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# Twofold of algebraic decomposition method used for the watermarking scheme with $LWT$ over medical images

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**Abstract**

Mathematics has always been of great importance in various sciences, especially computer science. The mechanism used to embed various types of information in a host medical images to safeguard the privacy of the patient including the patient's name, doctor's digital signature is called watermarking. There are a lot of improved watermark algorithms, however, this information is susceptible to attack when the data are transferred over universal internet channels. This paper proposed a robust watermark algorithm that uses a Lifting Wavelet Transform ($LWT$) and two times of the Hessenberg Matrix Decomposition Method ($HMDM$) to embed a watermark in a chosen channel of the host image after performing the transform. The experimental results demonstrate that the improvement appears (higher robustness against $JPEG$ compression attack) and good imperceptibility against some attacks, to evaluate the fineness of the original with watermarked images and the extracted watermark respectively.

*Keywords:* Hessenberg Matrix Decomposition Method (HMDM), Lifting Wavelet Transform (LWT), Image Watermarking.
*2010 MSC:* 49M27; 65T60, 94A08

## 1. Introduction

In this era, there is a rapid development of ways of communication between people and therefore the internet has become an important part of our daily life to transfer data and share with others

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such as photos, videos, documents and other, between doctors and hospital staff such as medical images these data need protection to protect the privacy of patient information or not tampering by an unauthorized party watermark is one of the methods used to protect patient data [1]. A watermark is a process of introducing an identifier within the host data in an intangible manner. The watermark is based on the controlled distortion of the host data, depending on the resistance of the watermark to the attacks. There are two types robust and fragile, Based on the information needed to extract the watermark the algorithm is based on three types non-blind, semi-blind and blind [2].

Mathematics and especially linear algebra include a lot of significant tools to be useful in all science. Matrix decomposition is one of the tools that factorize any matrix to be a multiplication of two or more matrices [3]. The most known and considerable one is the Singular value decomposition ($SVD$) that decomposes the matrix into a product of three matrices $U\Sigma V^T$ where $U$ and $V$ are orthogonal (unitary) matrices while $\Sigma$ is a diagonal matrix. The authors turned to other methods of matrix decomposition that had not previously been used in watermarking techniques. Or, combine other tools with the $SVD$ method like transformations and optimal algorithms. Using the $SVD$ method has much usefulness in particular in image watermarking. Joseph et al. in [4] proposed to use the $SVD$ method with the $DWT$ to embed two watermarks into the two mid channels of the transform applied on the cover image. Thakur et. al [5] utilized several tools with the $SVD$ method to suggest a watermarking approach to secure medical images based on chaos. On the other hand, Sabri et al. in [6] implemented the $LU$ decomposition method with fuzzy theory on the host image to build a watermarking system based on 2-level $LWT$, the mid channel obtained from the second level is selected to embed the secret bits. Furthermore, a fragile watermarking mechanism is presented by Nejati in [7] depending on $QR$ factorization method and Fourier Transform to hide images. While Pranab et al. in [8] used fan-beam transform with the $QR$ decomposition method to protect multimedia information. In [9], Mohammed et al. combine the Hessenberg decomposition method with the genetic algorithm to optimize the results. In addition, the quaternion $QR$ decomposition method is applied in [10] by Chen et al. to design a robust blind watermarking technique on color images without mark each color channel individually.

In this paper, a new linear algebra application is introduced depending on the algebraic decomposition method called Hessenberg using the twofold repetition. The $LWT$ is employed first to perform the embedding process in the $HL$ channel. Several attacks are used on images to show that the proposed scheme is robust and imperceptible.

This paper is organized as the following. The twofold algebraic decomposition method ($HMDM$) with $LWT$ is introduced in Section 2. In Section 3 the experimental results are discussed in detail. In Section 4 a comparison with other works is given and Section 5 concludes the proposed scheme.

## 2. Twofold of Algebraic Decomposition Method ($HMDM$) with $LWT$

A watermarking scheme using algebraic decomposition $HMDM$ and $LWT$ to embed and extract a watermark ($WT$) in a medical host image ($HI$) is introduced in this section.

### 2.1. Embedding Algorithm

1- Enter the host image ($HI$) of size $\rho \times \rho$. Switch $HI$ into a gray-scale form.
2- Section HI using 1-level $LWT$ into the subchannels $CA, CH, CV$ and $CD$.
3- Section channel $CH$ into $4 \times 4$ nonoverlapping blocks.
4- Perform the logistic map as: $\upsilon_{i+1} = \mu\upsilon_i(1 - \upsilon_i)$ to switch locations of the blocks.
5- Perform the $HMDM$ ($PHP^T$) to take out the Hessenberg matrix $H$ from each block.

6- Perform the $HMDM$ $(PHP^T)$ on each matrix $H$ in step 5.
7- Enter the watermark $WT$. Switch $WT$ to a binary form $BWT$.
8- Modify the watermark binary bits in $H$ sub-matrix produced from step 6.

$$H\{i,j\}(1,1) = max(H\{i,j\}(1,:)) + 2 \text{ if } BWT(i,j) = 1$$

$$H\{i,j\}(1,1) = min(H\{i,j\}(1,:)) - 2 \text{ if } BWT(i,j) = 0$$

where the coordinate $(1,1)$ is the base of the embedding process in the matrix $H$ which is the midvalue in $H$.
9- Perform the reverse of the $HMDM$ $(PHP^T)$ and logistic map to obtain the $WTedI$.

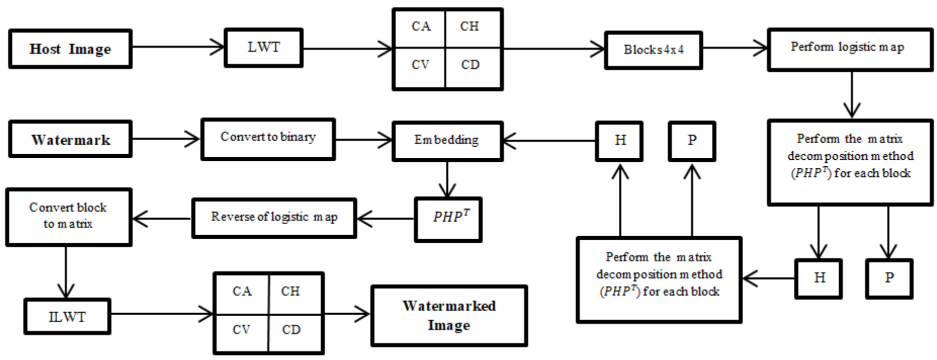



Figure 1: Embedding Algorithm

### 2.2. *Extracting Algorithm*

1- Enter the $WTedI$ and switch it into a grayscale form.
2- Section $WTedI$ using 1-level $LWT$ into the subchannels $CA, CH, CV$ and $CD$.
3- Section channel $CH$ into $4 \times 4$ nonoverlapping blocks.
4- Perform the logistic map on the blocks obtained in step 3 to restore the original locations.
5- Perform the $HMDM$ $(PHP^T)$ to take out the Hessenberg matrix $H$ from each block.
6- Perform the $HMDM$ $(PHP^T)$ on each matrix $H$ in step 5.
7- Modify the bits in $H$ sub-matrix produced from step 6 to extract the binary watermark $(BWT)$.

$$BWT(i,j) = 0 \text{ if } (H\{i,j\}(1,1)) < ave$$

$$BWT(i,j) = 1 \text{ if } (H\{i,j\}(1,1)) \geq ave$$

where the average of the first column elements of each block is represented by $ave$.

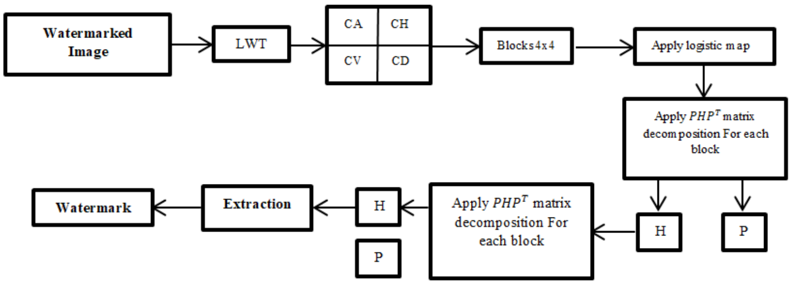



Figure 2: Extraction Algorithm

## 3. Experimental Results

Three types of medical images ($CT$ Image, $MRI$ Image, and $X$-RAY Image) are used to test the watermarking scheme proposed in this paper. Also, a watermark ($WT$) of size $512 \times 512$ is used to hide in the $HI$ to take out the watermarked image ($WTedI$) as in the following figure:

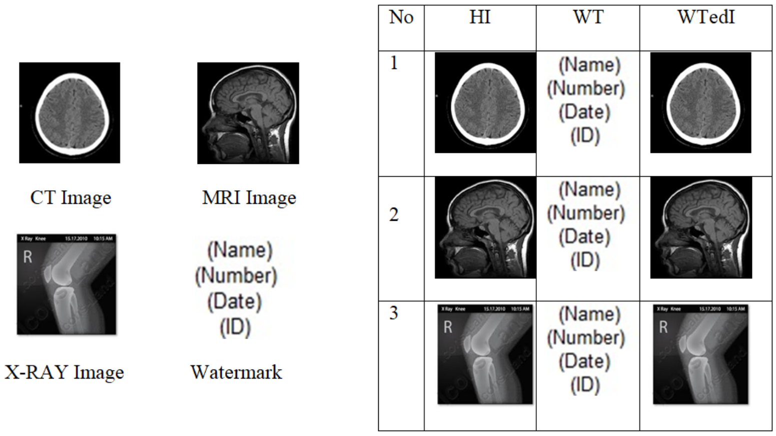



Figure 3: The Medical Host Images and the Watermarked Images

The effect of the $HMDM$ is illustrated according to the use of $LWT$. It is interesting to note that the results obtained are regarded as good on medical images after using twofold the $HMDM$. This is due to relying on the twofold $HMDM$ in the process of embedding and the selection of the hiding location within the matrix $H$ in the second stage. All of the above proves that $HMDM$ is the most influential and represents the main pivot point in the flow of the algorithm.
The measurements of the proposed scheme before the images attacked are given in the following Table:

Table 1: $PSNR - NC$ Values for $WTedI's$ Before Attacks

| Image | CT Image | MRI Image | X-RAY Image |
|---|---|---|---|
| PSNR | 45.453 | 43.783 | 45.057 |
| NC | 1 | 1 | 1 |

The robustness of the scheme has been measured after different attacks were utilized.

Table 2: The Values of the $PSNR - NC$ of the $WTedI$'s After Attacks

| Attacks | CT Image | | MRI Image | | X-RAY Image | |
|---|---|---|---|---|---|---|
| | PSNR | NC | PSNR | NC | PSNR | NC |
| Salt and Pepper %1 | 25.345 | 0.979 | 25.689 | 0.978 | 26.106 | 0.976 |
| Imadjust | Inf | 1 | 35.807 | 0.997 | Inf | 1 |
| JPEG Compression | 60.284 | 0.973 | 59.418 | 0.987 | 59.552 | 0.997 |
| Speckle Noise | 39.232 | 0.959 | 41.429 | 0.975 | 37.858 | 0.925 |
| Histogram equalization | 8.377 | 0.985 | 9.114 | 0.975 | 14.522 | 0.984 |

The medical attacked watermarked images and the extracted $WT$ are illustrated in Table 4 below.

## 4. Comparison

A comparison between the proposed scheme and two different references is given in Table 3.

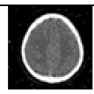
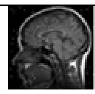
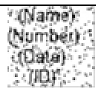
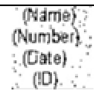
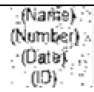
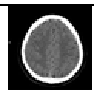
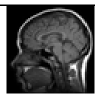
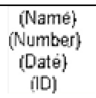
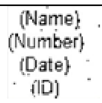
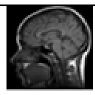
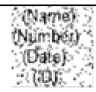
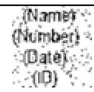
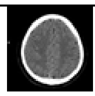
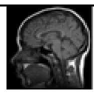
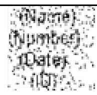
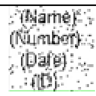
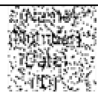
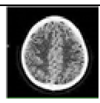
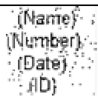
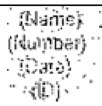
Table 3: $PSNR - NC$ Values Compared with the References [11] and [12]

| Attacks | CT Image | | | MRI Image | | | X-RAY Image | | |
|---|---|---|---|---|---|---|---|---|---|
| | P.S. | Ref [11] | Ref [12] | P.S. | Ref [11] | Ref [12] | P.S. | Ref [11] | Ref [12] |
| Salt and Pepper %1 | 0.97 | 0.95 | 0.91 | 0.97 | 0.99 | 0.90 | 0.97 | 0.90 | 0.91 |
| JPEG Compression | 1.00 | 0.99 | 0.98 | 0.99 | 1.00 | 0.97 | 1.00 | 092 | 0.98 |
| Gaussian noise | 0.85 | 0.71 | 0.86 | 0.89 | 0.80 | 0.81 | 0.87 | 0.69 | 0,93 |
| Histogram equalization | 0.98 | ------ | 0.97 | 0.98 | ------- | 0.96 | 0.98 | ------ | 0.96 |

## 5. Conclusions

A new linear algebra application is introduced depending on the algebraic decomposition method called Hessenberg ($HMDM$) using the twofold repetition. The scheme proposed in this paper utilized the $LWT$ with the twofold of the $HMDM$. The logo that contains the patient's information has been embedded into medical host images as a binary watermark ($BWT$). The $LWT$ is employed first to perform the embedding process in the $HL$ channel using the maximum value of the first row in each block. This process guarantees the optimal choice represented by the element $(1, 1)$. Several attacks are used on images to show that the proposed scheme is robust and imperceptible. The results and the comparison show that the measurements values are better than other systems that used the same algebraic decomposition. Almost, the better results appear with the $JPEG$ Compression attack.

Table 4: Three Medical Attacked Watermarked Images After Attacks

| No | | CT Image | MRI Image | X-RAY Image |
|---|---|---|---|---|
| Salt and Pepper %1 | Image |  |  |  |
| | E.WT. |   |   |   |
| Imadjust | Image |  |  |  |
| | E.WT. |   |   |   |
| JPEG Compression | Image |  |  |  |
| | E.WT. |   |   |   |
| Speckle Noise | Image |  |  |  |
| | E.WT. |  |   |  |
| Histogram Equalization | Image |  |  |  |
| | E.WT. |   |  |   |

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