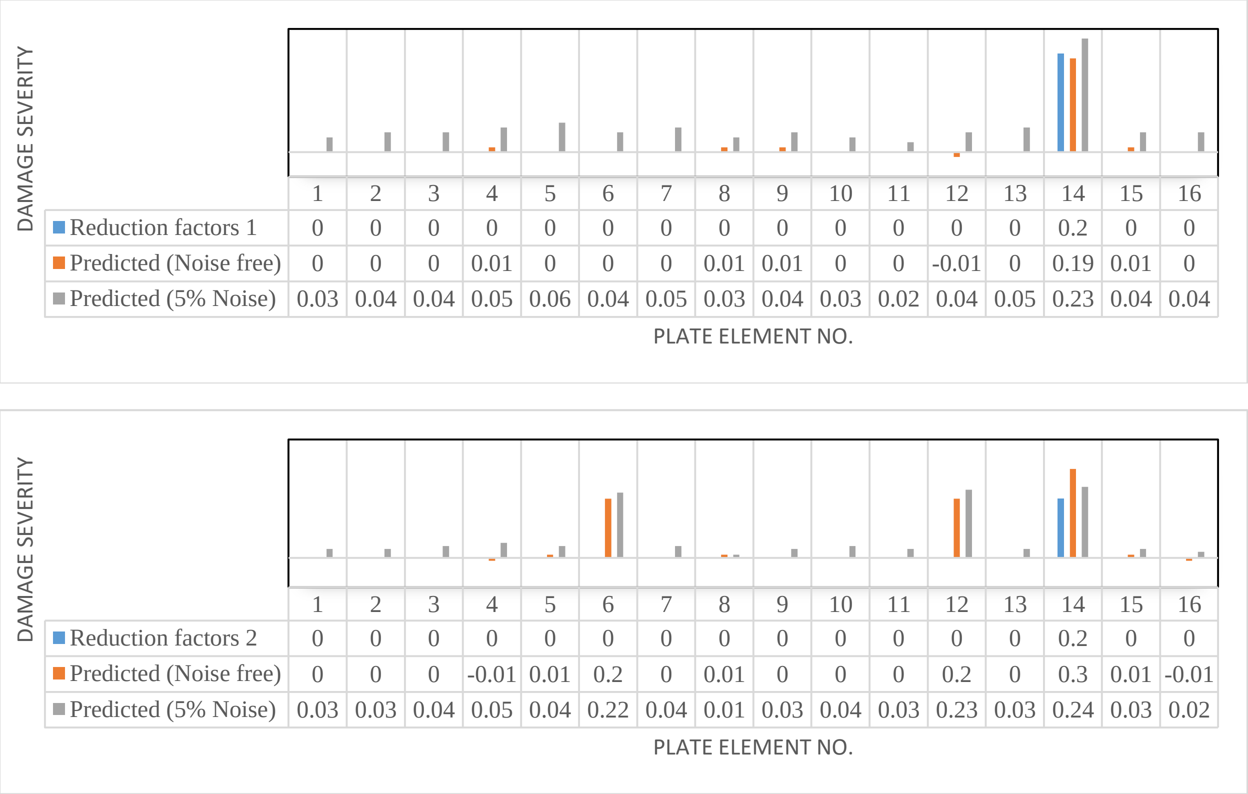
Fig. 4. Identified damage in the two-fixed supported plate using 400 sample and first mode shape and frequency.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
🟦 Reduction factors 1	0	0	0	0	0	0.2	0	0	0	0	0	0.2	0	0.3	0	0
🟧 Predicted (Noise free)	0	0	0	0	0.01	-0.01	0	0	0	0	0	0	0	0.2	-0.01	0
◼ Predicted (5% Noise)	0.04	0.05	0.06	0.06	0.05	0.05	0.03	0.03	0.04	0.03	0.05	0.04	0.04	0.23	0.05	0.06

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
🟦 Reduction factors 2	0	0	0	0	0	0.2	0	0	0	0	0	0.2	0	0.3	0	0
🟧 Predicted (Noise free)	-0.01	0	0	-0.01	0.01	0.2	0	0	0	0	0	0.2	0	0.3	-0.01	0.01
◼ Predicted (5% Noise)	0.03	0.04	0.03	0.02	0.04	0.22	0.02	0.05	0.03	0.02	0.02	0.22	0.04	0.33	0.05	0.04

Fig. 5. Identified damage in the two-fixed supported plate using 200 sample and first two mode shapes and frequencies.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
🟦 Reduction factors 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2	0	0
🟧 Predicted (Noise free)	0	0	0	0.01	0	0	0	0.01	0.01	0	0	-0.01	0	0.19	0.01	0
⬜ Predicted (5% Noise)	0.03	0.04	0.04	0.05	0.06	0.04	0.05	0.03	0.04	0.03	0.02	0.04	0.05	0.23	0.04	0.04

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
🟦 Reduction factors 2	0	0	0	0	0	0	0	0	0	0	0	0	0	0.2	0	0
🟧 Predicted (Noise free)	0	0	0	-0.01	0.01	0.2	0	0.01	0	0	0	0.2	0	0.3	0.01	-0.01
⬜ Predicted (5% Noise)	0.03	0.03	0.04	0.05	0.04	0.22	0.04	0.01	0.03	0.04	0.03	0.23	0.03	0.24	0.03	0.02

Fig. 6. Identified damage in the two-fixed supported plate using 400 sample and first two mode shapes and frequencies.

Figures 5 and 6 show the results based on first two modal data and using 200 and 400 data. As can be seen, the results presented in Figure 6 are more accurate than the other cases. The reason for this is related to the use of more modal data for GPR training.

3.2. Cantilever plate

The second example studied is a cantilever plate provided by Ghodrati et al. [18] as in Fig. 7, which the material properties and thickness are similar to the first example. The boundary conditions and the dimensions of plate are different in this example comparing to previous case study in order to demonstrate effectiveness of the proposed methods for various cases. Also, two damage scenarios are showed in Table 3.

Table 3. Two damage scenarios for cantilever plate.

Damaged element No.	8	1, 11, 16
Severity of damage	0.1	0.1, 0.1, 0.1

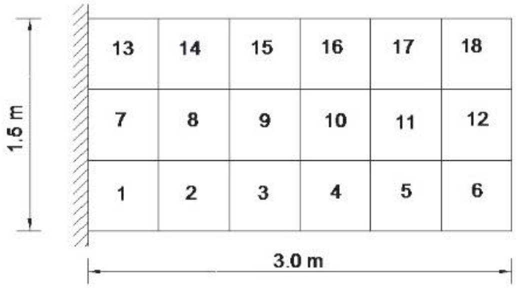


Fig. 7. The element number of cantilever plate.

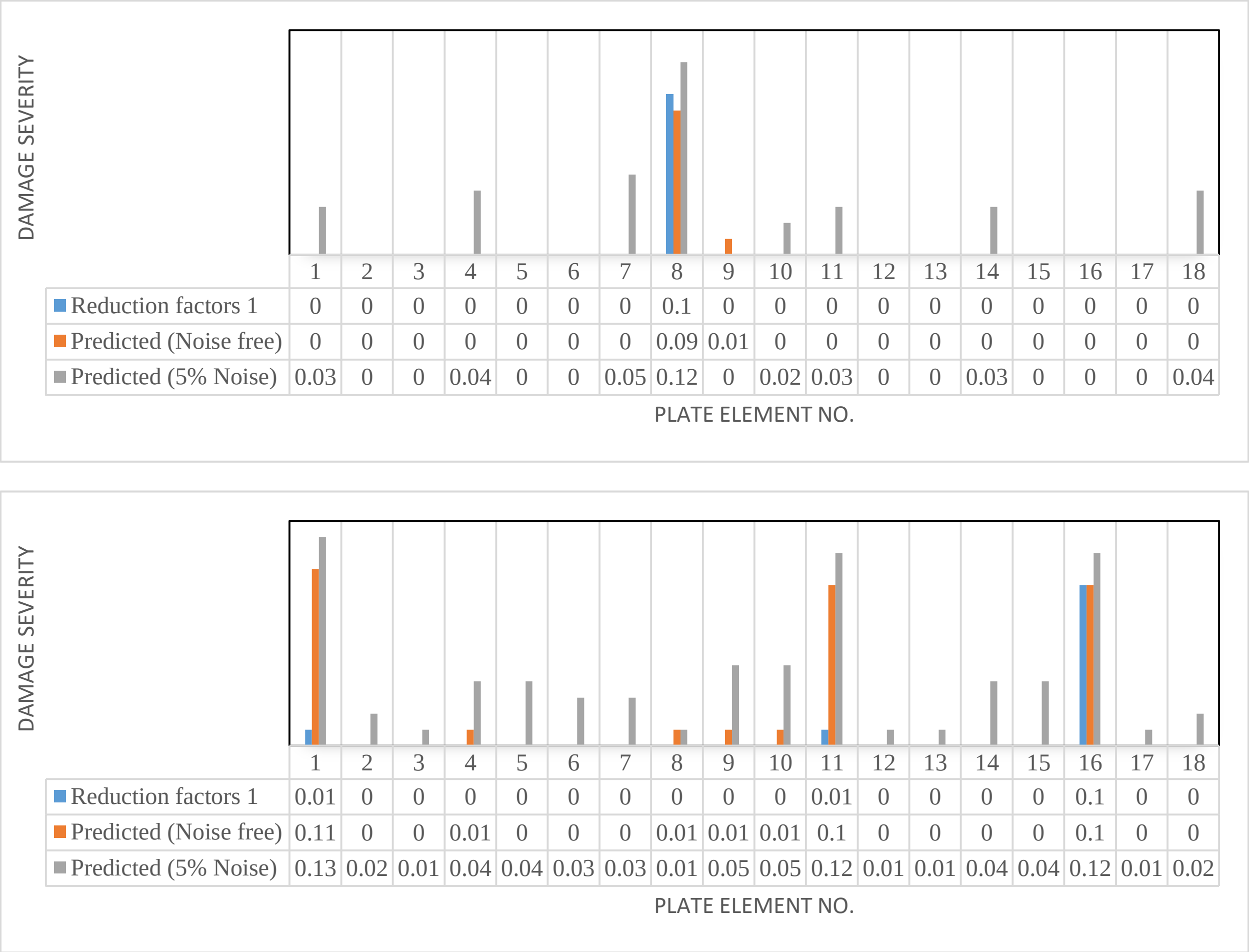
For training purpose different damage cases has been generated. Damage severities of zero and ten percent for all elements were considered. For training GPR, 400 and 800 random combinations of damage states have been assigned. According to Table 4, MSE values are very low in spite of low value of samples and 5% noise in modal data.

Table 4. MSE for training and testing for cantilever plate using modal data.

		Samples	Noise Free	5 % Noise
P=1	Training	360	6.098e-4	7.099e-4
	Testing	40	8.9234e-4	6.983e-4
P=1	Training	720	4.2430e-4	4.897e-4
	Testing	80	5.8711e-4	6.555e-4
P=2	Training	360	5.7347e-4	7.9769e-4
	Testing	40	1.5579e-4	3.087e-4
P=2	Training	720	3.3345e-4	2.076e-4
	Testing	80	1.0007e-4	1.569e-4

Figs. 8, 9 show the performance of GPR in detecting damaged elements in cantilever plate based on first mode data using 400 and 800 data, respectively. The results reveal high accuracy of the GPR for predicting damage in cantilever plate based on first mode data using only 400 samples which may be noisy.

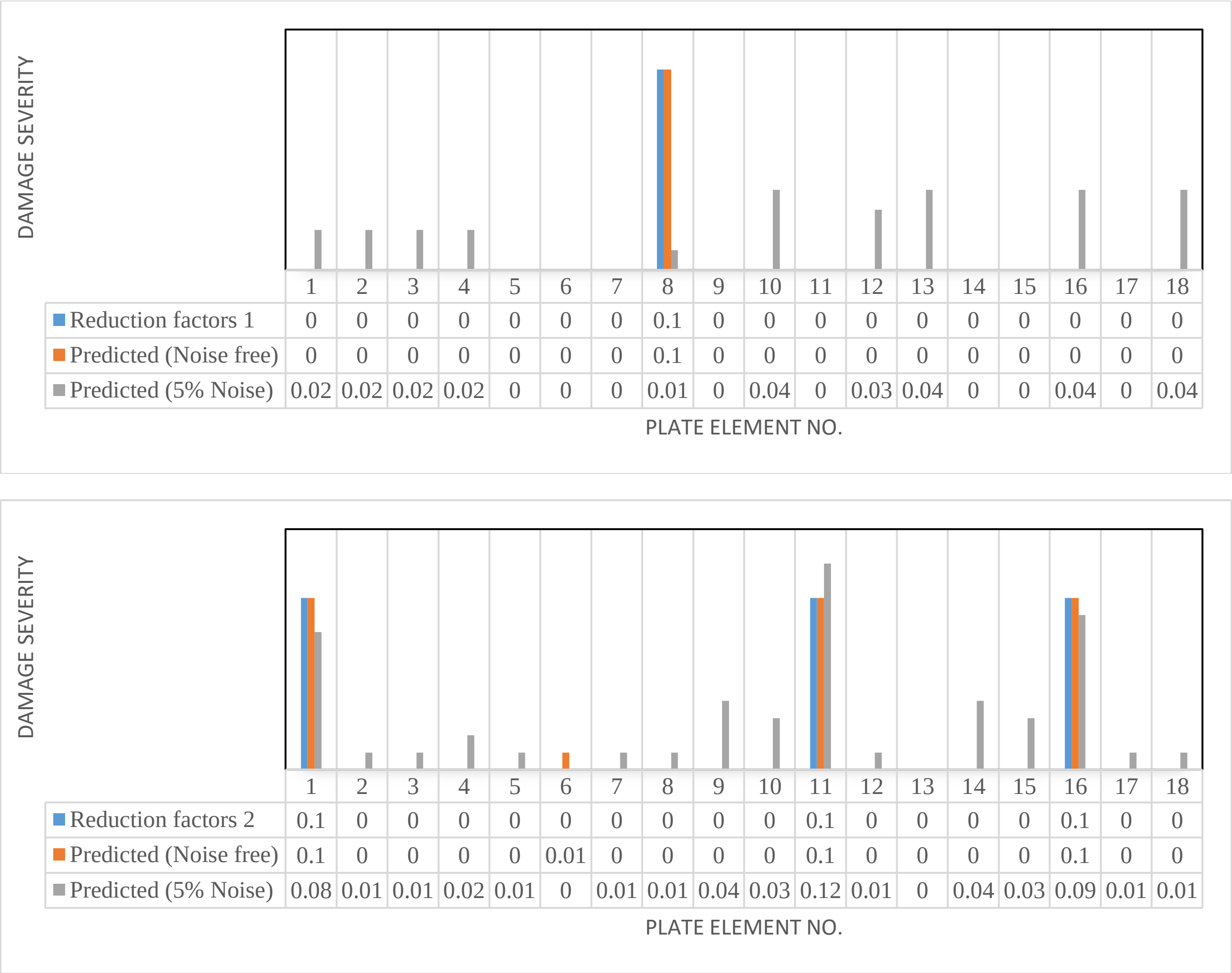
In other study, first two mode's data used to identify damage. Figures 10 and 11 show the results using 400 and 800 data, respectively. It can be seen that damaged elements detected correctly.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
🟦 Reduction factors 1	0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0
🟧 Predicted (Noise free)	0	0	0	0	0	0	0	0.09	0.01	0	0	0	0	0	0	0	0	0
⬜ Predicted (5% Noise)	0.03	0	0	0.04	0	0	0.05	0.12	0	0.02	0.03	0	0	0.03	0	0	0	0.04

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
🟦 Reduction factors 1	0.01	0	0	0	0	0	0	0	0	0	0.01	0	0	0	0	0.1	0	0
🟧 Predicted (Noise free)	0.11	0	0	0.01	0	0	0	0.01	0.01	0.01	0.1	0	0	0	0	0.1	0	0
⬜ Predicted (5% Noise)	0.13	0.02	0.01	0.04	0.04	0.03	0.03	0.01	0.05	0.05	0.12	0.01	0.01	0.04	0.04	0.12	0.01	0.02

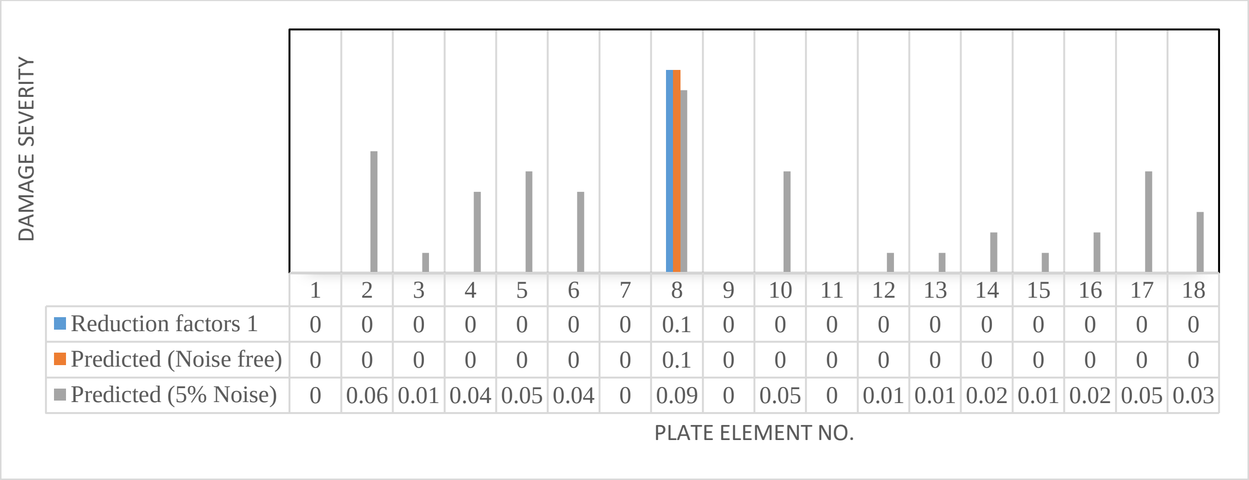
Fig. 8. Identified damage in cantilever plate using 400 sample and first mode data.



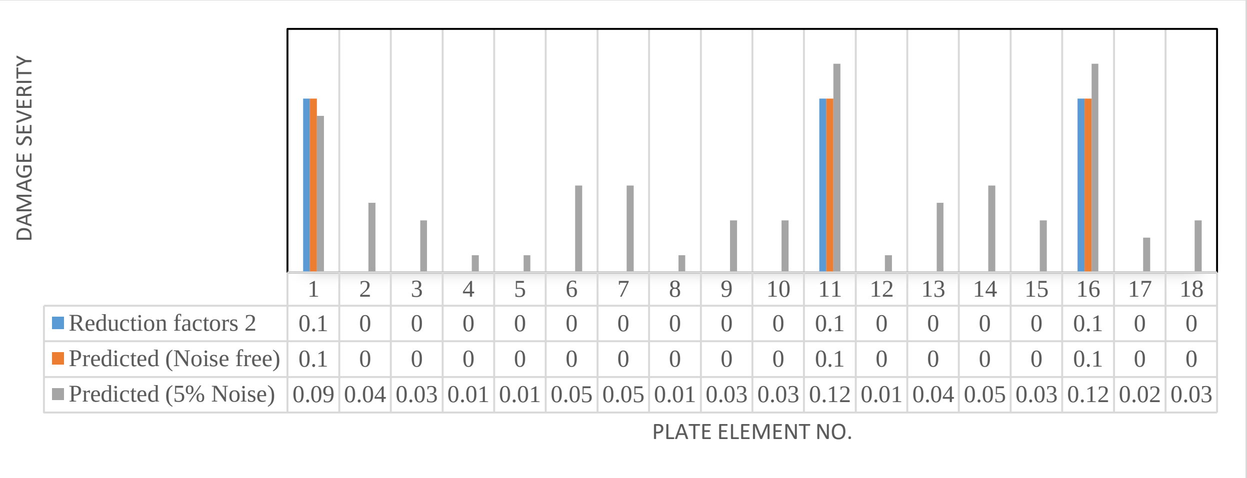
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
🟦 Reduction factors 1	0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0
🟧 Predicted (Noise free)	0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0
⬜ Predicted (5% Noise)	0.02	0.02	0.02	0.02	0	0	0	0.01	0	0.04	0	0.03	0.04	0	0	0.04	0	0.04

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
🟦 Reduction factors 2	0.1	0	0	0	0	0	0	0	0	0	0.1	0	0	0	0	0.1	0	0
🟧 Predicted (Noise free)	0.1	0	0	0	0	0.01	0	0	0	0	0.1	0	0	0	0	0.1	0	0
⬜ Predicted (5% Noise)	0.08	0.01	0.01	0.02	0.01	0	0.01	0.01	0.04	0.03	0.12	0.01	0	0.04	0.03	0.09	0.01	0.01

Fig. 9. Identified damage in cantilever plate using 800 sample and first mode data.

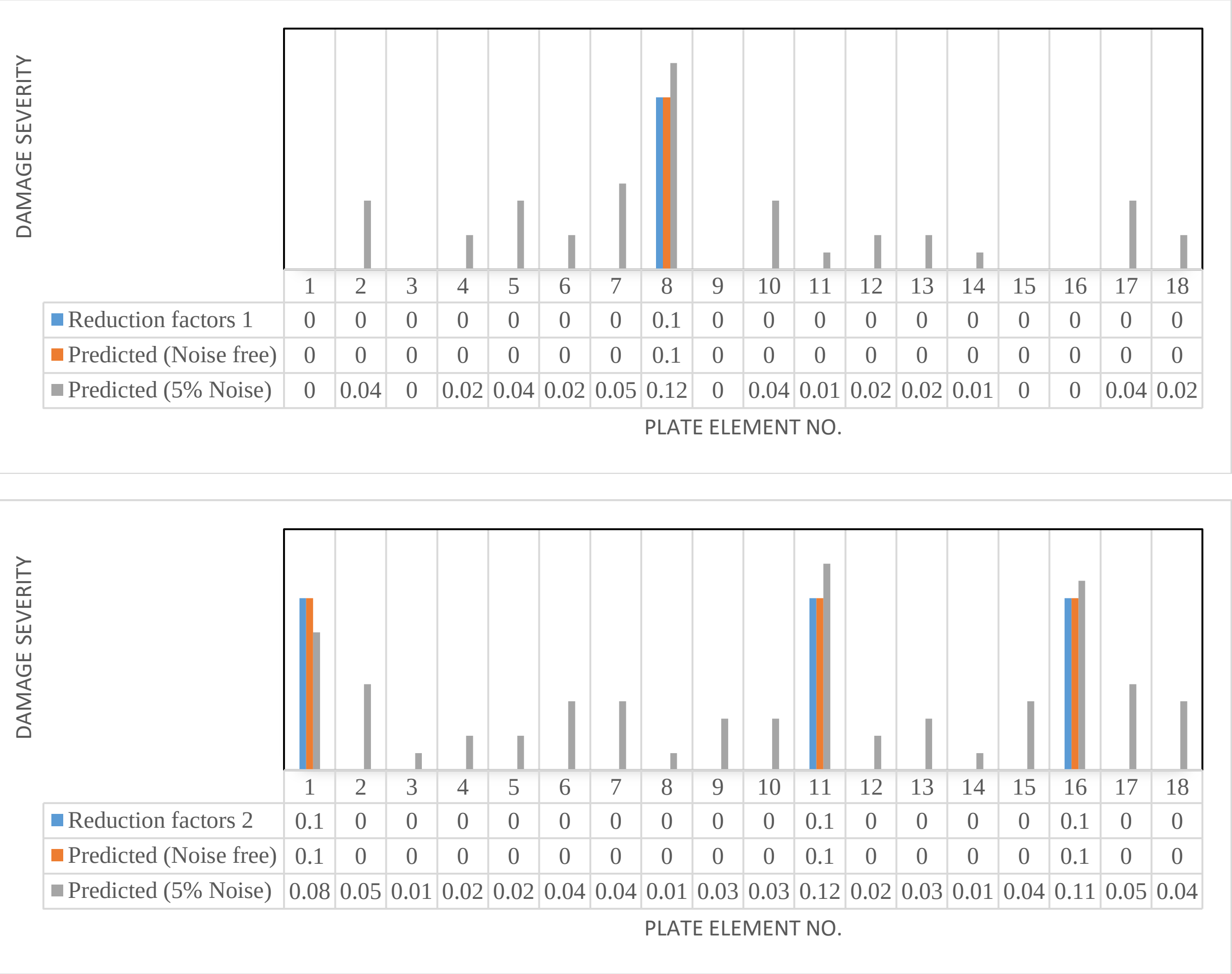


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
🟦 Reduction factors 1	0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0
🟧 Predicted (Noise free)	0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0
⬜ Predicted (5% Noise)	0	0.06	0.01	0.04	0.05	0.04	0	0.09	0	0.05	0	0.01	0.01	0.02	0.01	0.02	0.05	0.03



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
🟦 Reduction factors 2	0.1	0	0	0	0	0	0	0	0	0	0.1	0	0	0	0	0.1	0	0
🟧 Predicted (Noise free)	0.1	0	0	0	0	0	0	0	0	0	0.1	0	0	0	0	0.1	0	0
■ Predicted (5% Noise)	0.09	0.04	0.03	0.01	0.01	0.05	0.05	0.01	0.03	0.03	0.12	0.01	0.04	0.05	0.03	0.12	0.02	0.03

Fig. 10. Identified damage in cantilever plate using 400 sample and first two modes data.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
🟦 Reduction factors 1	0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0
🟧 Predicted (Noise free)	0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0
■ Predicted (5% Noise)	0	0.04	0	0.02	0.04	0.02	0.05	0.12	0	0.04	0.01	0.02	0.02	0.01	0	0	0.04	0.02

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
🟦 Reduction factors 2	0.1	0	0	0	0	0	0	0	0	0	0.1	0	0	0	0	0.1	0	0
🟧 Predicted (Noise free)	0.1	0	0	0	0	0	0	0	0	0	0.1	0	0	0	0	0.1	0	0
■ Predicted (5% Noise)	0.08	0.05	0.01	0.02	0.02	0.04	0.04	0.01	0.03	0.03	0.12	0.02	0.03	0.01	0.04	0.11	0.05	0.04

Fig. 11. Identified damage in cantilever plate using 800 sample and first two modes data.

To validate the proposed method, a comparative study has been done for cantilever plate proposed by Ghodrati amiri et al. [22]. In previous study an optimization based formulation presented based on first mode data (p=1) and first three modes data (p=3) and solved by Pattern search (PS) and Genetic algorithm (GA). In this study, the element number 3 is considered with 0.1 damage severity. Also, 3% noise has been considered. Fig. 12 show the results reported by Ghodrati amiri et al. [22] and obtained in this study. As can be seen, the damage obtained by the proposed method is more accurate and close to the actual damages.

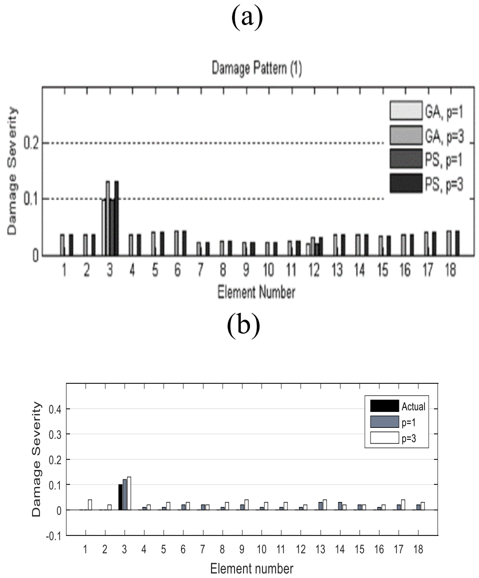


Fig. 12. Results reported by (a) Ghodrati amiri et al. [22] and obtained by (b) presented method.

4. Conclusions

In the presented work the damage identification problem in the plates was investigated using Gaussian process regression. To train Gaussian process regression model, first two natural frequencies and modes shapes used as input, in which the damage position and severity in plate elements as output. In this paper, two examples consist of two-fixed supported and cantilever plates have been studied. Also, a validation study has been done using a cantilever plates. To evaluate the performance of the presented method, effect of different sample numbers for training Gaussian process regression has been investigated.

The obtained results show:

- High accuracy of the presented method using only first mode data.
- Gaussian process regression can be trained using limited sample numbers for training.

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