



Some results for generalizations of semi-open sets in topological spaces

Ali J. Mahmood^{a,*}, A. I. Naser^a

^aDepartment of Mathematics, College of Education for Pure Science, Ibn Al-Haitham University of Baghdad, Iraq

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Abstract

This research generalized new types of semi-open sets using the concept of the grill and studied its properties, as well as the relationship between it and old concepts defined, as well as defining a new type of functions such as open and closed sets and determining the relationship between them, and providing many examples and properties that belong to this set, and this set will be a starting point to investigate the many futures of this set.

Keywords: Grill, semi-open sets, semi-closed sets, \mathbb{G}^* -s-o function, \mathbb{G}^* -s-c function.

1. Introduction

Choquet [2] established the notion of grilling on a topological space, and grilling has shown to be a useful tool for learning several topological issues. Subsets of a topological space (\mathcal{K}, τ) that are not empty collection \mathbb{G} that known as a grill whenever (a) $H \in \mathbb{G}$ and $H \subseteq M$ implying $M \in \mathbb{G}$, (b) H and M are subset for \mathcal{K} , also $H \cup M \in \mathbb{G}$ implying $H \in \mathbb{G}$ or $M \in \mathbb{G}$. A triple $(\mathcal{K}, \tau, \mathbb{G})$ a grill topological space is a kind of topological space.

Mukherjee also Roy [7] used a grill to establish a unique topology and researched topological ideas. For each topological space (\mathcal{K}, τ) point κ , The open neighborhoods of κ are represented by $\tau(\kappa)$. A mapping $\Psi: \mathcal{P}(\mathcal{K}) \rightarrow \mathcal{P}(\mathcal{K})$ is referred to as $\oint(H) = \{\kappa \in \mathcal{K}: H \cap Z \in \mathbb{G}, \text{ for every } Z \in \tau(\kappa) \text{ and } H \in \mathcal{P}(\mathcal{K})\}$. A mapping $\Psi: \mathcal{P}(\mathcal{K}) \rightarrow \mathcal{P}(\mathcal{K})$ is referred to as $\Psi(H) = H \cup \oint(H)$, for every $H \in \mathcal{P}(\mathcal{K})$. The map Ψ Kuratowski closure axioms are satisfied:

(a) $\Psi(\Psi) = \Psi$

*Corresponding author

Email addresses: ali.jamal1203a@ihcoedu.uobaghdad.edu.iq (Ali J. Mahmood), ahmed_math06@yahoo.com (A. I. Naser)

- (b) when $H \subseteq M$, then $\Psi(H) \subseteq \Psi(M)$,
- (c) when $H \subseteq K$, then $\Psi(\Psi(H)) = \Psi(H)$,
- (d) when $H, M \subseteq K$, then $\Psi(H \cup M) = \Psi(H) \cup \psi(M)$.

There are some types of a grill topological space as like as a cofinite topology and discreet topology [8]. On a topological space (K, τ) , this corresponds to a grill \mathbb{G} . There is a topology $\tau_{\mathbb{G}}$ on K that is unique provided by $\tau_{\mathbb{G}} = \{Z \subseteq K: \Psi(K-Z) = K-Z\}$, in which case $H \subseteq K$, $\Psi(H) = H \cup \phi(H)$. $\tau \subseteq \tau_{\mathbb{G}}$ and $\Psi(H) = \text{cl}(H)$. Using the following basis, we can find $\tau_{\mathbb{G}}$ on K provided by $\tau_{\mathbb{G}}$ by used the base as following $\beta(\tau_{\mathbb{G}}, K) = \{Y-H: Y \in \tau, H \notin \mathbb{G}\}$. [7] On any topological space (K, τ) , there is a grill \mathbb{G} , $\tau \subseteq \beta(\mathbb{G}, \tau) \subseteq \tau_{\mathbb{G}}$, where $\beta(\mathbb{G}, \tau)$ base for $\tau_{\mathbb{G}}$.

For instance, [4] to existence in topological space (K, τ) , $\tau_{\mathbb{G}} = \tau$ when $\mathbb{G} = \mathcal{P}(K) \setminus \phi$, implying $\tau_{\mathbb{G}} = \tau$. Semi-open is a subset H of a topological space (K, τ) , [4] if $H \subseteq \text{cl}(\text{Int}(H))$ [1]. Let (K, τ, \mathbb{G}) be a grill topological space. A subset H in K is called \mathbb{G} -semi-open if $H \subseteq \psi(\text{Int}(H))$, and every Ψ -semi-open is a semi-open. Many researchers have used this combinations to come up with generalizations [6, 5].

In this research used the symbol $\text{Int}(H)$ to interior of the set H and the symbol $\text{cl}(H)$ is the closure of H .

2. On semi-open sets in topological spaces

Definition 2.1. *The set H is said to be grill semi-open if there exists $z \in \tau$ such that $z - H \notin \mathbb{G}$ and $H - \text{cl}_{\mathbb{G}}(z) \notin \mathbb{G}$. And denoted by \mathbb{G} -semi-open. κ - \mathbb{G}^* -semi-open is a \mathbb{G} -semi-closed, and the set of all \mathbb{G} -semi-open presently by \mathbb{G}^* , so (κ) and the set of all \mathbb{G} -semi-closed presently by $\mathbb{G}^* \text{sc}(\kappa)$.*

Example 2.2. *Let $(\kappa, \tau, \mathbb{G})$ be a grill topological space and $\kappa = \{\kappa_1, \kappa_2, \kappa_3\}$, $\tau = \{\kappa, \phi, \{\kappa_1\}, \{\kappa_1, \kappa_2\}\}$, $\mathcal{F} = \{\kappa, \phi, \{\kappa_3\}, \{\kappa_3, \kappa_2\}\}$, $\mathbb{G} = \{Z \subseteq \kappa: \kappa_2 \in Z\}$, $\phi: \mathcal{P}(\kappa) \rightarrow \mathcal{P}(\kappa)$, $\phi(H) = \{\kappa \in \kappa: \forall z \in \tau_{\kappa}; z \cap H \in \mathbb{G}\}$, $\Psi(H) = H \cup \phi$, $\tau_{\mathbb{G}} = \{\kappa, \phi, \{\kappa_3, \kappa_2\}, \{\kappa_2\}, \{\kappa_1, \kappa_2\}\}$, $\mathcal{F}_{\mathbb{G}} = \{\kappa, \phi, \{\kappa_3, \kappa_2\}, \{\kappa_3\}, \{\kappa_1\}\}$, then $\mathbb{G}^* \text{so}(\kappa) = \{\kappa, \phi, \{\kappa_3\}, \{\kappa_1\}, \{\kappa_2\}, \{\kappa_1, \kappa_3\}, \{\kappa_1, \kappa_2\}, \{\kappa_3, \kappa_2\}\}$.*

Remark 2.3. (i) *Every set that is open is a \mathbb{G} -semi-open sets.*

(ii) *Every set that is closed is a \mathbb{G} -semi-closed.*

Proof . (i) *Let $H \in \tau$, then there exists $z \in H$ such that $z \subseteq \text{cl}_{\mathbb{G}}(z)$, but $z = H \in \tau$, so, $z - H = \phi \notin \mathbb{G}$ and $H - \text{cl}_{\mathbb{G}}(H) = \phi \notin \mathbb{G}$.*

(ii) *Let H be a closed set thus, $H^c \in \tau$, H^c is \mathbb{G}^* , so (κ) , then $H \in \mathbb{G}^* \text{sc}(\kappa)$. \square*

The converse of Remark 2.3 (i)-(ii) are not true, see Example 2.4.

Example 2.4. *Let $K = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$, $\tau = \{\kappa, \phi, \{\kappa_3\}, \{\kappa_2\}, \{\kappa_3, \kappa_2\}\}$, $\mathbb{G} = \mathcal{P}(\kappa) \setminus \{\emptyset\}$. It is clear that $\{\kappa_2, \kappa_4\} \in \mathbb{G}^*$, so (κ) , but $\{\kappa_2, \kappa_4\} \notin \tau$. And $\{\kappa_1, \kappa_3\} \in \mathbb{G}^* \text{sc}(\kappa)$, but $\{\kappa_1, \kappa_3\}$ it is not a closed set.*

Theorem 2.5. *Every \mathbb{G} -semi-open is a \mathbb{G} -semi-open.*

Proof . *Let H is an \mathbb{G} -semi-open but z is an open set; $z \subseteq H \subseteq \text{cl}_{\mathbb{G}}(z)$, thus $z - H = \emptyset \wedge H - \text{cl}_{\mathbb{G}}(z) = \emptyset$ and $z - H \notin \mathbb{G} \wedge H - \text{cl}_{\mathbb{G}}(z) \notin \mathbb{G}$, so $z - H \notin \mathbb{G} \wedge H - \text{cl}_{\mathbb{G}}(z) \notin \mathbb{G}$. \square*

The converse of Theorem 2.5 is not true see Example 2.6.

Example 2.6. *Let $(\kappa, \tau, \mathbb{G})$ be a grill topological space and $\kappa = \{\kappa_1, \kappa_2, \kappa_3\}$, $\tau = \{\kappa, \phi, \{\kappa_1\}\}$, $\mathbb{G} = \{Z \subseteq \kappa: \kappa_1 \in Z\}$, $\mathbb{G}^* \text{so}(\kappa) = \mathcal{P}(\kappa)$, then $\mathbb{G} \text{so}(\kappa) = \{\kappa, \phi, \{\kappa_1\}, \{\kappa_1, \kappa_2\}\}$, $\{\kappa_2\} \in \mathbb{G}^* \text{so}(\kappa)$ but $\{\kappa_2\} \notin \mathbb{G} \text{so}(\kappa)$.*

Proposition 2.7. For any grill topology $(\kappa, \tau, \mathbb{G})$, H is an \mathbb{G} -semi-open set if and only if H is a \mathbb{G}^* -semi-open set whenever $\mathbb{G} = \mathcal{P}(\kappa) \setminus \{\emptyset\}$.

Proof . Let H be an \mathbb{G} -semi-open set, there exists $z \in \tau$ such that $z \subseteq H \subseteq \text{cl}_{\mathbb{G}}(z)$. So $z - H = \emptyset \notin \mathbb{G}$ and $H - \text{cl}_{\mathbb{G}}(z) = \emptyset \notin \mathbb{G}$. So, H is a \mathbb{G}^* -semi-open set.

Conversely, let H be a \mathbb{G}^* -semi-open set so, there exists $z \in \tau$ such that $z - H \notin \mathbb{G} \wedge H - \text{cl}_{\mathbb{G}}(z) \notin \mathbb{G}$. Thus $z - H = \emptyset \wedge H - \text{cl}_{\mathbb{G}}(z) = \emptyset$, then $z \subseteq H$ and $H \subseteq \text{cl}_{\mathbb{G}}(z)$, so $z \subseteq H \subseteq \text{cl}_{\mathbb{G}}(z)$. Therefore H is an \mathbb{G} -semi-open. \square

Proposition 2.8. Let H be a \mathbb{G} -semi-open set and $M \subseteq \kappa$ such that $H \subseteq M \subseteq \text{cl}_{\mathbb{G}}(H)$, then M is a \mathbb{G} -semi-open set.

Proof . Since H is a \mathbb{G}^* -semi-open set, there exists $z \in \tau$ such that $z - H \notin \mathbb{G} \wedge H - \text{cl}_{\mathbb{G}}(z) \notin \mathbb{G}$, and since $H \subseteq M \subseteq \text{cl}_{\mathbb{G}}(H)$, $z - \text{cl}_{\mathbb{G}}(H) \subseteq z - M \notin \mathbb{G} \subseteq z - H$, then there exists $z \in \tau$ such that $z - M \notin \mathbb{G}$ and since $H \subseteq M$, $\text{cl}_{\mathbb{G}}(H) \subseteq \text{cl}_{\mathbb{G}}(M)$. So, $z - \text{cl}_{\mathbb{G}}(M) \notin \mathbb{G}$. Therefore M is a \mathbb{G} -semi-open set. \square

Remark 2.9. The two concepts \mathbb{G} -semi-open set and semi-open set are independent See Examples 2.10 and 2.11

Example 2.10. Let $(\kappa, \tau, \mathbb{G})$ be a grill topology and $\kappa = \{\kappa_1, \kappa_2, \kappa_3\}$, $\tau = \{\kappa, \phi, \{\kappa_1\}\}$, $\mathbb{G} = \{z : \kappa_1 \in z\}$, $\mathbb{G}^* o(\kappa) = \mathcal{P}(\kappa)$, and $so(\kappa) = \{\kappa, \phi, \{\kappa_1\}\} \cup \phi$, hence it is clear that $\{\kappa_2\} \in \mathbb{G}^* o(\kappa)$ but $\{\kappa_2\} \notin so(\kappa)$.

Example 2.11. Let $(\kappa, \tau, \mathbb{G})$ be a grill topology and $\kappa = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$, $\tau = \{\kappa, \phi, \{\kappa_1\}, \{\kappa_3\}, \{\kappa_1, \kappa_3\}\}$, $\mathbb{G} = \{\{\kappa_1\}, \{\kappa_2\}, \{\kappa_1, \kappa_3\}, \{\kappa_1, \kappa_2\}, \{\kappa_1, \kappa_4\}, \{\kappa_1, \kappa_2, \kappa_3\}, \{\kappa_1, \kappa_2, \kappa_4\}, \{\kappa_1, \kappa_3, \kappa_4\}, \{\kappa_2, \kappa_3\}, \{\kappa_2, \kappa_4\}, \{\kappa_3, \kappa_4\}, \kappa\}$, $H = \{\kappa_2, \kappa_3, \kappa_4\}$. Then we have

$$\tau_{\mathbb{G}} = \{\kappa, \phi, \{\kappa_1\}, \{\kappa_3\}, \{\kappa_1, \kappa_3\}, \{\kappa_1, \kappa_2\}, \{\kappa_1, \kappa_2, \kappa_4\}, \{\kappa_1, \kappa_2, \kappa_3\}\}.$$

It is clear that H is a semi-open set, but $H \notin \mathbb{G}^*$ -semi-open set, because there is not $z \in \tau$ such that $z - H \notin \mathbb{G}^*$ and $H - \text{cl}_{\mathbb{G}}(z) \notin \mathbb{G}$.

Corollary 2.12. Let $(\kappa, \tau, \mathbb{G})$ be a grill topology and H subset of κ . If $\mathbb{G} = \mathcal{P}(\kappa) \setminus \{\emptyset\}$, then H is a \mathbb{G} -semi-open set if and only if H is semi-open set.

Proof . Let H be a \mathbb{G}^* -semi-open set, then there exists $z \in \tau$ such that $z - H \notin \mathbb{G}$ and $H - \text{cl}_{\mathbb{G}}(z) \notin \mathbb{G}$, $z - H = \emptyset \notin \mathbb{G}$ and $H - \text{cl}_{\mathbb{G}}(z) = \emptyset$, $z \subseteq H$ and $H \subseteq \text{cl}_{\mathbb{G}}(z)$, so there exists $z \in \tau$, $z \subseteq H \subseteq \text{cl}_{\mathbb{G}}(z)$. Therefore H is a semi-open set.

Conversely, it is clear that $H \subseteq M \subseteq \text{cl}_{\mathbb{G}}(z)$, $z - H \notin \mathbb{G}$ and $H - \text{cl}_{\mathbb{G}}(z) = \emptyset \notin \mathbb{G}$, $z - H \notin \mathbb{G}$ and $H - \text{cl}_{\mathbb{G}}(z) = \emptyset \notin \mathbb{G}$. So, H is a \mathbb{G}^* -semi-open set. \square

Lemma 2.13. $\bigcup_{t \in I} (\text{cl}_{\mathbb{G}}(H_t)) \subseteq (\text{cl}_{\mathbb{G}}(\bigcup_{t \in I} H_t))$.

Proof . We have $H_t \subseteq \bigcup_{t \in I} H_t$. Then $\text{cl}_{\mathbb{G}}(H_t) \subseteq \text{cl}_{\mathbb{G}}(\bigcup_{t \in I} H_t)$. So $(\text{cl}_{\mathbb{G}}(H_t)) \subseteq (\text{cl}_{\mathbb{G}}(\bigcup_{t \in I} H_t))$. This implies that

$$\bigcup_{t \in I} (\text{cl}_{\mathbb{G}}(H_t)) \subseteq (\text{cl}_{\mathbb{G}}(\bigcup_{t \in I} H_t)).$$

\square

Theorem 2.14. Any union of \mathbb{G} -semi-open sets family is a \mathbb{G}^* -semi-open set.

Proof . Let H_t be a \mathbb{G}^* -semi-open set. We show that $\bigcup_{t \in I} H_t \in \mathbb{G}^*$ -semi-open set. Since $H_t \in \mathbb{G}^*$ -semi-open set, there exists $z_t \in \tau$ such that $z_t - H_t \notin \mathbb{G}$ and $(\text{cl}_{\mathbb{G}}(\bigcup_{t \in I} H_t) \cap z_t) \notin \mathbb{G}$, because by Lemma 2.13, $\bigcup_{t \in I} (z_t - H_t) \subseteq \bigcup_{t \in I} z_t - \bigcup_{t \in I} H_t \notin \mathbb{G}$. But $(z_t - H_t) \subseteq \bigcup_{t \in I} (z_t - H_t) \notin \mathbb{G} \supset (\bigcup_{i \in \Lambda} z_t - \bigcup_{i \in \Lambda} H_t) \notin \mathbb{G}$. So there exists an open set $w = \bigcup_{t \in I} z_t$ such that $(w - \bigcup_{t \in I} H_t) \notin \mathbb{G}$. Now to prove $\bigcup_{t \in I} H_t - (\text{cl}_{\mathbb{G}}(\bigcup_{t \in I} z_t)) \notin \mathbb{G}$. Hence, $H_t - (\text{cl}_{\mathbb{G}}(z_t)) \notin \mathbb{G}$ and $(\text{cl}_{\mathbb{G}}(z_t)) \subseteq \bigcup_{t \in I} (H_t - (\text{cl}_{\mathbb{G}}(\bigcup_{t \in I} z_t))) \notin \mathbb{G} \supset (\bigcup_{t \in I} H_t - \bigcup_{t \in I} (\text{cl}_{\mathbb{G}}(z_t))) \notin \mathbb{G}$, because $(\text{cl}_{\mathbb{G}}(z_t)) \subseteq (\text{cl}_{\mathbb{G}}(z_t))$. So $\bigcup_{t \in I} H_t - (\text{cl}_{\mathbb{G}}(\bigcup_{t \in I} z_t)) \subseteq \bigcup_{t \in I} H_t - \bigcup_{t \in I} (\text{cl}_{\mathbb{G}}(z_t))$. This implies that $\bigcup_{t \in I} H_t - (\text{cl}_{\mathbb{G}}(\bigcup_{t \in I} z_t)) \notin \mathbb{G}$. So $\bigcup_{t \in I} H_t - \text{cl}_{\mathbb{G}}(w) \notin \mathbb{G}$. Thus, $\bigcup_{t \in I} H_t \in \mathbb{G}^*$ -semi-open set. \square

Remark 2.15. Any two \mathbb{G} -semi-open sets intersecting need not be \mathbb{G} -semi-open set.

Example 2.16. Let $K = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$, $\tau = \{\kappa, \phi, \{\kappa_3\}, \{\kappa_2\}, \{\kappa_3, \kappa_2\}\}$, $\mathbb{G} = \mathcal{P}(K) \setminus \{\emptyset\}$, and let $H = \{\kappa_1, \kappa_3, \kappa_4\}$ and let $M = \{\kappa_3, \kappa_2\}$, so, $\{\kappa_2, \kappa_4\} \in so(\kappa) = \mathbb{G}^* so(K)$, but $\cap M = \{\kappa_4\} \notin so(\kappa) = \mathbb{G}^* so(K)$.

Remark 2.17. The family of all \mathbb{G} -semi-open sets is represented supra topology.

3. Some types of open functions

Definition 3.1. The function $\dot{f}: (\kappa, \tau, \mathbb{G}) \rightarrow (Y, \tau', \mathbb{G})$ is called:

- (1) \mathbb{G} -semi-open function, presently “ \mathbb{G} -s-o function” if $\dot{f}(z) \in \mathbb{G}^* so(Y)$, whenever $z \in \mathbb{G}^* so(\kappa)$.
- (2) \mathbb{G}^{**} -semi-open function, presently “ \mathbb{G}^{**} -s-o function” if $\dot{f}(z) \in \mathbb{G}^* so(Y)$, whenever $z \in \tau$.
- (3) \mathbb{G}^{***} -semi-open function, presently “ \mathbb{G}^{***} -s-o function” if $\dot{f}(z) \in \tau'$, whenever $z \in \mathbb{G}^* so(\kappa)$.

Proposition 3.2. Let $\dot{f}: (\kappa, \tau, \mathbb{G}) \rightarrow (Y, \tau', \mathbb{G})$ be a function. Then

- (1) \dot{f} is an open function, whenever \dot{f} is a \mathbb{G}^{***} -s-o function.
- (2) \dot{f} is \mathbb{G} -s-o function, whenever \dot{f} is a \mathbb{G}^{***} -s-o function.
- (3) \dot{f} is \mathbb{G}^{**} -s-o function, whenever \dot{f} is a \mathbb{G} -s-o function.
- (4) \dot{f} is \mathbb{G}^{**} -s-o function, whenever \dot{f} is an open function.

Proof . (1) Let $z \in \tau$, by Remark 2.3(i), $z \in \mathbb{G}^* so(\kappa)$. Since \dot{f} is a \mathbb{G}^{***} -s-o function, $\dot{f}(z)$ is an open function (Y, τ') . Hence, \dot{f} is an open function.

(2) Let $z \in \mathbb{G}^* so(\kappa)$. Since \dot{f} is a \mathbb{G}^{***} -s-o function, $\dot{f}(z)$ is an open function in (Y, τ') . By Remark 2.3(i), $\dot{f}(z) \in \mathbb{G}^* so(Y)$. Hence \dot{f} is a \mathbb{G} -semi-open function.

(3) Let $z \in \tau$. By Remark 2.3(i), $z \in \mathbb{G}^* so(\kappa)$. Since \dot{f} is a \mathbb{G} -s-o function, $\dot{f}(z) \in \mathbb{G}^* so(Y)$. So \dot{f} is a \mathbb{G}^{**} -s-o function.

(4) Let $z \in \tau$ and because \dot{f} is an open function so that $\dot{f}(z) \in \tau'$. By Remark 2.3(i), $(z) \in \mathbb{G}^* so(Y)$. So \dot{f} is a \mathbb{G}^{**} -s-o function. \square

The following diagram explains that how various nations’ ties were shown in Definition 3.1.

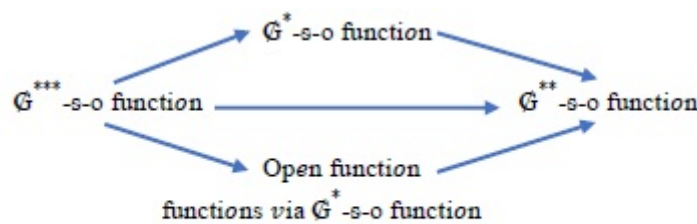


Diagram 1

Example 3.3. Let $K = \{\kappa_1, \kappa_2, \kappa_3\}$, $\tau = \{K, \phi, \{\kappa_1\}\}$, $\mathbb{G} = \mathcal{P}(K) \setminus \{\phi\}$, $\dot{f}: (K, \tau, \mathbb{G}) \rightarrow (K, \tau, \mathbb{G})$, $\dot{f}(\kappa) = \kappa$. It is clear that \dot{f} is an open function, $\mathbb{G}^* so(K) = \{z \subseteq K \text{ such that } \kappa_1 \in z\} \cup \{\phi\}$, there exists $\{\kappa_1, \kappa_2\} \in \mathbb{G}^* so(K)$, $\dot{f}(\{\kappa_1, \kappa_2\}) \notin \tau$. Then we observe that \dot{f} is not a \mathbb{G}^{***} -s-o function.

Example 3.4. Let $K = \{\kappa_1, \kappa_2, \kappa_3\}$, $\tau = \{\kappa, \phi, \{\kappa_1\}, \{\kappa_1, \kappa_2\}\}$, $\mathbb{G} = \{z \subseteq K: \kappa_3 \in z\}$, $\tau_{\mathbb{G}} = \mathcal{P}(K)$. Then $\mathbb{G}^* so(K) = \mathcal{P}(K)$, $\dot{f}: (\kappa, \tau, \mathbb{G}) \rightarrow (\kappa, \tau, \mathbb{G})$, $\dot{f}(\kappa_1) = \{\kappa_2\}$, $\dot{f}(\kappa_2) = \{\kappa_1\}$, $\dot{f}(\kappa_3) = \{\kappa_3\}$. We observe that the function is a \mathbb{G}^{**} -s-o function and \mathbb{G} -s-o function but it is not an open and not a \mathbb{G}^{***} -s-o function, because $\dot{f}(\kappa_1) = \{\kappa_2\} \notin \tau$.

Example 3.5. Let $\mathcal{K} = \{\kappa_1, \kappa_2, \kappa_3\}$, $\tau = \{\kappa, \phi, \{\kappa_1, \kappa_2\}\}$, $\mathbb{G} = \mathcal{P}(\mathcal{K}) \setminus \{\{\phi\} \cup \{\kappa_1\}\}$, $\dot{f}(\kappa_2) = \{\kappa_3\}$, $\dot{f}(\kappa_1) = \{\kappa_2\}$, $\dot{f}(\kappa_3) = \{\kappa_1\}$, $\tau_{\mathbb{G}} = \{\kappa, \phi, \{\kappa_2, \kappa_1\}, \{\kappa_3, \kappa_2\}, \{\kappa_2\}\}$, $\mathbb{G}^*so(\mathcal{K}) = \{\kappa, \phi, \{\kappa_2, \kappa_1\}, \{\kappa_3, \kappa_2\}, \{\kappa_2\}, \{\kappa_1\}\}$ \dot{f} is a \mathbb{G}^* -s-o function but f is not a \mathbb{G} -s-o function because there exists $\{\kappa_3, \kappa_2\} \in \mathbb{G}^*so(\mathcal{K})$, but $\dot{f}(\{\kappa_3, \kappa_2\}) = \{\kappa_3, \kappa_1\} \notin \mathbb{G}^*so(\mathcal{K})$.

Definition 3.6. The function $\dot{f}: (\kappa, \tau, \mathbb{G}) \rightarrow (Y, \tau', \mathbb{G})$ is reputed to:

- (1) \mathbb{G} -semi-closed function, presently “ \mathbb{G} -s-c function” if $\dot{f}(z) \in \mathbb{G}^*sc(Y)$, whenever, $z \in \mathbb{G}^*sc(\kappa)$.
- (2) \mathbb{G}^{**} -semi-closed function, presently “ \mathbb{G}^{**} -s-c function” if $\dot{f}(z) \in \mathbb{G}^*sc(Y)$, whenever, z is a closed set in (κ, τ) .
- (3) \mathbb{G}^{***} -semi-closed function, presently “ \mathbb{G}^{***} -s-c function” if $\dot{f}(z)$ is closed set in (Y, τ') , whenever, $z \in \mathbb{G}^*sc(\kappa)$.

Proposition 3.7. Let $\dot{f}: (\kappa, \tau, \mathbb{G}) \rightarrow (Y, \tau', \mathbb{G})$ be a function. Then

- (1) \dot{f} is a closed function, whenever \dot{f} is a \mathbb{G}^{***} -s-c function.
- (2) \dot{f} is a \mathbb{G} -s-c function, whenever \dot{f} is a \mathbb{G}^{***} -s-c function.
- (3) \dot{f} is a \mathbb{G}^{**} -s-c function, whenever \dot{f} is a \mathbb{G} -s-c function.
- (4) \dot{f} is a \mathbb{G}^{**} -s-c function, whenever \dot{f} is a closed function.

Proof . By way of Remark 2.3(i) and Definition 3.6, the proof is complete. \square

The inverse of Proposition 3.7 is not true, see Examples 3.3 and 3.4.

Remark 3.8. If \dot{f} is onto function then:

- (1) \mathbb{G}^* -s-c and \mathbb{G} -s-o function are equivalent.
- (2) \mathbb{G}^{**} -s-c and \mathbb{G}^{**} -s-o are equivalent.
- (3) \mathbb{G}^{***} -s-c and \mathbb{G}^{***} -s-o are equivalent.

Proof . As \dot{f} is an onto, hence the proof has been completed by Definition 3.1 and Definition 3.6. \square

The following diagram, describe the relationships between these different forms of closed functions.

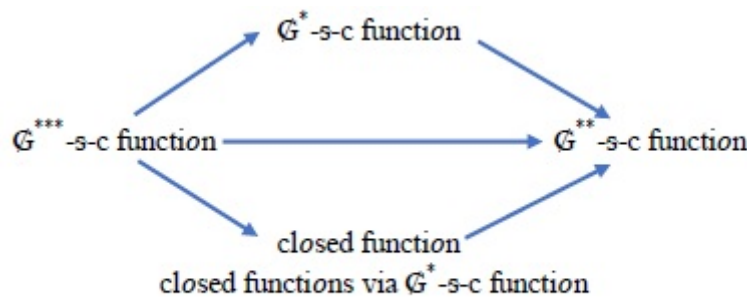


Diagram 2

4. Some types of continuous function

In the following, new type of continuous functions will present their definitions and the relationships between those functions will explain.

Definition 4.1. *The function $f:(\kappa, \tau, \mathbb{G}) \rightarrow (\mathcal{Y}, \tau', \mathbb{G})$ is reputed to:*

- (1) \mathbb{G}^* -s-continuous function, presently “ \mathbb{G}^* -s continuous function” if $f^{-1}(z) \in \mathbb{G}^*so(\mathcal{K})$, for all $z \in \tau$.
- (2) Strongly \mathbb{G}^* -s-continuous function, presently “strongly \mathbb{G}^* -s continuous function” if $f^{-1}(z) \in \tau$, for every $z \in \mathbb{G}^*so(\mathcal{Y})$.
- (3) \mathbb{G}^* -s-irresolute function, presently “ \mathbb{G}^* -s irresolute function” if $f^{-1}(z) \in \mathbb{G}^*so(\mathcal{K})$, for every $z \in \mathbb{G}^*so(\mathcal{Y})$.

Proposition 4.2. *Let $f:(\kappa, \tau, \mathbb{G}) \rightarrow (\kappa, \tau, \mathbb{G})$ be a function. Then*

- (1) f is $\mathbb{G}^*(\kappa)$ -s-irresolute function, whenever f is strongly- $\mathbb{G}^*(\kappa)$ -s-continuous function.
- (2) f is continuous function, whenever f is strongly- $\mathbb{G}^*(\kappa)$ -s-continuous function.
- (3) f is $\mathbb{G}^*(\kappa)$ s-continuous function, whenever f is continuous function.
- (4) f is a $\mathbb{G}^*(\kappa)$ -s-irresolute function, whenever f is $\mathbb{G}^*(\kappa)$ -s-continuous function.

Proof . (1) Let $z \in \mathbb{G}^*so(\mathcal{Y})$. Since f is a strongly $\mathbb{G}^*(\kappa)$ -s-continuous function, $f^{-1}(z) \in \tau$. By Remark 2.3(i), $f^{-1}(z) \in \mathbb{G}^*so(\mathcal{K})$. This implies that f is \mathbb{G}^* -s-irresolute function.

(2) Let z be an open set in $(, \tau)$. By Remark 2.3(i), $z \in \mathbb{G}^*so(\mathcal{Y})$. Since f is strongly- $\mathbb{G}^*(\kappa)$ -s-continuous function, $f^{-1}(z)$ is an open set in (κ, τ) . Thus, f is a continuous function.

(3) Let $z \in \tau$. Since f is a continuous function, $f^{-1}(z)$ is an open set in (κ, τ) . By Remark 2.3(i), $f^{-1}(z) \in \mathbb{G}^*so(\mathcal{K})$, so f is $\mathbb{G}^*(\kappa)$ s-continuous function.

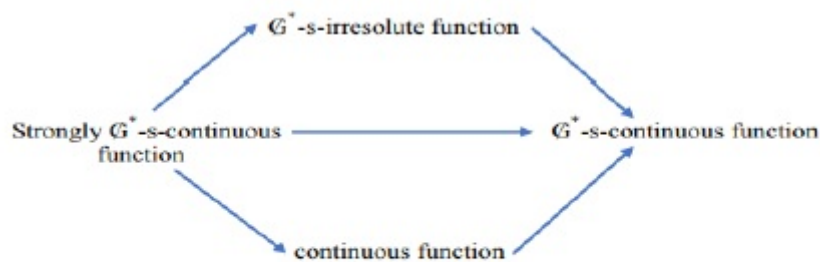
(4) Let $z \in \tau$. By Remark 2.3(i), $z \in \mathbb{G}^*so(\mathcal{Y})$, because f is \mathbb{G}^* -s-irresolute function. Then $f^{-1}(z) \in \mathbb{G}^*so(\mathcal{K})$. So f is $\mathbb{G}^*(\kappa)$ s-continuous function. \square

The inverse of Proposition 4.2 is not always hold, see the following example.

Example 4.3. *The function $f:(\kappa, \tau, \mathbb{G}) \rightarrow (\kappa, \tau, \mathbb{G}^\sim)$ such that $f(\kappa) = \kappa$, for each $\kappa \in \kappa$, where $\kappa = \{\kappa_1, \kappa_2, \kappa_3\}$, $\tau = \{\kappa, \phi, \{\kappa_1\}\}$, $\mathbb{G} = \mathcal{P}(\kappa) \setminus \{\phi\}$, $\mathbb{G}^\sim = \{z: \kappa_1 \in z\}$, $\mathbb{G}^*so(\mathcal{K}) = \{z: \kappa_1 \in z\} \cup \{\phi\}$, $\mathbb{G}^\sim^*so(\kappa) = \mathcal{P}(\kappa)$. So, f is \mathbb{G}^* -s-continues function and continuous function, but it is not a $\mathbb{G}^*(\kappa)$ s-irresolute function and it is not strongly $\mathbb{G}^*(\kappa)$ -s-continuous function because there exists $\{\kappa_2, \kappa_3\} \in \mathbb{G}^\sim^*so(\kappa)$, but $f^{-1}\{\kappa_2, \kappa_3\} = \{\kappa_2, \kappa_3\} \notin \mathbb{G}^*so(\mathcal{K})$.*

Example 4.4. *The function $f:(\kappa, \tau, \mathbb{G}) \rightarrow (\kappa, \tau, \mathbb{G}^\sim^*)$ such that $f(\{\kappa_2\}) = \{\kappa_1\}$, $f(\{\kappa_1\}) = \{\kappa_2\}$, $f(\{\kappa_3\}) = \{\kappa_3\}$, where $\kappa = \{\kappa_1, \kappa_2, \kappa_3\}$, $\tau = \{\kappa, \phi, \{\kappa_1\}\}$, $\mathbb{G} = \{z: \kappa_1 \in z\}$, $\mathbb{G}^\sim = \mathcal{P}(\kappa) \setminus \{\phi\}$, $\mathbb{G}^*so(\kappa) = \mathcal{P}(\kappa)$, $\mathbb{G}^\sim^*so(\kappa) = \{z: \kappa_1 \in z\} \cup \{\phi\}$, is \mathbb{G}^* , so (κ) continuous function and is not a $\mathbb{G}^*\kappa$ s-irresolute function, but it is not a continuous function and not a strongly $\mathbb{G}^*(\kappa)$ -s-continuous function, because $(\kappa_1) = \{\kappa_2\} \notin \tau$.*

The following diagram explains the relations between the concept in Definition 4.1.

**Diagram 3**Continuity via \mathcal{G}^* -s-open set**References**

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