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n-Jordan $*$-derivations in Fréchet locally C^*-algebras

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Abstract

Using the fixed point method, we prove the Hyers-Ulam stability and the superstability of n-Jordan $*$-derivations in Fréchet locally C^*-algebras for the following generalized Jensen-type functional equation

$$f\left(\frac{a+b}{2}\right)+f\left(\frac{a-b}{2}\right)=f(a).$$

Keywords: n-Jordan $*$-derivation; Fréchet locally C^*-algebra; Fréchet algebra; fixed point method; Hyers-Ulam stability
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1. Introduction and preliminaries

In this paper, assume that n is an integer greater than 1.

Definition 1.1. *Let $n \in \mathbb{N}-\{1\}$ and let A be a ring and B be an A-module. An additive map $D: A \to B$ is called n-Jordan derivation if*

$$D(a^n)=D(a)a^{n-1}+aD(a)a^{n-2}+\ldots+a^{n-2}D(a)a+a^{n-1}D(a),$$

for all $a \in A$.

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The concept of n-jordan derivations was studied by Eshaghi Ghordji.(see also [7, 8, 13]).

Definition 1.2. *Let A , B be C^*-algebras. A $\mathbb{C}$-linear mapping $D : A \to B$ is called n-Jordan $*$-derivation if*

$$D(a^n) = D(a)a^{n-1} + aD(a)a^{n-2} + \ldots + a^{n-2}D(a)a + a^{n-1}D(a),$$

$$D(a^*) = D(a)^*$$

for all $a \in A$.

We say functional equation (ξ) is stable if any function g satisfying the equation (ξ) approximately is near to the true solution of (ξ). We say that a functional equation is superstable if every approximate solution is an exact solution of it.

The stability of functional equations was first introduced by Ulam [28] in 1940. More precisely, he proposed the following problem: Given a group G_1, a metric group (G_2, d) and $\epsilon > 0$, does there exist a $\delta > 0$ such that if a mapping $f : G_1 \to G_2$ satisfies the inequality $d(f(ab), f(a)f(b)) < \delta$ for all $a, b \in G_1$, then there exists a homomorphism $T : G_1 \to G_2$ such that $d(f(a), T(a)) < \epsilon$ for all $a \in G_1$? As mentioned above, when this problem has a solution, we say that the homomorphisms from G_1 to G_2 are stable. In 1941, Hyers [16] gave a partial solution of *Ulam's* problem for the case of approximate additive mappings under the assumption that G_1 and G_2 are Banach spaces. In 1950, Aoki [2] generalized the Hyers' theorem for approximately additive mappings. In 1978, Rassias [27] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences.

Theorem 1.3. *[27] Let $f : E \to E'$ be a mapping from a normed vector space E into a Banach space E' subject to the inequality*

$$\|f(a + b) - f(a) - f(b)\| \leq \epsilon(\|a\|^p + \|b\|^p) \tag{1.1}$$

for all $a, b \in E$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then there exists a unique additive mapping $T : E \to E'$ such that

$$\|f(a) - T(a)\| \leq \frac{2\epsilon}{2 - 2^p}\|a\|^p \tag{1.2}$$

for all $a \in E$. If $p < 0$ then inequality (1.1) holds for all $a, b \neq 0$, and (1.2) holds for $a \neq 0$. Also, if the function $t \to f(ta)$ from $\mathbb{R}$ into E' is continuous for each fixed $a \in X$, then T is linear.

The result of the Rassias theorem was generalized by Forti [14] and Gavruta [15] who permitted the Cauchy difference to become arbitrary unbounded. Some results on the stability of functional equations in single variable and nonlinear iterative equations can be found in [1, 29]. During the last decades several stability problems of functional equations have been investigated by many mathematicians (see [6, 9, 10, 11, 12, 17, 18, 20, 21, 22, 23, 24, 25]).

Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a *generalized metric* on X if d satisfies

(1) $d(x, y) = 0$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(3) $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$.

We recall a fundamental result in fixed point theory.

Theorem 1.4. ([3, 5]) *Let (X, d) be a complete generalized metric space and let $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then for each given element $x \in X$, either*

$$d(J^n x, J^{n+1}x) = \infty$$

for all nonnegative integers n or there exists a positive integer n_0 such that

(1) $d(J^n x, J^{n+1}x) < \infty, \qquad \forall n \geq n_0$;

(2) *the sequence $\{J^n x\}$ converges to a fixed point y^* of J;*

(3) *y^* is the unique fixed point of J in the set $Y = \{y \in X \mid d(J^{n_0}x, y) < \infty\}$;*

(4) $d(y, y^*) \leq \frac{1}{1-L} d(y, Jy)$ *for all $y \in Y$.*

Definition 1.5. *A topological vector space X is a Fréchet space if it satisfies the following three properties:*

(1) *it is complete as a uniform space;*

(2) *it is locally convex;*

(3) *its topology can be induced by a translation invariant metric, i.e., a metric $d : X \times X \to \mathbb{R}$ such that $d(x, y) = d(x + a, y + a)$ for all $a, x, y \in X$.*

For more detailed definitions of such terminologies, we can refer to [9]. Note that a ternary algebra is called a ternary Fréchet algebra if it is a Fréchet space with a metric d.

Fréchet algebras, named after Maurice Fréchet, are special topological algebras as follows.

Note that the topology on A can be induced by a translation invariant metric, i.e. a metric $d : X \times X \to \mathbb{R}$ such that $d(x, y) = d(x + a, y + a)$ for all $a, x, y \in X$.

Trivially, every Banach algebra is a Fréchet algebra as the norm induces a translation invariant metric and the space is complete with respect to this metric.

A locally C^*-algebra is a complete Hausdorff complex $*$-algebra A whose topology is determined by its continuous C^*-seminorms in the sense that a net $\{a_i\}_{i\in I}$ converges to 0 if and if the net $\{p(a_i)\}_{i\in I}$ converges to 0 for each continuous C^*-seminorm p on A (see [19, 26]). The set of all continuous C^*-seminorms on A is denoted by $S(A)$. A Fréchet locally C^*-algebra is a locally C^*-algebra whose topology is determined by a countable family of C^*-seminorms. Clearly, any C^*-algebra is a Fréchet locally C^*-algebra.

For given two locally C^*-algebras A and B, a morphism of locally C^*-algebras from A to B is a continuous $*$-morphism φ from A to B. An isomorphism of locally C^*-algebras from A to B is a bijective mapping $\varphi : A \to B$ such that φ and φ^{-1} are morphisms of locally C^*-algebras.

Hilbert modules over locally C^*-algebras are generalization of Hilbert C^*-modules by allowing the inner product to take values in a locally C^*-algebra rather than in a C^*-algebra.

In this paper, using the fixed point method, we prove the Hyers-Ulam stability and the superstability of n-Jordan $*$-derivations in Fréchet locally C^*-algebras for the the following generalized Jensen-type functional equation

$$f\left(\frac{a+b}{2}\right) + f\left(\frac{a-b}{2}\right) = f(a).$$

2. Stability of n-Jordan $*$-derivations

Lemma 2.1. ([23]) *Let A, B be C^*-algebras, and let $D : A \to B$ be a mapping such that*

$$\|D\left(\frac{a+b}{2}\right) + D\left(\frac{a-b}{2}\right)\|_B \leq \|D(a)\|_B, \tag{2.1}$$

for all $a, b \in A$. Then D is Cauchy additive.

Proof . By putting $a = b = 0$ in (2.1) we get $\|2D(0)\| \leq \|D(0)\|$. So $D(0) = 0$, for all $a, b \in A$. Letting $x = \frac{a+b}{2}$,$y = \frac{a-b}{2}$ in (2.1), we conclude that D is additive

$$D(x) + D(y) = D(a) = D\left(\frac{a+b}{2} + \frac{a-b}{2}\right) = D(x+y).$$

□

Now, we prove the Hyers-Ulam stability problem for n-Jordan $*$-derivations in Fréchet locally C^*- algebras.

Theorem 2.2. *Let A, B be Fréchet locally C^*-algebras, and θ be nonnegative real number. Let $f : A \to B$ be a mapping such that*

$$\begin{aligned} \|\mu f(\frac{a+b}{2}) \ &+ \ \mu f(\frac{a-b}{2}) - f(\mu a) + f(c^n) - f(c)c^{n-1} + cf(c)c^{n-2} + \dots \\ &+ \ c^{n-2}f(c)c + c^{n-1}f(c) + f(d^*) - f(d)^*\|_B \leq \theta \end{aligned} \tag{2.2}$$

for all $\mu \in \mathbb{T}^1 := \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and all $a, b, c, d \in A$. Then the mapping $f : A \to B$ is an n-Jordan $*$-derivation.

Proof . Suppose that $\mu = 1$ and $c, d = 0$ in (2.2) by Lemma 2.1, the mapping $f : A \to B$ is additive. By putting $a = b$ and $c = d = 0$ in (2.2), we get

$$\|\mu f(\frac{2a}{2}) + \mu f(0) - f(\mu a)\| \leq \theta,$$

for all $a \in A$ and $\mu \in \mathbb{T}^1$. So

$$\mu f(a) = f(\mu a),$$

for all $a \in A$ and $\mu \in \mathbb{T}^1$.

By [4, Theorem 2.1], the mapping $f : A \to B$ is $\mathbb{C}$-Linear. Letting $a = b = d = 0$ in (2.2), we get

$$f(c^n) = f(c)c^{n-1} + cf(c)c^{n-2} + \dots + c^{n-2}f(c)c + c^{n-1}f(c),$$

for all $c \in A$ and by letting $a = b = c = 0$ in (2.2), we have

$$f(d^*) = f(d)^*,$$

for all $d \in A$. Hence the mapping $f : A \to B$ is a n-Jordan $*$-derivation. □

Theorem 2.3. *Let A, B be Frechet locally C^*-algebras and let θ be nonnegative real number. Let $f : A \to B$ be a mapping satisfying then the mapping $f : A \to B$ is a n-Jordan $*$-derivation*

Proof . The proof is similar to the proof of Theorem 2.2. □

Now we prove the Hyers-Ulam stability of n-Jordan derivations in C^*-algebras.

Theorem 2.4. *Let A, B be Fréchet locally C^*-algebras. Let $f : A \to B$ be a mapping for which there exists a function $\varphi : A^4 \to \mathbb{R}^+$ such that*

$$\psi(a, b, c, d) = \sum_{i=0}^{\infty} 2^{-i} \varphi(2^i a, 2^i b, 2^i c, 2^i d) < \infty, \tag{2.3}$$

$$\begin{aligned}\|\mu f(\frac{a+b}{2}) &+ \mu f(\frac{a-b}{2}) - f(\mu a) + f(c^n) - f(c)c^{n-1} + cf(c)c^{n-2} + \dots \\ &+ c^{n-2}f(c)c + c^{n-1}f(c) + f(d^*) - f(d)^*\|_B \le \varphi(a,b,c,d) \qquad (2.4)\end{aligned}$$

for all $a,b,c,d \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \le \psi(a,a,0,0) \qquad (2.5)$$

for all $a \in A$.

Proof . By putting $\mu = 1$ and $b = c = d = 0$ and replacing a by $2a$ in (2.4), we get

$$\|2f(\frac{2a}{2}) - f(2a)\|_B \le \varphi(2a,0,0,0) \qquad (2.6)$$

for all $a \in A$. Using the induction method, we have

$$\|f(a) - 2^{-n}f(2^n a)\|_B \le \frac{1}{2^n}\sum_{i=1}^{n}\varphi(2^i a,0,0,0) \qquad (2.7)$$

for all $a \in A$. Replace a by a^m in (2.6) and then divide by 2^m, we have

$$\|f(a^m) - 2^{-n-m}f(2^{n+m}a)\|_B \le \frac{1}{2^{n+m}}\sum_{i=m}^{m+n}\varphi(2^i a,0,0,0)$$

for all $a \in A$. Hence, $\{2^{-n}f(2^n a)\}$ is a Cauchy sequence. Since A is complete, then

$$D(a) = \lim_n 2^{-n}f(2^n a)$$

exists for all $a \in A$. By (2.4) one can show that

$$\begin{aligned}&\|D(\frac{a+b}{2}) + D(\frac{a-b}{2}) - D(a)\|_B \\ = &\lim_n \frac{1}{2^n}\|f(2^{n-1}(a+b)) + f(2^{n-1}(a-b)) - f(2^n a)\|_B \\ \le &\lim_n \frac{1}{2^n}\varphi(2^n a, 2^n b, 0, 0) \qquad (2.8)\end{aligned}$$

for all $a,b \in A$. So

$$D(\frac{a+b}{2}) + D(\frac{a-b}{2}) = D(a)$$

for all $a,b \in A$. Put $x = \frac{a+b}{2}$, $y = \frac{a-b}{2}$ in above equation, we have

$$D(x) + D(y) = D(a) = D(\frac{a+b}{2} + \frac{a-b}{2}) = D(x+y)$$

for all $x,y \in A$. Hence, D is Cauchy additive. On the other hand, we have

$$D(\mu a) - \mu D(a) = \lim_n \frac{1}{2^n}\|f(\mu 2^n a) - \mu f(2^n a)\|_B \le \lim_n \frac{1}{2^n}\varphi(2^n a, 2^n a, 0, 0) = 0$$

for all $\mu \in \mathbb{T}^1$, and all $a \in A$. So it is easy to show that D is linear. It follow from (2.4) that

$$\begin{aligned} & \|D(c^n) - D(c)c^{n-1} + cD(c)c^{n-2} + \ldots + c^{n-2}D(c)c + c^{n-1}D(c)\|_B \\ = & \lim_m \|\frac{1}{2^{mn}} f(2^m c)^n) - \frac{1}{2^{mn}}(f(2^m 2^{m(n-1)}c) + f(2^{2m}2^{m(n-2)}c) \\ + & f(2^{3m}2^{m(n-3)}c))^n + \ldots + f(2^{m(n-1)}2^m c)\|_B \leq \lim_m \frac{1}{2^{mn}}\varphi(0,0,0,2^m c) \\ \leq & \lim_m \frac{1}{2^m}\varphi(0,0,0,2^m c) \\ = & 0 \end{aligned} \tag{2.9}$$

for all $c \in A$. and we have

$$\begin{aligned} \|D(d^*) - D(d)^*\|_B &= \lim_n \|\frac{1}{2^n} f(2^n d^*) - \frac{1}{2^n}(f(2^n d))^*\|_B \\ &\leq \lim_n \frac{1}{2^{mn}}\varphi(0,0,0,2^n d) \\ &= 0 \end{aligned} \tag{2.10}$$

for all $d \in A$. Hence $D : A \to B$ is a unique n-Jordan $*$-derivation. □

Corollary 2.5. *Let A, B be Fréchet locally C^*-algebras, and let $f : A \to B$ be a mapping with $f(0) = 0$ for which there exist constants $\theta \geq 0$ and $p_1, p_2, p_3, p_4 \in (-\infty, 1)$ such that*

$$\begin{aligned} \|\mu f(\frac{a+b}{2}) &+ \mu f(\frac{a-b}{2}) - f(\mu a) + f(c^n) - f(c)c^{n-1} + cf(c)c^{n-2} + \ldots \\ &+ c^{n-2}f(c)c + c^{n-1}f(c) + f(d^*) - f(d)^*\|_B \\ &\leq \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4}) \end{aligned} \tag{2.11}$$

for all $a, b, c, d \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \leq \frac{2\theta\|a\|_A^{p_1}}{2 - 2^{p_1}} \tag{2.12}$$

for all $a \in A$.

Proof . By putting $\varphi(a, b, c, d) = \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4})$ in Theorem 2.3, we have

$$\|f(a) - D(a)\|_B \leq \frac{2\theta\|a\|_A^{p_1}}{2 - 2^{p_1}}$$

for all $a \in A$, as desired. □

Theorem 2.6. *Let A, B be Fréchet locally C^*-algebras. Let $f : A \to B$ be a mapping for which there exists a function $\varphi : A^4 \to \mathbb{R}^+$ such that*

$$\psi(a, b, c, d) = \sum_{i=0}^{\infty} 2^i \varphi(2^{-i}a, 2^{-i}b, 2^{-i}c, 2^{-i}d) < \infty, \tag{2.13}$$

$$\begin{aligned} \|\mu f(\frac{a+b}{2}) &+ \mu f(\frac{a-b}{2}) - f(\mu a) + f(c^n) - f(c)c^{n-1} + cf(c)c^{n-2} + \ldots \\ &+ c^{n-2}f(c)c + c^{n-1}f(c) + f(d^*) - f(d)^*\|_B \leq \varphi(a, b, c, d) \end{aligned} \tag{2.14}$$

for all $a, b, c, d \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \leq \psi(a, a, 0, 0) \tag{2.15}$$

for all $a \in A$.

Proof . Suppose that $\mu = 1$ and $b = c = d = 0$ in (2.14), we get

$$\|f(a) - 2f(2^{-1}a)\|_B \leq \varphi(a, 0, 0, 0) \tag{2.16}$$

for all $a \in A$. Using the induction method, we have

$$\|f(a) - 2^n f(2^{-n}a)\|_B \leq \sum_{i=1}^{n} 2^i \varphi(2^{-i}a, 0, 0, 0) \tag{2.17}$$

for all $a \in A$. Replace a by a^m in (2.17) and then divide by 2^m, we have

$$\|f(a^m) - 2^{n+m} f(2^{-n-m}a)\|_B \leq \sum_{i=m}^{m+n} 2^i \varphi(2^{-i}a, 0, 0, 0)$$

for all $a \in A$. Hence, $\{2^n f(2^{-n}a)\}$ is a Cauchy sequence. Since A is complete, then

$$D(a) = \lim_n 2^n f(2^{-n}a)$$

exists for all $a \in A$. By (2.14) one can show that

$$\begin{aligned} & \|D(\frac{a+b}{2}) + D(\frac{a-b}{2}) - D(a)\|_B \\ = & \lim_n 2^n \|f(2^{-n-1}(a+b)) + f(2^{-n-1}(a-b)) - 2f(2^{-n}a)\|_B \\ \leq & \lim_n 2^n \varphi(2^{-n}a, 2^{-n}b, 0, 0) \end{aligned} \tag{2.18}$$

for all $a, b \in A$. So

$$D(\frac{a+b}{2}) + D(\frac{a-b}{2}) = D(a)$$

for all $a, b \in A$. Put $x = \frac{a+b}{2}$, $y = \frac{a-b}{2}$ in above equation, we have

$$D(x) + D(y) = D(a) = D(\frac{a+b}{2} + \frac{a-b}{2}) = D(x + y)$$

for all $x, y \in A$. Hence, D is Cauchy additive.

The rest of proof is similar to the proof of Theorem 2.3. □

Corollary 2.7. *Let A, B be Fréchet locally C^*-algebras, and let $f : A \to B$ be a mapping with $f(0) = 0$ for which there exist constants $\theta \geq 0$ and $p_1, p_2, p_3, p_4 \in (-\infty, 1)$ such that*

$$\begin{aligned} \|\mu f(\frac{a+b}{2}) \ & + \ \mu f(\frac{a-b}{2}) - f(\mu a) + f(c^n) - f(c)c^{n-1} + cf(c)c^{n-2} + \dots \\ & + \ c^{n-2}f(c)c + c^{n-1}f(c) + f(d^*) - f(d)^*\|_B \\ & \leq \ \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4}) \end{aligned} \tag{2.19}$$

for all $a, b, c, d \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \leq \frac{r\theta\|a\|_A^{p_1}}{2 - 2^{p_1}} \tag{2.20}$$

for $r < 1$ and all $a \in A$.

Proof . Letting $\varphi(a,b,c,d) = \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4})$ in Theorem 2.5, we have

$$\|f(a) - D(a)\|_B \leq \frac{r\theta\|a\|_A^{p_1}}{2 - 2^{p_1}}$$

for $r < 1$ and all $a \in A$, as desired. □

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