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A study of g^*-fuzzy closed set via α^m in double fuzzy topological space

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Abstract

The aim of this research paper is to present and study a new class of generalized closed sets in double fuzzy topological spaces, by using the fuzzy α^m-closed set, that was previously presented. This new class is called the fuzzy $\alpha^m - g^*$-closed set. The relationship of the new concept with previous concepts is studied and the characteristics of this concept is investigated through important theorems that determine the position of this set in relation to sets that have been studied or that will be presented later. Also, generalizations of the functions are presented according to the concept, their properties are studied, and some necessary examples that show the properties of this concept and its relationships.

Keywords: DFT, DF$\alpha^m - g^* f$ closed set, DF-$\alpha^m - g^*$-continuous function.

1. Introduction

In 1965, the concept of fuzzy set was introduced and studied by Zadeh in his classical paper [11]. In 1968, the fuzzy topological space was first defined by Chang [4]. The concepts of fuzzy semi open (fuzzy semi closed) and fuzzy semi continuous mappings in fuzzy topological spaces was studied by Azad [3]. F. M. Mohammed, et.al. [5] in 2017 presented and studied the concept of $(m_1, n_1) - \alpha^m-$fuzzy closed sets in double fuzzy topological spaces. Then, in the same year, they generalized and studied some types of functions across $(m_1, n_1) - \alpha^m-$fuzzy closed sets [9, 2] where the notion of α^m-continuous and α^m-generalized continuous in the double fuzzy topological space was introduced.

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In this paper, we propose and develop the concept of $\alpha^m - g^* fuzzy$ closed set in double fuzzy topological space and introduce definition, relationships, theories and study some of their characteristics related to this concept.

2. Preliminaries

In this section, we describe some basics which are useful for the rest of this study.

Definition 2.1. *[5] A double fuzzy topology $(\mathfrak{L}_x, \mathfrak{L}_x^*)$ on a non-empty set X is a pair of functions $\mathfrak{L}_x, \mathfrak{L}_x^* : I^X \to I$, which satisfies the following properties:*

(i) $\mathfrak{L}_x(\upsilon) \leq \breve{1} - \mathfrak{L}_{x^*}(\upsilon)$ *for each* $\upsilon \in I^X$.

(ii) $\mathfrak{L}_x(\upsilon_1 \bigwedge \upsilon_2) \geq \mathfrak{L}_x(\upsilon_1) \bigwedge \mathfrak{L}_x(\upsilon_2)$ *and* $\mathfrak{L}_x^*(\upsilon_1 \bigwedge \upsilon_2) \leq \mathfrak{L}_x^*(\upsilon_1) \bigvee \mathfrak{L}_x^*(\upsilon_2)$ *for each* $\upsilon_1, \upsilon_2 \in I^X$.

(iii) $\mathfrak{L}_x(\bigvee_{i\in\mathfrak{z}} \upsilon_i) \geq \bigwedge_{i\in r} \mathfrak{L}_x(\upsilon_i)$ *and* $\mathfrak{L}_x^*(\bigvee_{i\in\mathfrak{z}} \upsilon_i) \leq \bigvee_{i\in\mathfrak{z}} \mathfrak{L}_x^*(\upsilon_i)$ *for each* $\upsilon_i \in I^X, i \in \mathfrak{Z}$.

The triplex $(X, \mathfrak{L}_x, \mathfrak{L}_{x^*})$ is called a double fuzzy topological spaces (dfts).

Definition 2.2. *[5, 10] Let X be a dfts, then for each $m_1 \in Im_1, n_1 \in In_1$ and $\upsilon \in I^X$, we define the double fuzzy closure and interior operator $C_{\mathfrak{L}_x,\mathfrak{L}_x^*} : I^X \ x \ Im_1 \ x \ In_1 \to I^X$ as follows:*
$C_{\mathfrak{L}_x,\mathfrak{L}_x^*}(\upsilon_1, m_1, n_1) = \bigwedge\{\Omega \in I^X : \upsilon \leq \Omega, \mathfrak{L}_x(\breve{1} - \Omega) \geq m_1, \mathfrak{L}_x^*(\breve{1} - \Omega) \leq n_1\}$.
$I_{\mathfrak{L}_x,\mathfrak{L}_x^*}(\upsilon_1, m_1, n_1) = \bigvee\{\Omega \in I^X : \Omega \leq \upsilon, \mathfrak{L}_x(\Omega) \geq m_1, \mathfrak{L}_x^*(\Omega) \leq n_1\}$.

Definition 2.3. *Let $(X, \mathfrak{L}_x, \mathfrak{L}_x^*)$ be a dfts,s $\upsilon, \Omega \in I^X, m_1 \in I_{m_1}$ and $n_1 \in I_{n_1}$, a fuzzy set υ is called:*

(i) *[10] An (m_1, n_1) fuzzy open set $((m_1, n_1) - fo)$ if $\mathfrak{L}_x(\upsilon) \geq m_1$ and $\mathfrak{L}_x^*(\upsilon) \leq n_1$. A fuzzy set υ is called an (m_1, n_1)-fuzzy closed set (for short, (m_1, n_1)–fc), whenever $\mathfrak{L}_x(\breve{1} - \upsilon) \geq m_1$ and $\mathfrak{L}_x^*(\breve{1} - \upsilon) \leq n_1$.*

(ii) *[6, 8] An (m_1, n_1)-fuzzy α-open set $((m_1, n_1) - f\alpha - open)$,*
if $\upsilon \leq I_{\mathfrak{L}_x,\mathfrak{L}_x^*}(C_{\mathfrak{L}_x,\mathfrak{L}_x^*}(I_{\mathfrak{L}_x,\mathfrak{L}_x^*}(\upsilon, m_1, n_1), m_1, n_1), m_1, n_1)$
and an (m_1, n_1)-fuzzy α-closed set $((m_1, n_1) - f\alpha - closed)$,
if $C_{\mathfrak{L}_x,\mathfrak{L}_x^*}(I_{\mathfrak{L}_x,\mathfrak{L}_x^*}(C_{\mathfrak{L}_x,\mathfrak{L}_x^*}(\upsilon, m_1, n_1), m_1, n_1), m_1, n_1) \leq \upsilon$.

(iii) *An (m_1, n_1)-generalized fuzzy closed $((m_1, n_1) - gf\ closed)$, if $C_{\mathfrak{L}_x,\mathfrak{L}_x^*}(\upsilon, m_1, n_1) \leq \Omega$ whenever $\upsilon \leq \Omega, \mathfrak{L}_x(\Omega) \geq m_1$ and $\mathfrak{L}_x^*(\Omega) \leq n_1$. υ is called (m_1, n_1) generalized fuzzy open $((m_1, n_1) - gf\ open)$ if $(\breve{1} - \upsilon)$ is an (m_1, n_1)-gfc set.*

Definition 2.4. *Let $(X, \mathfrak{L}_x, \mathfrak{L}_{x^*})$ be a dfts, $\upsilon \in I^X, m_1 \in I_{m_1}$ and $n_1 \in I_{n_1}$, a fuzzy set υ is called:*

(i) *[9, 2, 7] An (m_1, n_1)-fuzzy α^m-closed set $((m_1, n_1) - f\alpha^m - closed)$, if $I_{\mathfrak{L}_x,\mathfrak{L}_x^*}(C_{\mathfrak{L}_x,\mathfrak{L}_x^*}(\upsilon, m_1, n_1), m_1, n_1) \leq \Omega$ whenever $\upsilon \leq \Omega$ and Ω is $(m_1, n_1) - f\alpha - open$. υ is called (m_1, n_1)-fuzzy α^m-open set iff $\breve{1} - \upsilon$ is (m_1, n_1)-fuzzy α^m-closed set.*

(ii) *[9, 2] An (m_1, n_1)- α^m-generalized fuzzy-closed set $((m_1, n_1) - f\alpha^m - gf\ closed)$. if $\alpha^m C_{\mathfrak{L}_x,\mathfrak{L}_x^*}(\upsilon, m_1, n_1) \leq \Omega$ such that $\upsilon \leq \Omega$ and Ω is $(m_1, n_1) - f\alpha^m$-open set. υ is called $(m_1, n_1) - \alpha^m$-generalized fuzzy-open set $((m_1, n_1) - \alpha^m - gf - open)$ iff $\breve{1} - \upsilon$ is an $(m_1, n_1) - \alpha^m - gf - closed$ set.*

Definition 2.5. *[9, 7] If X is a dfts, for each $\upsilon, \Omega \in I^X, m_1 \in I_{m_1}$ and $n_1 \in I_{n_1}$ then, the α^m Closure of υ is defined as:*
$\alpha^m C_{\mathfrak{L}_x,\mathfrak{L}^*_x}(\upsilon, m_1, n_1) = \bigwedge\{\Omega \in I^X : \upsilon \leq \Omega, \Omega is (m_1, n_1) - f\alpha^m - closed\}$.

Definition 2.6. *Let $(X, \mathfrak{L}_x, \mathfrak{L}^*_x)$ and $(Y, \mathfrak{L}_y, \mathfrak{L}^*_y)$ be dfts,s. A function $h : (X, \mathfrak{L}_x, \mathfrak{L}^*_x) \to (Y, \mathfrak{L}_y, \mathfrak{L}^*_y)$ is called.*

(i) *[1] Double fuzzy continuous function iff $\mathfrak{L}_x(h^{-1}(\upsilon)) \geq \mathfrak{L}_y(\upsilon)$ and $\mathfrak{L}^*_x(h^{-1}(\upsilon)) \leq \mathfrak{L}^*_y(\upsilon)$ for each $\upsilon \in I^y$.*

(ii) *Double fuzzy α^m-continuous functions [9, 2] $(df - \alpha^m - c)$ if $h^{-1}(\Omega)$ is an $(m_1, n_1) - f\alpha^m - open$ such that $\mathfrak{L}_y(\Omega) \geq m_1$ and $\mathfrak{L}^*_y(\Omega) \leq n_1$.*

(iii) *[9, 2] Double fuzzy α^m-generalized -continuous function $(df - \alpha^m - g\ c)$ if the $h^{-1}(\Omega)$ is an $(m_1, n_1) - \alpha^m - gf - closed$ set in X for each $\mathfrak{L}_y(\breve{1} - \Omega) \geq m_1$ and $\mathfrak{L}^*_y(\breve{1} - \Omega) \leq n_1$.*

3. $(m_1, n_1) - \alpha^m - g^*$ Fuzzy Closed Set in DFTS

Definition 3.1. *Let $(X, \mathfrak{L}_x, \mathfrak{L}_{x^*})$ be a dfts, $\upsilon \in I^X, m_1 \in I_{m_1}$ and $n_1 \in I_{n_1}$, then υ is called:*

(i) *An $(m_1, n_1) - g^*$ fuzzy closed set $((m_1, n_1) - g^* f$ closed$)$ if $C_{\mathfrak{L}_x,\mathfrak{L}^*_x}(\upsilon, m_1, n_1) \leq \Omega$ whenever $\upsilon \leq \Omega$ and Ω is $(m_1, n_1) - g\ f$ open set in X.*

(ii) *An $(m_1, n_1) - \alpha^m - g^*$ fuzzy closed set $((m_1, n_1) - \alpha^m - g^* f$ closed$)$ if $\alpha^m C_{\mathfrak{L}_x,\mathfrak{L}^*_x}(\upsilon, m_1, n_1) \leq \Omega$ whenever $\upsilon \leq \Omega$ and Ω is $(m_1, n_1) - g\ f$ open set in X. υ is called an $((m_1, n_1) - \alpha^m - g^* f$ open$)$ if it is $(\breve{1} - \upsilon)$ is an $((m_1, n_1) - \alpha^m - g^* f$ closed$)$ set.*

Remark 3.2. *Every $(m_1, n_1) - fg$ closed set is an $(m_1, n_1) - fg^*$ closed set, the converse need not true. The following example show this case.*

Example 3.3. *Let $X = \{b, d\}$ and we define $(\mathfrak{L}_x(\mathfrak{G}), \mathfrak{L}^*_x(\mathfrak{G}))$ on X by*

$$\mathfrak{L}_x(\mathfrak{G}) = \begin{cases} \breve{1}, & if \mathfrak{G} \in \{\breve{0}, \breve{1}\}, \\ \frac{1}{2}, & \mathfrak{G}(x) = \mathfrak{G}, \\ \breve{0}, & otherwise \end{cases} \qquad \mathfrak{L}^*_x(\mathfrak{G}) = \begin{cases} \breve{0}, & if \mathfrak{G} \in \{\breve{0}, \breve{1}\}, \\ \frac{1}{2}, & \mathfrak{G}(x) = \mathfrak{G}, \\ \breve{1}, & otherwise \end{cases}$$

Such that, $\mathfrak{G}_1(b) = 0.4, \mathfrak{G}_1(d) = 0.5, \mathfrak{G}_2(b) = 0.7, \mathfrak{G}_2(d) = 0.5$
$\mathfrak{G}_1 \leq \mathfrak{G}_2, \mathfrak{G}^c_2 \leq \mathfrak{G}_1$ *is* $\left(\frac{1}{2}, \frac{1}{2}\right) - f\alpha - open \to C_{\mathfrak{L}_x,\mathfrak{L}^*_x}(\mathfrak{G}^c_1, \frac{1}{2}, \frac{1}{2}) \leq \mathfrak{G}_1, \mathfrak{G}^c_2 \leq \mathfrak{G}_1$. *So, $\mathfrak{G}^c_2$ is an $\left(\frac{1}{2}, \frac{1}{2}\right) - gf - closed \to \mathfrak{G}_2$ is an $\left(\frac{1}{2}, \frac{1}{2}\right) - gf - open$. $\mathfrak{G}_1 \leq \mathfrak{G}_2$, $C_{\mathfrak{L}_x,\mathfrak{L}^*_x}(\mathfrak{G}_1, \frac{1}{2}, \frac{1}{2}) \leq \mathfrak{G}_2$, $\beta^c_1 \leq \mathfrak{G}_2 \to \mathfrak{G}_1$ is an $\left(\frac{1}{2}, \frac{1}{2}\right) - g^* f - closed$. But, $\mathfrak{G}_1$ is not an $\left(\frac{1}{2}, \frac{1}{2}\right) - gf - closed$, since $\mathfrak{G}_2$ is not $\left(\frac{1}{2}, \frac{1}{2}\right) - f$ open.*

Definition 3.4. *Let $(X, \mathfrak{L}_x, \mathfrak{L}^*_x)$ and $(Y, \mathfrak{L}_y, \mathfrak{L}^*_y)$ be dfts's. A function $h : (X, \mathfrak{L}_x, \mathfrak{L}^*_x) \to (Y, \mathfrak{L}_y, \mathfrak{L}^*_y)$ is called double fuzzy$-\alpha^m - g^*$ continuous function $(df - \alpha^m - g^* c)$ if $h^{-1}(\Omega)$ is $(m_1, n_1)\alpha^m - g^* f$ open set in X, such that $\mathfrak{L}_y(\Omega) \geq m_1$ and $\mathfrak{L}^*_y(\Omega) \leq n_1$ whenever $m_1 \in I_{m1}$ and $n_1 \in I_{n1}$.*

Theorem 3.5. *Let $h : (X, \mathfrak{L}_x, \mathfrak{L}^*_x) \to (Y, \mathfrak{L}_y, \mathfrak{L}^*_y)$ is $(df - \alpha^m - g^* c)$ function iff $h^{-1}(\Omega)$ is $(m_1, n_1)\alpha^m - g^* f$ closed set. Such that $\mathfrak{L}_y(\breve{1} - \Omega) \geq m_1$ and $\mathfrak{L}^*_y(\breve{1} - \Omega) \geq n_1$.*

Proof . Suppose that $h : (X, \mathfrak{L}_x, \mathfrak{L}^*_x) \to (Y, \mathfrak{L}_y, \mathfrak{L}^*_y)$ is $(df - \alpha^m - g^* c)$ function $\to h^{-1}(\Omega)$ is $(m_1, n_1)\alpha^m - g^* f$ open set, such that $\mathfrak{L}_y(\breve{1} - \Omega) \geq m_1$ and $\mathfrak{L}^*_y(\breve{1} - \Omega) \leq n_1$. $h^{-1}(\breve{1} - \Omega) = \breve{1} - h^{-1}(\Omega)$ is $(m_1, n_1)\alpha^m - g^*$fuzzy open set in Y. $h^{-1}(\Omega)$ is $(m_1, n_1)\alpha^m - g^* f$ closed set. Suppose that $h^{-1}(\Omega)$ is $(m_1, n_1)\alpha^m - g^*$fuzzy closed set in Y. Put $\upsilon = \breve{1} - \Omega$, $\mathfrak{L}_y(\breve{1} - (\breve{1} - \upsilon)) \geq m_1$ and $\mathfrak{L}^*_y(\breve{1} - (\breve{1} - \upsilon)) \leq n_1$. Since $h^{-1}(\upsilon) = h^{-1}(\breve{1} - \Omega) = \breve{1} - h^{-1}(\Omega)$. □

Theorem 3.6. *$h:(X,\mathfrak{L}_x,\mathfrak{L}_x^*)\to(Y,\mathfrak{L}_y,\mathfrak{L}_y^*)$ is $df-c$ function then h is $df-\alpha^m-g^*c$ function, but not conversely we can show by following example.*

Proof . Suppose that $\mathfrak{L}_y(\upsilon)\geq m_1$ and $\mathfrak{L}_y^*(\upsilon)\leq n_1$. . Since h is $df-c$ function, then $\mathfrak{L}_x(h^{-1}(\upsilon))\geq m_1$ and $\mathfrak{L}_x^*(h^{-1}(\upsilon))\leq n_1$. So $h^{-1}(\upsilon)$ is $(m_1,n_1)\alpha^m-g^*f$ open set in X. Therefore, h is $df-\alpha^m-g^*c$ function. □

Example 3.7. *Let $X=\{b,d\}$, $Y=\{r,s\}$ and we define $(\mathfrak{L}_x(\mathfrak{G}),\mathfrak{L}_x^*(\mathfrak{G}))$ on X by:*

$$\mathfrak{L}_x(\mathfrak{G})=\begin{cases}\breve{1}, & if\mathfrak{G}\in\{\breve{0},\breve{1}\},\\ \frac{1}{2}, & \mathfrak{G}(x)=\mathfrak{G}1,\\ \frac{1}{4}, & \mathfrak{G}(x)=\mathfrak{G}2,\\ \breve{0}, & otherwise\end{cases}\qquad \mathfrak{L}_x^*(\beta)=\begin{cases}\breve{0}, & if\mathfrak{G}\in\{\breve{0},\breve{1}\},\\ \frac{1}{2}, & \mathfrak{G}(x)=\mathfrak{G}1,\\ \frac{3}{4}, & \mathfrak{G}(x)=\mathfrak{G}2,\\ \breve{1}, & otherwise\end{cases}$$

Such that, $\mathfrak{G}_1(b)=0.3,\mathfrak{G}_1(d)=0.4,\mathfrak{G}_2(b)=0.7,\mathfrak{G}_2(d)=0.6$
Also, we define $(\mathfrak{L}_y(\gamma),\mathfrak{L}_y^*(\gamma))$ on Y by:

$$\mathfrak{L}_y(\gamma)=\begin{cases}\breve{1}, & if\gamma\in\{\breve{0},\breve{1}\},\\ \frac{1}{2}, & \gamma(y)=\gamma1,\\ \frac{1}{4}, & \gamma(x)=\gamma2,\\ \breve{0}, & otherwise\end{cases}\qquad \mathfrak{L}_y^*(\gamma)=\begin{cases}\breve{0}, & if\gamma\in\{\breve{0},\breve{1}\},\\ \frac{1}{2}, & \gamma(y)=\gamma1,\\ \frac{3}{4}, & \gamma(y)=\gamma2,\\ \breve{1}, & otherwise\end{cases}$$

$\gamma_1(r)=0.7,\gamma_1(s)=0.8,\gamma_2(r)=0.3,\gamma_2(s)=0.2$
When, the function h between two dfts $(\mathfrak{L}_x(\mathfrak{G}),\mathfrak{L}_x^*(\mathfrak{G}))$ and $(\mathfrak{L}_y(\gamma),\mathfrak{L}_y^*(\gamma))$ is defined by: $h:(X,\mathfrak{L}_x,\mathfrak{L}_x^*)\to(Y,\mathfrak{L}_y,\mathfrak{L}_y^*)$ as, $h(b)=r,h(d)=s$.
So, $h^{-1}(\gamma_1)=(0.7,0.8),h^{-1}(\gamma_1)\leq\mathfrak{G}_2$, $\mathfrak{G}_2$ is an $\left(\frac{1}{4},\frac{1}{2}\right)-f\alpha^m$-closed set. $\alpha^mC_{\tau x,\tau x^*}(h^{-1}(\gamma_1),\frac{1}{4},\frac{1}{2})=\bigwedge\{\mathfrak{G}_2\in I^X,h^{-1}(\gamma_1)\leq\mathfrak{G}_2\}=\mathfrak{G}_2\leq\mathfrak{G}_2$, $h^{-1}(\gamma_1)$ is an $(\frac{1}{4},\frac{1}{2})-\alpha^m-gf$closed set $\to h$ is $df-\alpha^m-gc$. Since $h^{-1}(\gamma_1)\notin(\mathfrak{L}_x,\mathfrak{L}_x^*)\to h$ is not $df-c$. And, $h^{-1}(\gamma_1)\nleq C_{\mathfrak{L}_x,\mathfrak{L}_x^*}(I_{\mathfrak{L}_x,\mathfrak{L}_x^*}(h^{-1}(\gamma_1),\frac{1}{4},\frac{1}{2})\frac{1}{4},\frac{1}{2})$, since, $C_{\mathfrak{L}_x,\mathfrak{L}_x^*}(I_{\mathfrak{L}_x,\mathfrak{L}_x^*}(h^{-1}(\gamma_1),\frac{1}{4},\frac{1}{2})\frac{1}{4},\frac{1}{2})=\breve{0}$, $h^{-1}(\gamma_1)\nleq\breve{0}$. So $h^{-1}(\gamma_1)$ is not $\left(\frac{1}{4},\frac{1}{2}\right)-f\alpha^m$- open set $\to h$ is not $df-\alpha^m-c$.

Theorem 3.8. *$(X,\mathfrak{L}_x,\mathfrak{L}_x^*)$ and $(Y,\mathfrak{L}_y,\mathfrak{L}_y^*)$ be dfts's. $h:(X,\mathfrak{L}_x,\mathfrak{L}_x^*)\to(Y,\mathfrak{L}_y,\mathfrak{L}_y^*)$ is $df-\alpha^m-gc$ function then h is $df-\alpha^m-g^*c$ function.*

Proof . Let $h:(X,\mathfrak{L}_x,\mathfrak{L}_x^*)\to(Y,\mathfrak{L}_y,\mathfrak{L}_y^*)$ is $df-\alpha^m-gc$ function and $\mathfrak{L}_y(\upsilon)\geq m_1$ and $\mathfrak{L}_y^*(\upsilon)\leq n_1$ whenever $m_1\in I_{m1}$ and $n_1\in I_{n1}$, such that $h^{-1}(\upsilon)$ is an $(m_1,n_1)-\alpha^mgf$open set in X, $\alpha^mC_{\mathfrak{L}_y,\mathfrak{L}_y^*}(h(\upsilon),m_1,n_1)\leq\Omega$, whenever $\upsilon\leq\Omega$.
Since, every an $(m_1,n_1)-fg$ closed set is an $(m_1,s_1)-fg^*$ closed set, and $\alpha^mC_{\mathfrak{L}_x,\mathfrak{L}_x^*}(h^{-1}(\upsilon),m_1,n_1)\leq C_{\mathfrak{L}_x,\mathfrak{L}_x^*}(h^{-1}(\upsilon),m_1,n_1)\leq\omega$, put $\omega=\breve{1}-\Omega$. $h^{-1}(\upsilon)$ is an $(m_1,n_1)-\alpha^m-g^*f$ open set in X, such that ω is an $(m_1,n_1)-g$ fuzzy open set. □

Definition 3.9. *Let $(X,\mathfrak{L}_x,\mathfrak{L}_x^*)$ be a dfts's is called $df-(\mathfrak{L}_x,\mathfrak{L}_x^*)\tau\alpha^m$ space if every $(m_1,n_1)-\alpha^m-g^*$ fuzzy closed set is an $(m_1,n_1)-f$ closed set.*

Theorem 3.10. *Let $(X,\mathfrak{L}_x,\mathfrak{L}_x^*)\to(Y,\mathfrak{L}_y,\mathfrak{L}_y^*)$ is $df-\alpha^m-g^*c$ function, $(X,\mathfrak{L}_x,\mathfrak{L}_x^*)$ is $df-(\mathfrak{L}_x,\mathfrak{L}_x^*)\mathfrak{L}\alpha^m$ space.*
Then f is $df-c$ function.

Proof . Let $\mathfrak{L}_y(\breve{1}-\upsilon)\geq m_1$ and $\mathfrak{L}_x^*(\breve{1}-\upsilon)\leq n_1$, since f is $df-\alpha^m-g^*c$ function. Then $h^{-1}(\upsilon)$ is an $(m_1,n_1)\alpha^mg^*$fuzzy closed set in X. Since $(X,\mathfrak{L}_x,\mathfrak{L}_x^*)$ is $df-(\mathfrak{L}_x,\mathfrak{L}_x^*)\mathfrak{L}\alpha^m$ space, so $\mathfrak{L}_x(\breve{1}-h^{-1}(\upsilon))\leq n_1$. Thus h is $df-c$ function. □

Theorem 3.11. *Let $h : (X, \mathfrak{L}_x, \mathfrak{L}^*_x) \to (Y, \mathfrak{L}_y, \mathfrak{L}^*_y)$ is $df - \alpha^m - c$ function, $(X, \mathfrak{L}_x, \mathfrak{L}^*_x)$ is $df - (\mathfrak{L}_x, \mathfrak{L}^*_x)\mathfrak{L}\alpha^m$ space.*
Then h is $df - \alpha^m - g^* - c$ function.

Proof . Similar proof of theorem 3.10. □

Theorem 3.12. *Let $(X, \mathfrak{L}_x, \mathfrak{L}^*_x), (Y, \mathfrak{L}_y, \mathfrak{L}^*_y)$ and $(Z, \mathfrak{L}_z, \mathfrak{L}^*_z)$ be dfts's. $h : (X, \mathfrak{L}_x, \mathfrak{L}^*_x) \to (Y, \mathfrak{L}_y, \mathfrak{L}^*_y)$ and $k : (Y, \mathfrak{L}_y, \mathfrak{L}^*_y) \to (Z, \mathfrak{L}_z, \mathfrak{L}^*_z)$ are both $df - \alpha^m - g^*$ continuous function, $(Y, \mathfrak{L}_y, \mathfrak{L}^*_y)$ is $df - (\tau_y, \tau^*_y)\tau\alpha^m$. Then $hok : (X, \mathfrak{L}_x, \mathfrak{L}^*_x) \to (Z, \mathfrak{L}_z, \mathfrak{L}^*_z)$ is $df - \alpha^m - g^*$ continuous function.*

Proof . Assume that $\mathfrak{L}_z(\bar{1} - \upsilon) \geq m_1$ and $\mathfrak{L}^*_z(\bar{1} - \upsilon) \leq n_1$. Then $k^{-1}(\upsilon)$ is $(m_1, n_1) - \alpha^m - g^*$fuzzy closed set in Y. Since $(Y, \mathfrak{L}_y, \mathfrak{L}^*_y)$ is $df - (\mathfrak{L}_y, \mathfrak{L}^*_y)\tau\alpha^m$ space, then $\mathfrak{L}_y(k^{-1}(\upsilon)) \geq m_1$ and $\mathfrak{L}^*_y(k^{-1}(\upsilon)) \leq n_1$ And, since f is $df - \alpha^m - g^*$continuous function, so $h^{-1}(k^{-1}(\upsilon))$ is $(m_1, n_1) - \alpha^m - g^*$fuzzy closed set in X. That is hok is $df - \alpha^m - g^*$continuous function. □

4. Conclusions

In the presented study, we investigated a new class of generalized closed sets in double fuzzy topological spaces called fuzzy $\alpha^m - g^*$fuzzy, by using the fuzzy$-\alpha^m$closed set. In this way, the relationship of the new concept with previous concepts as well as the characteristics of this concept is investigated through important theorems. Moreover, generalizations of the functions are presented according to the concept, their properties are studied, and some necessary examples that show the properties of this concept and its relationships.

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