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Maximum likelihood for fuzzy pure spatial autoregressive model

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Abstract

This paper deals with the study about the formulation of spatial regression models for independent and dependent fuzzy variables, while the parameters crisp values, which are estimated by the maximum likelihood method. In this paper, has been formulated Fuzzy Pure Spatial Autoregressive Model (FPSAM) from the fuzzy general spatial model, and applied for Trapezoidal fuzzy number in the domain traffic accidents for several cities in Iraq for the year 2018 and that after converting the Trapezoidal fuzzy number into crisp values by centroid method, calculations the results by Matlab language.

Keywords: Fuzz spatial regression models, fuzzy pure spatial, autoregressive model, centroid method
2010 MSC: 15Bxx

1. Introduction

Spatial econometrics is one of the concepts of traditional econometrics because it deals with the spatial phenomenon of each variable on the basis of place, these phenomena are specific and known measurements and the errors resulting from them are random variables that can be controlled by studying their behavior [9], As for fuzzy statistics, it has recently emerged after the emergence of the theory of fuzzy aggregates to be concerned with phenomena whose variables cannot be measured in points but rather measured in periods, or what is described as uncertain cases or cases with fuzzy data because of its characteristics that make them unclear such as variables that belong in certain

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proportions to their aggregates. It has no complete affiliation, as well as linguistic variables that cannot be measured numerically, and there are variables that are measured roughly, but in fact, they are ambiguous. As a result, fuzzy logic has become applied in many fields. The Artificial Intelligent model, especially in the field of artificial intelligence, is a technique that has a mechanical ability to find solutions to various scientific and applied problems, This is one of the reasons that prompted us to study fuzzy logic in the general linear spatial regression, we get a fuzzy pure spatial autoregressive model for fuzzy trapezoid data by centroid method, and we use least squares to estimated parameters.

2. Basic concepts in fuzzy logic

2.1. Fuzzy Set

It is a set whose components have a value of belonging, called the degree of membership, which are real numbers within the closed interval $[0, 1]$, and the degree membership is expressed as $\mu_A(x)$ that represents the degree of belonging the element from the variable X to The fuzzy set A is written as:

$$A = (x, \mu_A(x)) : \mu_A(x) : x \to [0, 1].$$

The memberships change from full or complete to non-membership, or partial membership.

2.2. Crisp Set

They are the elements that have a specific characteristic, which takes one of the two values, (1) when the element belongs to the set and (0) when the particular element does not belong to the set. It is called crisp set to distinguish it from the fuzzy set in concepts, let we have a set A known as a function and called the characteristic function as:

$$\chi_A(x) = \begin{cases} 0 & if \quad x \notin A \\ 1 & if \quad x \in A \end{cases}$$

2.3. Alpha Cat Set (α-cat set)

Let A be a fuzzy subset in universal set X, then we define an α-cat of A as:

$$A_{[\alpha]} = \{x \in x : \mu_A(x) \geq \alpha\}, \qquad \alpha \in [0, 1].$$

2.4. Strong α-cat set

Let A be a fuzzy subset in universal set X, then we define a strong α-cat of A as:

$$A_{[\alpha^+]} = \{x \in x : \mu_A(x) > \alpha\}, \qquad \alpha \in [0, 1].$$

2.5. Normalized Fuzzy Set (Core)

A fuzzy subset A in universal set X is called normalized (Core) if:

$$\sup_{x \in X} \mu_A(x) = 1.$$

2.6. Convex Fuzzy Set

A fuzzy subset A in universal set X is called convex if:

$$\mu_A(t) \geq \min[\mu_A(r), \mu_A(s)] \;\; and \;\; t = \lambda r + (1 - \lambda)s \;\; where \;\; r, s \in R, \;\; \lambda \in [0, 1].$$

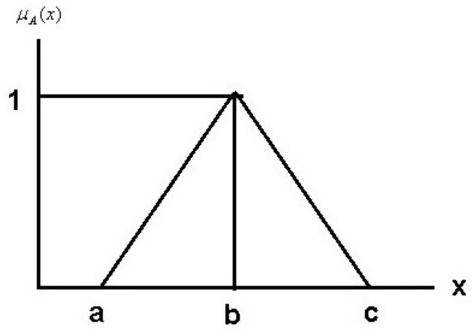

Figure 1: Traingular Fuzzy Number

2.7. Fuzzy number

A fuzzy subset A in universal set X is called fuzzy number if satisfy following condition:

1. convex fuzzy set.
2. normalized fuzzy set (maximum membership value is 1).
3. it's membership function is piecewise continuous.
4. it is defined in the real number [8, 5, 12].

2.8. Triangular fuzzy number

A fuzzy subset A in universal set X is called triangular fuzzy number that expressed as where if has membership functions as:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & if \quad a \leq x \leq b \\ \frac{c-x}{c-b} & if \quad b \leq x \leq c \\ 0 & otherwise \end{cases} \tag{2.1}$$

2.9. Trapezoidal fuzzy number

A fuzzy subset A in universal set X is called trapezoidal fuzzy number that expressed as $A = (a, b, c, d)$ where $a < b < c < d$ if has membership functions as:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & if \quad a \leq x \leq b \\ 1 & if \quad b \leq x \leq c \\ \frac{d-x}{d-c} & if \quad c \leq x \leq d \\ 0 & otherwise \end{cases} \tag{2.2}$$

2.10. Convert fuzzy number to crisp number (Defuzzification)

Let A be a fuzzy number, we can transform A to crisp by centroid method this process is called defuzzification, the centroid method has the following formula:

$$A_c(x) = \frac{\int x\mu_{\tilde{A}}(x)\,dx}{\int \mu_{\tilde{A}}(x)\,dx} = \frac{1}{3}(x_L + x_M + x_R) \quad if \quad A \quad Triangular \tag{2.3}$$

$$A_c(x) = \frac{\int x\mu_{\tilde{A}}(x)\,dx}{\int \mu_{\tilde{A}}(x)\,dx} = \frac{1}{4}(x_L + x_M + x_R) \quad if \quad A \quad Trapezoidal \tag{2.4}$$

This method purposed by Sugeno in 1985 is the most commonly used technique and it is very accurate [10, 13, 6].

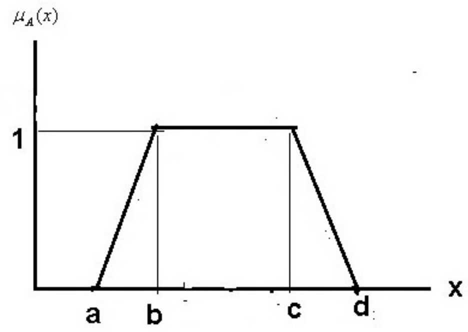

Figure 2: Trapezoidal Fuzzy Number

2.11. Convert crisp number to fuzzy number (Fuzzyfication)

The convert process crisp number to fuzzy is called fuzzyfication, and use membership function in convert which requires have range from zero and one, as shown in the following figure:

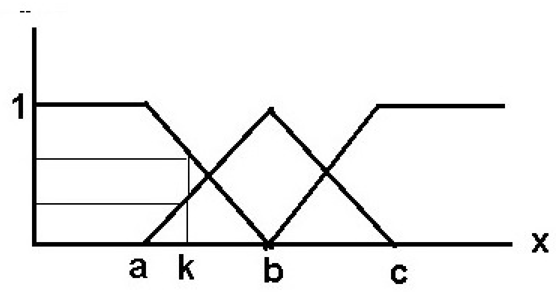

Figure 3: Fuzzyfication

3. Mathematical model of fuzzy multiple linear regression

The mathematical model for fuzzy linear regression is defined as:

$$\begin{aligned} &\tilde{y}_i = \beta_0 + \beta_1\tilde{x}_{1i} + \beta_2\tilde{x}_{2i} + \beta_3\tilde{x}_{3i} + \ldots + \beta_n\tilde{x}_{ni}, \quad i = 1, , \ldots, m \\ &\tilde{Y} = \beta\tilde{X} + \tilde{E} \end{aligned} \tag{3.1}$$

where, $\tilde{X} = [1, \tilde{x}_{1i}, \tilde{x}_{2i}, \tilde{x}_{3i}, \ldots, \tilde{x}_{ni}]$ vector of independent Triangular fuzzy variables, expressed as follows: $\tilde{x} = (x_L, x_M, x_R)$ where x_L left side, x_R right side and x_M center value.
$\beta = [\beta_0, \beta_1, \beta_2, \ldots, \beta_n]$ vector of parameters of fuzzy regression model are real value.
$\tilde{Y} = [\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \ldots, \tilde{y}_m]$ vector of dependent Triangular fuzzy variables, expressed as follows: $\tilde{y} = (y_L, y_M, y_R)$.
$\tilde{E} = [\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \ldots, \tilde{e}_m]$ vector represents the Triangular fuzzy errors of the model and written as: $\tilde{e} = (e_L, e_M, e_R)$.[4]

3.1. Parameter Estimation For Fuzzy Regression Models By Centroid Method

In traditional general linear regression

$$Y = X\beta + E \tag{3.2}$$

where $X = \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1p} \\ 1 & x_{21} & x_{22} & \dots & x_{2p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & \dots & x_{Np} \end{bmatrix}$, $\beta = [\beta_0, \beta_1, \beta_2, \dots, \beta_n]^T$, $E = [e_1, e_2, e_3, \dots, e_m]^T$, Then the lest squares estimator of β is : $\tilde{\beta} = (X^T X)^{-1} X^T Y$. As for fuzzy general linear regression that has model in Eq. (3.1) Where $(\tilde{x}_{ij})$, $i = 1, 2, \dots, n$ *and* $j = 1, 2, \dots, p$ are observational data set of fuzzy input and output and all observations are triangular fuzzy numbers. And $\beta_0, \beta_1, \beta_2, \dots, \beta_n$ are real value, $\tilde{e}_i$ error terms are also fuzzy number.

Let $\tilde{Y} = [\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \dots, \tilde{y}_m]^T$, $\tilde{E} = [\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \dots, \tilde{e}_m]$, $\tilde{X} = \begin{bmatrix} 1 & \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1p} \\ 1 & \tilde{x}_{21} & x_{22} & \dots & \tilde{x}_{2p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \tilde{x}_{n1} & \tilde{x}_{n2} & \dots & \tilde{x}_{Np} \end{bmatrix}$

The model in (3.3), we can written as matrix form :

$$\tilde{Y} = \tilde{X}\beta + \tilde{E}. \tag{3.3}$$

Since $\tilde{X}_{ij}, \tilde{Y}_i$, and $\tilde{E}_i$ are fuzzy triangular number, it can be written as $\tilde{x} = (x_L, x_M, x_R), \tilde{y} = (y_L, y_M, y_R)$, and $\tilde{e} = (e_L, e_M, e_R)$, the fuzzy data are transformed into crisp data by the centroid method with formal:

$$x_c(x) = \frac{\int x\mu_x(x)\,dx}{\int \mu_x(x)\,dx} = \frac{1}{3}(x_L + x_M + x_R)$$
$$y_c(x) = \frac{\int y\mu_y(y)\,dy}{\int \mu_y(y)\,dy} = \frac{1}{3}(y_L + y_M + y_R)$$

where x_c and y_c crisp data and $\mu_A(x), \mu_A(y)$, triangular membership function for x_{ij} and y_{ij}, then the estimator for β is: $\hat{\beta} = (X_c^T X_c)^{-1} X_c^T Y_c$, where

$Y_c = [Y_{1c}, Y_{2c}, \dots, Y_{mc}]^T$, $X_c = \begin{bmatrix} 1 & x_{11c} & x_{12c} & \dots & x_{1pc} \\ 1 & x_{21c} & x_{22c} & \dots & x_{2pc} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1c} & x_{n2c} & \dots & x_{Npc} \end{bmatrix}$ [14, 1].

4. Spatial Linear Regression Models

The form of general spatial model that contain both spatially lagged and error structure spatial correlation, as shown in the following formula

$$\begin{aligned} y &= \lambda W y + X\beta_1 + WX\beta_2 + u \qquad |\lambda| < 1 \\ u &= \rho W u + e \qquad |\rho| < 1 \end{aligned} \tag{4.1}$$

where X is a matrix of non stochastic regression, W is a weight matrix, $e/X = i.i.d.N(0, \sigma_{en}^2 I_n)$, β_1, β_2 are the vectors (n*1) of parameters that required estimate them, λ, ρ are parameters of spatial regression, u is the residue of spatially associated and e is the (n*1) random error vector. The model (4.2) can be written as

$$\begin{aligned} y &= \lambda W y + Z\beta_1 + u \qquad |\lambda| < 1 \\ u &= \rho W u + e \qquad |\rho| < 1 \end{aligned} \tag{4.2}$$

where $z = [X, WX]$ and $\beta = [\beta_1, \beta_2]$. This model is called spatial Autoregressive by Autoregressive error structure and Includes many spatial econometric models they are as follows:

1. $\beta = 0$ and $\lambda = 0$ or $\rho = 0$ is called pure spatial autoregressive model.
2. $\lambda = 0$ and $\rho = 0$ is called lagged independent variable model.
3. $\lambda \neq 0$ and $\rho = 0$ is called spatial lag model.
4. $\lambda = 0$ and $\rho \neq 0$ is called spatial error model [3, 11].

4.1. Pure Spatial Autoregressive Model (PSAM)

In model (4.3) let $\beta = 0$ and $\lambda = 0$ or $\rho = 0$ and we assuming again $e/X = i.i.d.N(0, \sigma^2_{en} I_n)$ and matrix W non stochastic, so the model given simple spatial autoregression. Thus when $\rho = 0$ we have model :

$$y = \lambda W y + e \qquad |\lambda| < 1. \tag{4.3}$$

By maximum likelihood we can estimate the parameter as:

$$L(y/\lambda, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{n/2}} |I_n - \lambda W| \exp\left[-\frac{1}{2\sigma^2}(y - \lambda W y)'(y - \lambda W y)\right]. \tag{4.4}$$

Taking the logarithm which produces the following equation.

$$\ln L = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 + \ln|I - \lambda W| - \frac{1}{2\sigma^2}(y - \lambda W y)'(y - \lambda W y). \tag{4.5}$$

where calculation the determinant $|I - \lambda W|$ as below

$$|I - \lambda W| = \Pi_{i=1}^{n}(I - \lambda w_i)$$
$$\ln|I - \lambda W| = \sum_{i=1}^{n}\ln(I - \lambda w_i)$$

w_i: is eigen value for weight matrix W.

$$\ln L = -\frac{n}{2}(y - \lambda W y)'(y - \lambda W y) + \sum \ln(I - \lambda w_i).$$

By using iterative methods for likelihood function we get to parameter λ, and by derive quantity (4.5) with respect to and equal to zero we get [9, 2, 5, 6]

$$\sigma^2 = \frac{(y - \lambda W y)'(y - \lambda W y)}{n}. \tag{4.6}$$

4.2. Parameter Estimation for Fuzzy Pure Spatial Autoregressive Model by centroid method

The Fuzzy Pure Spatial Autoregressive Model by centroid when $\rho = 0$, it can be written as

$$y_c = \lambda W y_c + e_c \qquad |\lambda| < 1 \tag{4.7}$$

where
y_c: vector (n*1) is centroid of fuzzy number are depend variable,
λ: parameter spatial regression,
W : spatial weights matrix,
e_c: vector (n*1) is centroid of fuzzy number are random error.
By Maximum Likelihood we can estimate λ as

$$L(y_c/\lambda, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{n/2}} |I_n - \lambda W| \exp\left[-\frac{1}{2\sigma^2}(y_c - \lambda W y_c)'(y_c - \lambda W y_c)\right]. \tag{4.8}$$

Taking the logarithm which produces the following equation.

$$\ln L = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln \sigma^2 + \ln|I - \lambda W| - \frac{1}{2\sigma^2}(y_c - \lambda W y_c)'(y_c - \lambda W y_c). \tag{4.9}$$

where calculation the determinant $|I - \lambda W|$ as below

$$|I - \lambda W| = \Pi_{i=1}^{n}(I - \lambda w_i)$$

$$\ln|I - \lambda W| = \sum_{i=1}^{n}\ln(I - \lambda w_i)$$

w_i: is eigen value for weight matrix W.

$$\ln L = -\frac{n}{2}(y_c - \lambda W y_c)'(y_c - \lambda W y_c) + \sum \ln(I - \lambda w_i).$$

By using iterative methods for Likelihood function we get to parameter λ, and by derive quantity (4.9) with respect to σ^2 and equal to zero we get

$$\sigma^2 = \frac{(y_c - \lambda W y_c)'(y_c - \lambda W y)}{n} \tag{4.10}$$

5. Spatial weights matrix (Rook contiguity)

It is a square matrix which elements positive value, and it is not necessary to be symmetric, and it is based on the geographic arrangement of the observations, or contiguity, i.e. the relations among location with other locations in one row of the matrix and the diagonal elements in the matrix are equal to zero, the Spatial weights matrix by Rook contiguity is equal to 1 if the two areas (locations) neighbour by limited and have relationships between the two areas (locations) in any side, and it is equal to 0 otherwise. This matrix is used in applications more than the others [14, 10].

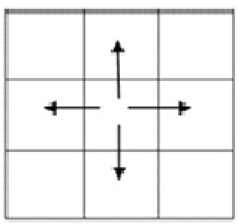

Figure 4: Shows the Rook weight matrix

6. Moran Test for Spatial Regression [6]

It is a general measure that depends on the general linear model GLM and uses for autocorrelation coefficient (called the Moran coefficient because Moran is the name of the Scientist that find the test). The Moran formula is:

$$I = \frac{n(e'we)}{S_0(e'e)} \tag{6.1}$$

where $S_0 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}$ sum of all elements of matrix W, W is Weights Matrix (neighborhood) it is square $n \times n$, n is a number of observation (size sample) and e is a vector residual dimensions. We use row-standardized where sum of row equal to 1 in this case $(n = S_0)$ that is work to simplify the Moran's formula as follows :

$$I = \frac{e'we}{e'e}. \tag{6.2}$$

To know if the value of the Moran coefficient is indicator statistics in a certain degree of confidence we must use Moran (Z) test with hypotheses:

$H_0 : \lambda = 0, \quad \theta = 0$ there is no spatial dependence

$H_1 :$ at lest one of $\lambda \neq 0$ *or* $\theta \neq 0$ there is spatial dependence

$$\begin{aligned} Z &= \frac{I - E(I)}{\sqrt{V(I)}} \\ E(I) &= \frac{n(tr(MW))}{S_0(n-k)} \\ V(I) &= \frac{tr(MWMW') + tr(MW)^2 + (tr(MW))^2}{(n-k)(n-k+1)} \left(\frac{n}{S_0}\right)^2 - (E(I))^2 \end{aligned} \tag{6.3}$$

where

$M = I - X(X'X)^{-1}X'$ Idempotent Matrix that is and symmetric,

tr: Sum diagonal element.

k: Number of explanatory variables.

The calculated value is compared with value of Z table for $(\alpha/2)$, and where Moran test is significant that is mean relation between geographic location that refers to use spatial regression model and not enough general linear model (GLM) and we have spatial dependency.

7. Practical Part

7.1. Estimate initial value

In this part of the paper, we used, real data these data represent the number of deaths $\tilde{Y}_i$ resulting from traffic accidents in Iraq for six governorates (Baghdad, Anbar, Diyala, Salah al-Din, Kirkuk, Nineveh) by 38 observations. And these traffic accidents as Crash accidents $\tilde{X}_1$ and Overturn accidents $\tilde{X}_2$ and Run over accidents $\tilde{X}_3$, this data represent a trapezoidal fuzzy number and has membership function, in this paper explains given an idea about dealing with such data, in this paper transform fuzzy data into Crisp data, by the formula

$$\begin{aligned} x_c(x) &= \frac{1}{4}(x_L + x_{M_1} + x_{M_2} + x_R) \\ y_c(x) &= \frac{1}{4}(y_L + y_{M_1} + y_{M_2} + y_R). \end{aligned}$$

So the fuzzy multiple regression formula for this data is

$$y_{ci} = \beta_0 + \beta_1 X_{c1i} + \beta_2 X_{c2i} + \beta_3 X_{c3i} + e_{ci}$$

Where:

y_{ci} : the number deaths in traffic accidents (D.A)

X_{c1i}: Crash accidents (C.A)
X_{c2i}: Overturn accidents (O.A)
X_{c3i}: Run over accidents (R.A).
We get to β_i by ordinary least square

Table 1: Estimated initial value of beta.

Model	*Constant*	*C.A*	*O.A*	*R.A*
Beta	*7.3078*	*0.0544*	*0.7951*	*0.2653*

7.2. Moran Test

The initial values that estimated are used in the Moran test, which we obtained the following result:

Table 2: Result Moran test.

I	*Z(I)*
-0.0372	*0.7878*

Since the value of Z(I) is less, the tabular value of Z in terms of $(\alpha/2)$, then we accept the null hypothesis, which states that the data is spatially dispersed and randomly.

7.3. Estimate parameter FPSAM

To estimate the parameters of the fuzzy autoregressive model, we calculated spatial matrix weights between locations with 38×38 by (Rook contiguity), and we are have two cases, the first case when $\rho = 0$, then the form of the model is as follows:

$$y_c = \lambda W y_c + e_c$$

Second case when $\lambda = 0$

$$y_c = \rho W y_c + e_c$$

and after estimating the parameter λ and ρ by the Maximum Likelihood method as in formula (4.7) and (4.8).

Table 3: Estimate coefficient.

Estimate coefficient FPSAM	
$\hat{\rho} = 0$	$\hat{\lambda} = 0.0192$
$\hat{\lambda} = 0$	$\hat{\rho} = 0.0192$

Since $\hat{\rho} = \hat{\lambda}$, the fuzzy pure spatial autoregressive model (FPSAM) is

$$y_{ci} = 0.0192 \sum_{j=1}^{38} W_{ij} y_{ci}, \qquad i = 1, 2, \ldots, 38.$$

This model is also called the spatial autoregressive model of first-order and as it does not include any independent variable, that means it is based on the idea of explaining the phenomenon for itself by considering the independent variable as the same dependent variable y, but only with a spatial lag.

8. Recommendations

1. Using the ordinary least square method in estimating parameters and comparing them with Maximum Likelihood to determine the best method for estimating parameters by Root Mean Squares Error or Mean Absolute Percentage Error.
2. Fuzzyfication the data into triangular fuzzy numbers and comparing it with the trapezoidal fuzzy number for showing a difference between them in the estimate.
3. There are other methods than the centroid method in dealing with fuzzy data such average method and alpha cat.
4. Study and application of real data on other fuzzy spatial models.

Table 4: Centroid data and results error.

	Location Cities	Centroid Fuzzy number y	Centroid Fuzzy number X_1	Centroid Fuzzy number X_2	Centroid Fuzzy number X_3	$\hat{y}$	e
1	Mosul	52	6	1	68	8.4622	43.5378
2	AL-Hamdaniya	16	26	5	20	8.4622	7.5378
3	Telkaif	16	6	1	4	8.4622	7.5378
4	Sinjar	4	6	1	4	8.4622	- 4.4622
5	Tel afar	16	6	5	4	8.4622	7.5378
6	AL-Hatra	16	6	1	4	10.1546	5.8454
7	Maqmoor	16	6	1	4	8.4622	7.5378
8	Kirkuk	40	46	13	52	7.0775	32.9225
9	Daquq	28	26	21	20	7.0775	20.9225
10	Debes	28	6	13	4	7.0775	20.9225
11	Tikrit	16	66	1	52	28.9253	-12.9253
12	Tuz kurmato	4	6	5	4	28.9253	-24.9253
13	Samara	16	6	5	4	28.9253	-12.9253
14	Baled	4	6	1	4	28.9253	-24.9253
15	AL- Dor	4	6	1	4	28.9253	-24.9253
16	AL- shargat	4	6	5	4	28.9253	-24.9253
17	Baquba	88	186	13	132	11.2316	76.7684
18	AL- meqdadia	16	46	9	4	11.2316	4.7684
19	AL-Kalus	76	66	17	20	11.2316	64.7684
20	Kanaqeen	4	46	13	4	11.2316	-7.2316
21	Baladrouz	16	6	9	4	11.2316	4.7684
22	Al-Rasafa	64	106	17	132	12.4625	51.5375
23	AL-aadamia	16	46	13	52	12.4625	3.5375
24	AL-sader2	16	26	9	36	12.4625	3.5375
25	AL-sader1	4	26	5	52	12.4625	-8.4625
26	AL-Karek	40	126	17	116	12.4625	27.5375
27	AL-Kademia	4	66	9	68	12.4625	-8.4625
28	AL-Mahmoodia	40	26	5	20	12.4625	27.5375
29	Abu-griab	16	6	1	4	12.4625	3.5375
30	AL-Taremia	4	6	1	4	12.4625	-8.4625
31	AL-Madayn	40	6	1	20	12.4625	27.5375
32	AL-Rumadi	16	146	5	20	16.4628	-0.4628
33	Heet	4	6	1	4	16.4628	-12.4628
34	AL-Faloga	4	26	9	4	16.4628	-12.4628
35	Anah	4	6	1	4	16.4628	-12.4628
36	Haditha	4	6	5	4	16.4628	-12.4628
27	AL- Rutba	16	6	5	4	16.4628	-0.4628
38	AL-qaem	28	6	5	4	16.4628	11.5372

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